FUNDAMENTALS OF TIME SERIES ANALYSIS FOR TRANSFER FUNCTION ESTIMATION

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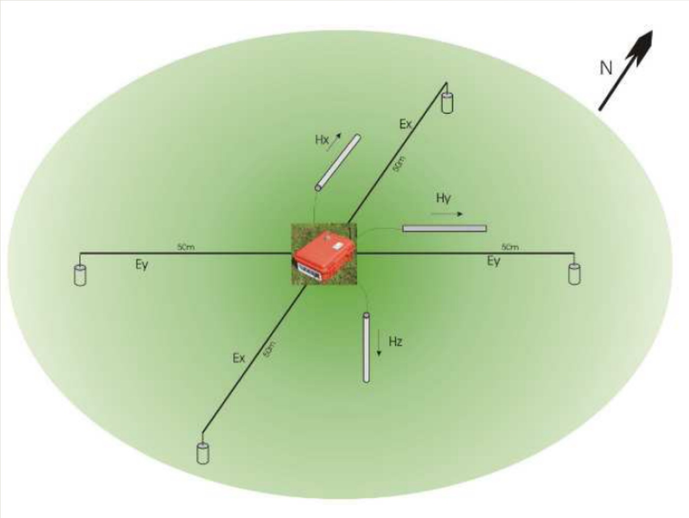
FUNDAMENTAL PROBLEM

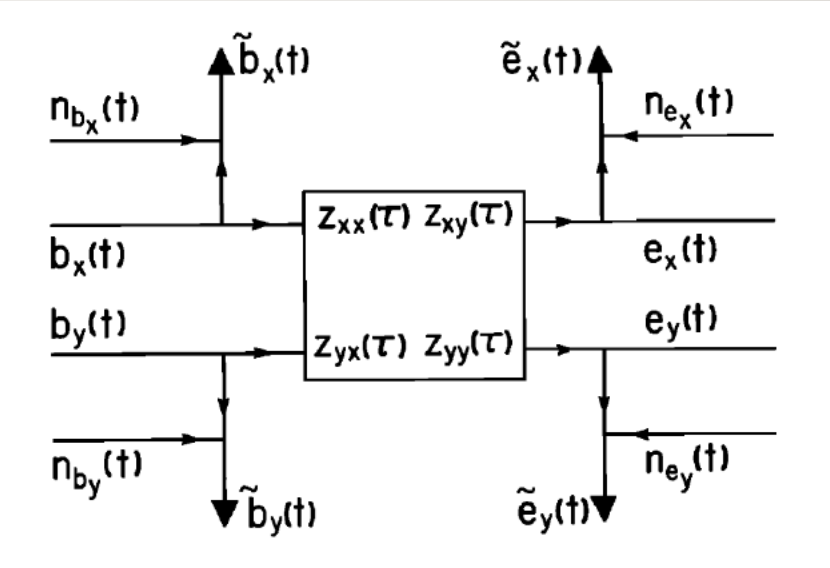


$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix}$$

↓

$$E_x = Z_{xx} \cdot B_x + Z_{xy} \cdot B_y$$
$$E_x = Z_{yx} \cdot B_x + Z_{yy} \cdot B_y$$

- electrical impedance *Z* describes subsurface conductivity structure
- data always contains source signal and noise

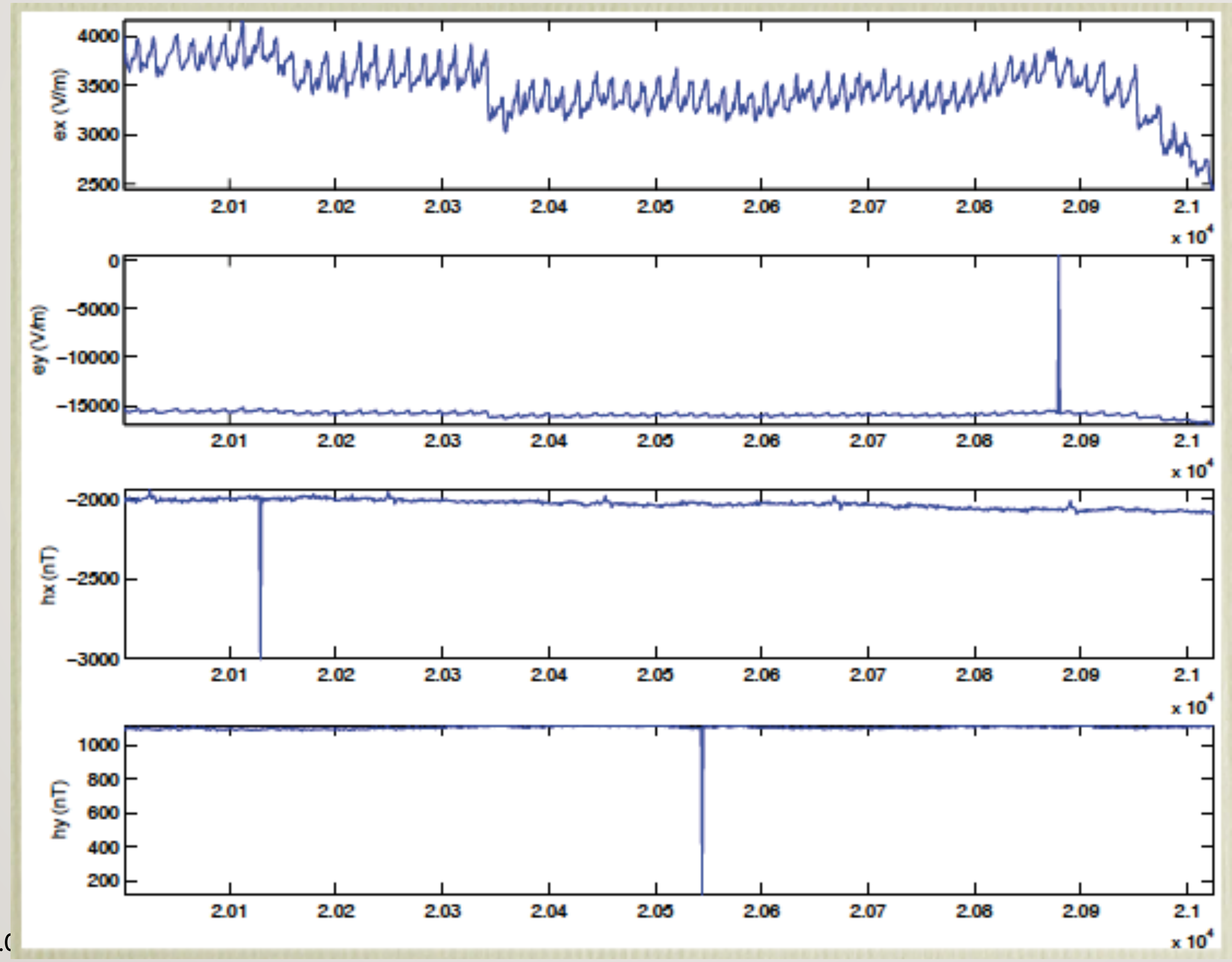


ROADMAP

1. Pre-processing
2. Spectral Estimation
3. Impedance Estimation
4. Uncertainty Estimation

1. PRE-PROCESSING

LOOK AT THE DATA
(GARCIA ET AL., 1999; WECKMANN ET AL., 2005)



DATA CONDITIONERS

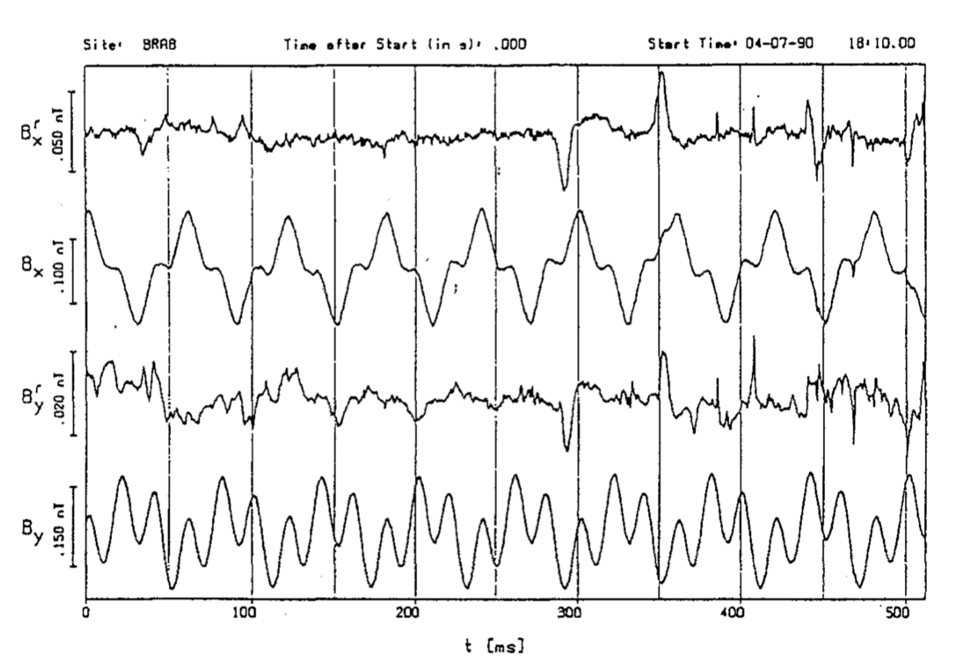
- Motion Inducted noise: adaptive cancellation filter
- Filters, including Notch filters
- Trend removal: AR filter
- Resampling, interpolation

DELAY LINE FILTER (SCHNEGG AND FISCHER, 1980; BRASSE, 1993)

Delay by one:

$$y_n = x_n - x_{n-1}$$

16 2/3 Hz & 50 Hz:



B_x after denoising

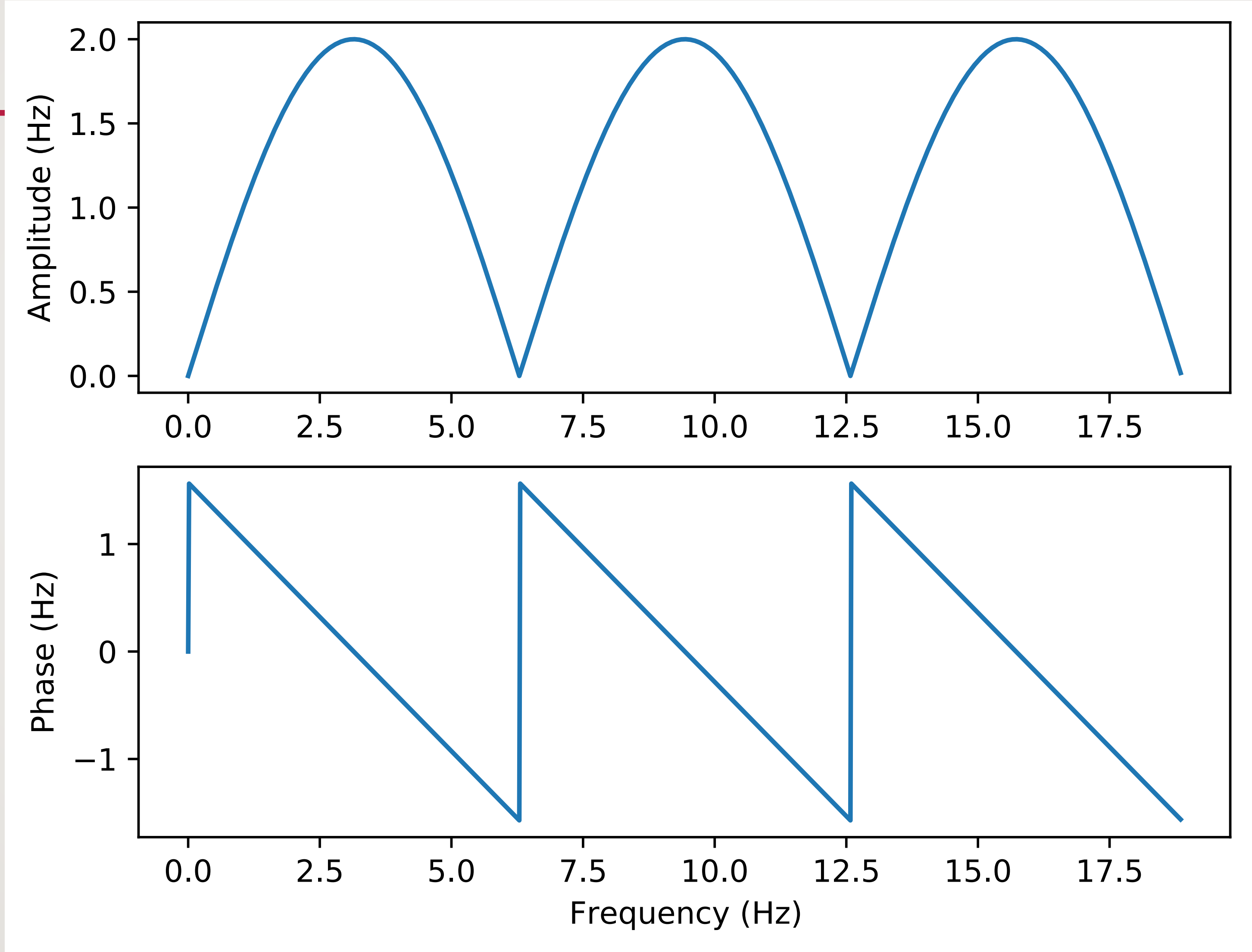
B_x before denoising

B_y after denoising

B_y before denoising

Transfer function:

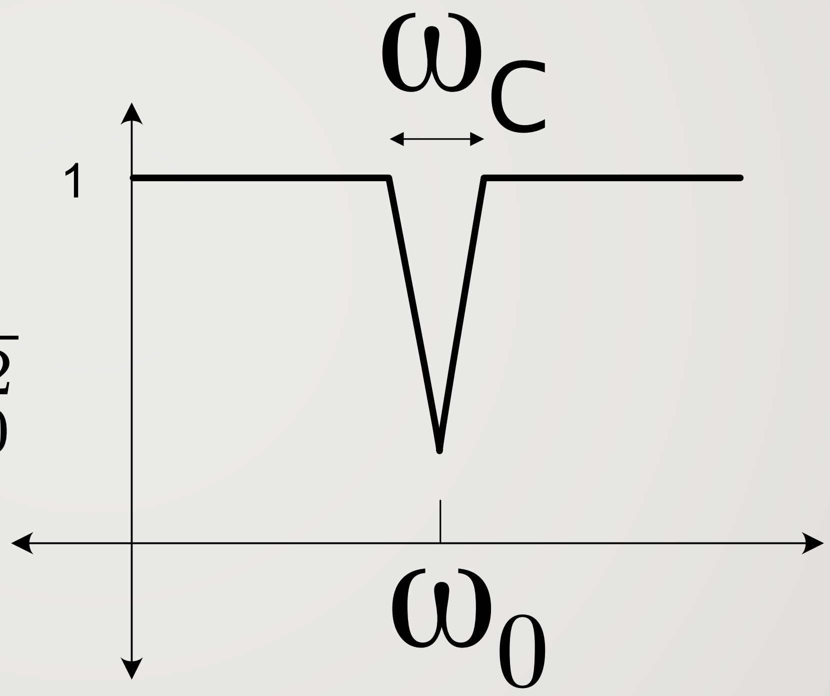
$$H(z) \stackrel{\text{def}}{=} \sum_{n=-\infty}^{\infty} h[n] \cdot z^{-n}$$



NOTCH FILTERS

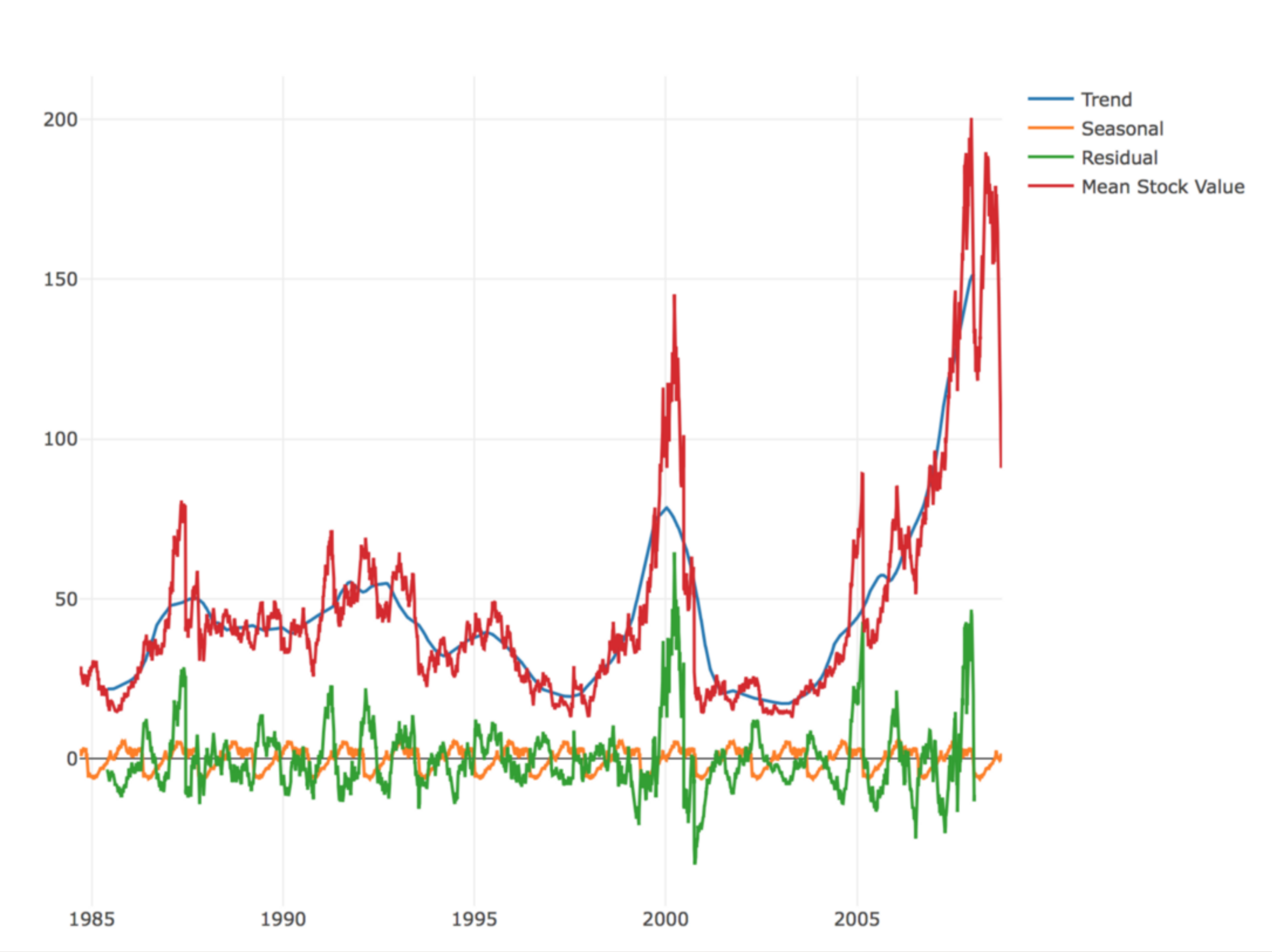
Transfer function:

$$H(z) = \frac{z^2 + \omega_0^2}{z^2 + \omega_c z + \omega_0^2}$$



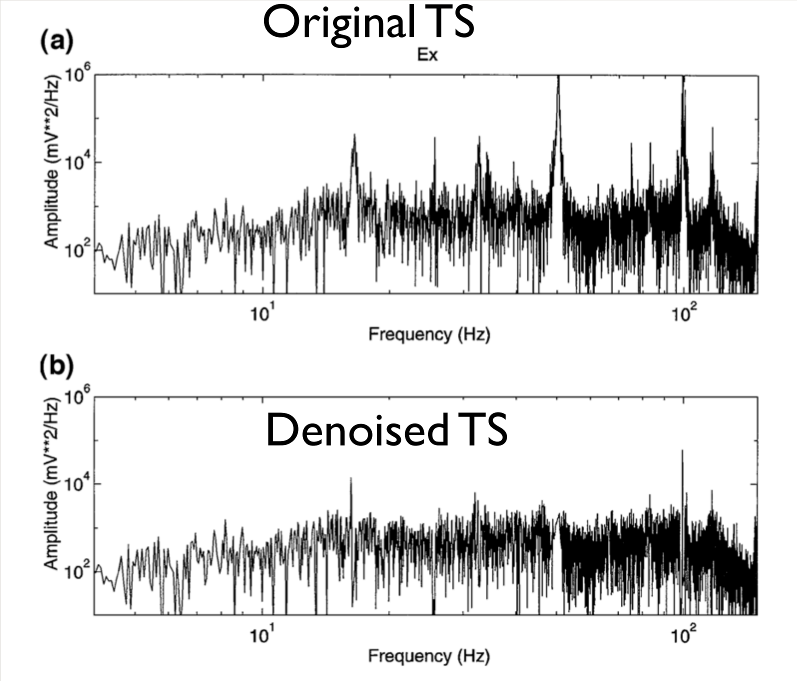
AR TREND REMOVAL (CHAVE, 2012)

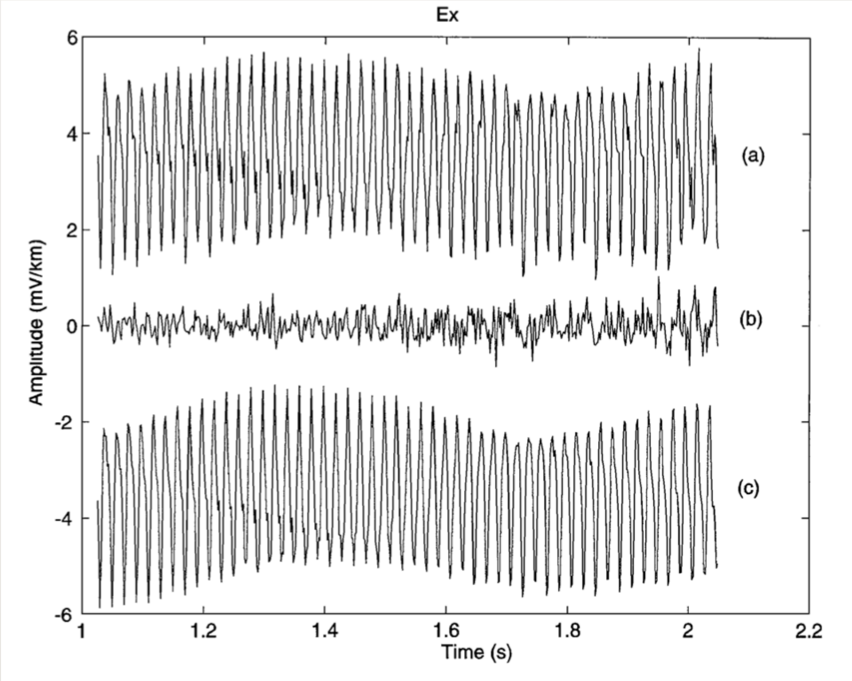
- AR (auto regression): is a representation of a type of random process; as such, it is used to describe certain time-varying processes in nature



WAVELET DENOISING
(TRAD & TRAVASSOS, 2000)

- Hypothesis: noise and signal can be sepparated at different scales
- Using Thomson weights, search for noisy segments and eliminate them





Signal + noise

Signal

Noise

USE OF ANALYTICAL SIGNAL (JONES AND SPRATT, 2002)

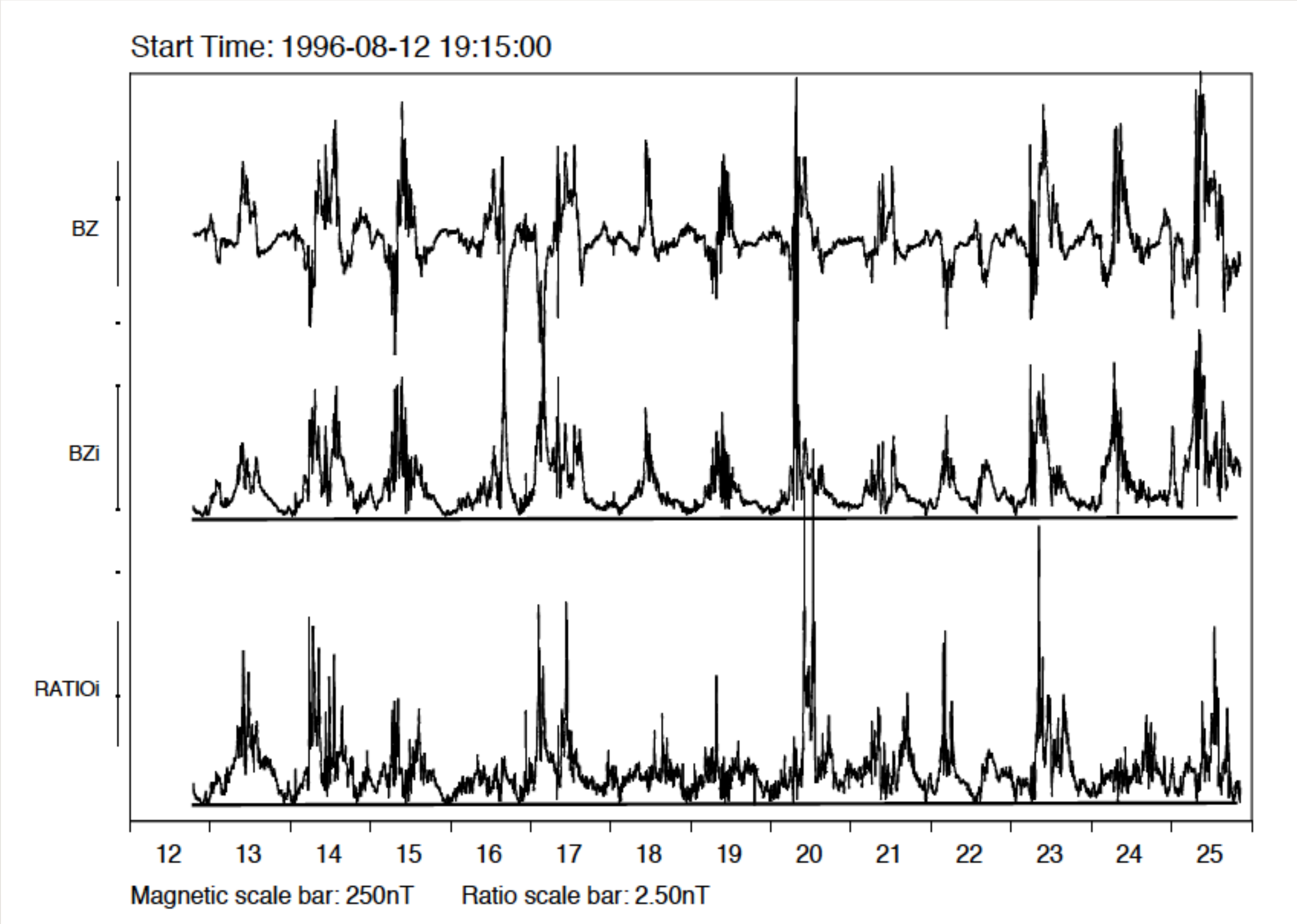
- Derive analytic signal:

$$z(t) = x(t) + i\mathcal{H}[x(t)]$$

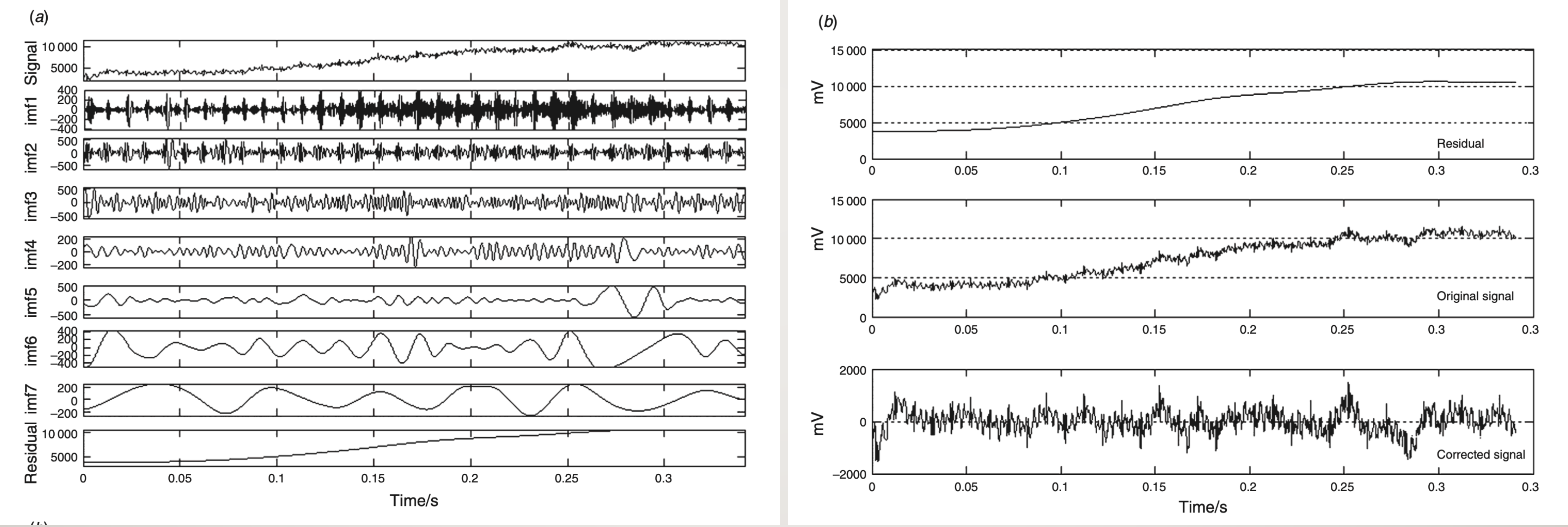
- Estimate ratio:

$$ratio = \frac{B_{zi}}{(B_{xi}^2 + B_{yi}^2)^{1/2}}$$

- Select segments
- Process TS

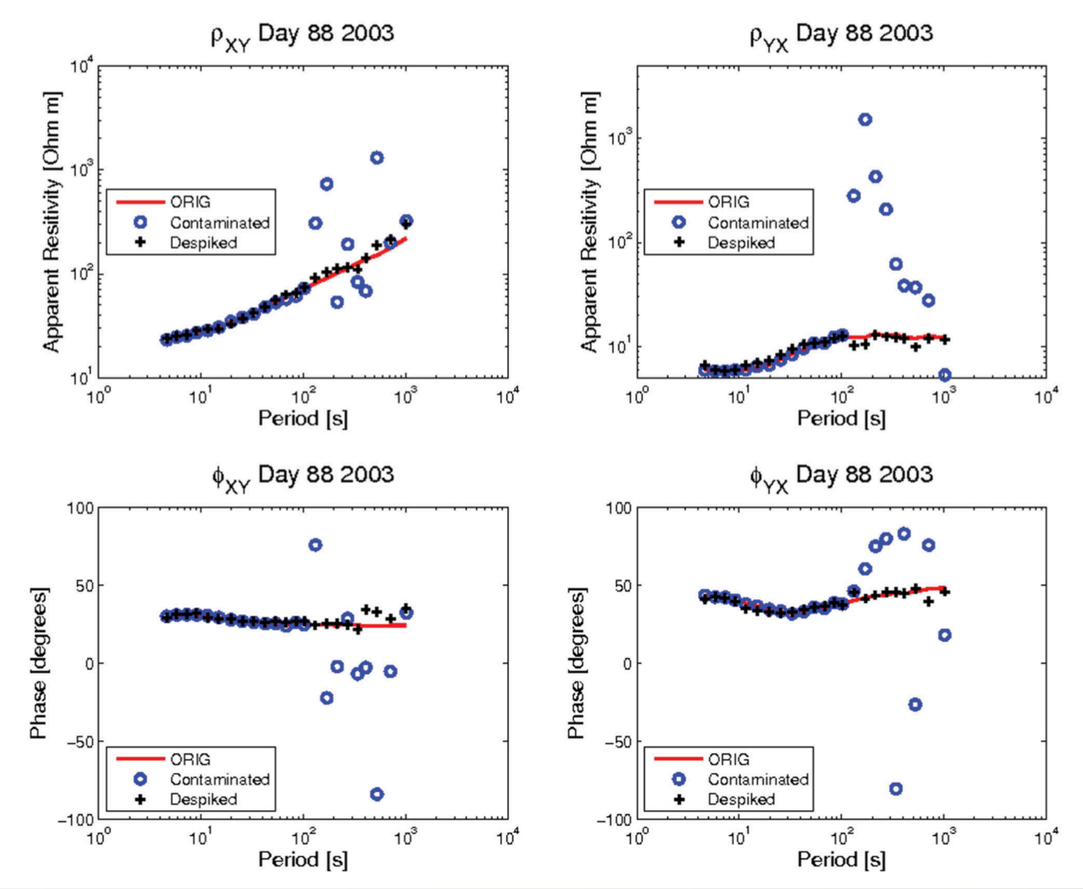


EMD DENOISING (CAI ET AL., 2019)



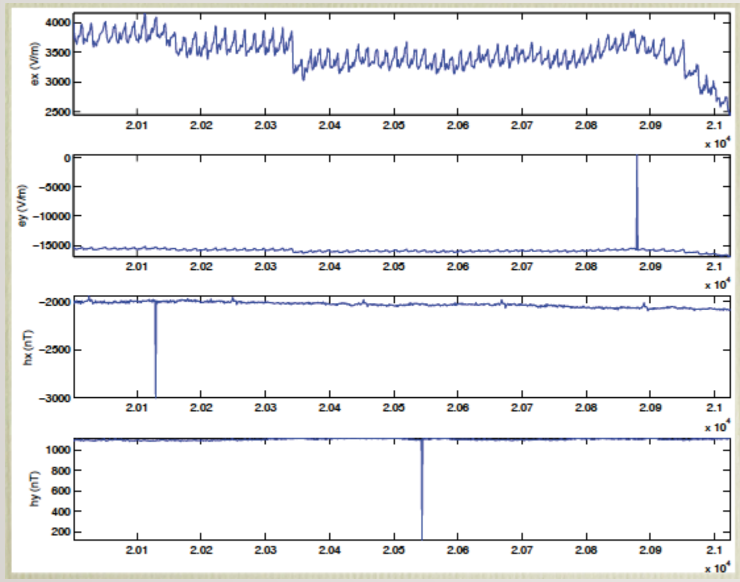
DATA VARIANCE TECHNIQUE FOR DESPIKING (KAPLER, 2012)

- Select clean segments
- Train a Wiener filter in clean segments with the aid of a remote reference
- Locate noisy sections
- Predict signal in noisy sections using Wiener filter and the remote reference
- Reconstruct signal and process

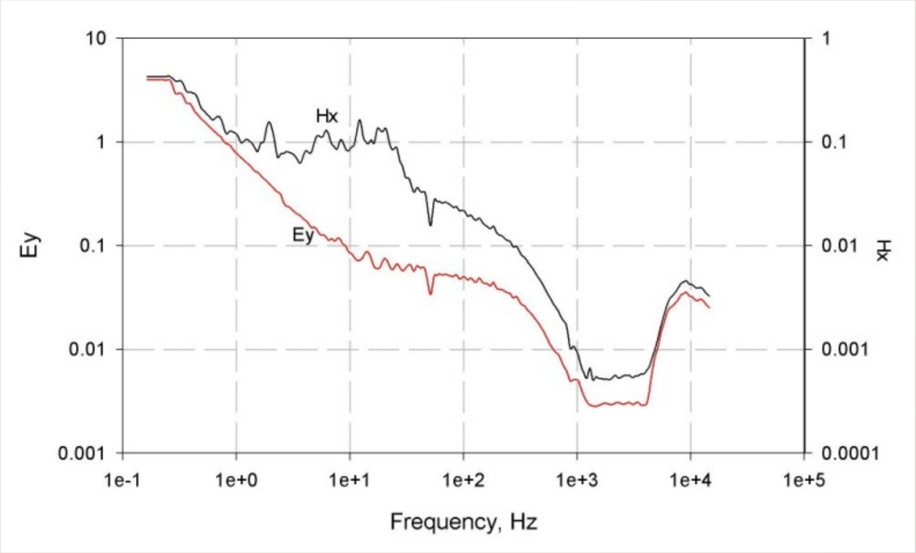


2. SPECTRAL ESTIMATION

It is desirable to be able to obtain a reasonable spectrum, with little or no bias and small uncertainties (Prieto et al., 2007)



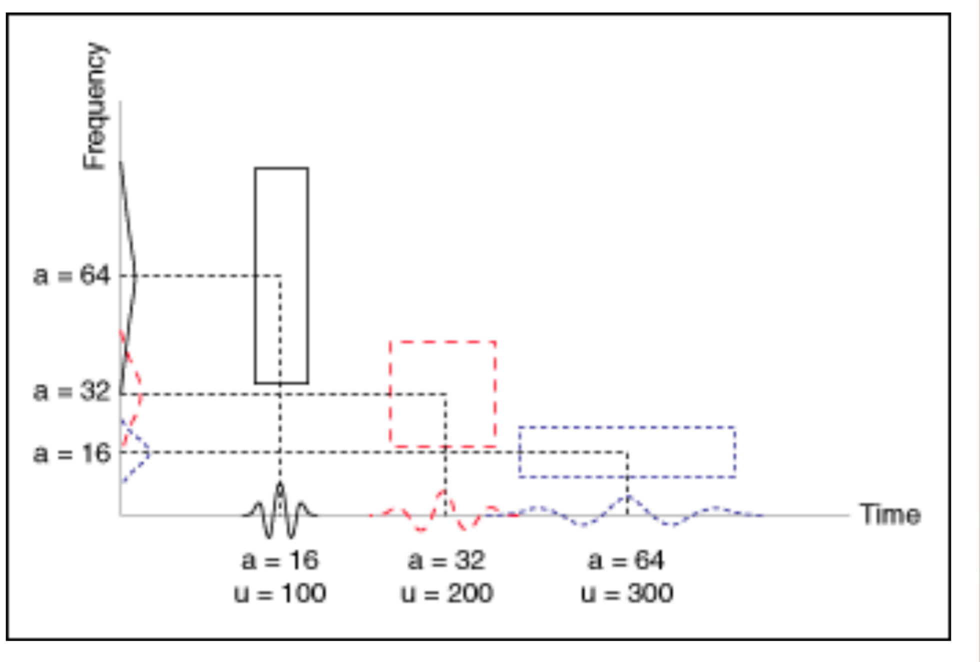
Fourier
Wavelet
EMD



NOTE

- The frequency information you obtain at one time point is a weighted sum of the frequency information at surrounding time points.
- In interpreting time-frequency results, each time point is an estimate of instantaneous activity influenced by neighboring activity.

$$\Delta t \Delta f \geq 1$$



FOURIER TECHNIQUES

- Conventional methods → Short-time Fourier Transform (STFT)

$$\mathbf{STFT}\{\mathbf{x}(\mathbf{t})\}(\tau,\omega) \equiv \mathbf{X}(\tau,\omega) = \int_{\infty}^{\infty} \mathbf{x}(\mathbf{t})\mathbf{w}(\mathbf{t}-\tau)\mathbf{e}^{-\mathbf{i}\omega\mathbf{t}}\mathbf{dt}$$

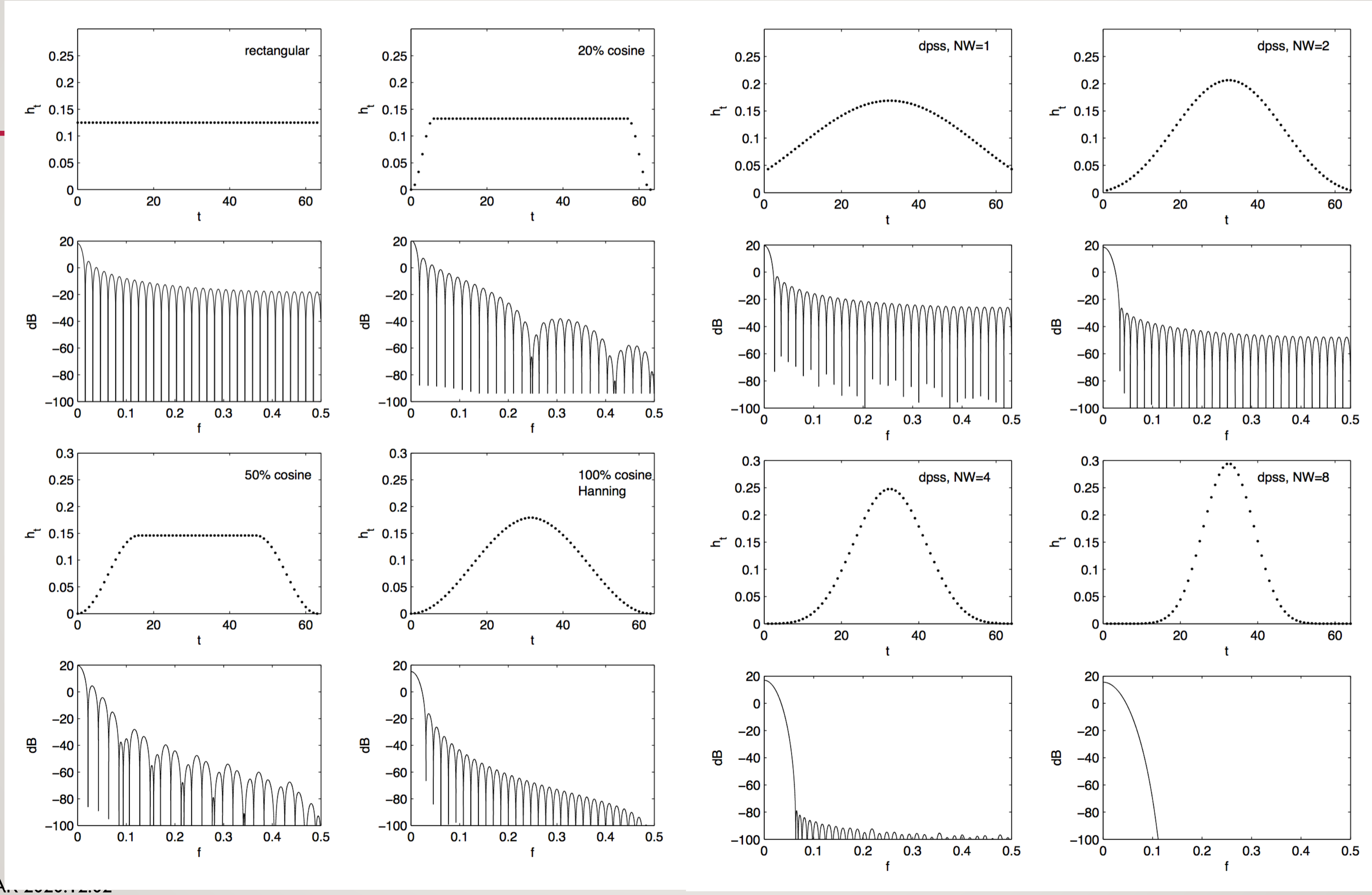
- Non-conventional methods → Cascade decimation (Wight and Bostick, 1979)
  - Use segments of 32 (64 or 128 …) points
  - Pre-condition with Hanning window
  - Use DFT to estimate spectra and select the 6th and 8th harmonic
  - Low pass data and decimate (by 2), start all over

FOURIER: WINDOWING (TAPER)

Reduce spectral leakage and bias caused by limited TS or by spectral peaks

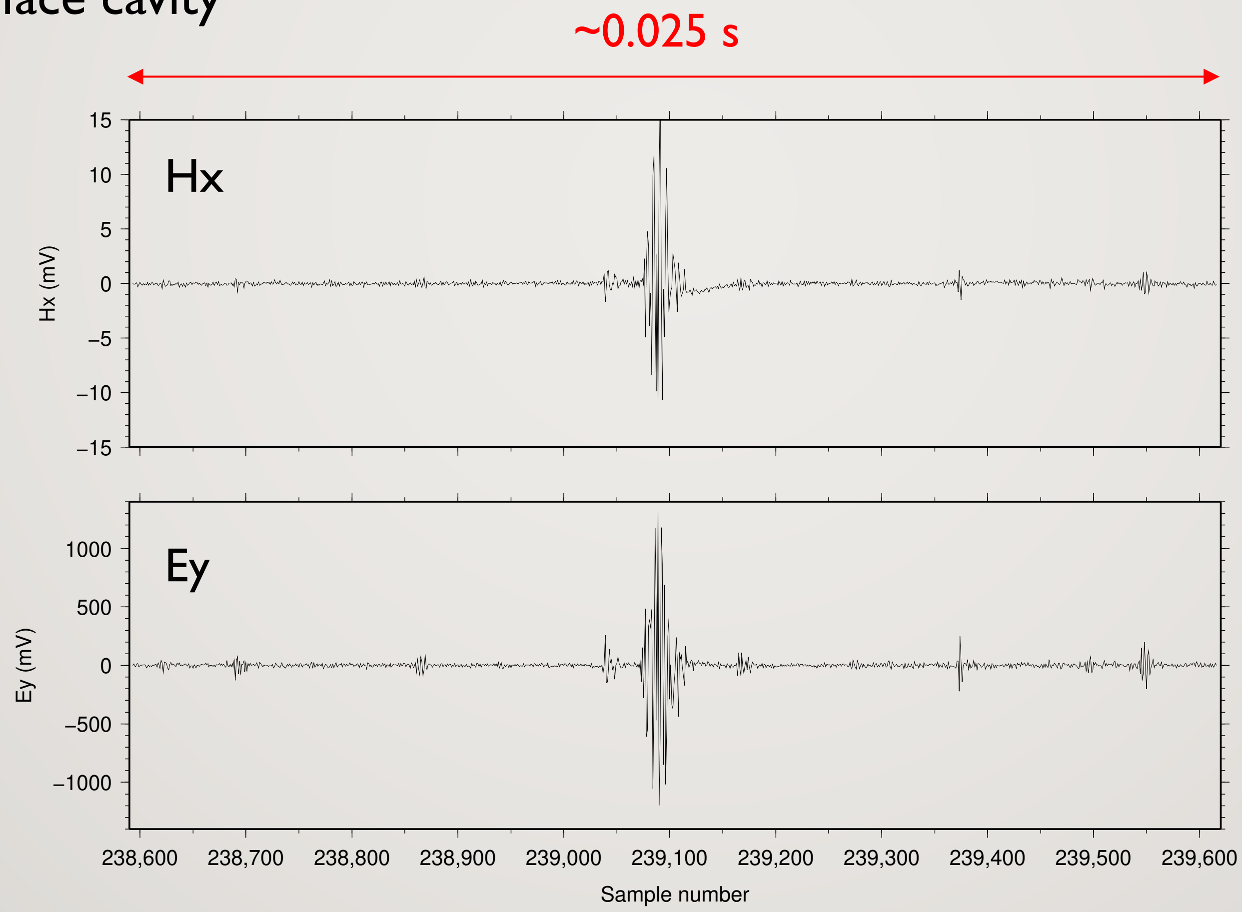
- Single window: Hann, Hamming, Parzen, Welch, Blackman, … (see wikipedia for more info)
- Multitaper: Discrete Prolate Spectral Sequence

TAPER


rectangular
20% cosine
dpss, NW=1
dpss, NW=2
50% cosine
100% cosine
Hanning
dpss, NW=4
dpss, NW=8
h_t
dB
t
f

WAVELET SPECTRAL ESTIMATION (GARCIA AND JONES, 2008)

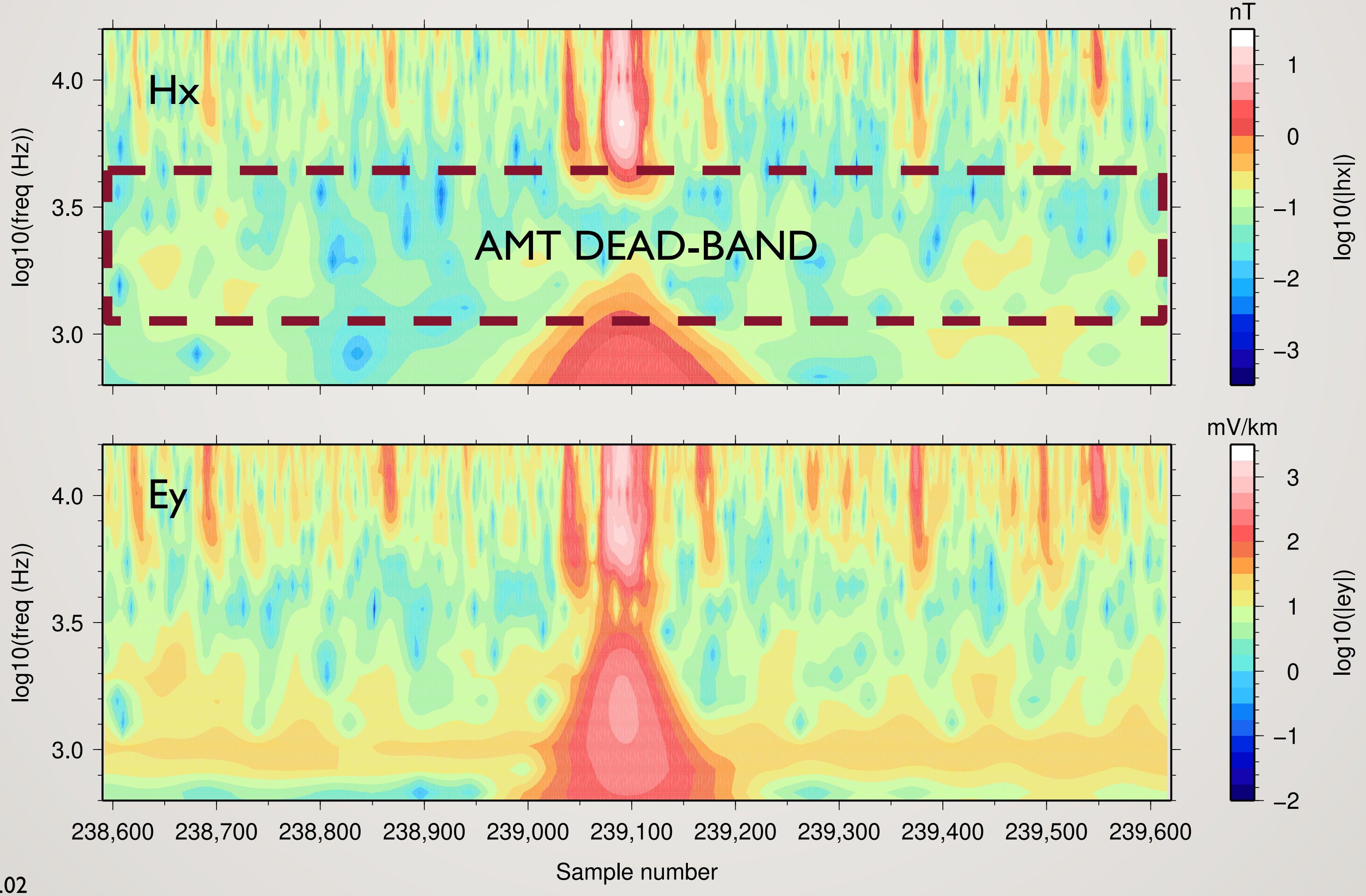
- AMT signal: global lightning discharges create plane waves that travel on the ionosphere-Earth's surface cavity



AMT SPECTRA

Blue: low amplitude

Red: high amplitude



CONTINUOUS WAVELET TRANSFORM

- Goal: locate high energy events
- Varying parameters allow for Time-Frequency tradeoffs in precision
- Similar to decomposition with sine waves of different frequencies in FFT: Time-Frequency decomposition involves wavelets of different frequencies
- To make a family you change the frequency of the sine wave while leaving other parameters unchanged

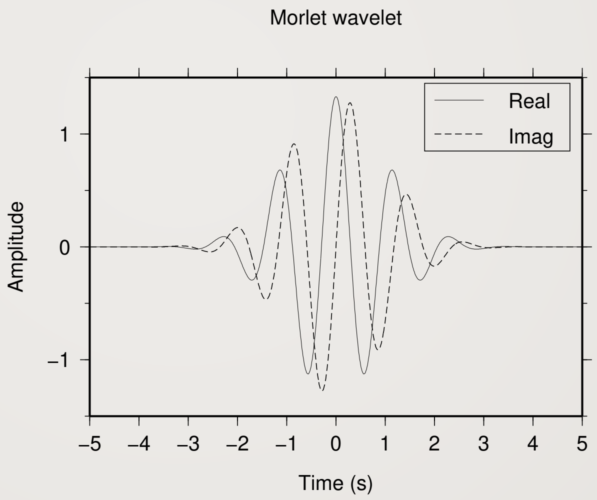
MORLET WAVELET

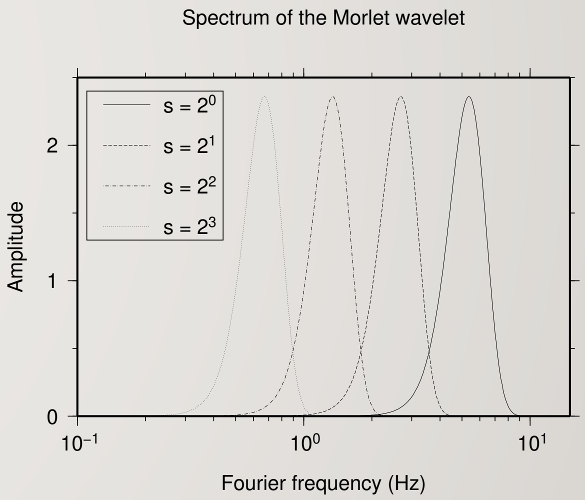
$$\psi_0(s) = \pi^{1/4} e^{i\omega_0 s} e^{-s^2/2}$$

$$\omega_0 = \pi\sqrt{2/\log(2)}$$

Period definition:

$$T = \frac{4\pi s}{\omega_0 + \sqrt{2 + \omega_0}}$$





STATIONARITY OF MT TS

- Uman and Rakov (2007), lightning discharges:
  - fast and transient leader-return stroke sequences
  - slow and quasi-stationary continuing currents
  - **perturbations and surges on the continuing currents**

May be viewed as being stationary on a section with some dynamic length confined by the recurrent transient strokes

- Liu and Fujimoto (2011):
  - magnetospheric current is nonlinearly driven by the dynamic solar wind but behaves in a static manner for high magnetospheric pressure conditions

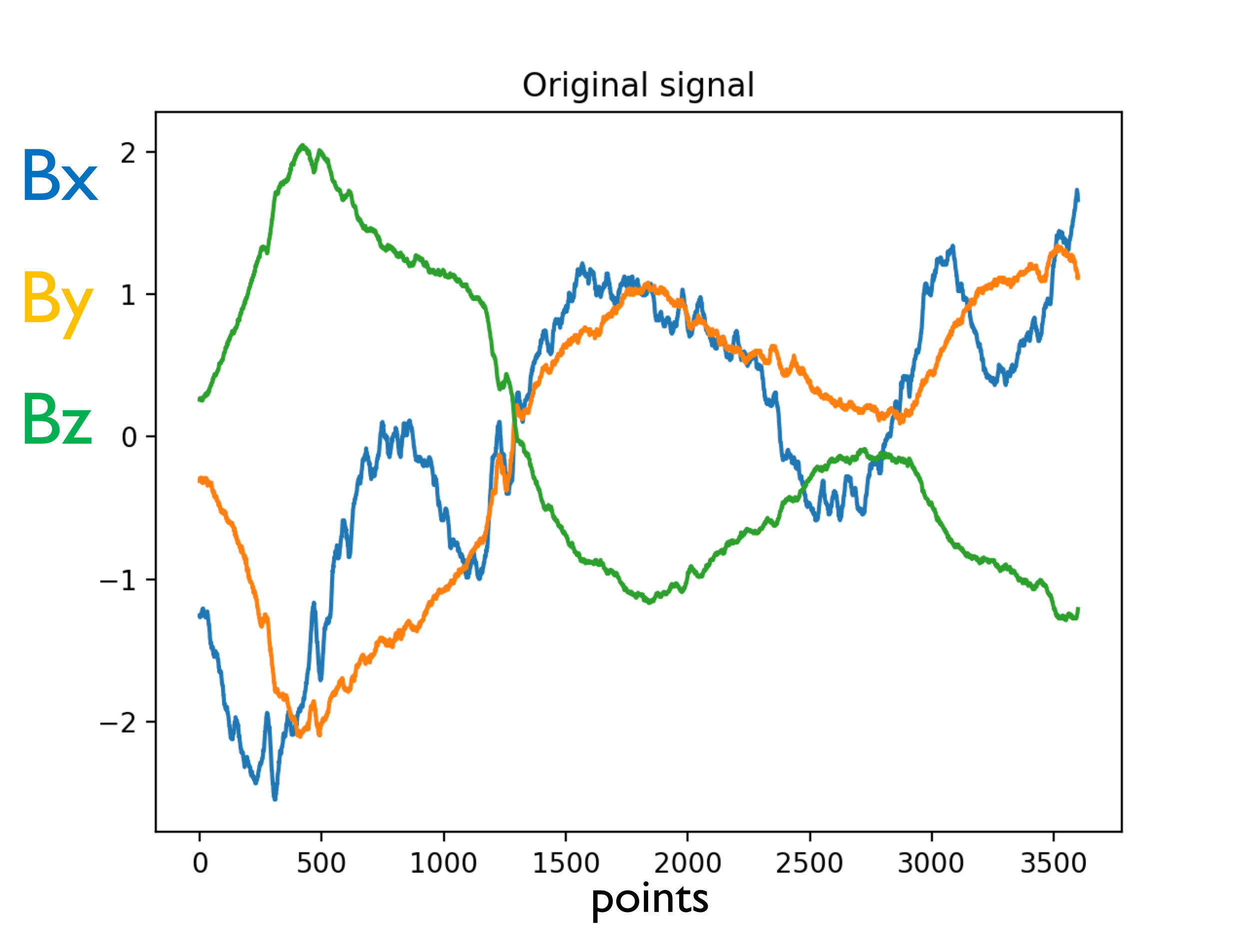
STATIONARITY OF MT TS (2) (NEUKIRCH AND GARCIA, 2014)

- EM sources are naturally nonstationary, since both, lightning strokes and magnetospheric pressure conditions, are very dynamic and thus strictly limit the duration of any stationary electromagnetic signal (Neukirch and Garcia, 2014)
- Noise sources do not need to be stationary, e.g., a station near a road (Adam et al., 1986)
- Try a tool that does not assume stationarity of TS

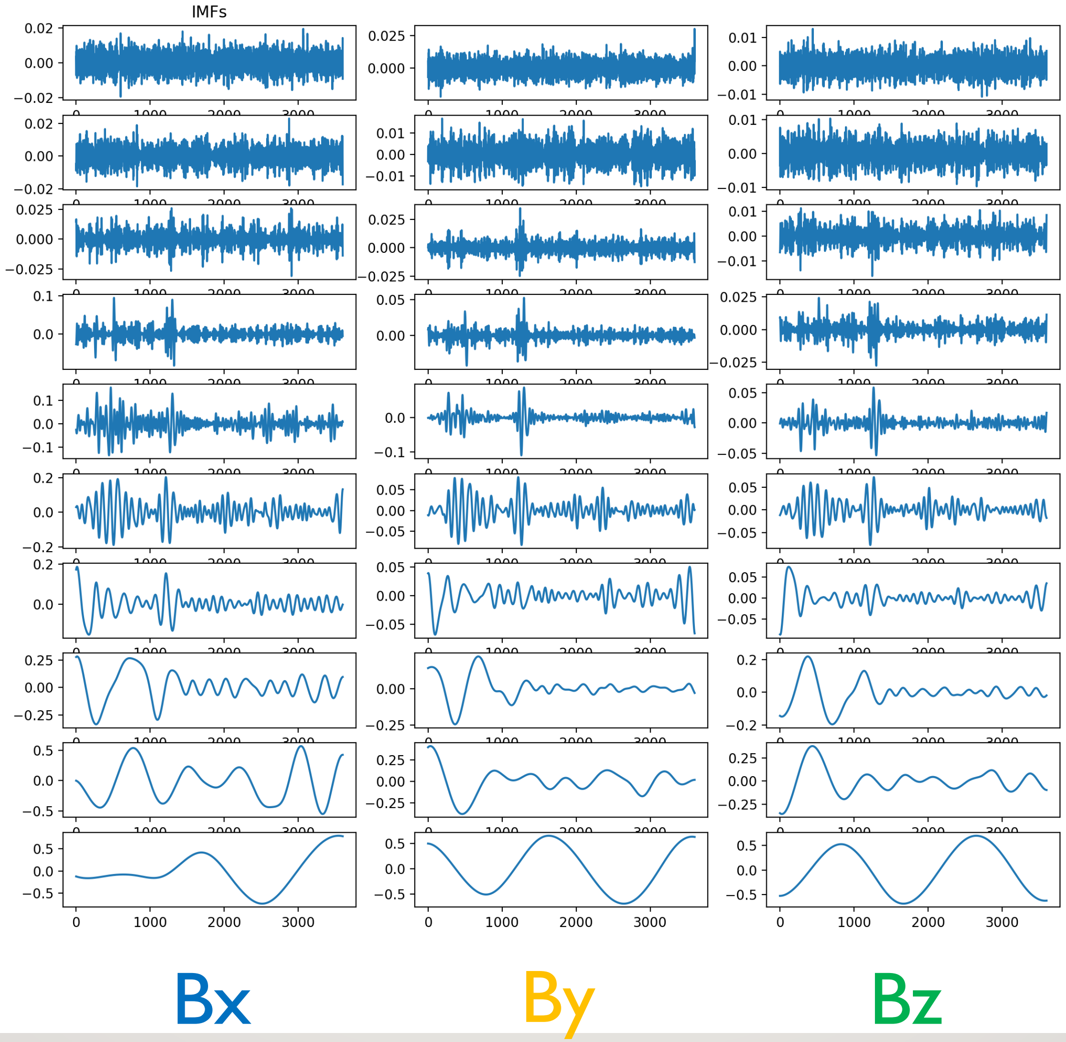
MEMD: MULTIEMPIRICAL MODE DECOMPOSITION

- Data based
- Does not make assumptions on data stationarity
- Based on the Hilbert-Huang transform
- Multivariate version of EMD
- Decompose TS into Intrinsic Mode Functions (IMF)
- IMF is a time series with a dynamic and locally narrow banded IF (*Flandrin and Rilling*, 2004)
- Offers great time-frequency localization
- BUT it suffers from high sensitivity to the signal-to-noise ratio
- EMD and MEMD spectra equivalent to Fourier spectra (*Neukirch and Garcia*, 2013)

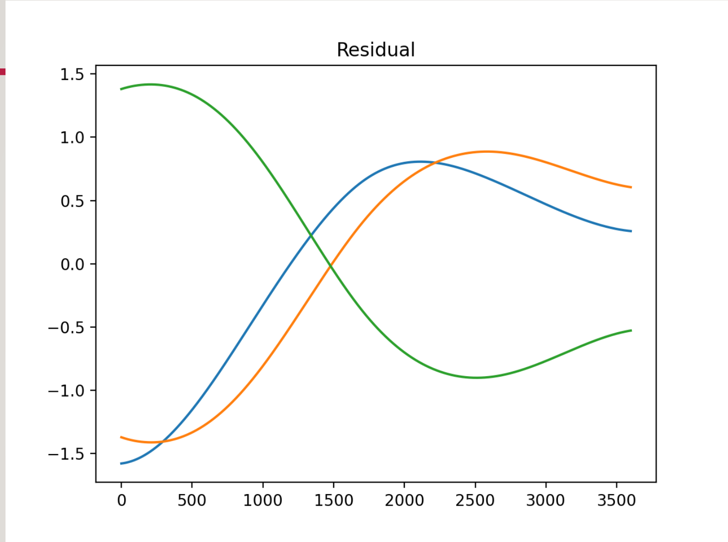
EXAMPLE: TIPPER DATA FROM LEMI INSTRUMENT MEASURED IN NATAL (BRAZIL)

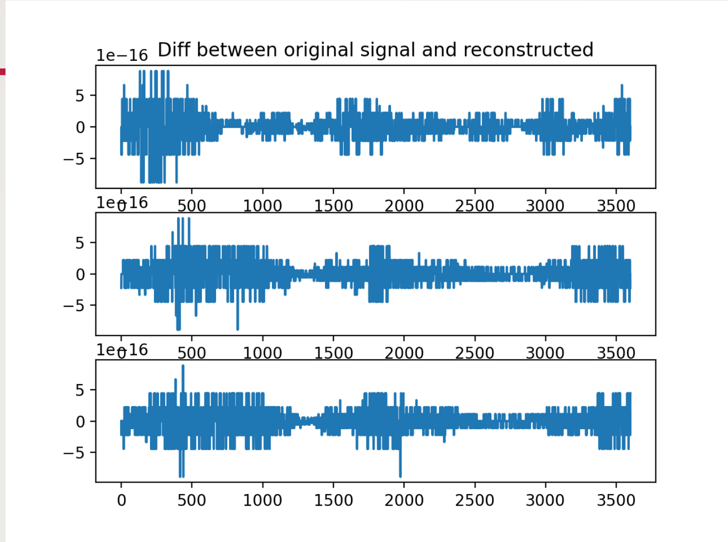


IMF DECOMPOSITION



RECONSTRUCTED SIGNAL …

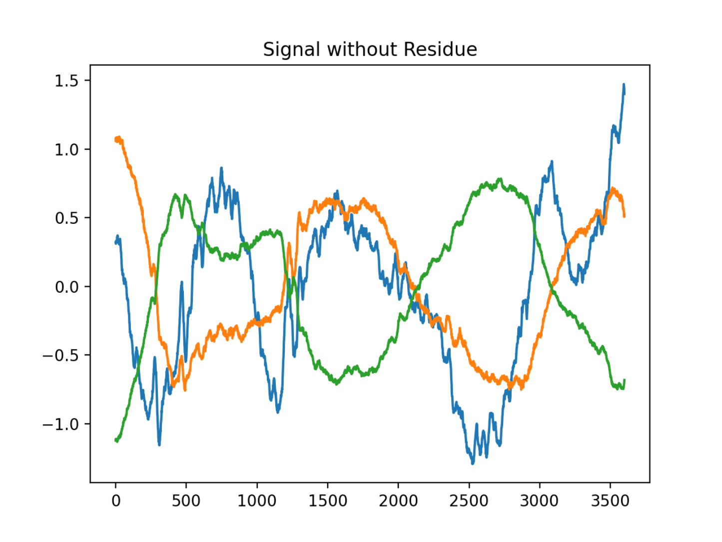




Bx

By

Bz



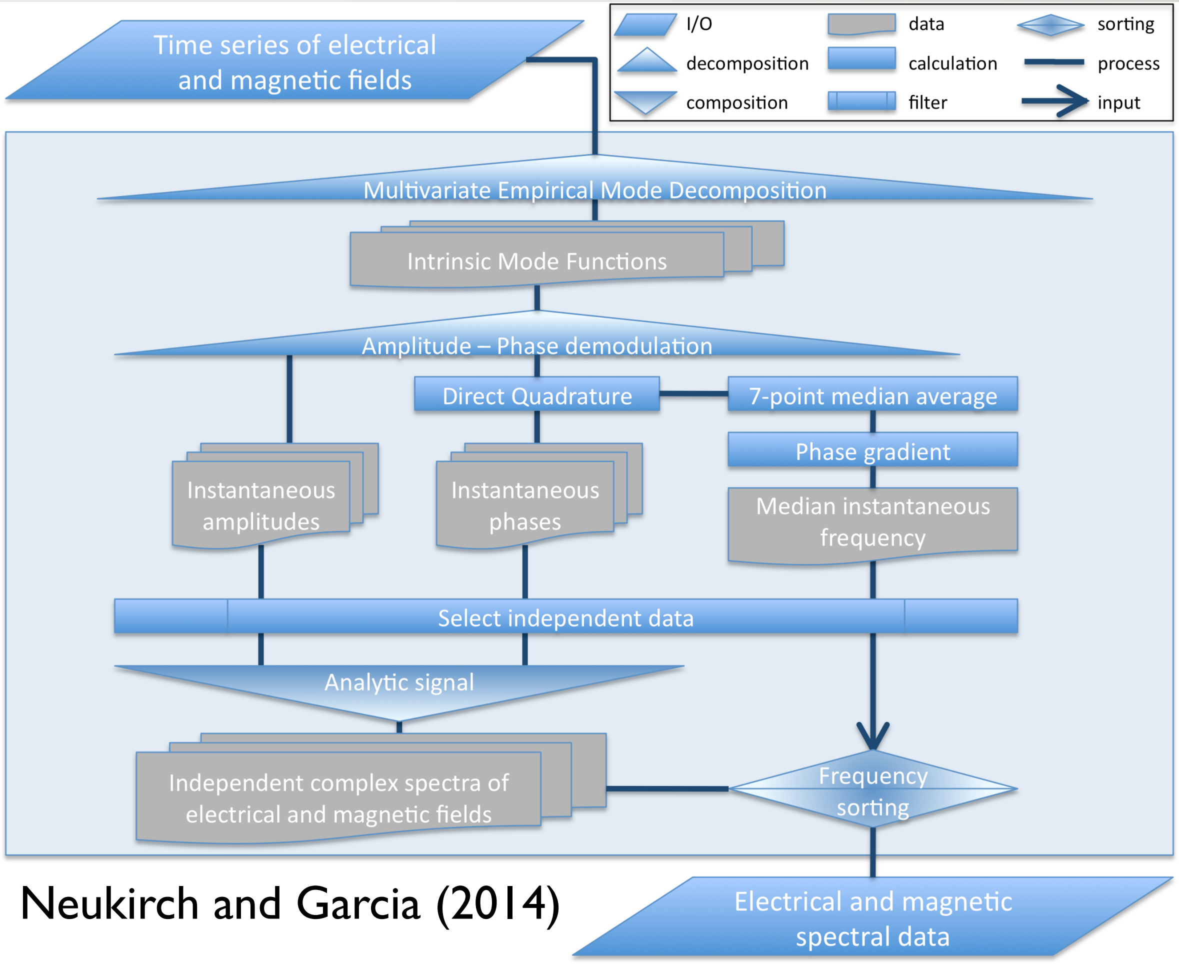
IMF → SIGNAL + FREQUENCY

EMD

- Chen and Jegen (2008) suggest to use the marginal spectra for each IMF
- Chen et al (2012) use instantaneous parameters (Huang et al, 2008): common Instantaneous Frequency in all channels

MEMD

- Neukirch and Garcia (2014) use multivariate EMD and then similar strategy as Chen et al (2012)



Neukirch and Garcia (2014)

3. IMPEDANCE ESTIMATION

- Tranformed TS into spectral domain, solve:

$$\mathbf{e} = \mathbf{z}\mathbf{b}$$

- Can be solved using OLS, converting it to a *linear regression model*

$$\mathbf{e_i} = \mathbf{b_i}\mathbf{z} + \epsilon_\mathbf{i}$$

- And solving the objective function *S*:

$$\hat{z} = \underset{z}{\arg\min}\, S(z)$$

$$S = \|\mathbf{e} - \mathbf{b}\mathbf{z}\|$$

OLS

- Minimize *S* by solving the normal equations

$$\left(\mathbf{b}^{\mathbf{T}}\mathbf{b}\right)\hat{\mathbf{z}} = \mathbf{b}^{\mathbf{T}}\mathbf{e}$$

$$\hat{\mathbf{z}} = \left(\mathbf{b}^{\mathbf{T}}\mathbf{b}\right)^{-1}\mathbf{b}^{\mathbf{T}}\mathbf{e}$$

- but $\boldsymbol{z}$ is correlated with ε, therefore OLS in inconsistent

REMOTE REFERENCE (AKA, INSTRUMENTAL VARIABLES METHOD)

- Generally used as presented by Gamble et al (1979) using magnetic channels
- Magnetic fields suffer less noise than electric ones and are less affected by local inhomogeneities
- **Requirements:**
  1. Number of RR has to be equal or larger than independent variables, for MT: $\geq$ **2**
  2. RR must be exogenous (valid):

$$Cov(\mathbf{r}, \epsilon) = \mathbf{0}$$

  3. RR must be relevant: must be correlated to each of the endogenous regressors ***b***

2SLS: 2-STAGE LEAST SQUARES

- Solve the RR problem by first solving the regression between local channels and the remotes (1st stage), then substituting these predicted values back into the normal equation (2nd stage)

$$\hat{\mathbf{b}}_i = \mathbf{r_i}\pi + \nu_\mathbf{i}$$

Since ***r· ε = 0*** and assuming any number of RR larger or equal than 2:

$$\mathbf{H_r} = \mathbf{r} \cdot (\mathbf{r^T} \cdot \mathbf{r})^{-1} \mathbf{r^T}$$

$$\breve{\mathbf{b}} = \mathbf{r} \cdot (\mathbf{r^T} \cdot \mathbf{r})^{-1} \mathbf{r^T} \cdot \mathbf{b}$$

$$\hat{\mathbf{z}}_{RR} = (\breve{\mathbf{b}}^\mathbf{T} \cdot \mathbf{b})^{-1} \breve{\mathbf{b}}^\mathbf{T} \cdot \mathbf{e}$$

WARNING

Chave and Thomson (1989):

The Remote Reference method can help in MT data processing but is not ***robust****, and in the presence of severe contamination or uncorrelated channels it will fail at providing reliable responses*

ROBUST LEAST SQUARES

- **ROBUST**: Identify and remove or damp the effect of outliers to make estimate insensitive to their presence

  →that means that it is not greatly affected by outliers and responds slowly to addition of more data

- Introduced by Huber in late 60s
- MT:
  - Jones and Jödicke (1984): LTS
  - Egbert and Booker (1986): regression M-estimator
  - Chave et al (1987): regression M-estimator
  - Larsen (1989): D+ time-series event picking

Iterative Reweighted Least Squares (IRWLS)

IRWLS – ITERATIVE REWEIGHTED LEAST SQUARES

- Recall that OLS minimizes:

$$\hat{z} = \arg\min_{z} S(z)$$

$$S = \|\mathbf{e} - \mathbf{bz}\|$$

- In IRWLS, the function to minimize is:

$$\arg\min_{z} \sum_{i=1}^{n} |y_i - f_i(z)|^p$$

- Solved iteratively as:

$$z^{t+1} = \arg\min_{z} \sum_{i=1}^{n} \omega(z^t)\, |y_i - f_i(z)|^2$$

IRWLS MATRIX FORM

- For no RR, only single site processing:

$$\hat{\mathbf{z}} = (\mathbf{b}^{\mathbf{T}} \cdot \omega \cdot \mathbf{b})^{-\mathbf{1}} \mathbf{b}^{\mathbf{T}} \cdot \omega \cdot \mathbf{e}$$

where ω is a diagonal matrix containing the Robust weights

IRWLS

- IRWLS is used to find the maximum likelihood of a generalized linear model, or what is the same, to find an M-estimator that minimizes the effect of outliers
- For MT, the solution would be:

$$z^{t+1} = \arg\min_{z} \sum_{i=1}^{n} \omega(z^t) \left| e_i - h_i z \right|^2 = \left(h^T W^{(t)} h \right)^{-1} h^T W^{(t)} e$$

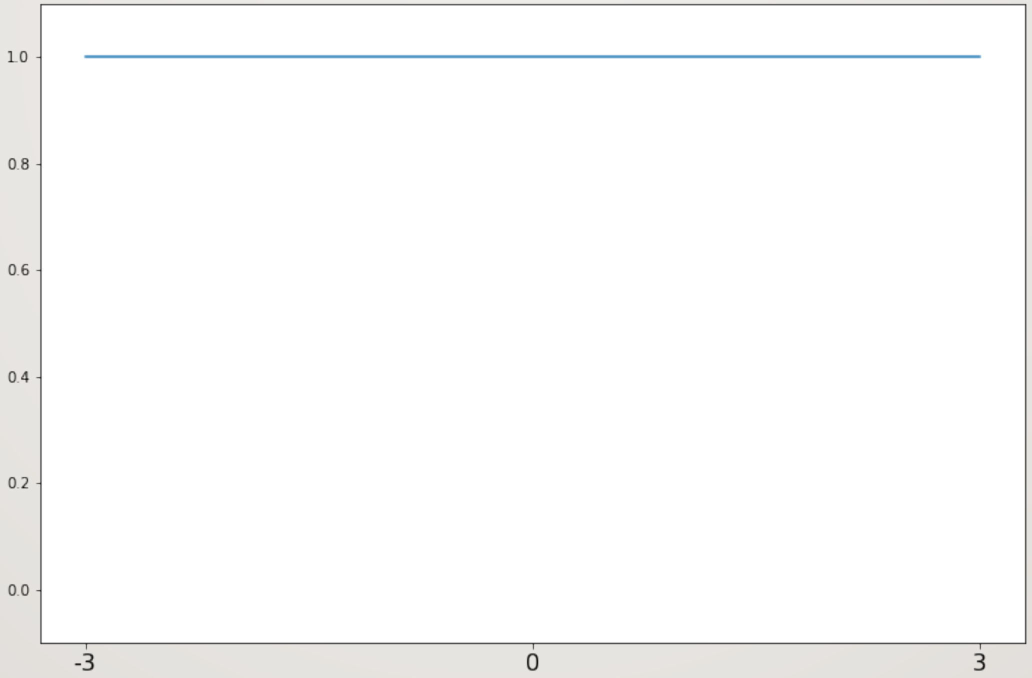
$$\omega^{(0)} = 1$$

$$\omega^{(t)} = \sum_i \rho \left(\frac{e_i}{s} \right)$$

ρ : weight function of choice

WEIGHT FUNCTIONS: OLS

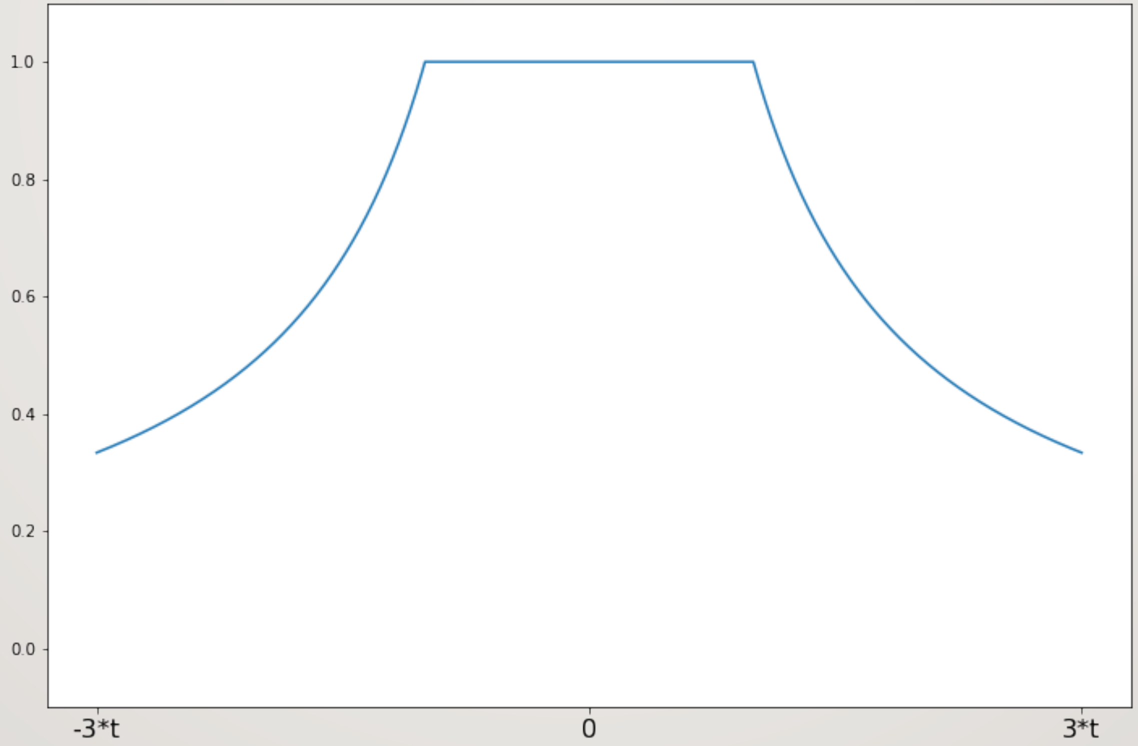
$$\omega = 1$$



WEIGHT FUNCTIONS: HUBER'S T

$$\omega^{(t+1)} = \begin{cases} 1 & |e^{(t)}| = t \\ \frac{t}{|e^{(t)}|} & |e^{(t)}| > t \end{cases}$$

t = 1.345 or 1.5

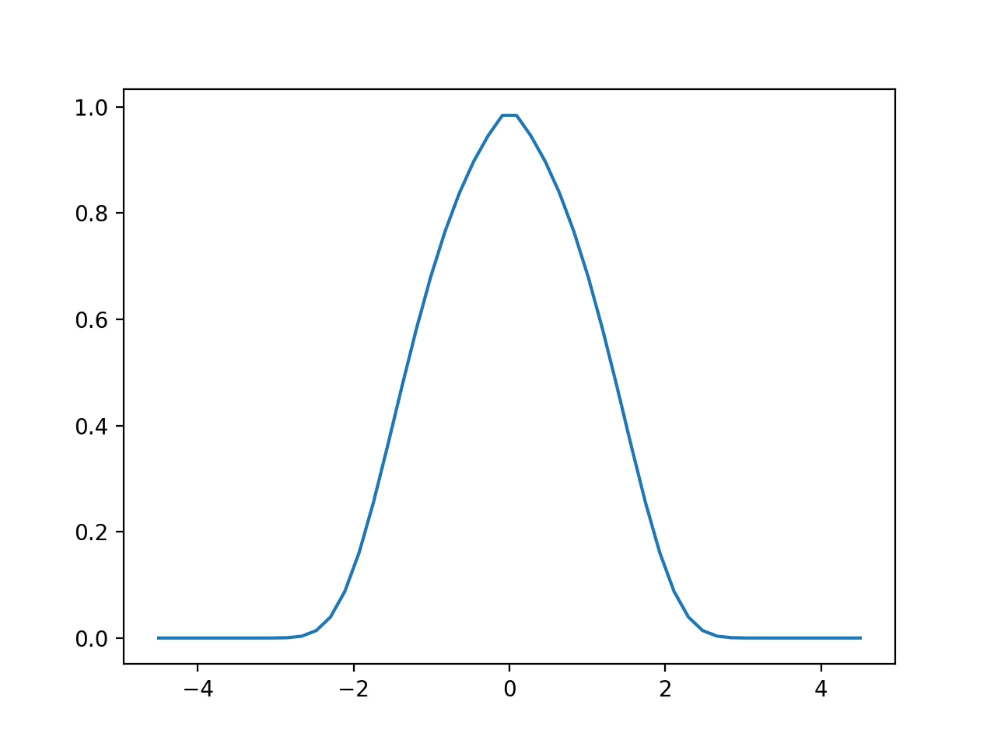


WEIGHT FUNCTION: THOMSON

$$\omega^{(t+1)} = \omega^{(t)} exp(e^{-\xi^2}) exp(-e^{\xi(|\epsilon^{(t)}|-\xi)})$$

$$\xi = \sqrt{2log(2N)}$$ N: *Nth quantile of the Rayleigh distribution*

$$\omega(e)^{(0)} = 1$$



IRWLS + RR

- IRWLS:

$$\hat{\mathbf{z}} = (\mathbf{b}^{\mathbf{T}} \cdot \omega \cdot \mathbf{b})^{-\mathbf{1}} \mathbf{b}^{\mathbf{T}} \cdot \omega \cdot \mathbf{e}$$

- RR:

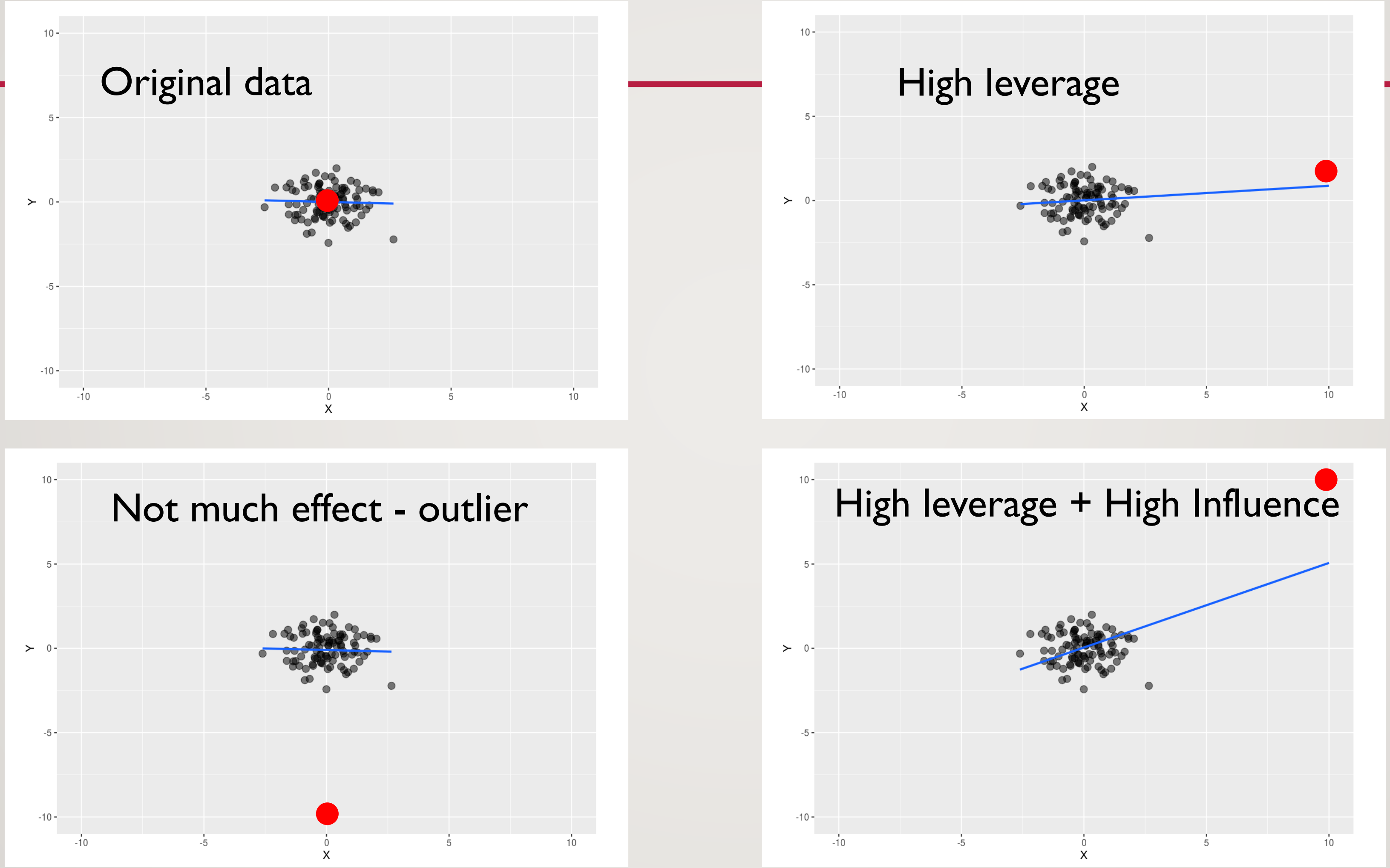
$$\hat{\mathbf{z}}_{RR} = (\breve{\mathbf{b}}^{\mathbf{T}} \cdot \mathbf{b})^{-\mathbf{1}} \breve{\mathbf{b}}^{\mathbf{T}} \cdot \mathbf{e}$$

- IRWLS + RR:

$$\hat{\mathbf{z}}_{RR} = (\breve{\mathbf{b}}^{\mathbf{T}} \cdot \omega \cdot \mathbf{b})^{-\mathbf{1}} \breve{\mathbf{b}}^{\mathbf{T}} \cdot \omega \cdot \mathbf{e}$$

$$\breve{\mathbf{b}} = \mathbf{r} \cdot (\mathbf{r}^{\mathbf{T}} \cdot \mathbf{r})^{-\mathbf{1}} \mathbf{r}^{\mathbf{T}} \cdot \mathbf{b}$$

LEVERAGE AND INFLUENCE



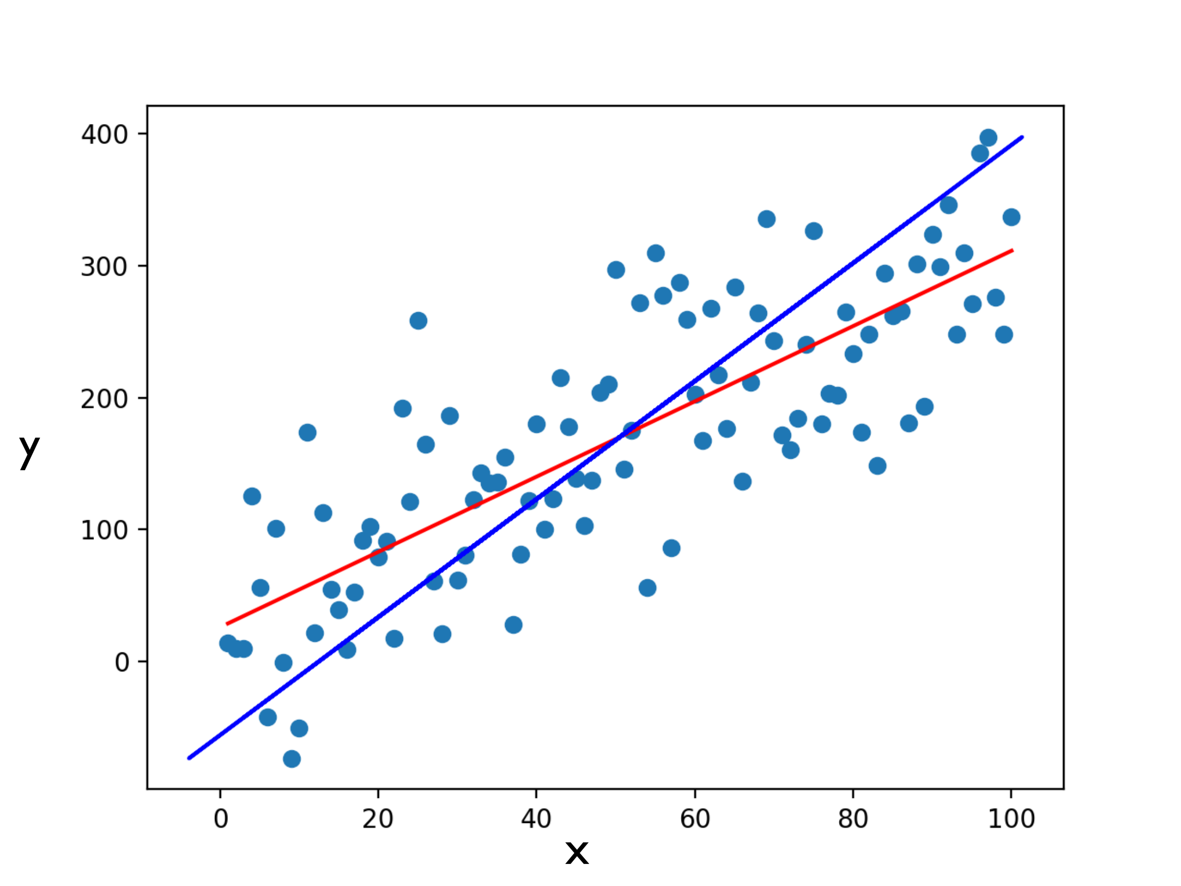
https://omaymas.github.io/InfluenceAnalysis/

WHAT ABOUT PCA?

(*EGBERT*, 1997, 2002; *SMIRNOV AND EGBERT*, 2012; NEUKIRCH AND GARCIA, 2014)

- *Egbert* (1997) shows that MT sources are well described by two electromagnetic field polarizations
- The entire data vector space of all channels in a data set can be represented by the combination of two polarization vectors
- Find the 2 PCs that better represent the source
- BUT the fields are a combination of signal and noise
- **Multivariate-Multisite** approach helps

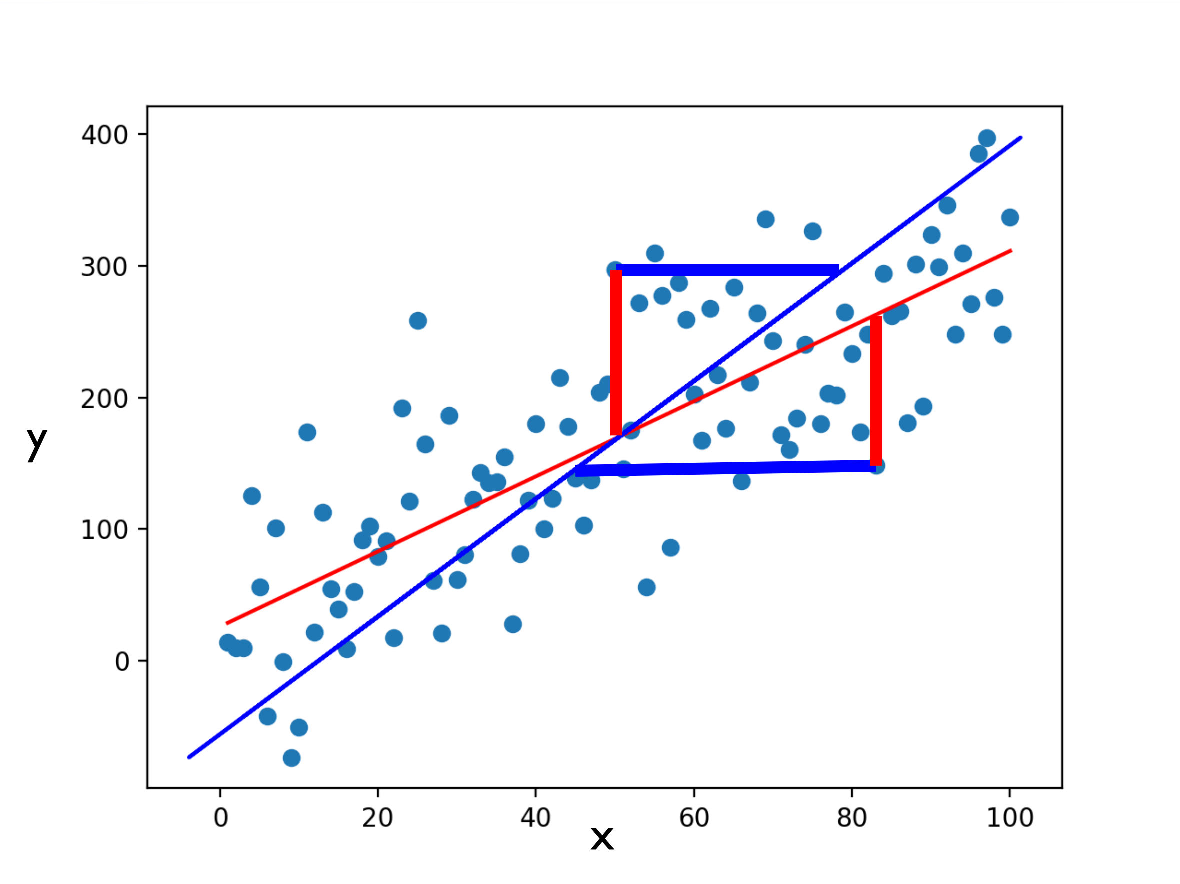
PCA VS OLS:
I – OLS



$$y = c \cdot x + m$$

$$x = d \cdot y + n$$

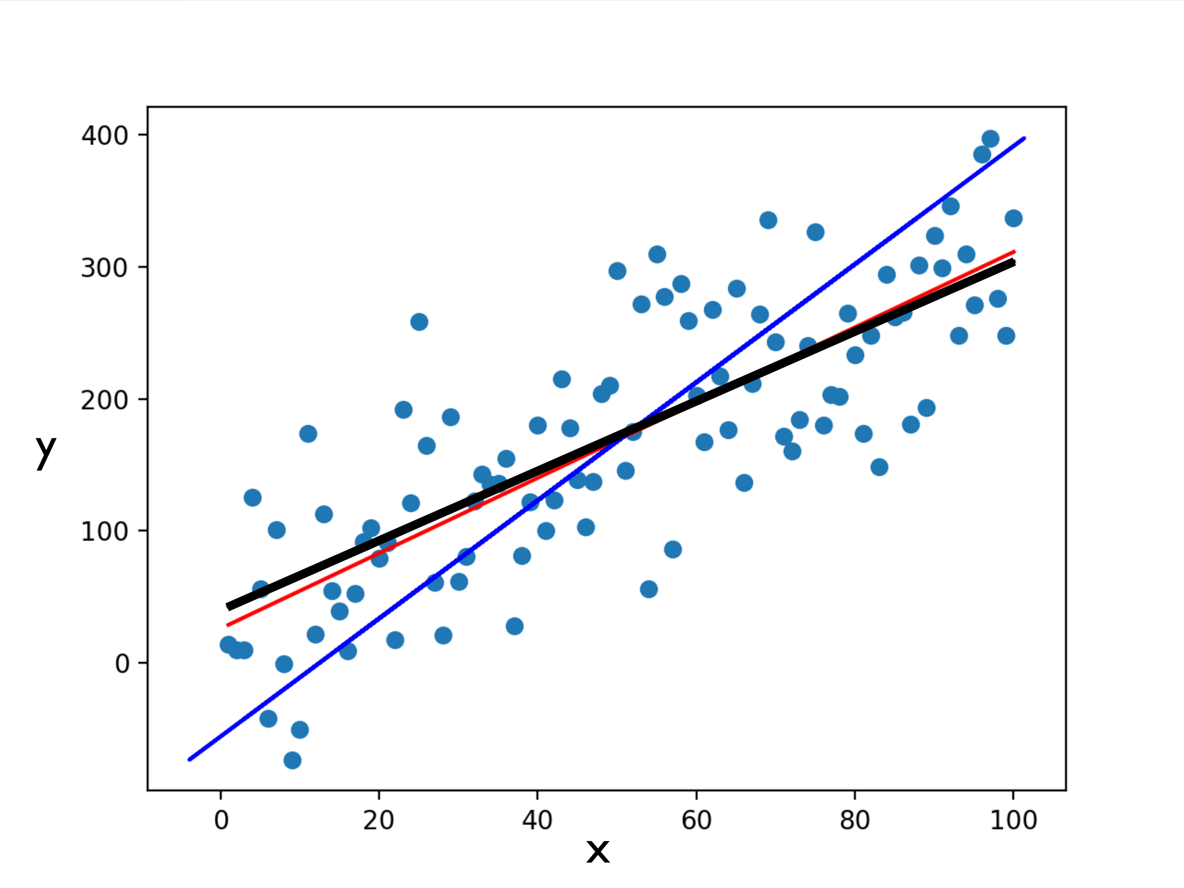
PCA VS OLS:
I – OLS (2)



$$y = c \cdot x + m$$

$$x = d \cdot y + n$$

PCA VS OLS:
2 – PCA



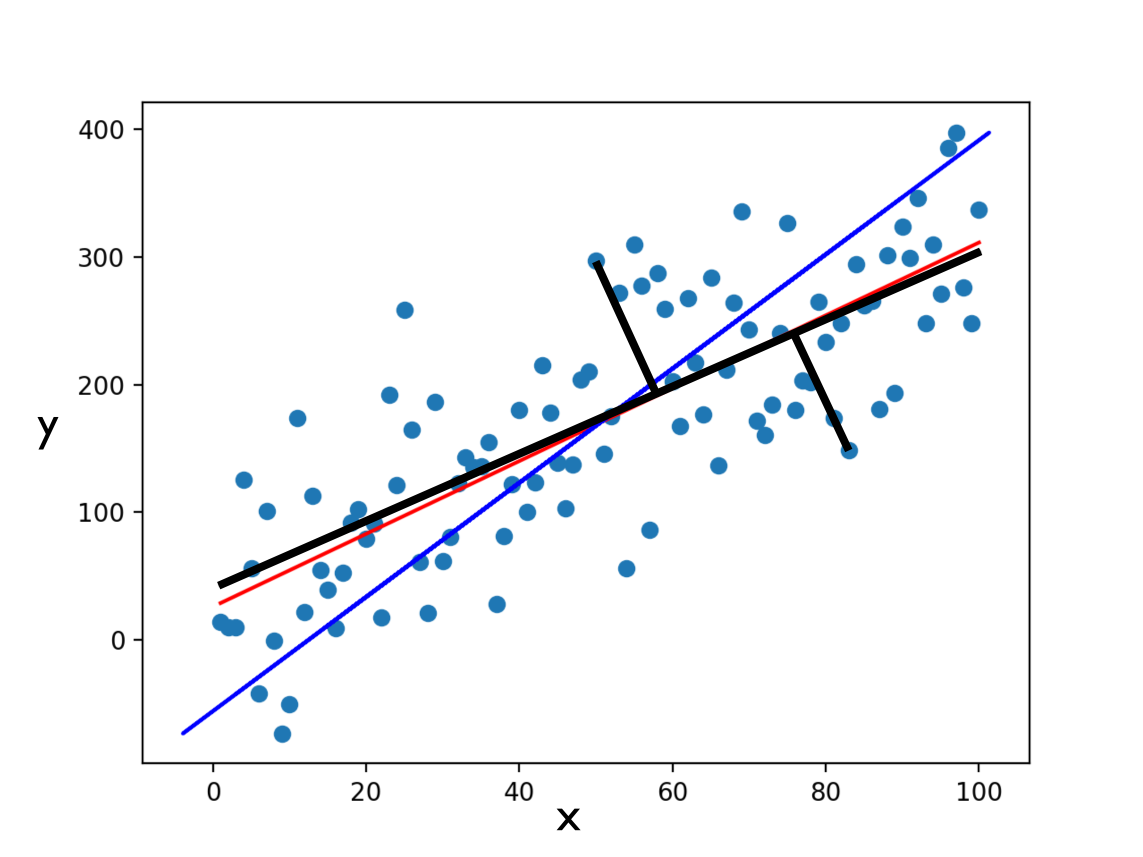
$$y = c \cdot x + m$$

$$x = d \cdot y + n$$

PCA regression:

- Scale input variables
- Estimate Covariance
- SVD on cov
- Choose largest eigenvalue
- Predict y

PCA VS OLS:
2 – PCA(2)



$$y = c \cdot x + m$$

$$x = d \cdot y + n$$

PCA regression:

- Scale input variables
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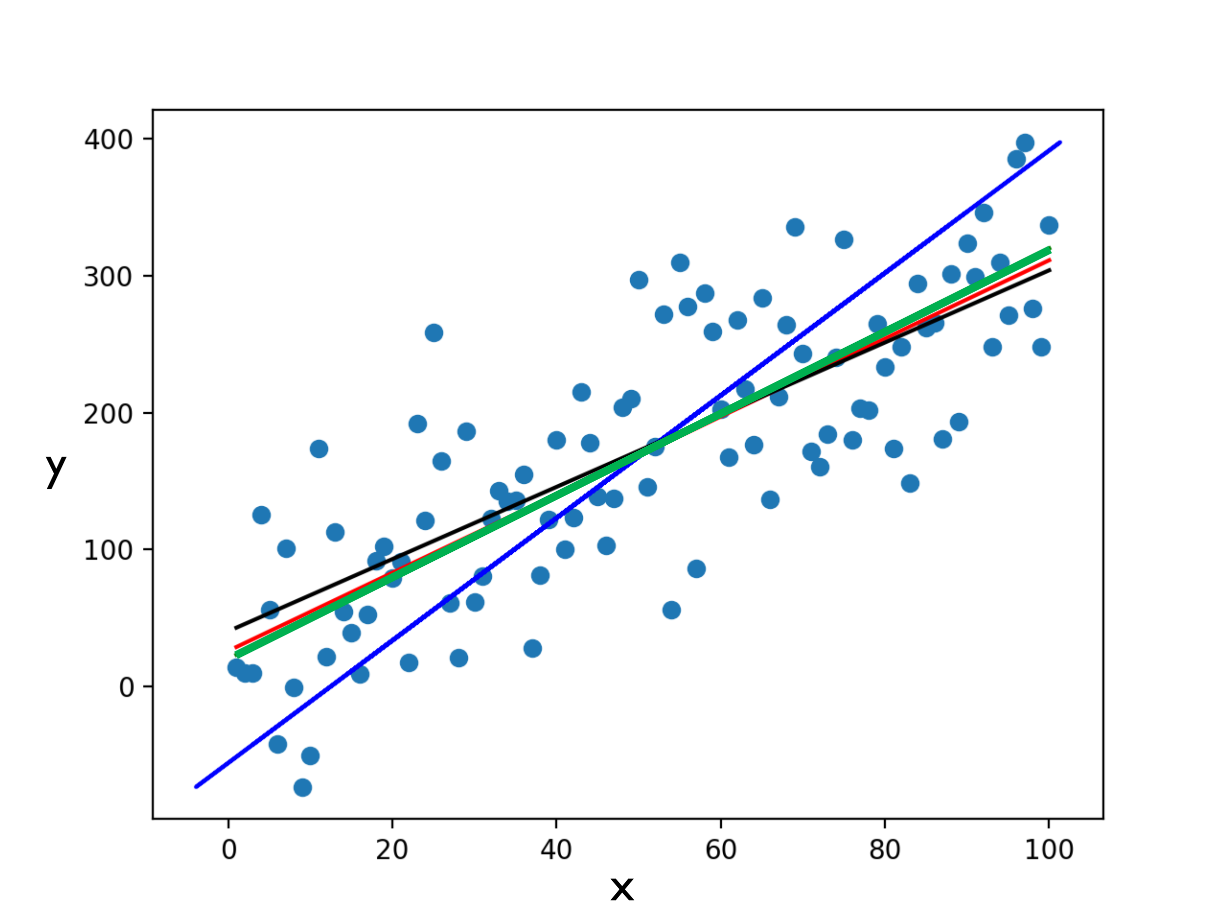
PCA VS OLS



TRUE CURVE

$$y = c \cdot x + m$$

$$x = d \cdot y + n$$

PCA regression:

- Scale input variables
- Estimate Covariance
- SVD on cov
- Choose largest eigenvalue
- Predict y

4. UNCERTAINTY ESTIMATION

- Parametric methods:
  - Huber, 1967: assymptotically normal, only valid for large number of regressors
  - Chave and Lezaeta, 2002: non-central Chi-square (2 dof) for rho, short-tailed Gaussian for phases
- Non-parametric methods:
  - Bootstrap
  - Jackknife

JACKKNIFE

- Let $\{x_i\}$ be an independent sample of size N
- Let ξ be a statistical parameter using the estimator Z:

$$\xi = Z(x_1, ..., x_N)$$

- Create N groups of N-1 samples, always leaving one of the samples from $\{x_i\}$ out:

$$\xi_{\setminus i} = Z(x_1, ..., x_{i-1}, x_{i+1}, ..., x_N)$$

- For example, estimate variance:

$$\sigma_J^2 = \frac{N-1}{N} \sum_{i=1}^{N} (\xi_{\setminus i} - \xi)^2$$

BOOTSTRAP

- Let $\{x_i\}$ be an independent sample of size N
- Let ξ be a statistical parameter using the estimator Z:

$$\xi = Z(x_1, ..., x_N)$$

- Create K groups of M samples, chosen randomly with replacement from $\{x_i\}$:

$$\xi_i = Z(M\,elements) \quad i = 1, ..., K$$

- For example, estimate variance:

$$\sigma_B^2 = \frac{K-1}{K} \sum_{i=1}^{K} (\xi_i - \xi)^2$$

NON-PARAMETRIC APPLIED TO MT

JACKKNIFE: BIRRP, Chave and Thomson, 2003, 2004:

$$\sigma_J^2 = \frac{N}{N-P}\sum_{i=1}^{N}(1-h_{ii})^2(z-z_{\setminus i})\cdot(z-z_{\setminus i})^T$$

BOOTSTRAP: EMT, Neukirch and Garcia, 2014:

$$\sigma_B^2 = \frac{K-1}{K}\sum_{i=1}^{K}(z-z_i)\cdot(z-z_i)^T$$

FINAL NOTES

- LOOK at the data
- There are plenty of freely available ROBUST processing codes
- Lots of developments in TS analysis and regression

THANK YOU!