Magnetotelluric Response Function Analysis

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MOTIVATION

Basic steps in the Magnetotelluric (MT) Method:

- Data acquisition
- Time series processing → response functions
- Modelling, inversion
- Interpretation

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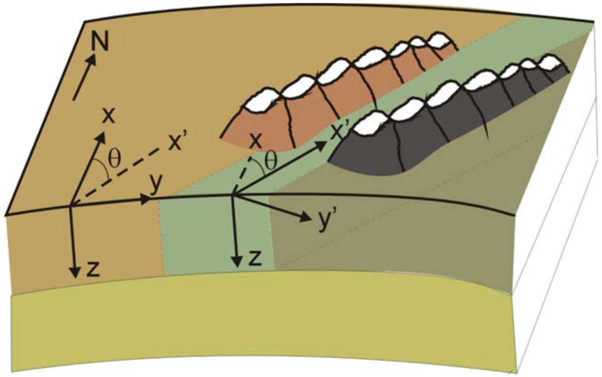
Basic steps in the Magnetotelluric (MT) Method:

- Data acquisition
- Time series processing → response functions
- **Response function analysis**
- Modelling, inversion
- Interpretation

Importance of response function analysis:

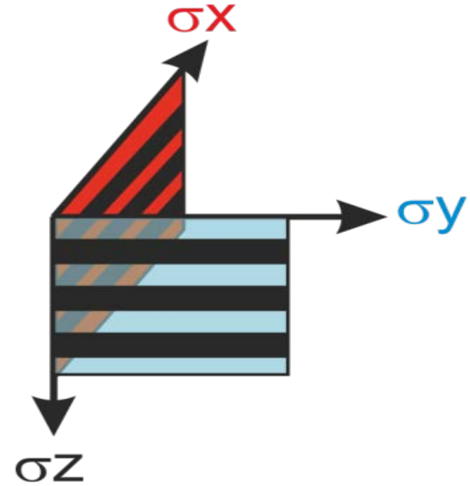
- to know the dimensionality of the structures before modelling/inverting data



- to correct data from

static shift, distortion, and rotate to strike direction

- to identify hints of anisotropy

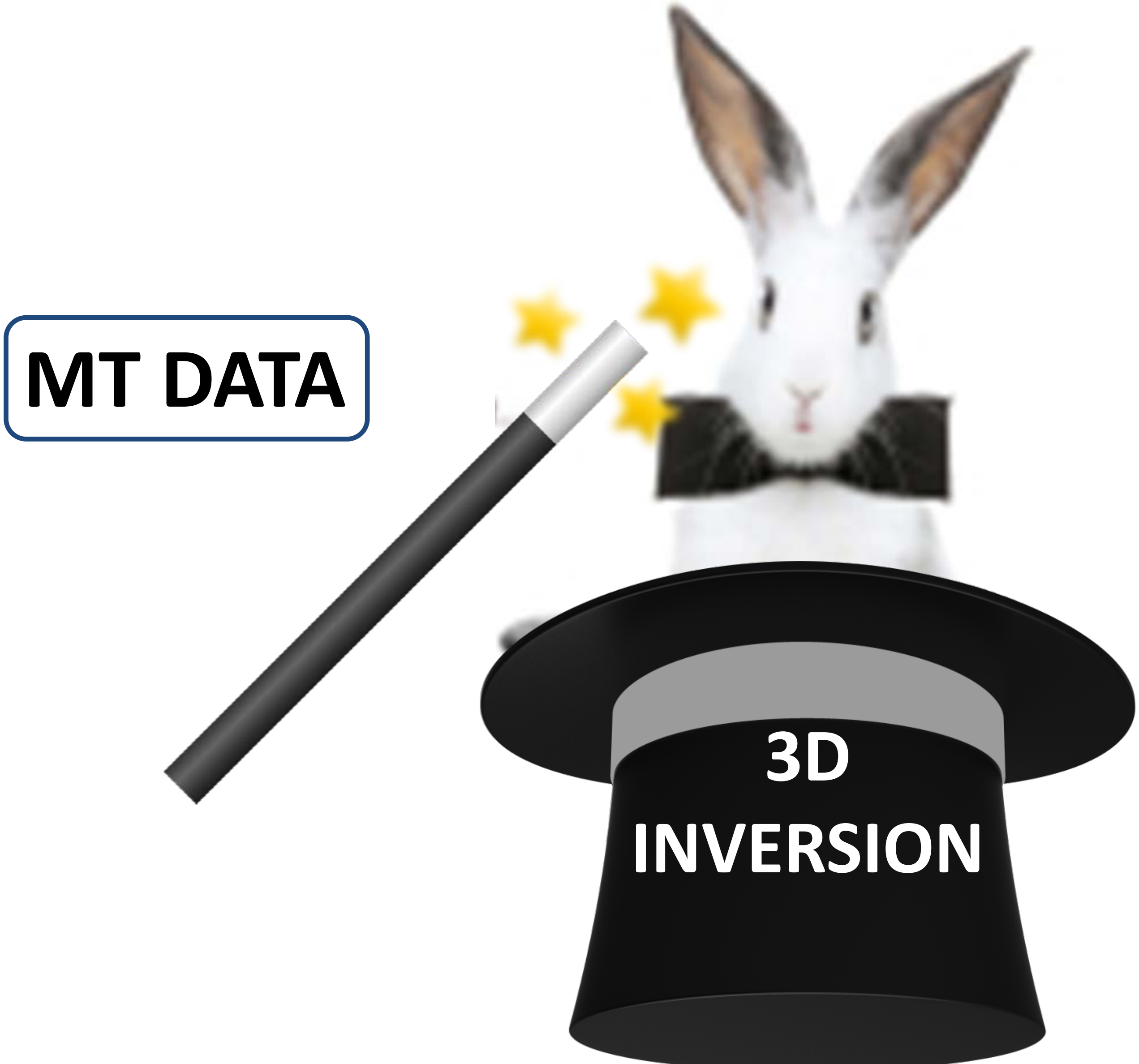


Ignoring this step might led to wrong models

- Ledo et al. (2002); Ledo (2005): limitations of 3D data modelled as 2D

- Miensopust and Jones (2011): artifacts due to anisotropic data modelled as isotropic

We might think that 3D inversion solves everything …



Instead …

- Information from data analysis can be used as constraints for 3D models: preferred orientations, distorting bodies …

About this EMinar:

I will focus on the analysis of MT responses, based on my experience:

1) Explain what information about dimensionality and galvanic distortion is in the impedance tensor and tipper.

2) Describe methods and tools to analyse it. WAL invariants (WALDIM), Phase Tensor, G&B (Strike code)

3) Extend this analysis to anisotropic structures

Contents:

- **MT responses, dimensionality and distortion**
- **Dimensionality analysis methods**
- **Decomposition methods**
- **Extension to an anisotropic earth**

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- MT responses, dimensionality and distortion
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MT responses

$E(t)$, $H(t)$ → cross spectra → **MT transfer functions (freq. domain)**

Impedance Tensor $\underline{Z}$ (Ohms)
(complex components)

$$\begin{pmatrix} E_x(\omega) \\ E_y(\omega) \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \cdot \begin{pmatrix} H_x(\omega) \\ H_y(\omega) \end{pmatrix}$$

Magnetotelluric tensor $\underline{M}$ (m/s): relates ***E*** and ***B***: $\underline{Z}=\mu_0\underline{M}$

Apparent Resistivity and Phase:

$$\rho_{ij}(\omega) = \frac{1}{\mu_0\omega}\left|Z_{ij}(\omega)\right|^2 \; (\Omega\cdot m)$$

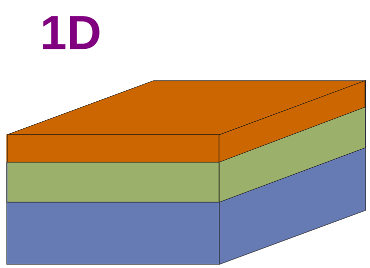
$$\varphi_{ij}(\omega) = \arctan\left(\operatorname{Im}\left(Z_{ij}(\omega)\right) / \operatorname{Re}\left(Z_{ij}(\omega)\right)\right)$$

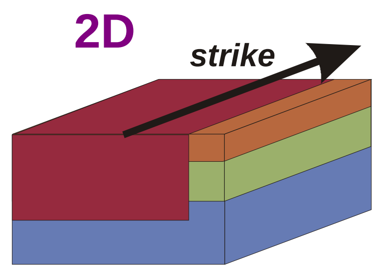
Tipper *vector* ***T***
(complex components)

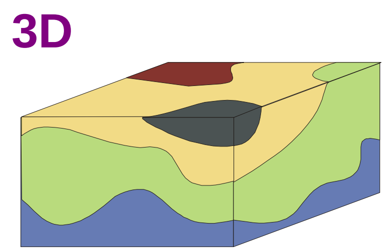
$$H_z(\omega) = \left(T_x(\omega), T_y(\omega)\right)\begin{pmatrix} H_x(\omega) \\ H_y(\omega) \end{pmatrix}$$

Geoelectrical Dimensionality

Earth MT dimensionality types (for isotropic conductivity):







$$\underline{Z} = \begin{pmatrix} 0 & Z_{xy} \\ -Z_{xy} & 0 \end{pmatrix}$$

$$\vec{T} = (0,0)$$

$$\underline{Z} = \begin{pmatrix} 0 & Z_{xy} \\ Z_{yx} & 0 \end{pmatrix} = \begin{pmatrix} 0 & Z_{TE} \\ Z_{TM} & 0 \end{pmatrix}$$

$$\vec{T} = (0, T_y)$$

$$\underline{Z} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}$$

$$\vec{T} = (T_x, T_y)$$

2D cases not measured along the principal directions:

$$\underline{Z} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \qquad \vec{T} = \left(T_x, T_y\right)$$

$$\underline{Z}' = R_\theta Z R_\theta^T \qquad \vec{T}' = R_\theta \vec{T}$$

where $R_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

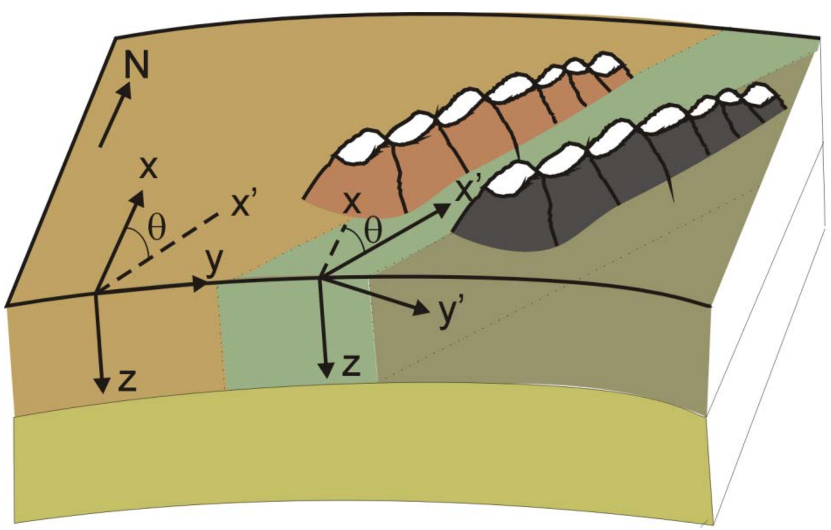
x = measurement direction

x' = strike direction

θ = strike angle

$$\underline{Z}' = \begin{pmatrix} 0 & Z_{x'y'} \\ Z_{y'x'} & 0 \end{pmatrix} = \begin{pmatrix} 0 & Z_{TE} \\ Z_{TM} & 0 \end{pmatrix}$$

$$\vec{T} = \left(0, T_{y'}\right)$$



Galvanic distortion:

Distortion: caused by shallow and local (3D) bodies << target of interest

→ Induction effects (in MT $\sigma >> \omega\varepsilon$), decay rapidly with period and can be ignored.

→ Galvanic effects (important in MT): accumulation of charges at the surface of the local bodies → anomalous electric field $\boldsymbol{E}_a$.

$$\vec{E}' = \vec{E}_R + \vec{E}_A \approx C\vec{E}_R$$

$$\vec{E}' = C(\underline{Z}_R \vec{H}_R) = \underline{Z}_{meas} \vec{H}_R$$

Mathematical representation:

C: Matrix of real components:

$$C = \begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix}$$

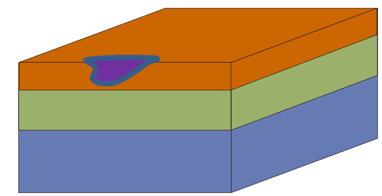
Groom and Bailey (1989):

$C = g \cdot T \cdot S \cdot A$ (site gain, Twist, Shear, Anisotropy)

$$C = g \cdot \begin{pmatrix} 1 & -t \\ t & 1 \end{pmatrix} \begin{pmatrix} 1 & s \\ s & 1 \end{pmatrix} \begin{pmatrix} 1+a & 0 \\ 0 & 1-a \end{pmatrix}$$

If the regional structure is 1D:

3D/1D

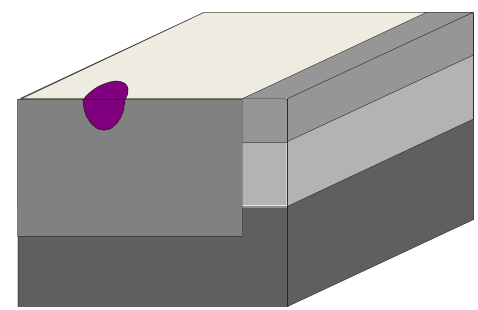


$$Z_{meas} = C \cdot Z_{meas} = \begin{pmatrix} -C_{xy}Z_{xy} & C_{xx}Z_{xy} \\ -C_{yy}Z_{xy} & C_{yx}Z_{xy} \end{pmatrix}$$

If the regional structure is 2D:

$$Z_{meas} = C \cdot Z_{2D} = \begin{pmatrix} C_{xy}Z_{yx} & C_{xx}Z_{xy} \\ C_{yy}Z_{yx} & C_{yx}Z_{xy} \end{pmatrix}$$

Case 3D/2D



If not measured along strike direction:

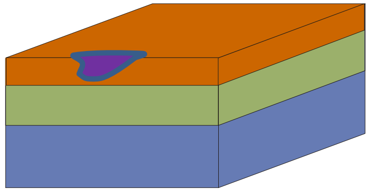
$$Z_{meas} = R_\theta \cdot C \cdot Z_{2D} \cdot R_\theta^T$$

3D/1D and 3D/2D:

matrix C affects regional tensor components amplitudes (but not phases):

g and A are undeterminable: **Static Shift**

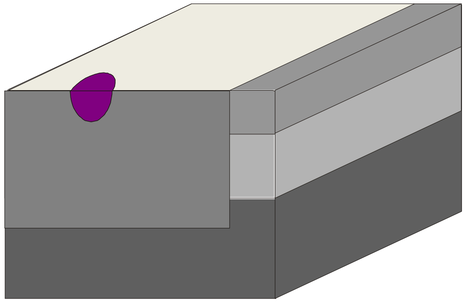
3D/1D



2D cases with equal phases in TE and TM:

3D/2D is undistinguishable from 3D/1D

Case 3D/2D



Static shift correction:

MT data alone:

E.g. Ledo et al. 2002. Use ***T*** and $\underline{Z}$ relationships

Additional information:

TDEM data. E.g. Pellerin & Hohhmann, 1990; Pace et al. (under revision)

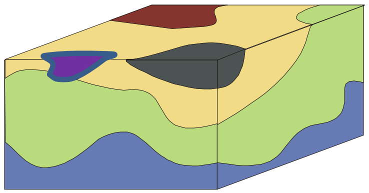
Static shift as part of the inversion process: E.g. Avdeeva et al. 2015 (inverts for the full distortion)

3D/3D:

Undistinguishable from general 3D

$$Z_{meas} = C \cdot Z_{3D} = \begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix} \cdot \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix}$$

3D or 3D/3D



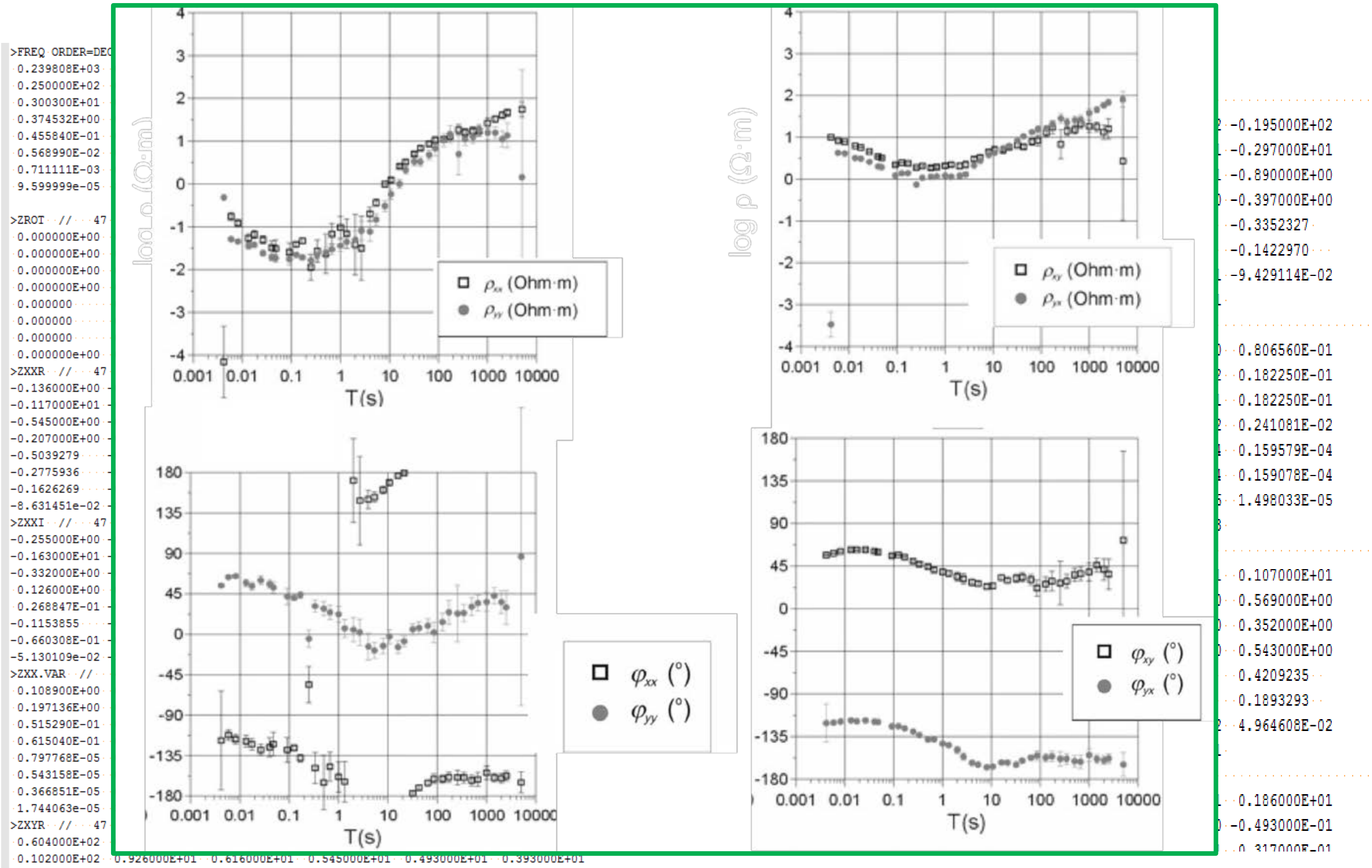
With all this information … We can look at the data …

```
>FREQ ORDER=DEC  //   47
 0.239808E+03  0.170068E+03  0.123001E+03  0.751880E+02  0.571429E+02  0.380228E+02
 0.250000E+02  0.210084E+02  0.110011E+02  0.800000E+01  0.598802E+01  0.400000E+01
 0.300300E+01  0.200000E+01  0.149925E+01  0.100000E+01  0.751880E+00  0.500000E+00
 0.374532E+00  0.250000E+00  0.187617E+00  0.125000E+00  0.934579E-01  0.625000E-01
 0.455840E-01  0.341880E-01  0.227596E-01  0.170697E-01  0.113798E-01  0.853485E-02
 0.568990E-02  0.426743E-02  0.284444E-02  0.213333E-02  0.142222E-02  0.106667E-02
 0.711111E-03  0.533333E-03  3.555557e-04  2.666668e-04  1.923077e-04  1.600000e-04
 9.599999e-05  8.000001e-05  4.800003e-05  3.999998e-05  2.400001e-05

>ZROT  //   47
 0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
 0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
 0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
 0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00  0.000000E+00
 0.000000      0.000000      0.000000      0.000000      0.000000      0.000000
 0.000000      0.000000      0.000000      0.000000      0.000000      0.000000
 0.000000      0.000000      0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
 0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00  0.000000e+00
>ZXXR  //   47
-0.136000E+00 -0.456000E+01 -0.390000E+01 -0.219000E+01 -0.233000E+01 -0.191000E+01
-0.117000E+01 -0.961000E+00 -0.741000E+00 -0.738000E+00 -0.876000E+00  0.266000E+00
-0.545000E+00 -0.468000E+00 -0.599000E+00 -0.647000E+00 -0.489000E+00 -0.306000E+00
-0.207000E+00 -0.434000E+00 -0.520000E+00 -0.744000E+00 -0.748000E+00 -0.895000E+00
-0.5039279    -0.4790654    -0.4454106    -0.4100896    -0.3566264    -0.3221351
-0.2775936    -0.2502640    -0.2225565    -0.2090112    -0.1935553    -0.1794896
-0.1626269    -0.1441109    -1.315160e-01 -1.227659e-01 -1.114016e-01 -1.062367e-01
-8.631451e-02 -8.311318e-02 -6.265154e-02 -9.339188e-02  2.204277e-02
>ZXXI  //   47
-0.255000E+00 -0.113000E+02 -0.775000E+01 -0.394000E+01 -0.368000E+01 -0.240000E+01
-0.163000E+01 -0.153000E+01 -0.928000E+00 -0.100000E+01 -0.794000E+00 -0.395000E+00
-0.332000E+00 -0.125000E+00 -0.385000E+00 -0.253000E+00 -0.140000E+00  0.475000E-01
 0.126000E+00  0.249000E+00  0.269000E+00  0.261000E+00  0.148000E+00  0.563000E-01
 0.268847E-01 -0.179701E-01 -0.717417E-01 -0.990736E-01 -0.1197412    -0.1204945
-0.1153855    -0.1074353    -0.926990E-01 -0.836689E-01 -0.756749E-01 -0.702834E-01
-0.660308E-01 -0.651813E-01 -5.691488e-02 -5.074219e-02 -5.081308e-02 -4.517944e-02
-5.130109e-02 -5.835883e-02 -6.057418e-02 -7.555395e-02  7.554711e-02
>ZXX.VAR  //   47
 0.108900E+00  0.228010E+01  0.923521E+00  0.388129E+00  0.330625E+00  0.142884E+00
 0.197136E+00  0.242064E+00  0.114921E+00  0.732736E-02  0.112360E-01  0.396010E-01
 0.515290E-01  0.924160E-01  0.585640E-01  0.585640E-01  0.566440E-01  0.900000E-01
 0.615040E-01  0.112360E-01  0.473344E-02  0.440896E-02  0.350464E-02  0.184900E-02
 0.797768E-05  0.307478E-05  0.258194E-05  0.257618E-05  0.391403E-05  0.507379E-05
 0.543158E-05  0.364137E-05  0.385044E-05  0.352291E-05  0.493168E-05  0.486148E-05
 0.366851E-05  0.549304E-05  5.771235e-06  7.396844e-06  5.635689e-06  1.253703e-05
 1.744063e-05  3.146644e-05  1.525336e-04  1.681691e-03  2.131458e-02
>ZXYR  //   47
 0.604000E+02  0.438000E+02  0.341000E+02  0.226000E+02  0.190000E+02  0.137000E+02
 0.102000E+02  0.926000E+01  0.616000E+01  0.545000E+01  0.493000E+01  0.393000E+01
```

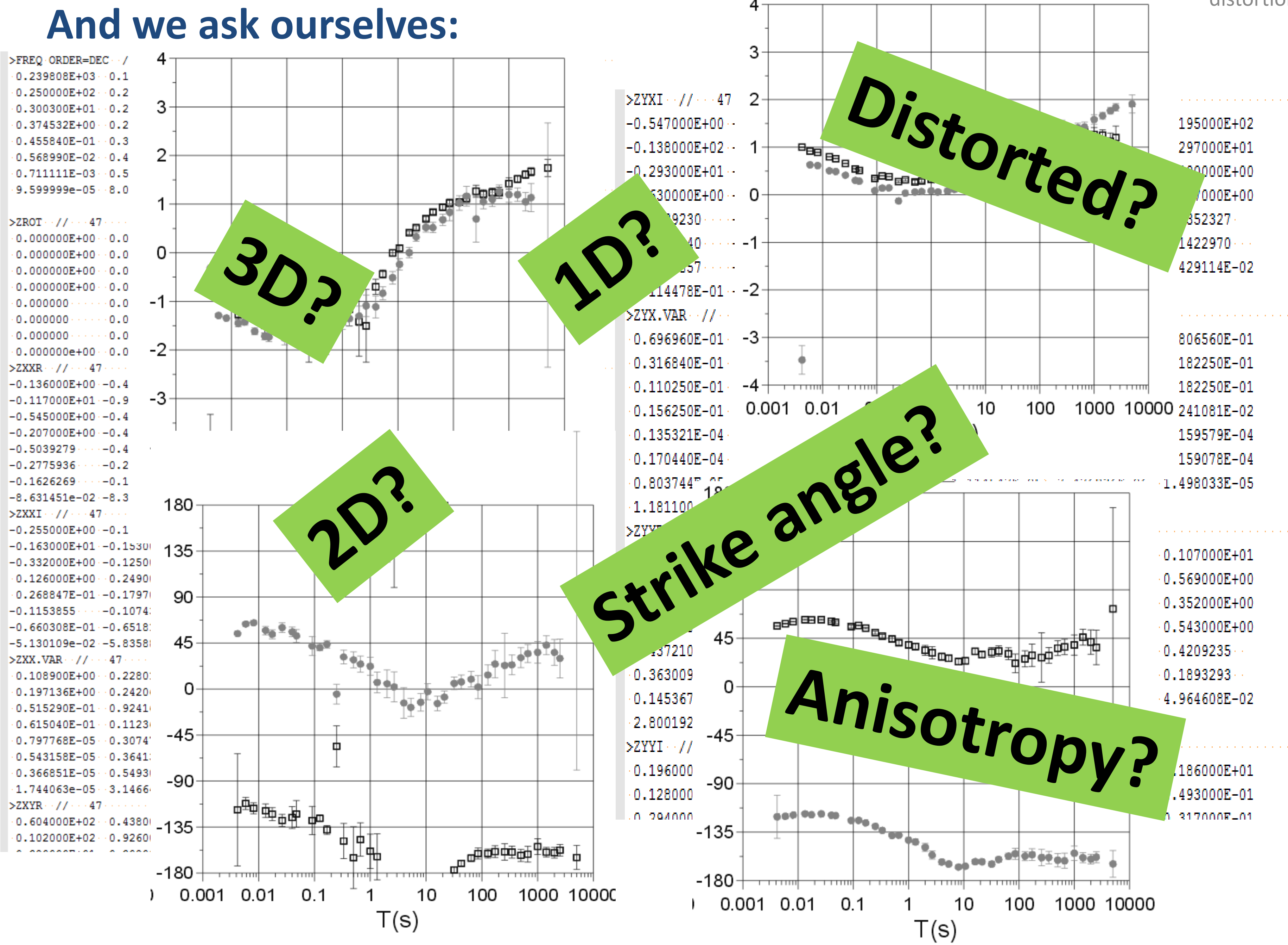
```
>ZYXI  //   47
-0.547000E+00 -0.521000E+02 -0.440000E+02 -0.306000E+02 -0.260000E+02 -0.195000E+02
-0.138000E+02 -0.124000E+02 -0.679000E+01 -0.617000E+01 -0.521000E+01 -0.297000E+01
-0.293000E+01 -0.226000E+01 -0.199000E+01 -0.150000E+01 -0.122000E+01 -0.890000E+00
-0.630000E+00 -0.480000E+00 -0.427000E+00 -0.335000E+00 -0.326000E+00 -0.397000E+00
-0.4639230    -0.4684913    -0.4440483    -0.4143425    -0.3724505    -0.3352327
-0.2857440    -0.2508257    -0.2089823    -0.1817412    -0.1581490    -0.1422970
-0.1289857    -0.1265638    -1.110858E-01 -1.041131E-01 -1.016429E-01 -9.429114E-02
-1.114478E-01 -1.124974E-01 -9.479300E-02 -1.068946E-01 -1.055695E-01
>ZYX.VAR  //   47
 0.696960E-01  0.494209E+00  0.205209E+00  0.304704E+00  0.222784E+00  0.806560E-01
 0.316840E-01  0.428490E-01  0.353440E-01  0.318096E-02  0.426409E-02  0.182250E-01
 0.110250E-01  0.116640E-01  0.112360E-01  0.106090E-01  0.118810E-01  0.182250E-01
 0.156250E-01  0.586756E-02  0.235225E-02  0.291600E-02  0.252004E-02  0.241081E-02
 0.135321E-04  0.584795E-05  0.638357E-05  0.700498E-05  0.123673E-04  0.159579E-04
 0.170440E-04  0.109698E-04  0.121363E-04  0.116187E-04  0.148185E-04  0.159078E-04
 0.803744E-05  0.850931E-05  1.042686E-05  1.114547E-05  7.476836E-06  1.498033E-05
 1.181100E-05  7.592026E-06  8.469698E-05  5.291419E-04  6.796081E-03
>ZYYR  //   47
 0.141000E+02  0.293000E+01  0.225000E+01  0.197000E+01  0.195000E+01  0.107000E+01
 0.865000E+00  0.859000E+00  0.730000E+00  0.713000E+00  0.551000E+00  0.569000E+00
 0.482000E+00  0.438000E+00  0.431000E+00  0.393000E+00  0.405000E+00  0.352000E+00
 0.390000E+00  0.302000E+00  0.353000E+00  0.427000E+00  0.519000E+00  0.543000E+00
 0.4372101     0.4844561     0.5097550     0.4892580     0.4408619     0.4209235
 0.3630095     0.3355713     0.3024104     0.2679833     0.2293016     0.1893293
 0.1453673     0.1209964     8.601386E-02  6.288213E-02  5.038427E-02  4.964608E-02
 2.800192E-02  3.393156E-02  8.498555E-03  7.440713E-03 -1.566439E-01
>ZYYI  //   47
 0.196000E+02  0.592000E+01  0.478000E+01  0.306000E+01  0.265000E+01  0.186000E+01
 0.128000E+01  0.110000E+01  0.662000E+00  0.611000E+00  0.527000E+00 -0.493000E-01
```

We can plot it …



…

And we ask ourselves:



The answer is not directly evident

- Rotations and distortion modify the regional geoelectrical responses

- Data errors affect the data

- Geological structures do not exactly fit to the ideal 1D, 2D, 3D cases

→ In general, all the measured tensors will look like:

$$\underline{Z}_{meas} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yz} & Z_{yy} \end{pmatrix}$$

But we can unveil the regional dimensionality by using dimensionality and decomposition analysis methods:

Dimensionality Analysis:

-Type of dimensionality

- θ (if 2D or 3D/2D)

- Galvanic Distortion parameters
(if 3D/1D or 3D/2D)

Decomposition

-Assume a specific regional structure

- Identify and quantify distortion (and strike direction if 2D)

- Retrieval of $Z_{\underline{R}}$

Contents:

- MT responses, dimensionality and distortion
- Dimensionality analysis methods
- Decomposition methods
- Extension to an anisotropic earth

Dimensionality analysis:

Based on the study of:

- the relationships between impedance tensor components and derived parameters (such as rotational invariants)

Tipper: not explicitly included, but is crucial to solve ambiguities in strike direction and to identify current concentrations or hints for anisotropy

- Based on rotational invariants:

WALDIM dimensionality analysis

- Based on the phase: Phase Tensor

Rotational Invariants of the Impedance tensor:

7 real independent rotational invariants (Szarka, Menvielle, 1997):

Trace: **S1 = Zxx + Zyy** (2)

$$\begin{bmatrix} reZ_{xx} & reZ_{xy} \\ reZ_{yz} & reZ_{yy} \end{bmatrix} \quad \begin{bmatrix} imZ_{xx} & imZ_{xy} \\ imZ_{yz} & imZ_{yy} \end{bmatrix}$$

Diff. non diagonals: **D2 = Zxy - Zyx** (2)

$$\begin{bmatrix} reZ_{xx} & reZ_{xy} \\ reZ_{yz} & reZ_{yy} \end{bmatrix} \begin{bmatrix} imZ_{xx} & imZ_{xy} \\ imZ_{yz} & imZ_{yy} \end{bmatrix}$$

Determinant: **det (Re(Z)), det(Im(Z)), Im (det(Z))**) → $\rho_{det}, \varphi_{det}$

$$\begin{vmatrix} reZ_{xx} & reZ_{xy} \\ reZ_{yz} & reZ_{yy} \end{vmatrix} \quad \begin{vmatrix} imZ_{xx} & imZ_{xy} \\ imZ_{yz} & imZ_{yy} \end{vmatrix} \quad im \begin{vmatrix} Z_{xx} & Z_{xy} \\ Z_{yz} & Z_{yy} \end{vmatrix}$$

$$\underline{Z}_{meas} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yz} & Z_{yy} \end{pmatrix}$$

re(S1) im(S1)

re(D2) im(D2)

det(Re(Z)) det(Im(Z))

im(det(Z))

direction

8 components of Z ←→ 7 invariants + 1 direction

Different sets of invariants can be constructed from combinations of these ones and establish dimensionality criteria

Dimensionality criteria based on different sets of invariants:

Swift, 1967

Berdichevski and Dmitriev, 1976

Bahr, 1988, 1991

Lilley, 1993, 1998a, 1998b

Szarka and Menvielle, 1997

Weaver et al., 2000

Martí et al., 2005

Lilley and Weaver, 2010

WAL Invariants (Weaver et al., 2000)

$$\underline{M} = \begin{pmatrix} \xi_1 + \xi_3 & \xi_2 + \xi_4 \\ \xi_2 - \xi_4 & \xi_1 - \xi_3 \end{pmatrix} + i \begin{pmatrix} \eta_1 + \eta_3 & \eta_2 + \eta_4 \\ \eta_2 - \eta_4 & \eta_1 - \eta_3 \end{pmatrix}$$

$\underline{M} = \underline{Z} / \mu_0$

$$I_1 = \left(\xi_1^2 + \xi_4^2\right)^{1/2} \text{ (m/s)} \qquad I_2 = \left(\eta_1^2 + \eta_4^2\right)^{1/2} \text{ (m/s)}$$

→ ρ_{1D}, φ_{1D}

$$I_3 = \frac{\left(\xi_2^2 + \xi_3^2\right)^{1/2}}{I_1} \qquad I_4 = \frac{\left(\eta_2^2 + \eta_3^2\right)^{1/2}}{I_2}$$

$$I_5 = \frac{\xi_4\eta_1 + \xi_1\eta_4}{I_1 I_2} \qquad I_6 = \frac{\xi_4\eta_1 - \xi_1\eta_4}{I_1 I_2} = d_{41}$$

$$I_7 = \left(d_{41} - d_{23}\right) / \mathrm{Q}$$

I_3 to I_7 are dimensionless and normalised

their vanishing corresponds to a particular property of the tensor

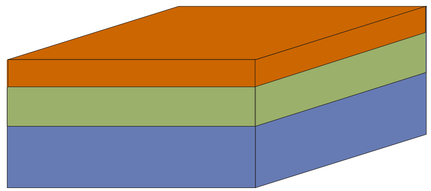
Dependent invariant:

$$\mathrm{Q} = \left[\left(d_{12} - d_{34}\right)^2 + \left(d_{13} + d_{24}\right)^2\right]^{1/2} \qquad \left(d_{ij} = \frac{\xi_i \eta_j - \xi_j \eta_i}{I_1 I_2}\right)$$

WAL dimensionality criteria

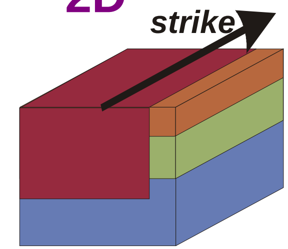
1D

$I_3 - I_6 = 0$
$I_7 = 0$ *or* $Q = 0$

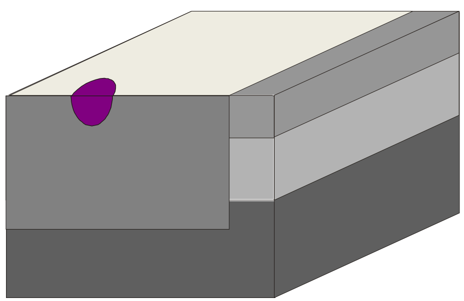


2D

I_3 *or* $I_4 \neq 0$
$I_5, I_6 = 0$
$I_7 = 0$ *or* $Q = 0$

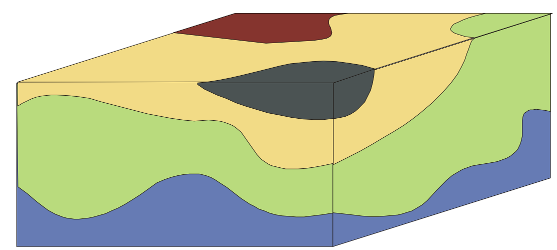


Cases 3D/1D or 3D/2D



I_3 *or* $I_4 \neq 0$
Different combinations of $I_5 - I_7$ *and* Q

3D



$I_7 \neq 0$

WAL dimensionality criteria: what parameters can we determine

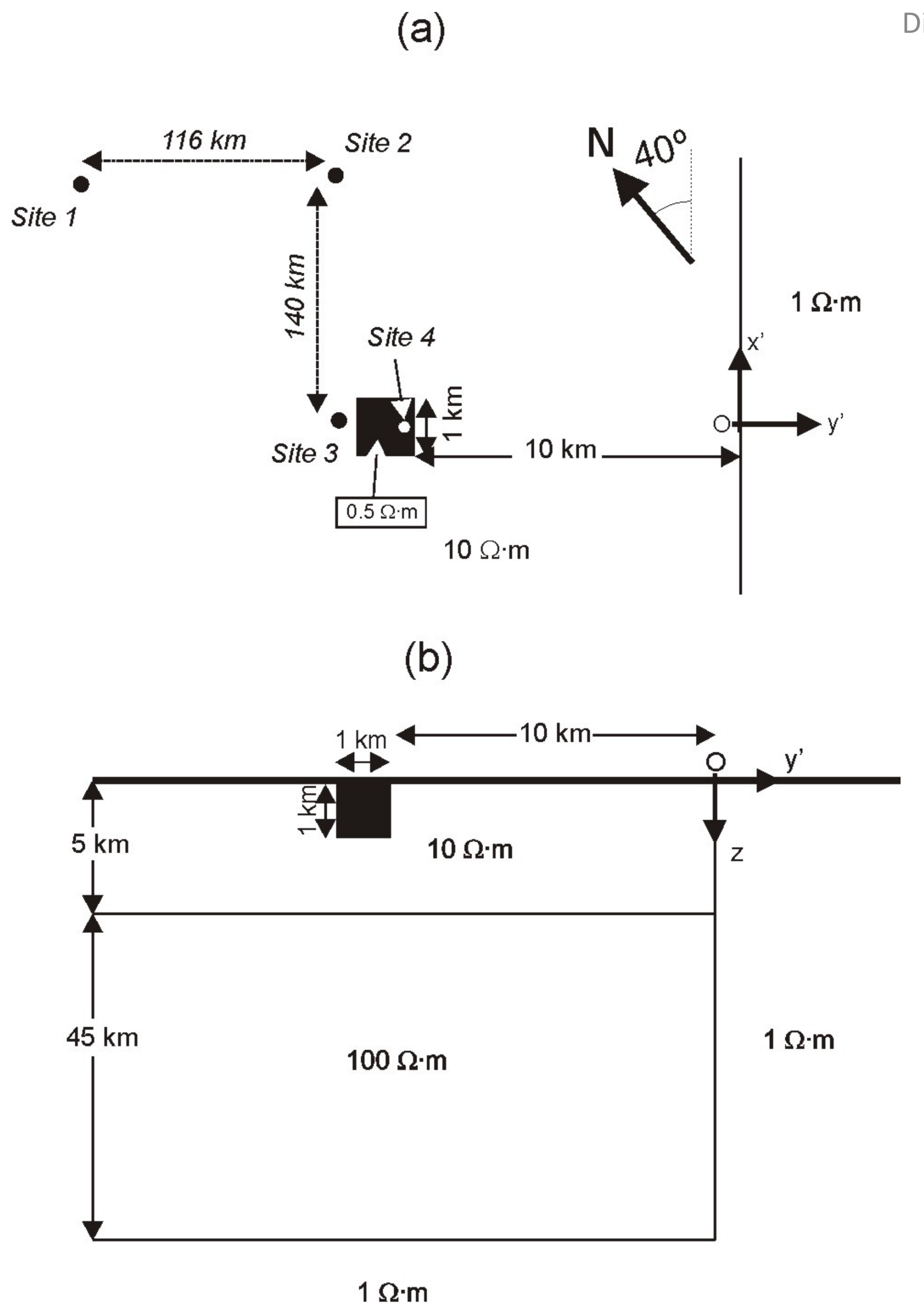
$$1D : \rho_{app,1D} , \varphi_{app,1D}$$

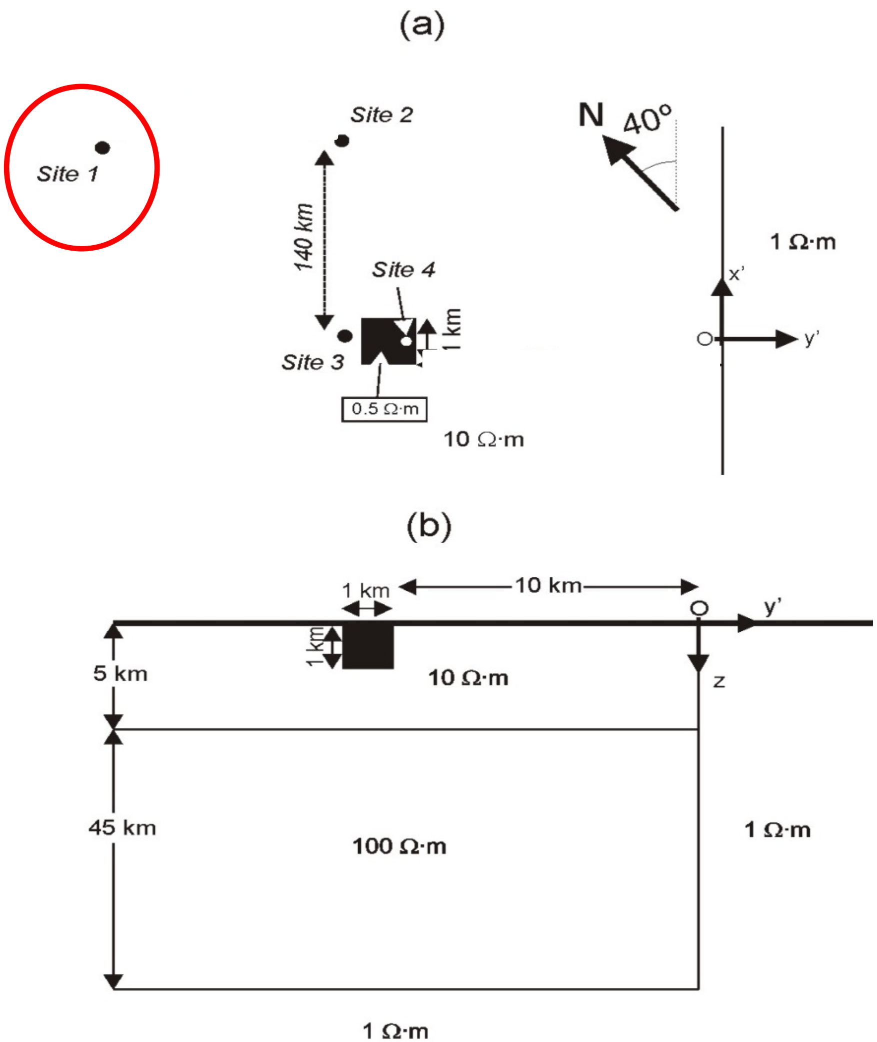
$$2D : \theta_{2D} = \theta_1('' real'') = \theta_2('' imag'')$$

$$3D / 2D : \theta_3 \qquad ; \varphi_{twist} ; \varphi_{shear}$$

Synthetic example

(Weaver et al., 2000)





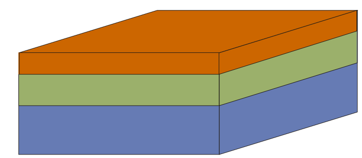
Site 1, 100 s

$$M = \begin{pmatrix} -2.28 - i\,2.94 & 1070 + i\,576 \\ -1070 - i\,575 & 2.28 + i\,2.94 \end{pmatrix}$$

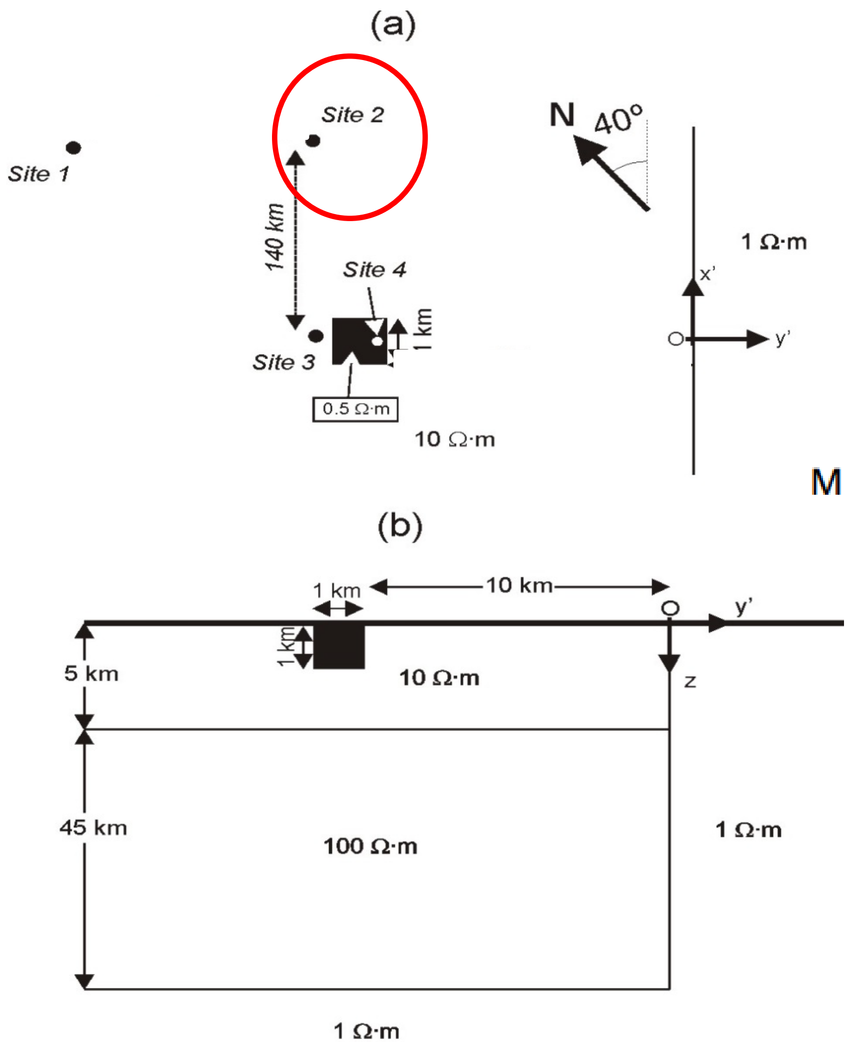
$I_3 - I_6 = 0$

$I_7 = 0$ or $Q = 0$

1D



$$\rho_{1D} = 29.5\ ohm \cdot m, \varphi_{1D} = 28°$$



Site 2, 1000 s

$$\mathbf{M} = \begin{pmatrix} 39.7 + i\,76.9 & 118 + i\,240 \\ -132 - i\,267 & -39.7 - i\,76.9 \end{pmatrix}$$

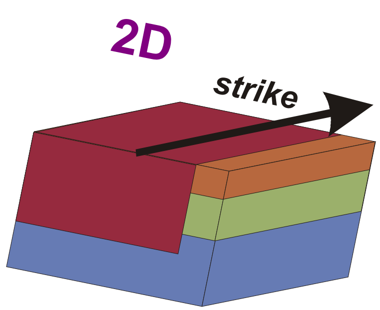
Site 2, 100 s

$$\mathbf{M} = \begin{pmatrix} 228 - i\,54.2 & 812 + i\,619 \\ -892 - i\,600 & -228 + i\,54.2 \end{pmatrix}$$

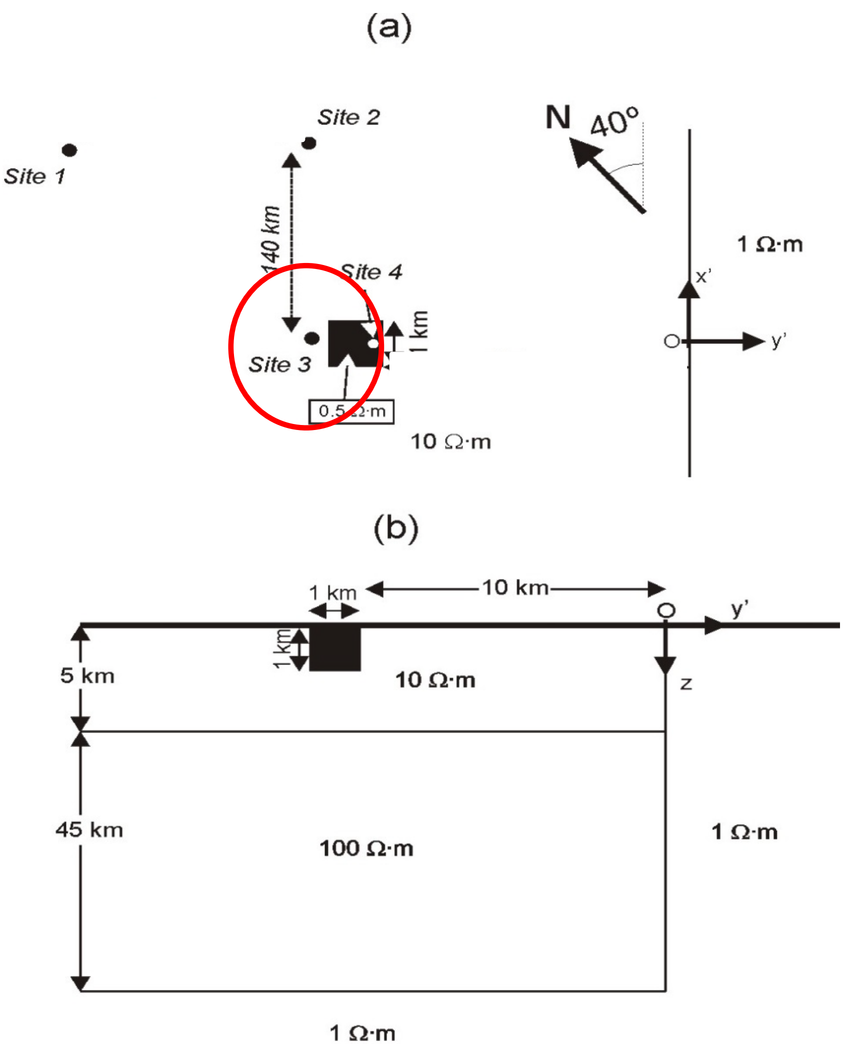
I_3 or $I_4 \neq 0$

$I_5, I_6 = 0$

$I_7 = 0$ or $Q = 0$



$$\theta_1 = \theta_2 = 40°$$



Site 3, 1000 s

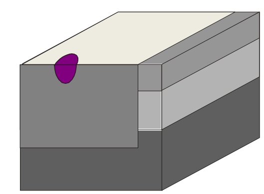
$$M = \begin{pmatrix} 68.9 + i\,131 & 174 + i\,353 \\ -86.5 - i\,178 & -34.7 - i\,64.9 \end{pmatrix}$$

TE and TM same phases

$I_3 - I_6 \neq 0$

$Q = 0$

3D/1D2D



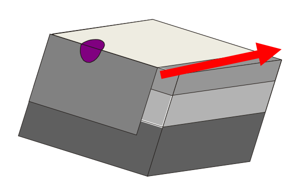
Site 3, 100 s

$$M = \begin{pmatrix} 392 - i\,39.9 & 1190 + i\,863 \\ -588 - i\,390 & -200 - i\,4.24 \end{pmatrix}$$

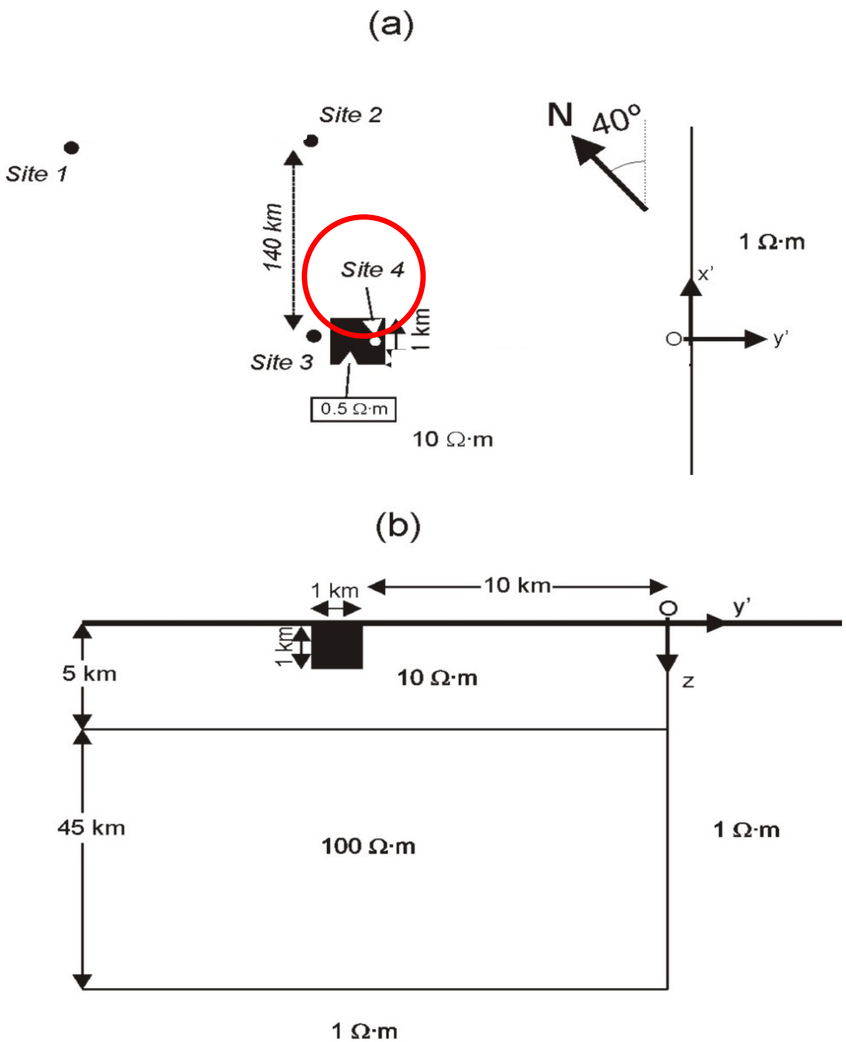
3D/2D

$I_3 - I_6 \neq 0$

$I_7 = 0$



$$\theta_3 = 40°, \varphi_{twist} = -0{,}1°, \varphi_{shear} = -20°$$

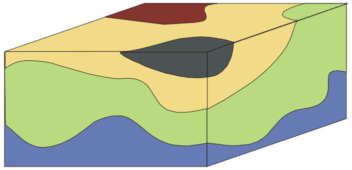


Site 4, 1 s

$$M = \begin{pmatrix} -3420 - i\,1550 & 3800 + i\,4220 \\ -6700 - i\,5010 & 1750 + i\,942 \end{pmatrix}$$

$I_7 \neq 0$

3D



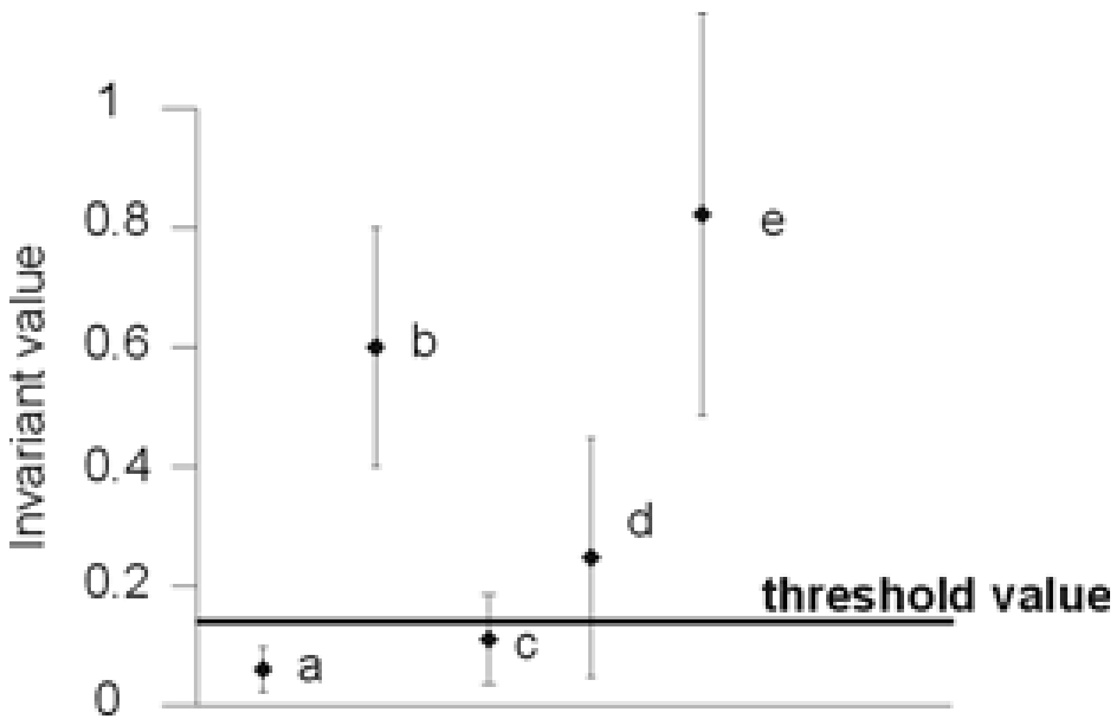
WAL analysis for real data

- impedance errors propagate to invariants

- geological structures ≠ ideal models described

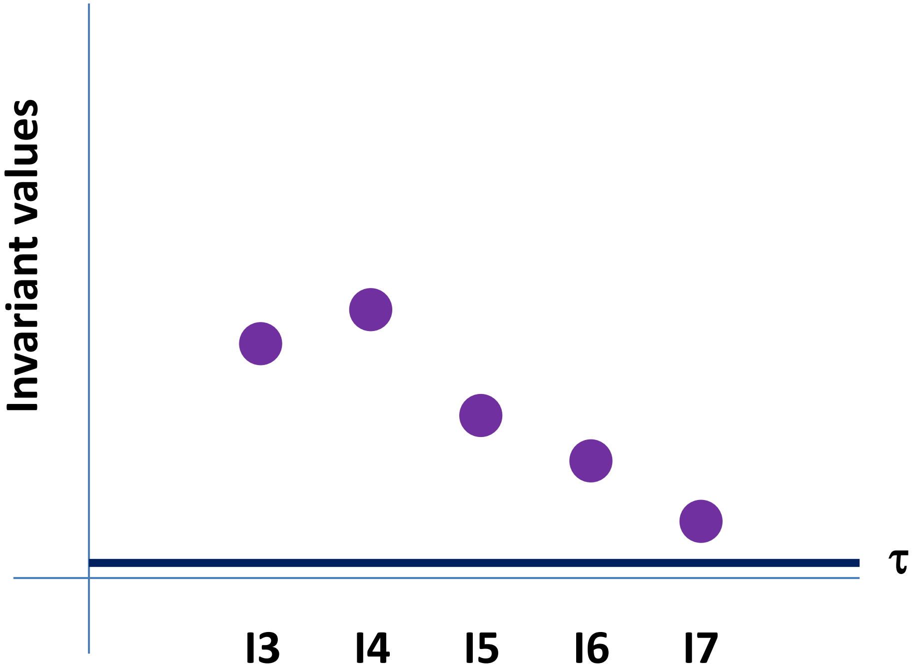
→ **invariant values never vanish**

Error estimation of the invariants and related parameters (strike, distortion):



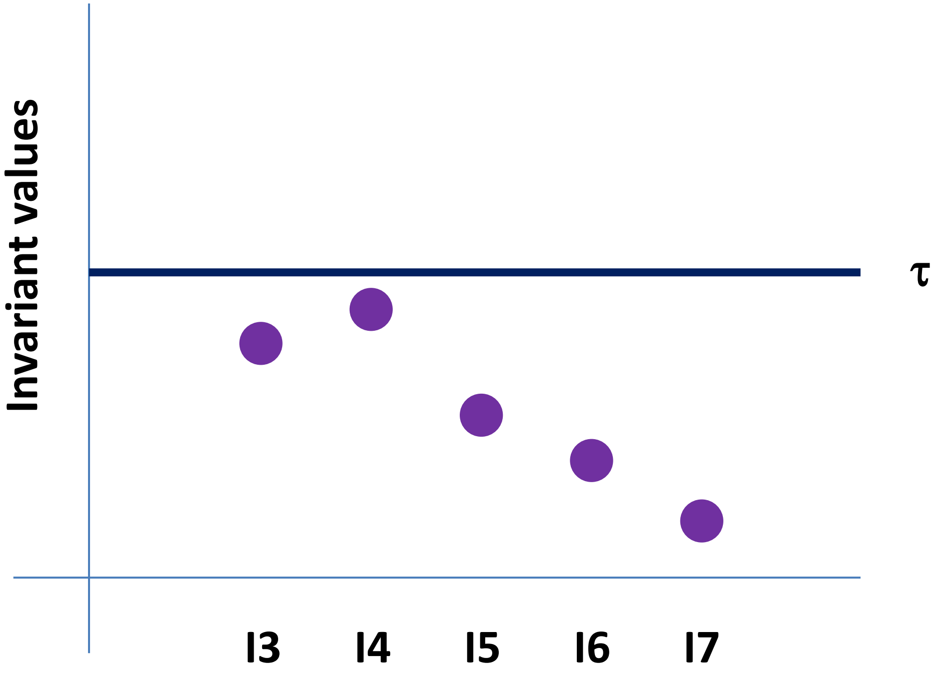
Threshold values (τ) ?

τ too low: everything tends to be 3D



Threshold values (τ) ?

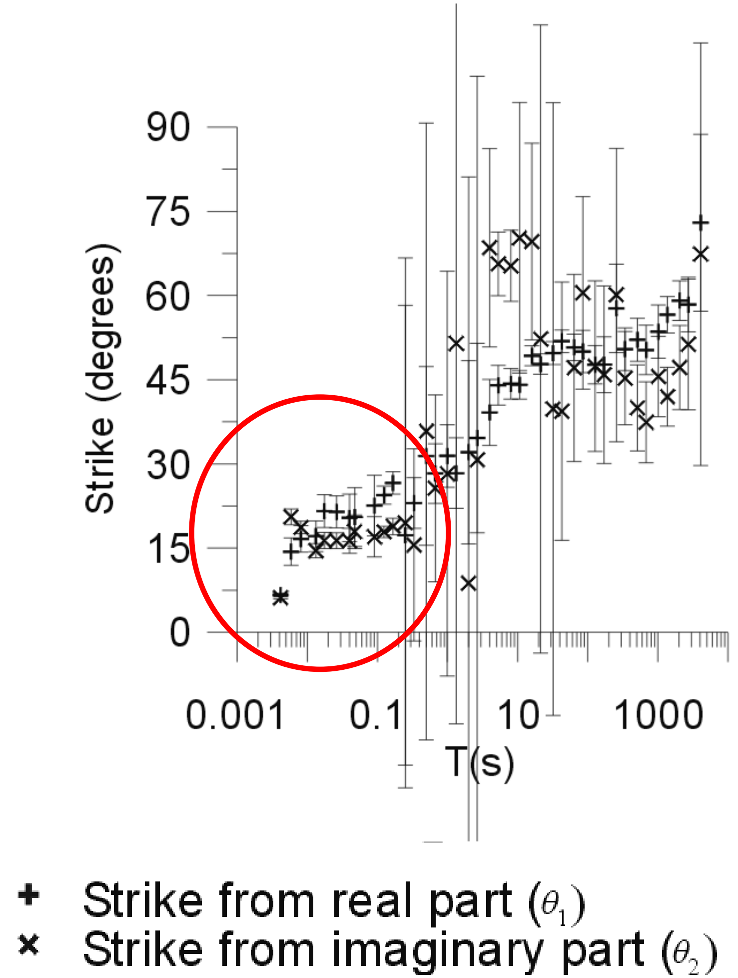
τ too high: everything tends to be 1D



Choice of τ:
Dimensionality must be constistent with the determination of strike and distortion angles

Example:

If 2D: $\theta_1 \sim \theta_2$ with small errors

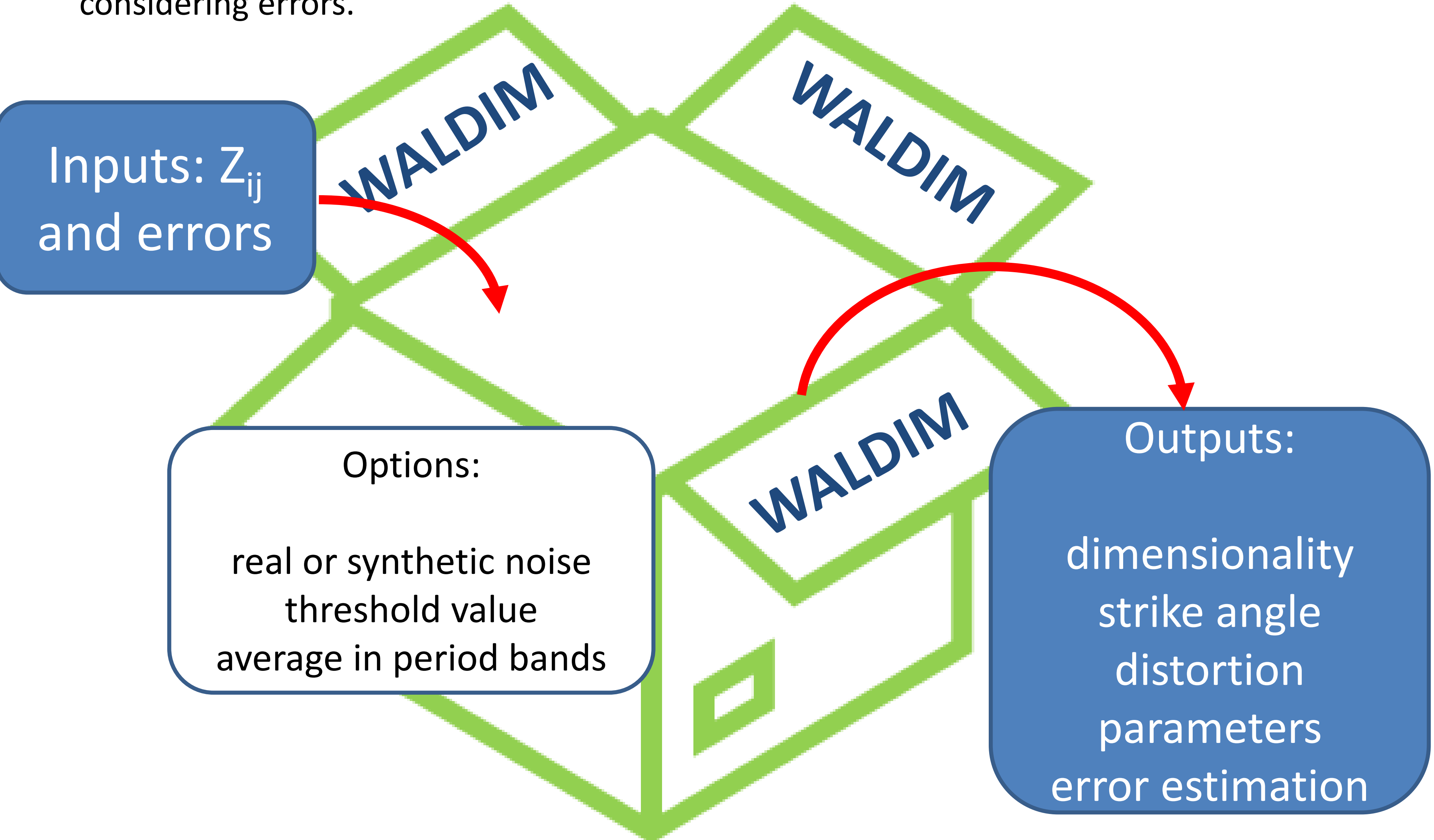


→ τ = 0.1 - 0.2 for data with up to 30 % error

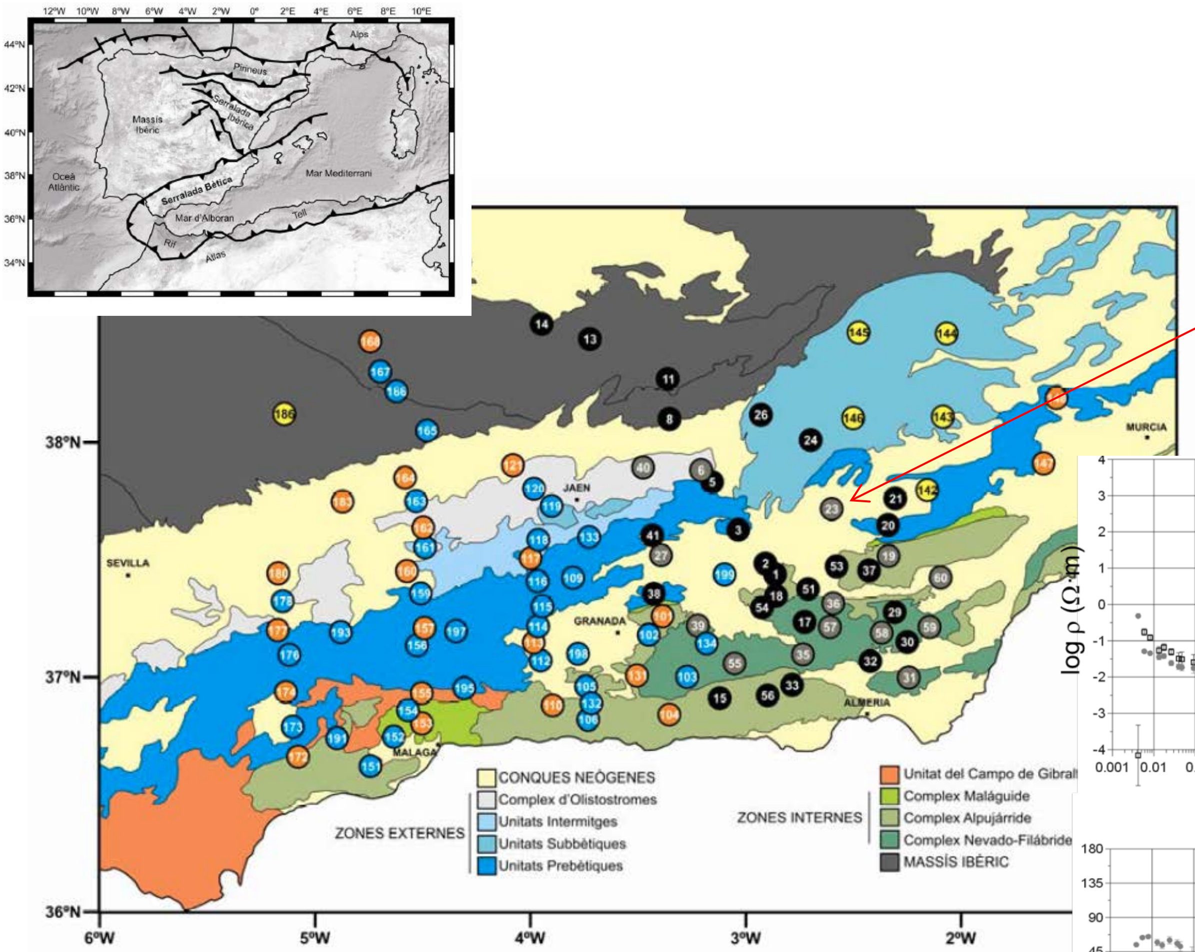
WALDIM Code

(Martí et al., 2009. Computers and Geosciences)

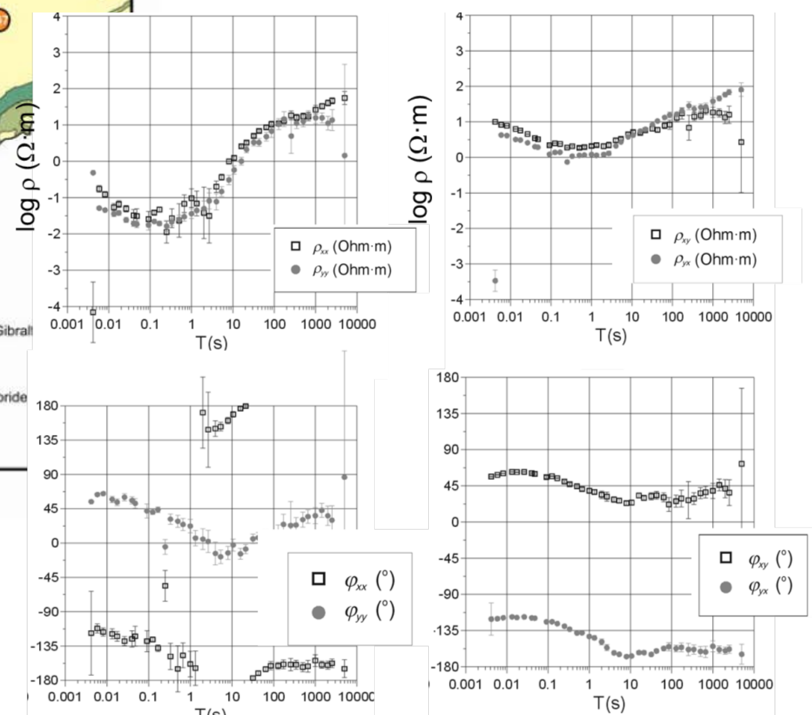
Dimensionality analysis of raw or synthetic data using WAL criteria, considering errors.



Example with real data: The Betics dataset



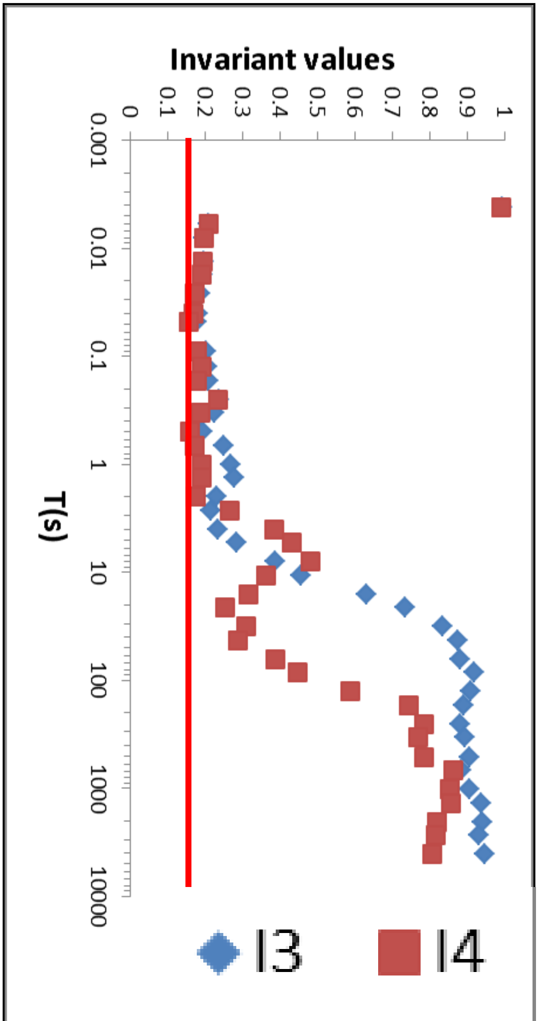
Site b23



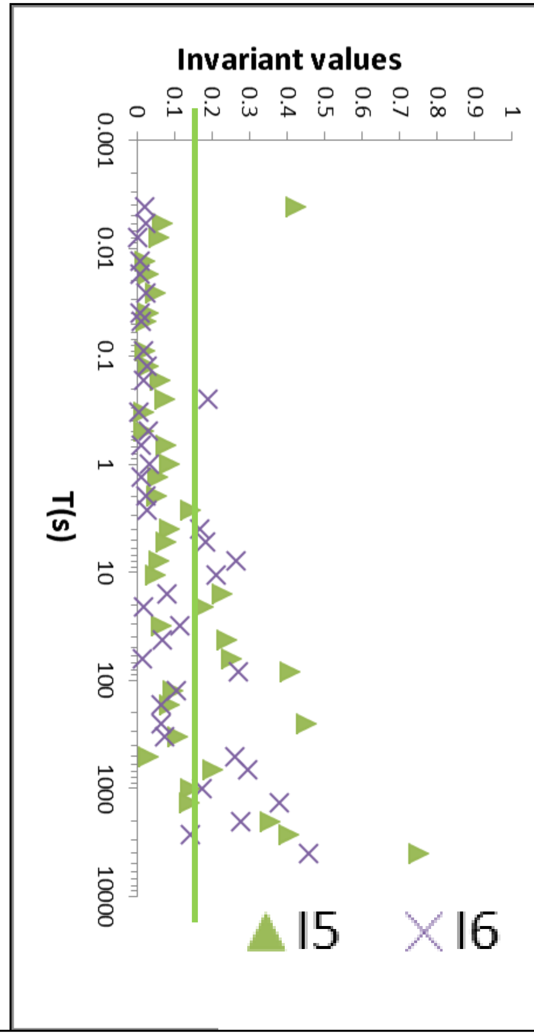
Rosell, 2013

Site b23

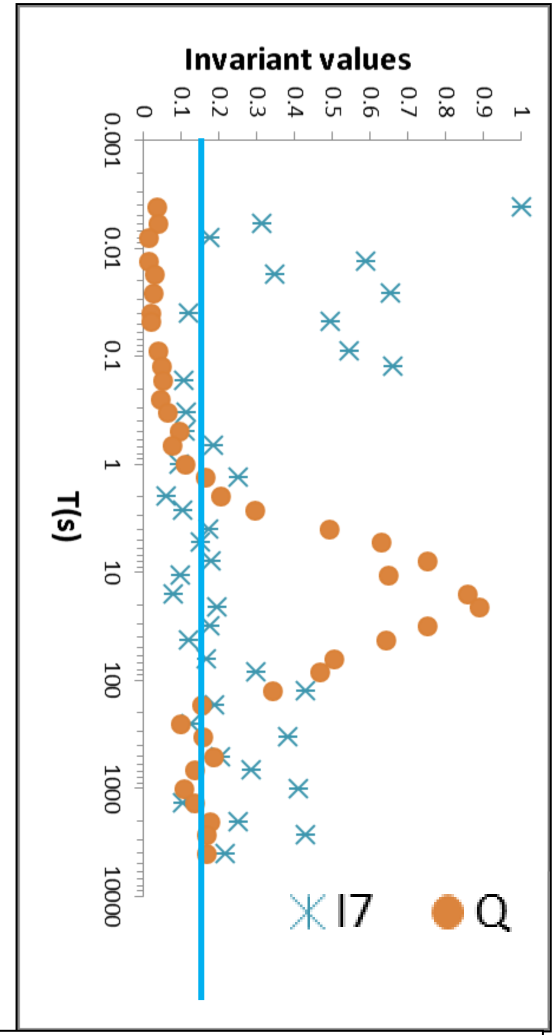
I3, I4



I5, I6



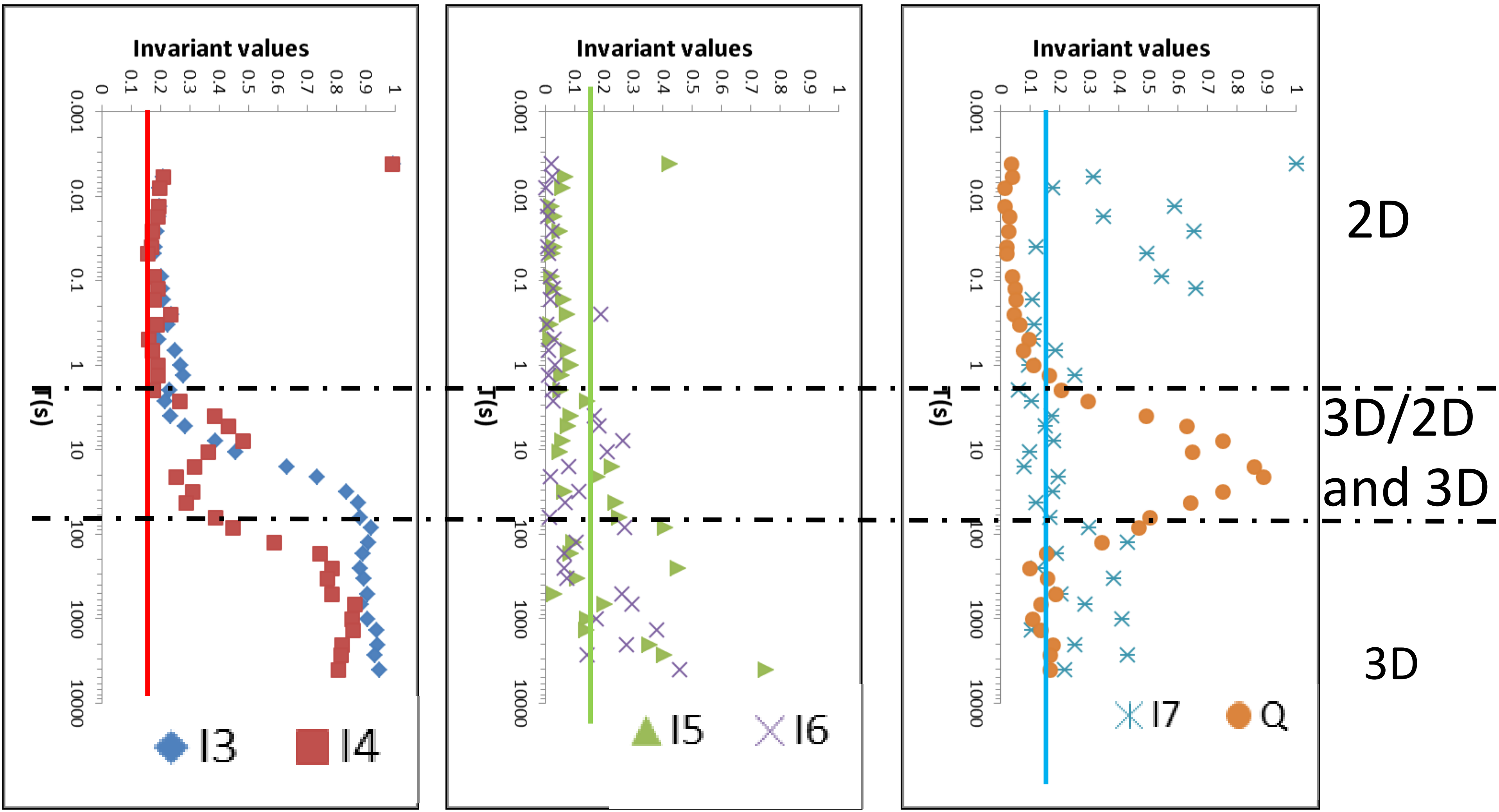
I7, Q



→ τ = 0.1 - 0.2 for data with up to 30 % error

Site b23

I3, I4 | I5, I6 | I7, Q



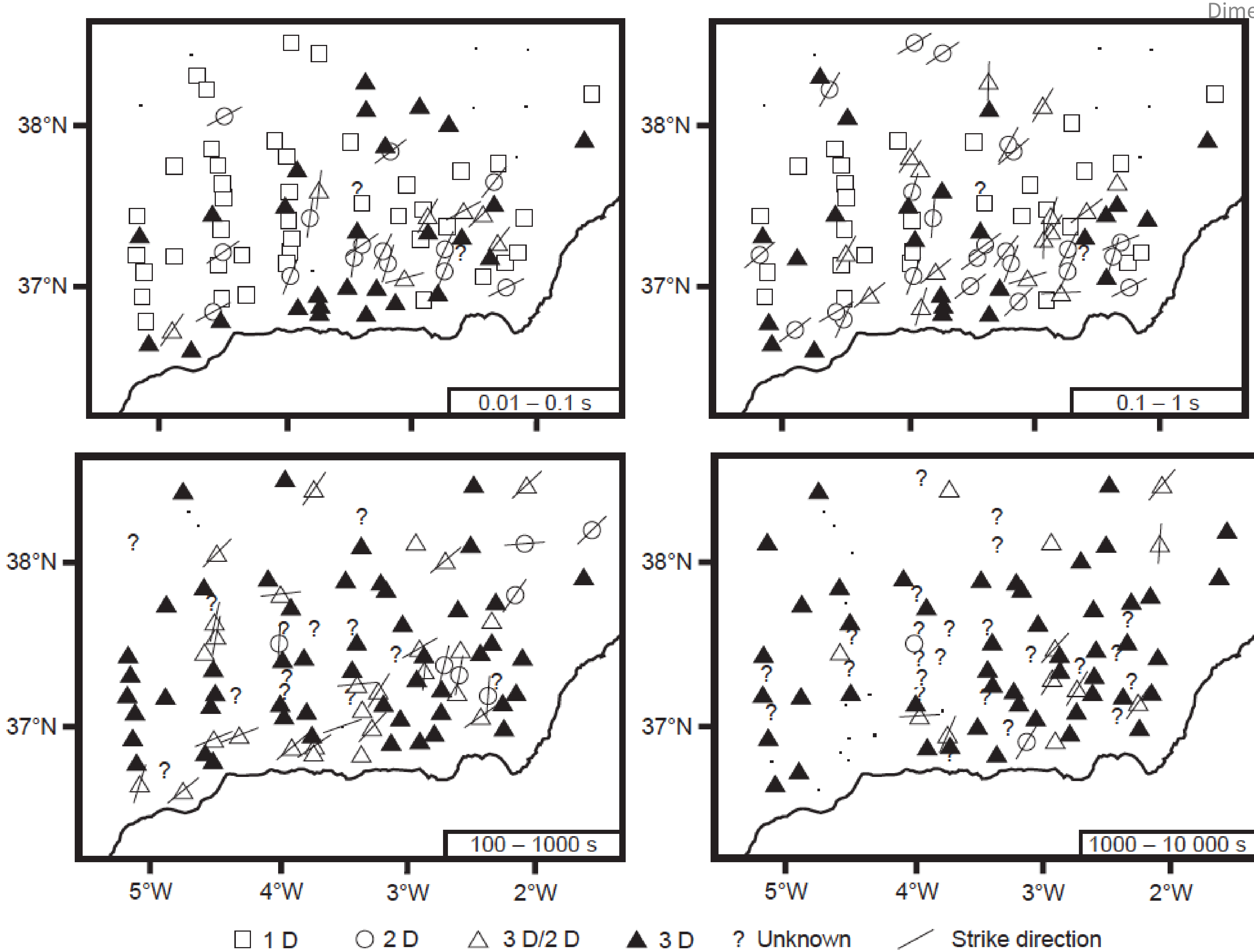
Limitations:

-No statistical framework

But:

- dimensionality results can be grouped into bands

- dimensionality maps provide information on regions with preferred orientations or with more or less complexity



Rosell et al. 2011

The Phase tensor

Caldwell et al. (2004)

Real tensor: product of the inverse of the real part of $\underline{Z}$ (X) with the imaginary part of $\underline{Z}$ (Y):

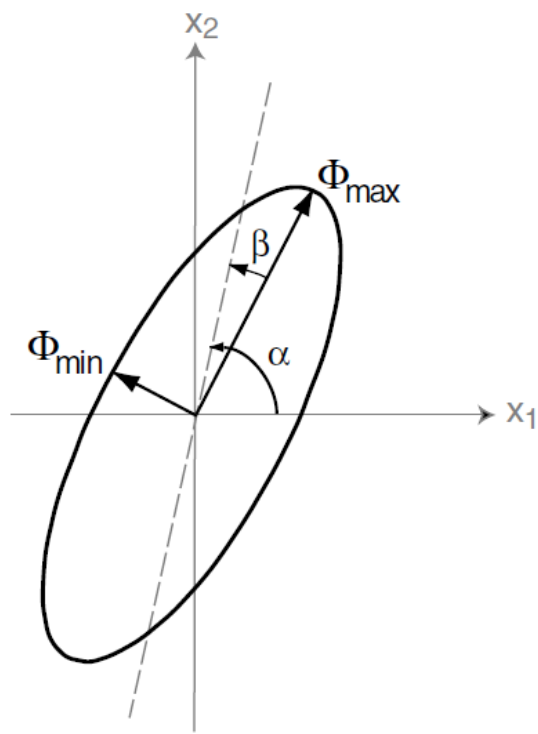
$$\mathbf{\Phi} = X^{-1}\, Y$$

This tensor is unaffected by any galvanic distortion (C) regardless of the nature of Z:

$$\mathbf{\Phi'} = (C\, X_R)^{-1}\, (C\, Y_R) = X_R^{-1}\, C^{-1}\, C\, Y_R = X_R^{-1}\, Y_R = \mathbf{\Phi}$$

The phase tensor can be decomposed in the form:

$$\Phi = \mathbf{R}^{T}(\alpha - \beta)\begin{bmatrix}\Phi_{max} & 0 \\ 0 & \Phi_{min}\end{bmatrix}\mathbf{R}(\alpha + \beta),$$



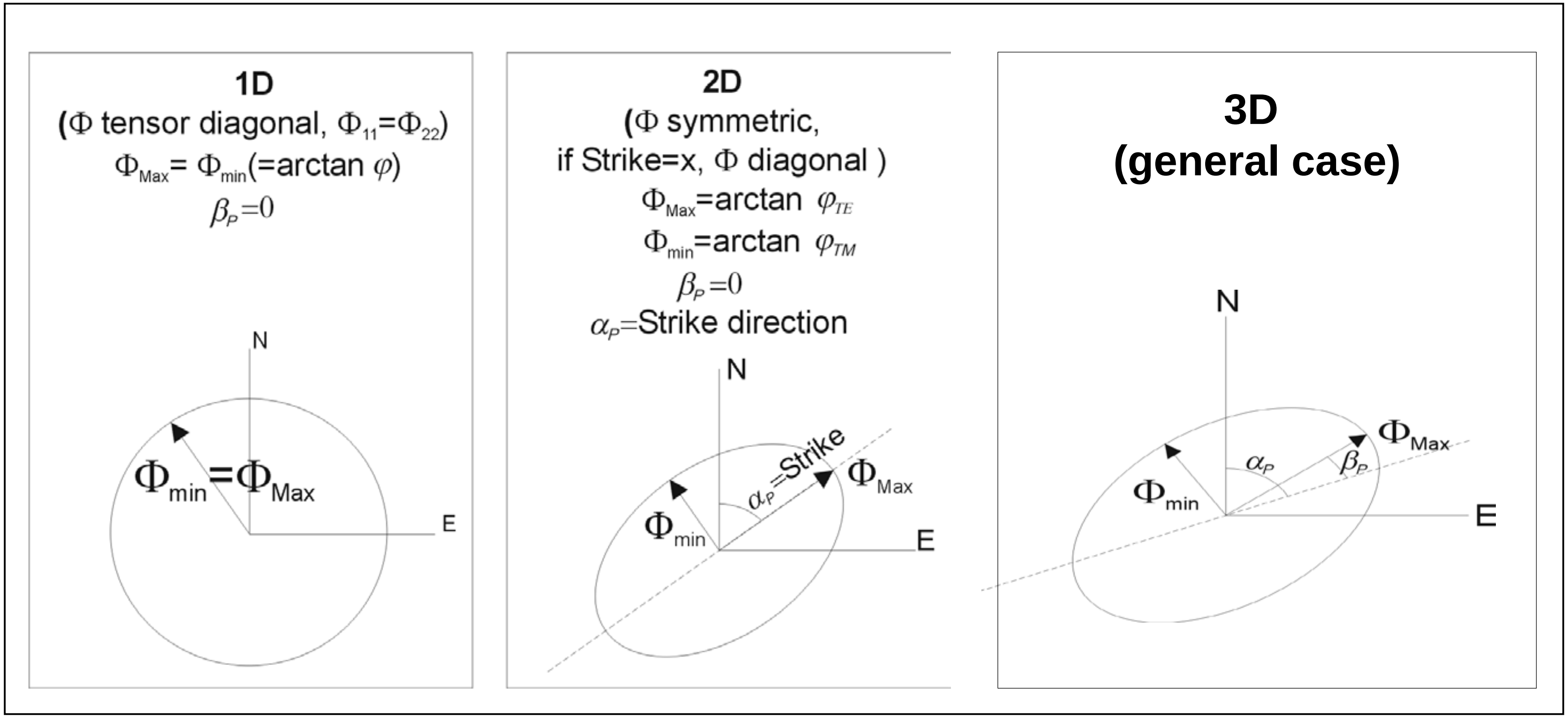
In 1D: $\Phi_{max} = \Phi_{min}$

(but in 2D with equal phases as well)

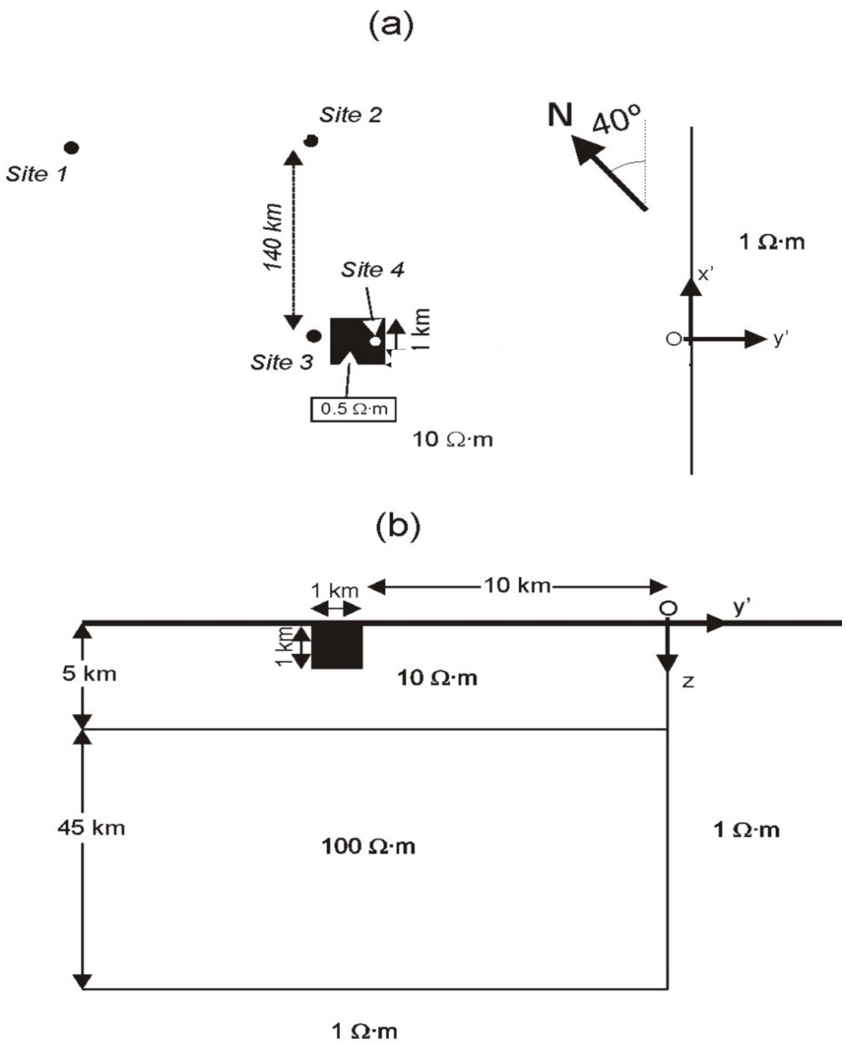
In 2D: $\beta = 0$ and $\alpha =$ strike direction

In 3D: $\beta \neq 0$

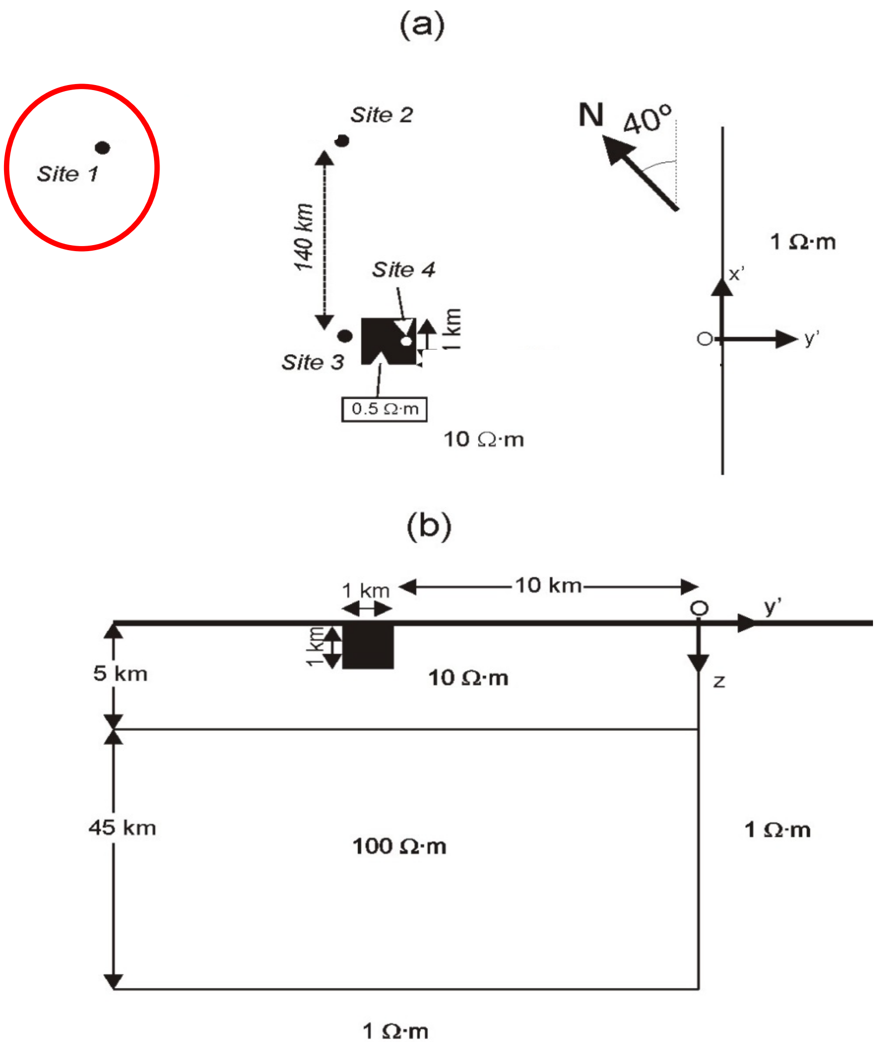
Phase tensor representation



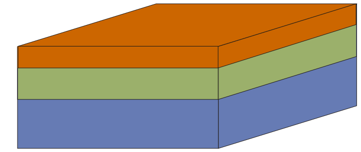
Caldwell et al., 2004
Moorkamp, 2007; Weaver et al., 2006
Booker, 2013: Review of the phase tensor

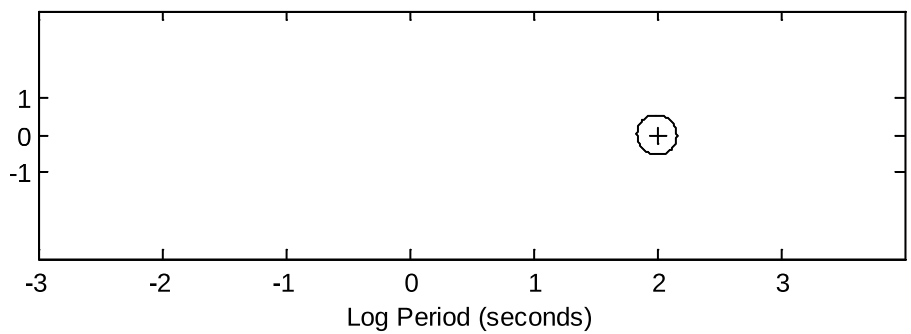


(a)
Site 1
Site 2
140 km
Site 4
1 km
Site 3
0.5 Ω·m
10 Ω·m
N
40°
1 Ω·m
x'
O
y'
(b)
1 km
10 km
O
y'
1 km
5 km
10 Ω·m
z
45 km
100 Ω·m
1 Ω·m
1 Ω·m



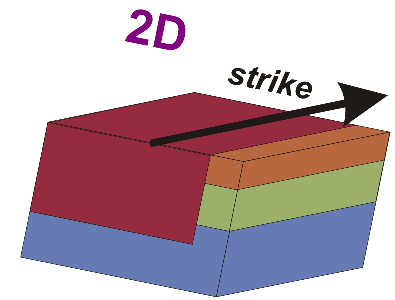
1D

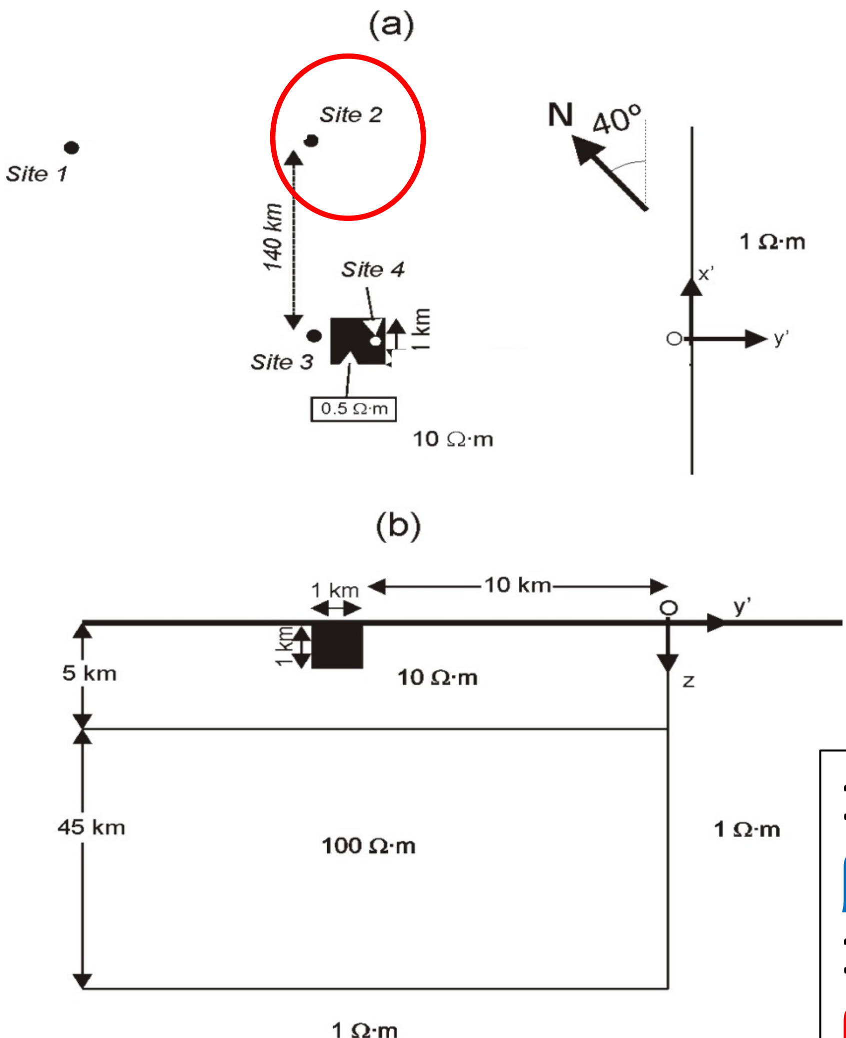


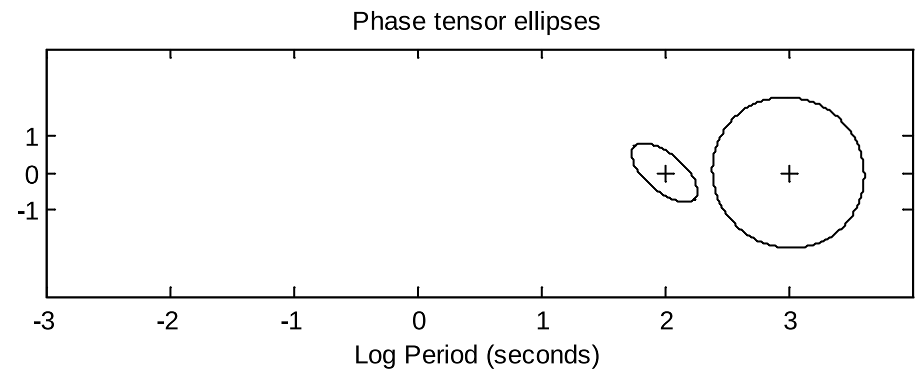


$\Phi_{max} = \Phi_{min}$

$(\alpha = 32^{o})$; $\beta = 0 \rightarrow$ 1D





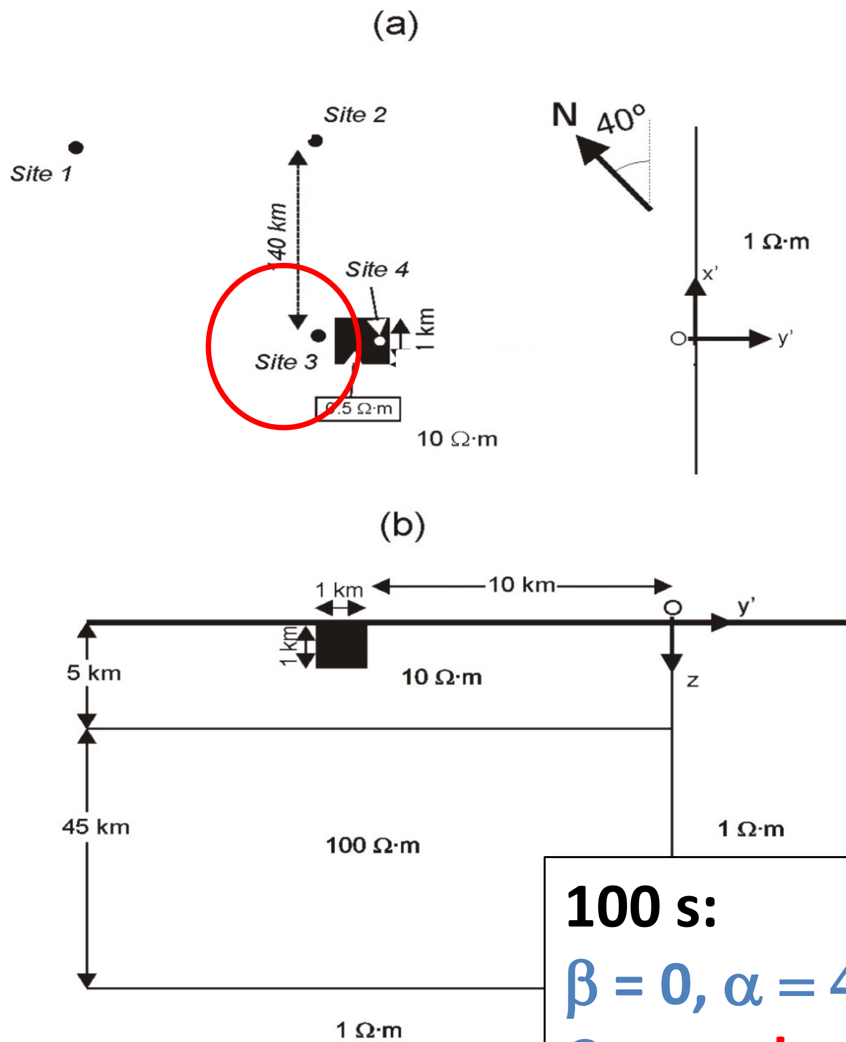


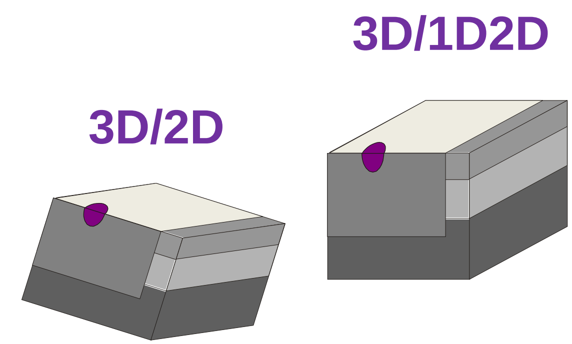
100 s:

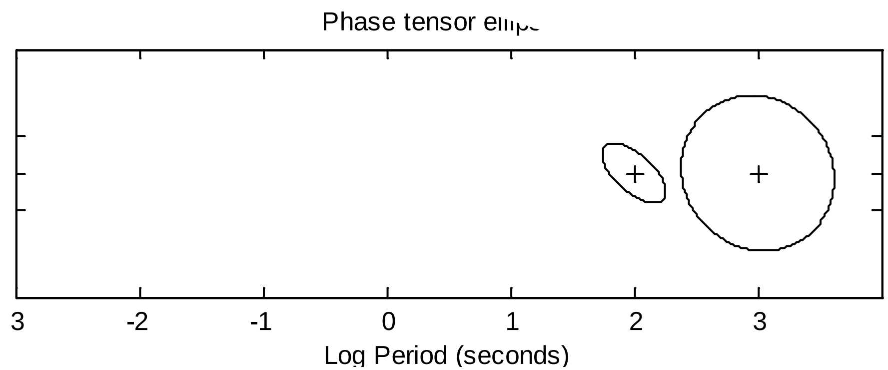
$\beta = 0$, $\alpha = 40$ °= strike → 2D

1000 s: $\Phi_{max} \sim \Phi_{min}$

$\beta = 0$, $\alpha = 40$ = strike → 2D BUT GRAPHICALLY IT LOOKS LIKE 1D because TE and TM phases are equal





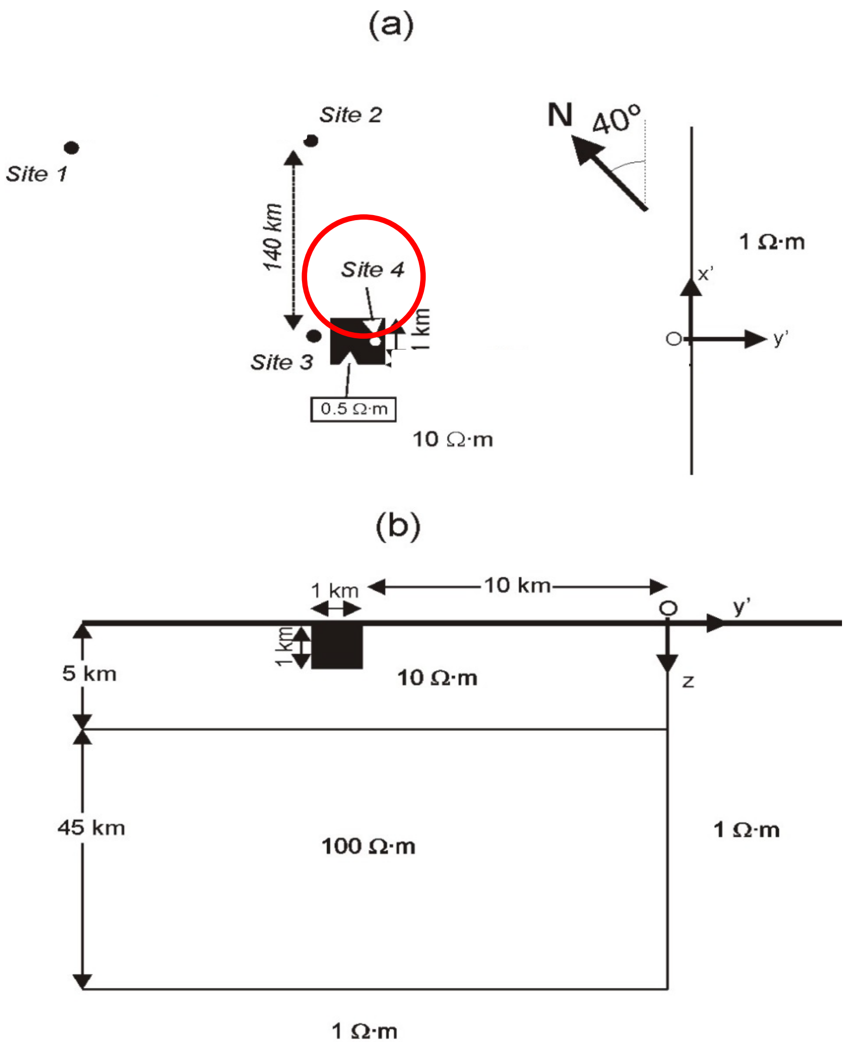


100 s:

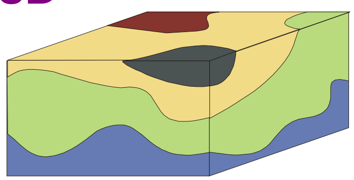
$\beta = 0$, $\alpha = 42°$= strike → 2D

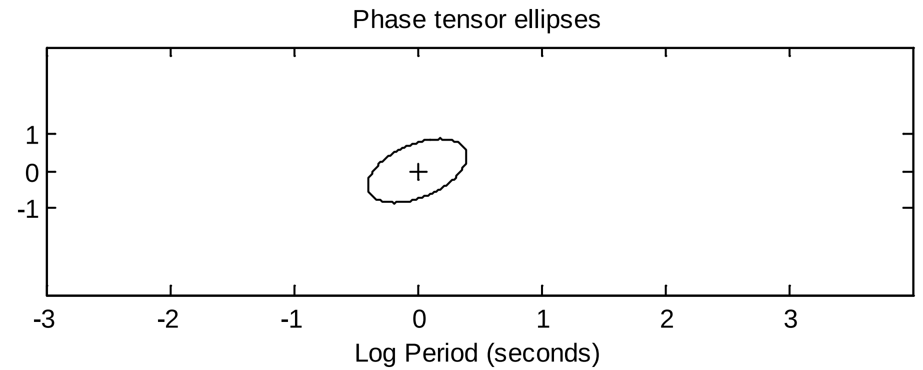
Correct but no info about distortion

1000 s: $\Phi_{max} \sim \Phi_{min}$, $\beta = 0$ → 1D. Equal phases. No infor about distortion



3D





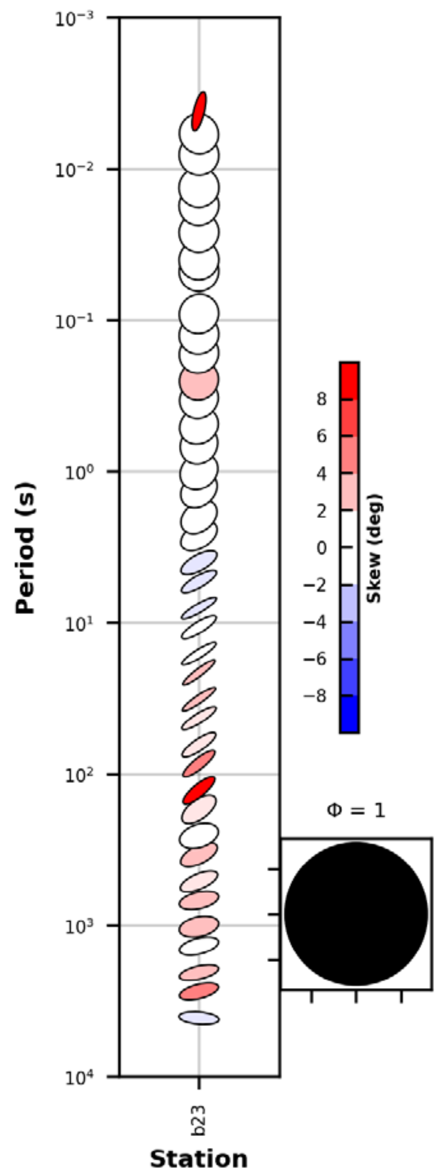
1 s:

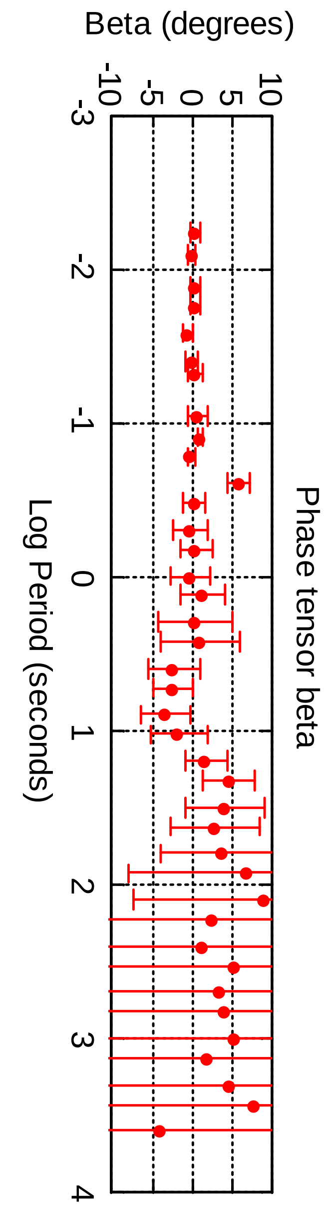
β= -2º , α = -25º= strike ?

BE CAREFUL WITH β THRESHOLD !!!

Site b23

Period bands	WAL analysis
T<0.01 s	2D θ= 20º±5º
0.01 s-0.1 s	
0.1 s-1 s	
1 s-10 s	3D/2D θ= 57º±9º φ_t =2º±0.5º φ_e = -9º±2º
10 s-100 s	3D/2D θ= 59º±3º φ_t =3º±1º φ_e = -16º±2º
100 s-1000 s	3D
T>1000 s	3D





Contents:

- **MT responses, dimensionality and distortion**

- **Dimensionality analysis methods**

- **Decomposition methods**

- **Extension to an anisotropic earth**

Decomposition methods:

Historical perspective:

Chave and Jones (2012), Chapter 6, and references therein

3D/1D: Larsen (1977)

Based on Groom and Bailey (G&B, 1989):

3D/2D: McNeice and Jones (2001): Strike code

3D/3D: Ledo et al. (1998); Garcia and Jones (2002)

3D/2D G&B decomposition:

$$Z_{meas} = R_\theta \cdot C \cdot Z_{2D} \cdot R_\theta^T$$

Z_{meas}: 8 known parameters

Strike, g, t, s, a, Z_{xyre}, Z_{xyim}, Z_{yxre}, Z_{yxim}: **9 unknowns**

g·A = scale factor: static shift : $Z_{2D}' = g \cdot A \cdot Z_{2D}$

Strike, t, e, Z_{xyre}, Z_{xyim}, Z_{yxre}, Z_{yxim} : **7 unknowns**

For a single frequency the problem will be solved by fitting 8 data to 7 parameter model

Strike code

McNeice and Jones (2001):

- G&B decomposition of the MT tensor data supported by statistical methods: fitting data to a 3D/2D model
- Can be performed for single sites, single frequencies or grouping sites / frequencies
- It allows to fix or not the model parameters (e.g. Strike, distortion)

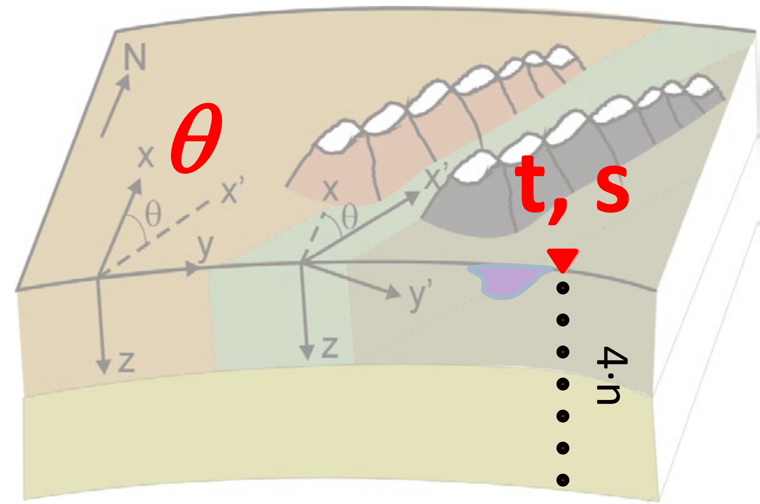
Multisite – multifrequency:

1 site, n freq: fitting 7 parameter model to 8n data →

same θ, t and s

$\boldsymbol{Z}_{2D}$ (4·n)

<u>8n data and 3+4n unknowns</u>



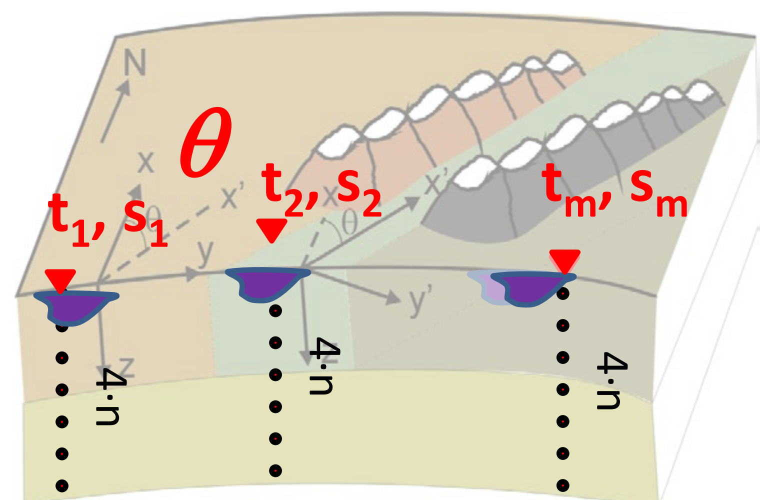
m sites, n freq: fitting 7 parameter model to 8n data →

same θ

different t and s at each site (2·m)

$\boldsymbol{Z}_{2D}$ (4·n)

<u>8·n·m data and 1 + 2·m+4·m·n unknowns</u>



- Tool to investigate if a dataset or a subset can be decomposed as 2D in views of 2D modelling or to obtain apriori information for a 3D.

- Cannot be used as a black box, it is recommended to perform it in steps to fully understand the dataset.

- It is very important to quantify the rms or the misfit: **how well does my data fit the 3D/2D model assumption?**

Site b23

<table>
<tr><th rowspan="2">Period bands</th><th rowspan="2">WAL analysis</th><th colspan="4">Strike analysis</th></tr>
<tr><th>1) All parameters free - analysis per bands</th><th>2) All parameters free - all periods</th><th colspan="2">3) Two period bands:
a) Non distortion
b_1) Free – one band
b_2) Free – two bands</th></tr>
<tr><td>T<0.01 s</td><td rowspan="3">2D
θ = 20°±5°</td><td>θ=2°
φ_t= -1.5°
φ_e= 9°</td><td rowspan="7">θ = 60°
φ_t =0°
φ_e = -11°</td><td colspan="2" rowspan="3">a)
θ=18°</td></tr>
<tr><td>0.01 s-0.1 s</td><td>θ=50°
φ_t = 0°
φ_e = -9°</td></tr>
<tr><td>0.1 s-1 s</td><td>θ=60°
φ_t = -1°
φ_e = -11°</td></tr>
<tr><td>1 s-10 s</td><td>3D/2D
θ = 57°±9°
φ_t =2°±0.5°
φ_e = -9°±2°</td><td>θ = 59°
φ_t = -0.5°
φ_e = -10°</td><td rowspan="4">b_1)
θ = 57°
φ_t = 0°
φ_e = -10°</td><td rowspan="2">b_2)
θ = 57°
φ_t = 0°
φ_e = -10°</td></tr>
<tr><td>10 s-100 s</td><td>3D/2D
θ = 59°±3°
φ_t =3°±1°
φ_e = -16°±2°</td><td>θ = 55°
φ_t = -0.5°
φ_e= -9°</td></tr>
<tr><td>100 s-1000 s</td><td>3D</td><td>θ = -17°
φ_t = 15°
φ_e = 39°</td><td rowspan="2">θ = -13°
φ_t = 12°
φ_e = 39°</td></tr>
<tr><td>T>1000 s</td><td>3D</td><td>θ = -13°
φ_t = 12.5°
φ_e = 40°</td></tr>
</table>

: Large misfits

Contents:

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- Extension to an anisotropic earth

Anisotropy

Electrical anisotropy: Property of a medium/material in which its electrical conductivity depends upon orientation.

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} \xrightarrow[\alpha_S,\ \alpha_D,\ \alpha_L]{\text{Diagonalization}} \sigma = \begin{bmatrix} \sigma'_{xx} & 0 & 0 \\ 0 & \sigma'_{yy} & 0 \\ 0 & 0 & \sigma'_{zz} \end{bmatrix}$$

-symmetric
-positive-definite

How does anisotropy affect the magnetotelluric responses?

1D:

$$Z_{1D-anis} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & -Z_{xx} \end{bmatrix}$$

Not necessarily diagonalizable

$$\vec{T} = 0$$

All sites the same responses

3D:

General solution

2D:

Decoupled eqs:

$$Z_{2D-anis} = \begin{bmatrix} 0 & f(\sigma_{xx}) \\ g(\sigma_{yy}, \sigma_{zz}) & 0 \end{bmatrix}$$

$$\vec{T}_{2D-anis} = (0, h(\sigma_{xx}))$$

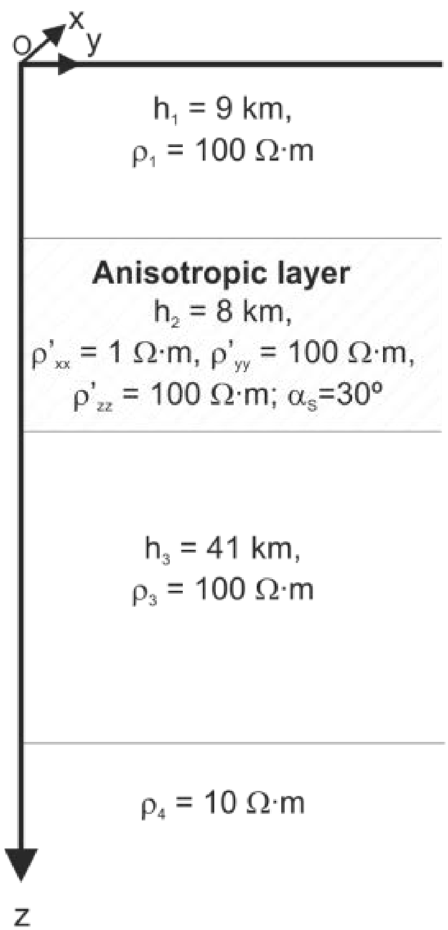
TE and TM sense different conductivity components

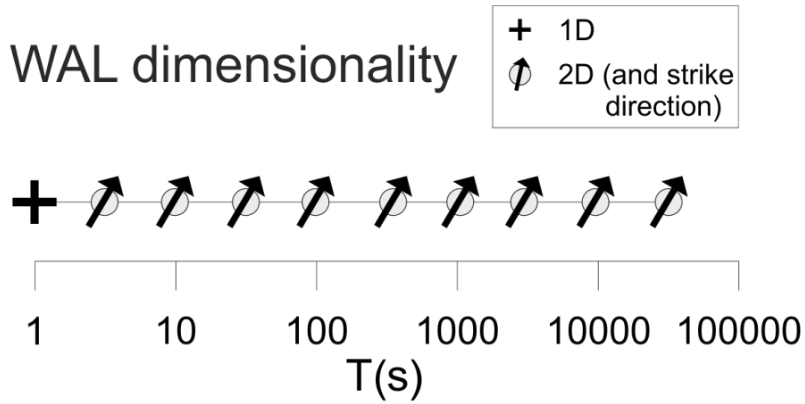
Same pattern should repeat along structural strike direction

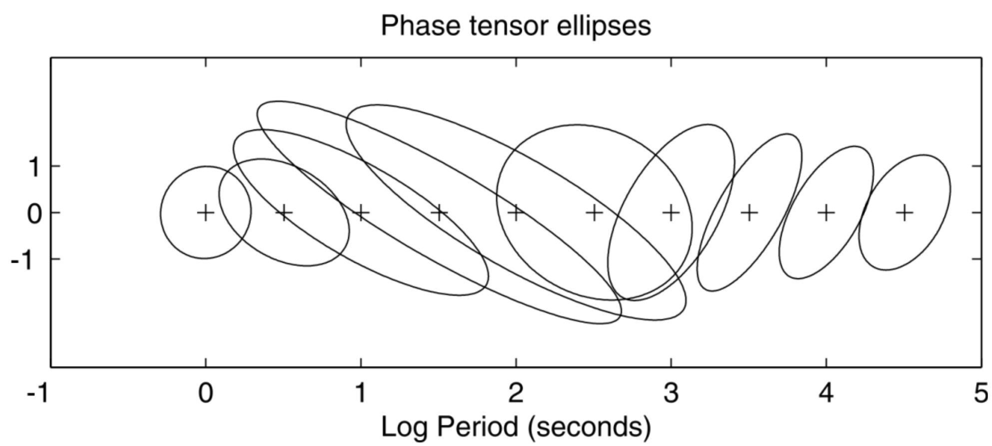
Coupled eqs:

Apparently 3D

One anisotropic layer:





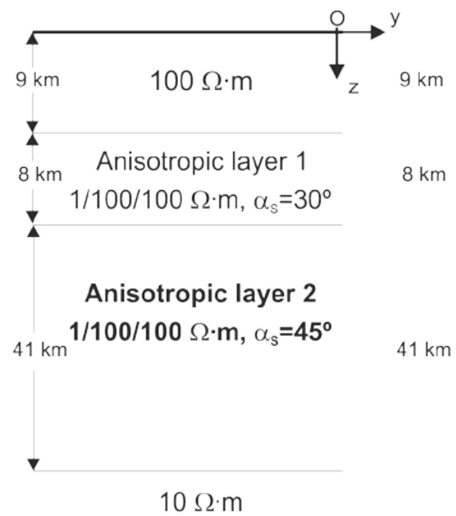


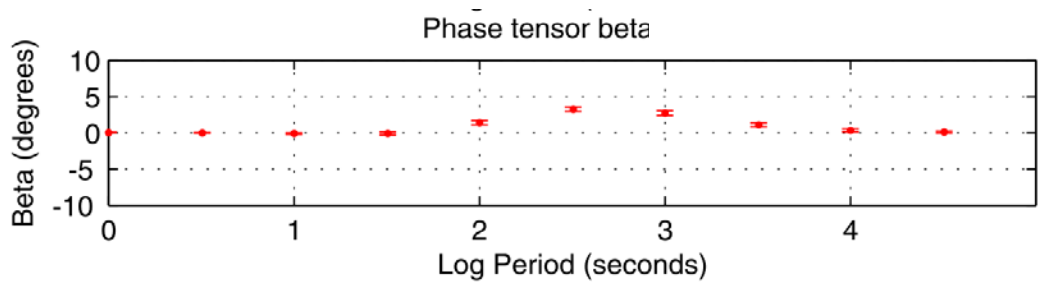
β =0

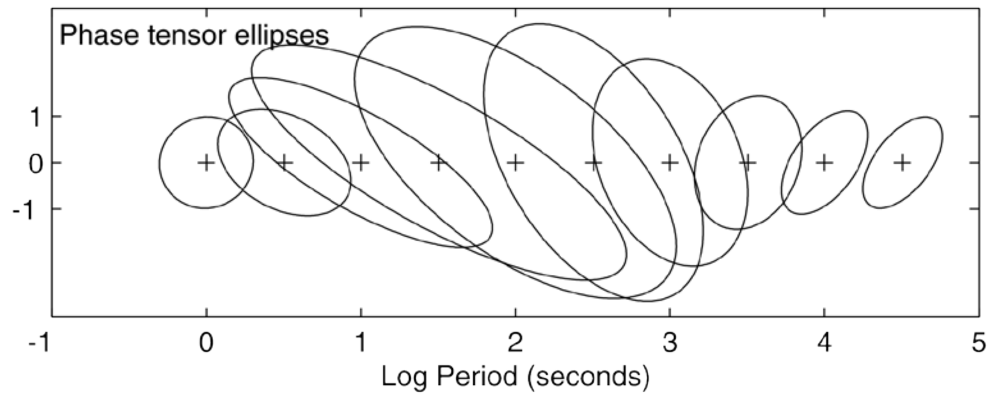
$$\vec{T} = 0$$

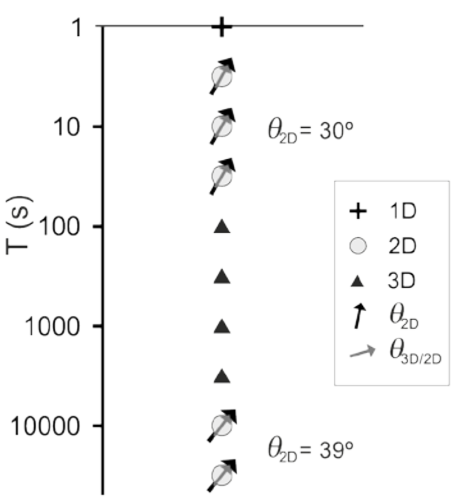
Same responses for all sites

Two anisotropic layers:









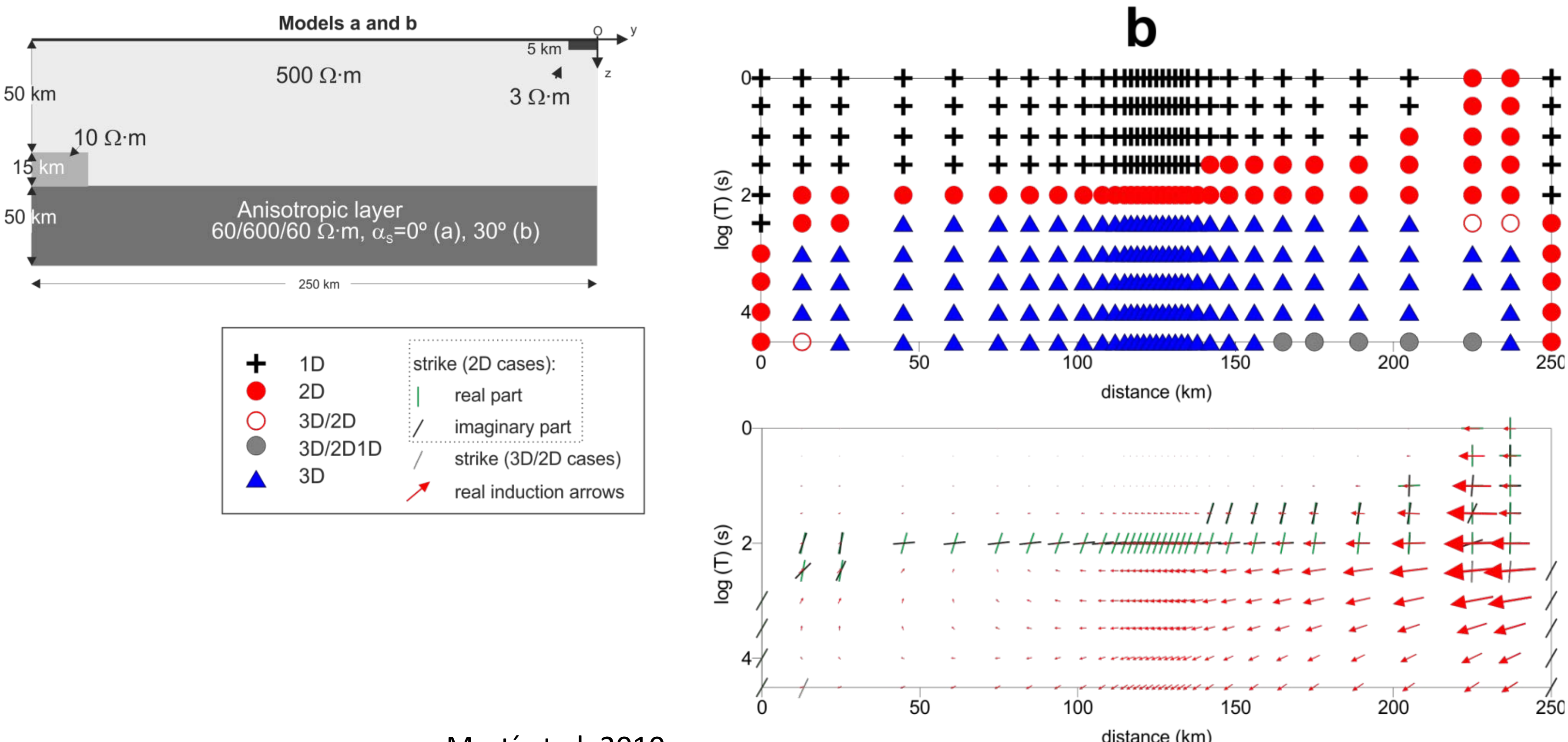
$$\vec{T} = 0$$

Same responses for all sites

2D model with an anisotropic layer

Azimuthal anisotropy (30º):

- 2D cases with different strike directions from real and imaginary parts.
- Some induction arrows non perpendicular to strike directions
- 3D cases



Martí et al. 2010

Heise et al. (2006): Anisotropy and phase splits in magnetotellurics

Martí et al. (2010): WALDIM criteria extended to anisotropic structures

Jones (2012): 3D/1Danis decomposition scheme

See Martí (2014) for an extended review

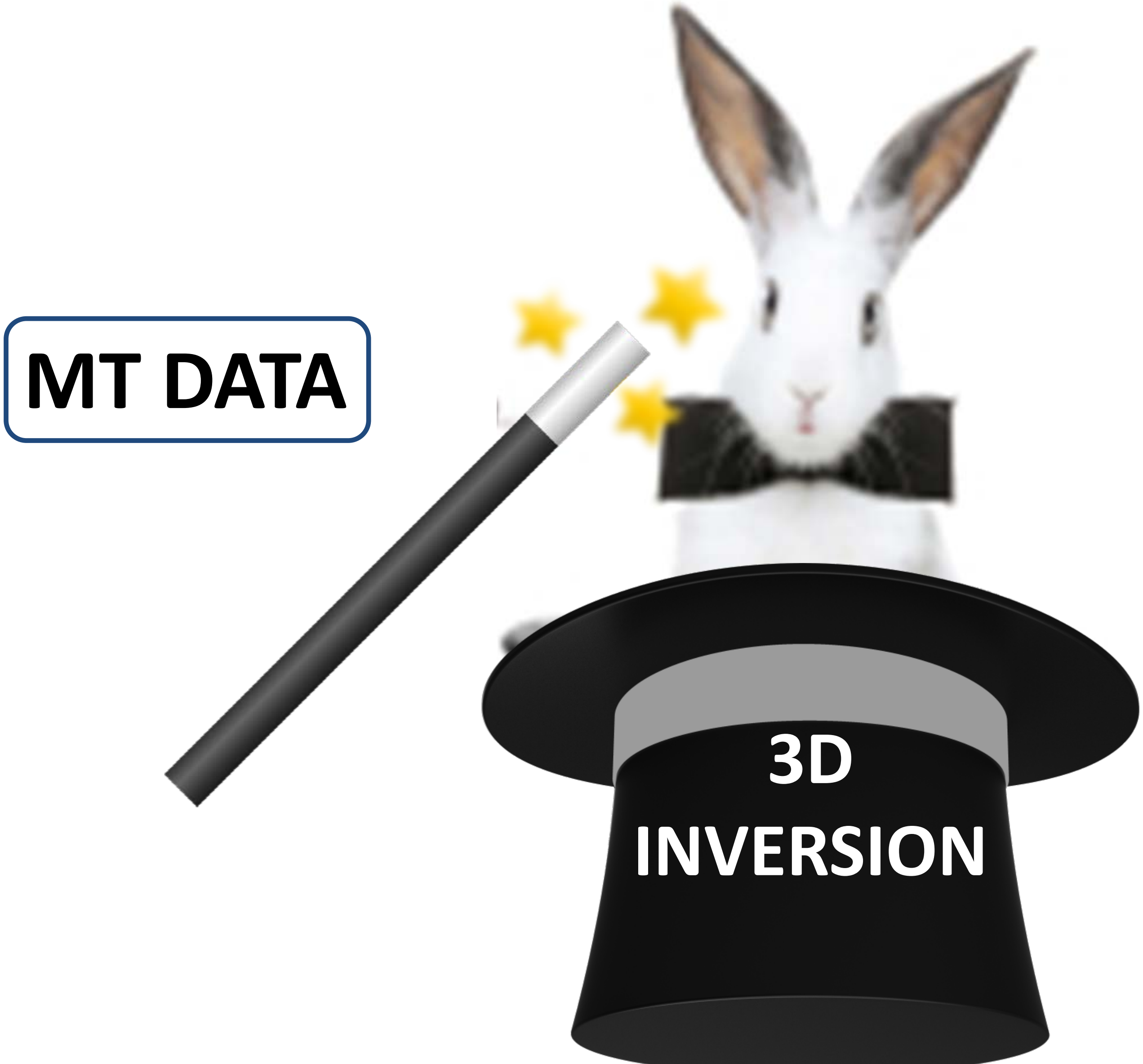
Summing up …

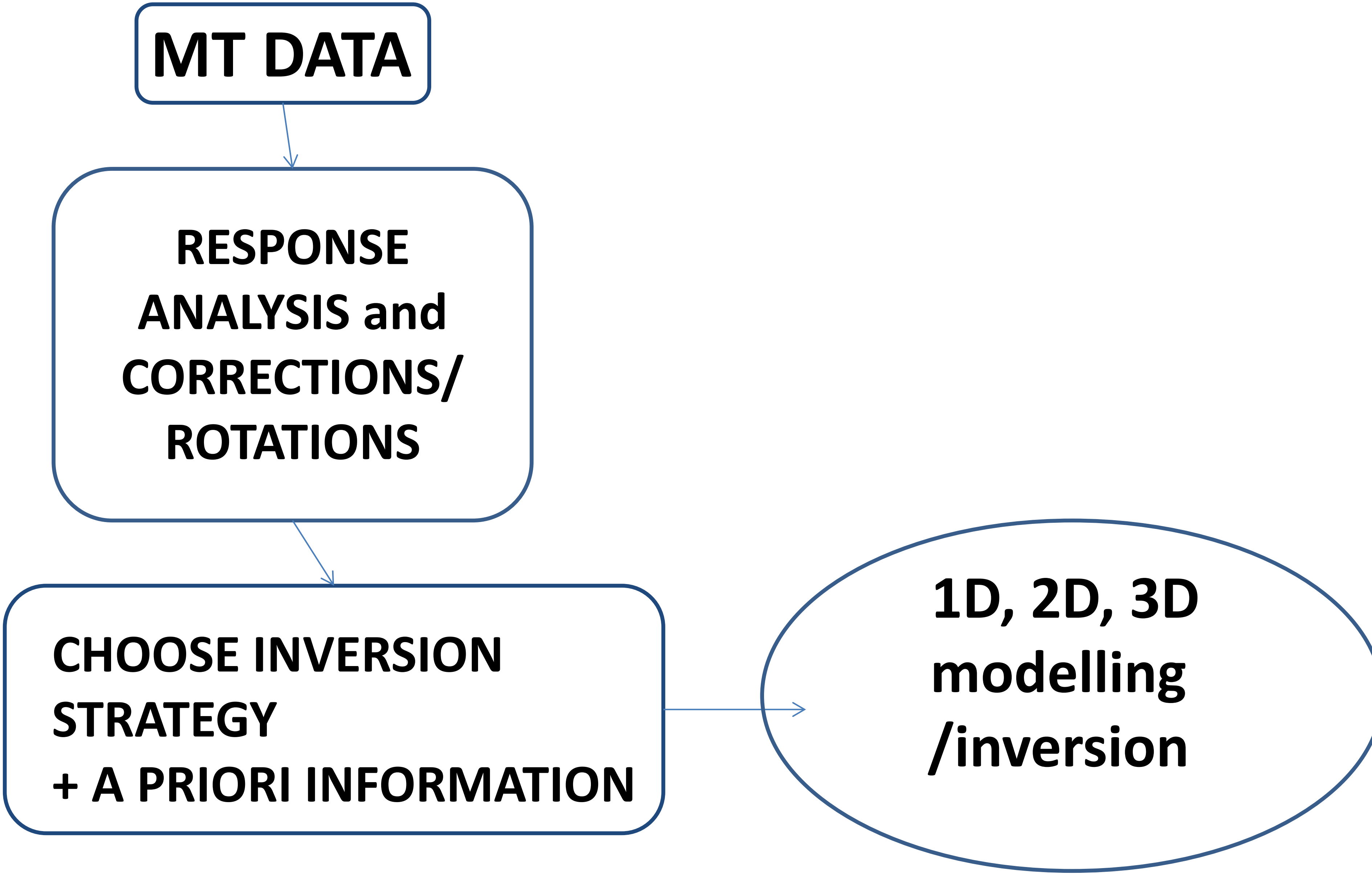
MT response analysis is a powerful tool to assess the data dimensionality and have a first glance at the structures below before modelling.

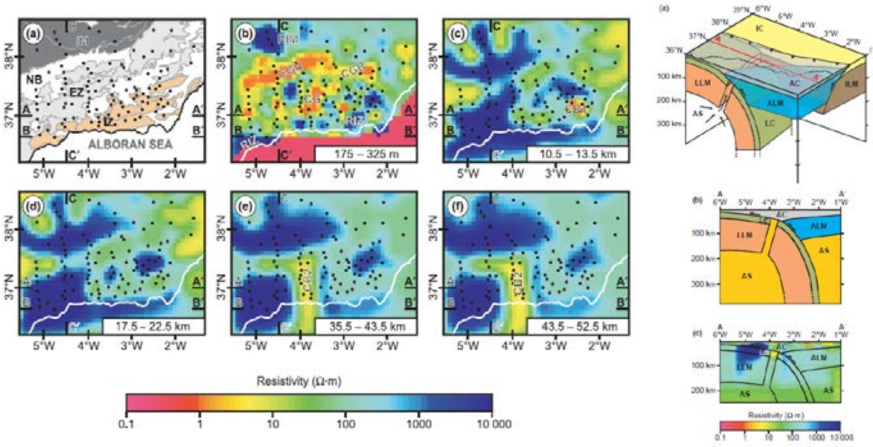
Here we have reviewed some of the most used tools: Strike, WALDIM and phase tensor. All valid and complementary if taken with caution.

Error data must be included in the analysis

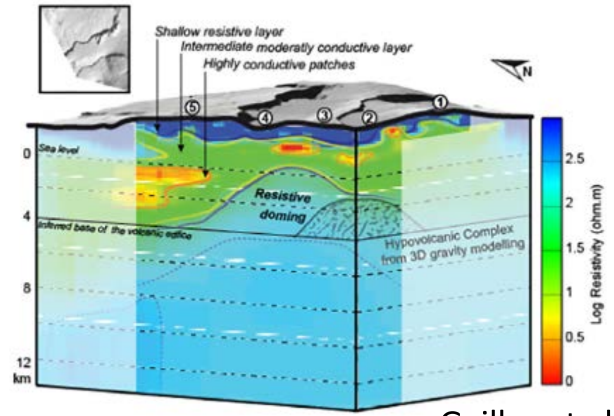
So … if we don’t want to get this …



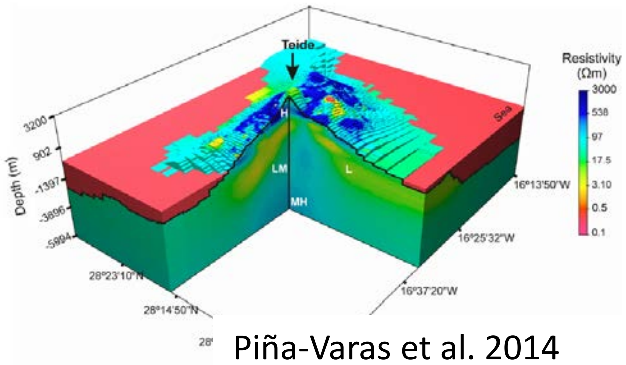

MT DATA
RESPONSE ANALYSIS and CORRECTIONS/ ROTATIONS
CHOOSE INVERSION STRATEGY
+ A PRIORI INFORMATION
1D, 2D, 3D modelling /inversion



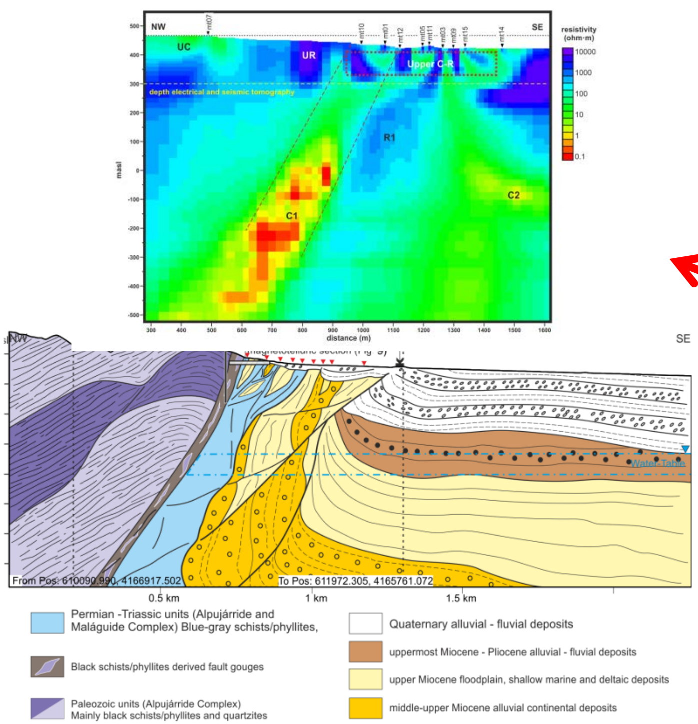
Rosell et al. 2011



Gailler et al. 2018



Piña-Varas et al. 2014



Martí et al. 2020

1D, 2D, 3D modelling /inversion

Thanks for your attention !!!!

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