

capturing knowledge in code

simpeg and geosci

Lindsey Heagy, Seogi Kang, Doug Oldenburg
& the simpeg + geosci teams

collaborators



Seogi Kang
Stanford



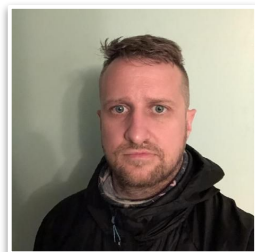
Thibaut Astic
UBC GIF



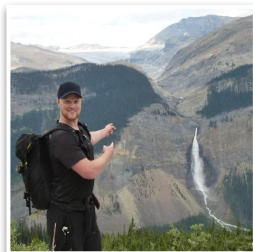
Dom Fournier
Mira Geosciences



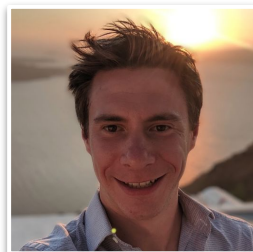
Joe Capriotti
UBC GIF



John Kuttai
Dias Geophysical



Devin Cowan
UBC GIF



Rowan Cockett
Curvenote



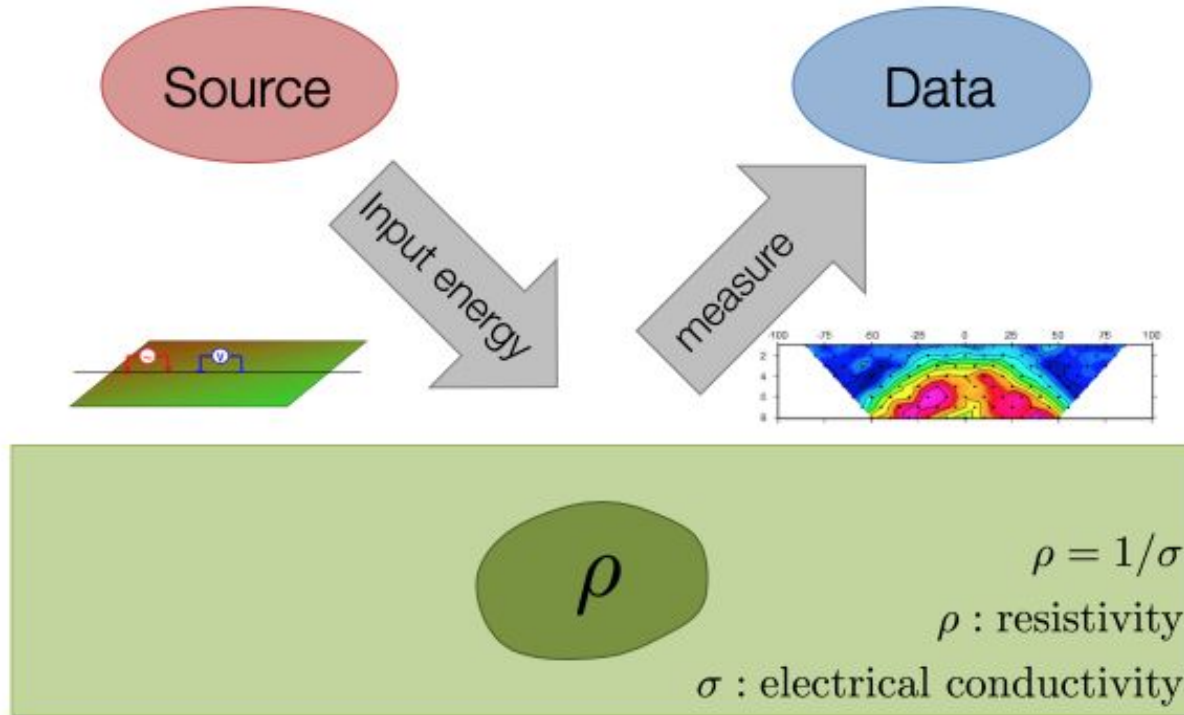
Doug Oldenburg
UBC GIF



Last week:
Fundamentals of Inversion
Doug Oldenburg

<http://www.mtnet.info/EMinars/EMinars.html>

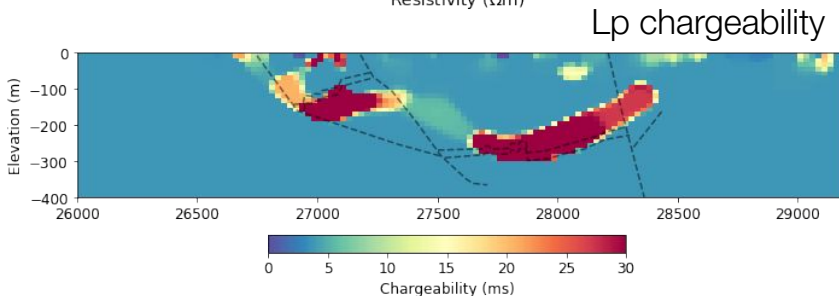
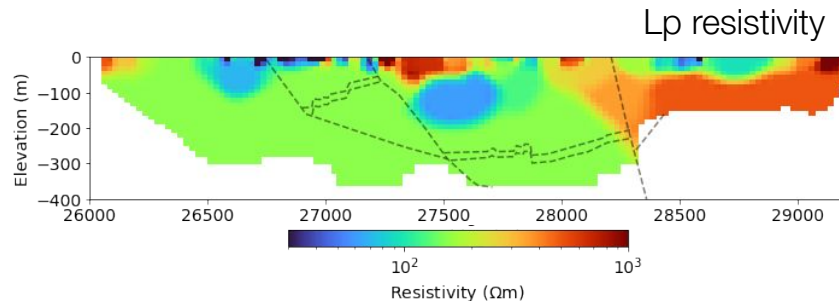
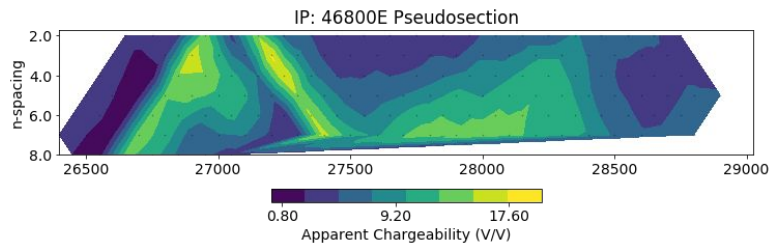
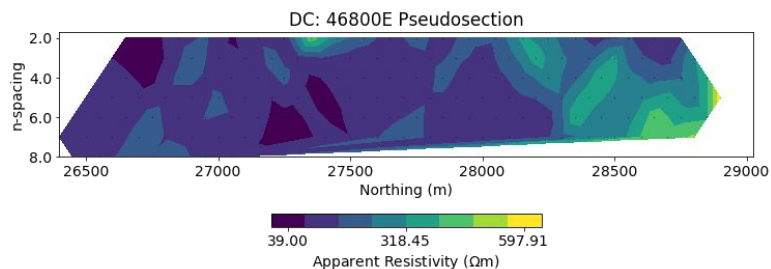
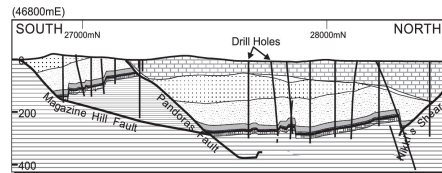
Geophysical experiments & physical properties



Last week: DC resistivity and IP at century

Last week: Century Deposit

- IP: linear inverse problem
- DC: non-linear inverse problem



DC resistivity

Governing PDE

$$\nabla \cdot \frac{1}{\rho} \nabla \phi = -I \delta(r)$$

DC is an illustrative problem

- foundation for the physics of EM
- poisson equation, starting point: numerical simulations, finite volume
- inverse problem: non-linear, ideal example for showing impacts of: model norm, constraints, ...

Finite Volume Tutorial ([Cockett et al., 2016](#))

SEG LIBRARY
Volume 35, Issue 8
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10.1190/tle35080703.1

The Leading Edge

Pixels and their neighbors: Finite volume

Rowan Cockett, Lindsey J. Heagy, and Douglas W. Oldenburg

$\nabla \cdot \vec{j}(p) = \lim_{v \rightarrow (p)} \iint_{S(v)} \frac{\vec{j} \cdot \vec{n}}{|v|} dS$

Labels in diagram: flux, around a point, normal, area, volume, enclosing surface, a cell, \vec{j}_3 , \vec{j}_1 , \vec{j}_2 , \vec{j}_0 , a_0 , a_1 , a_2 , a_3 , $\frac{1}{v} [-1 \ +1 \ -1 \ +1]$, normal points out of cell, \mathbf{Dj} , $\begin{bmatrix} j_0 \\ j_1 \\ j_2 \\ j_3 \end{bmatrix}$

Inversions with SimPEG ([Heagy, 2020](#))

TRANSFORM 2020
virtual conference

Launcher 1-century-dcip-inversion.ipynk

```
[ ]: import SimPEG
```

Inversion of DC & IP data at the Century Deposit

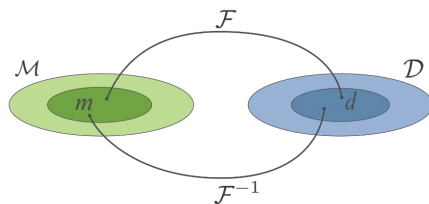
The Century Deposit is a Zinc-lead-silver deposit is located 250 km to the NNW of the Mt Isa region in Queensland Australia (Location: 18° 43' 15"S, 138° 35' 54"E).

Focus for today: electromagnetics

Theory: Maxwell's equations, inversion

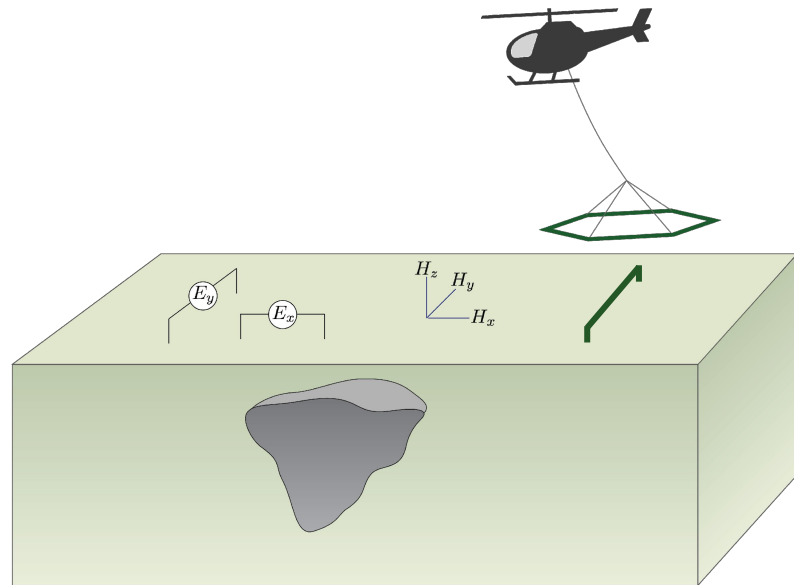
$$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$$

$$\nabla \times \vec{h} = \vec{j} + \frac{\partial \vec{d}}{\partial t}$$

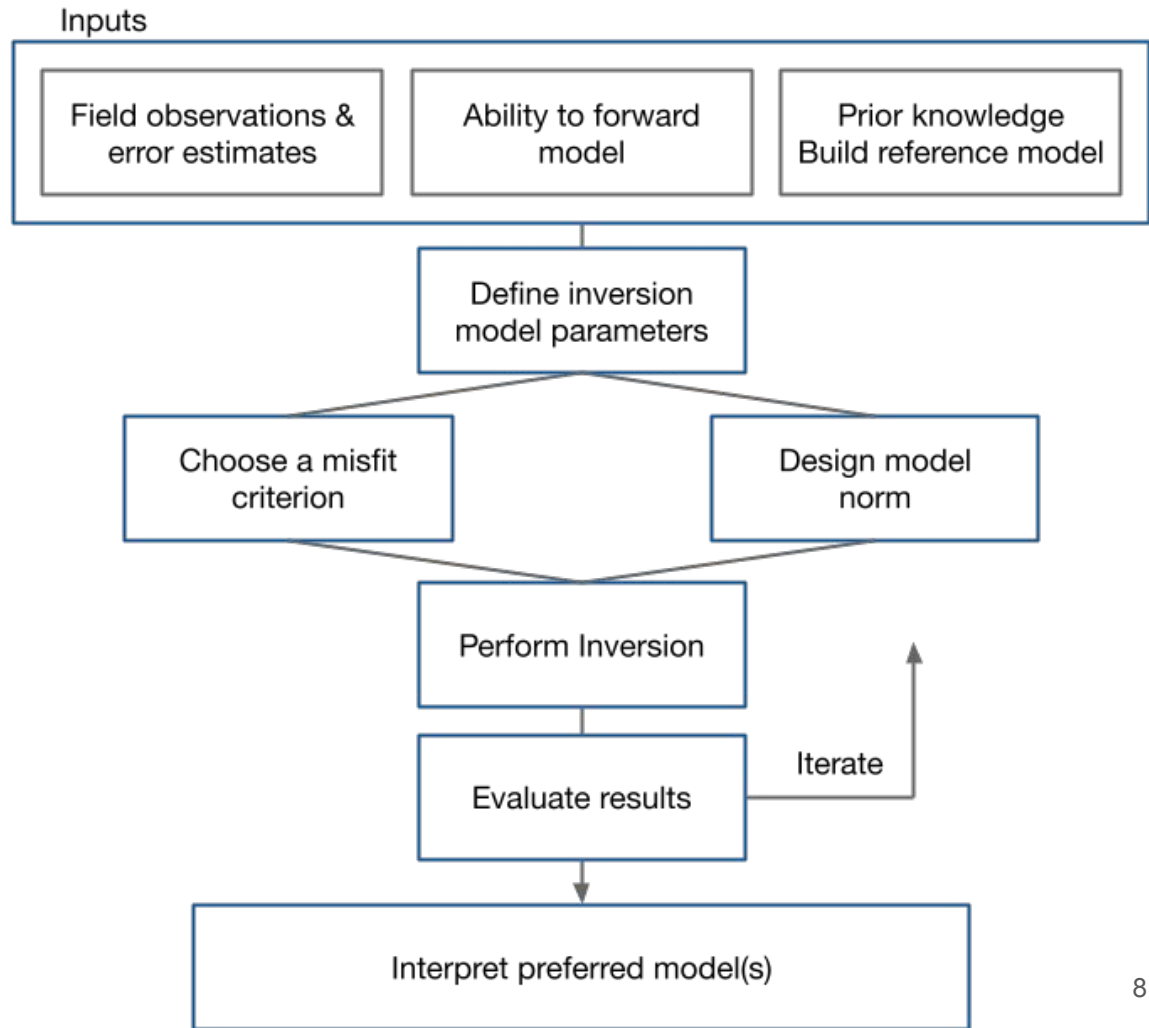


Captured in code

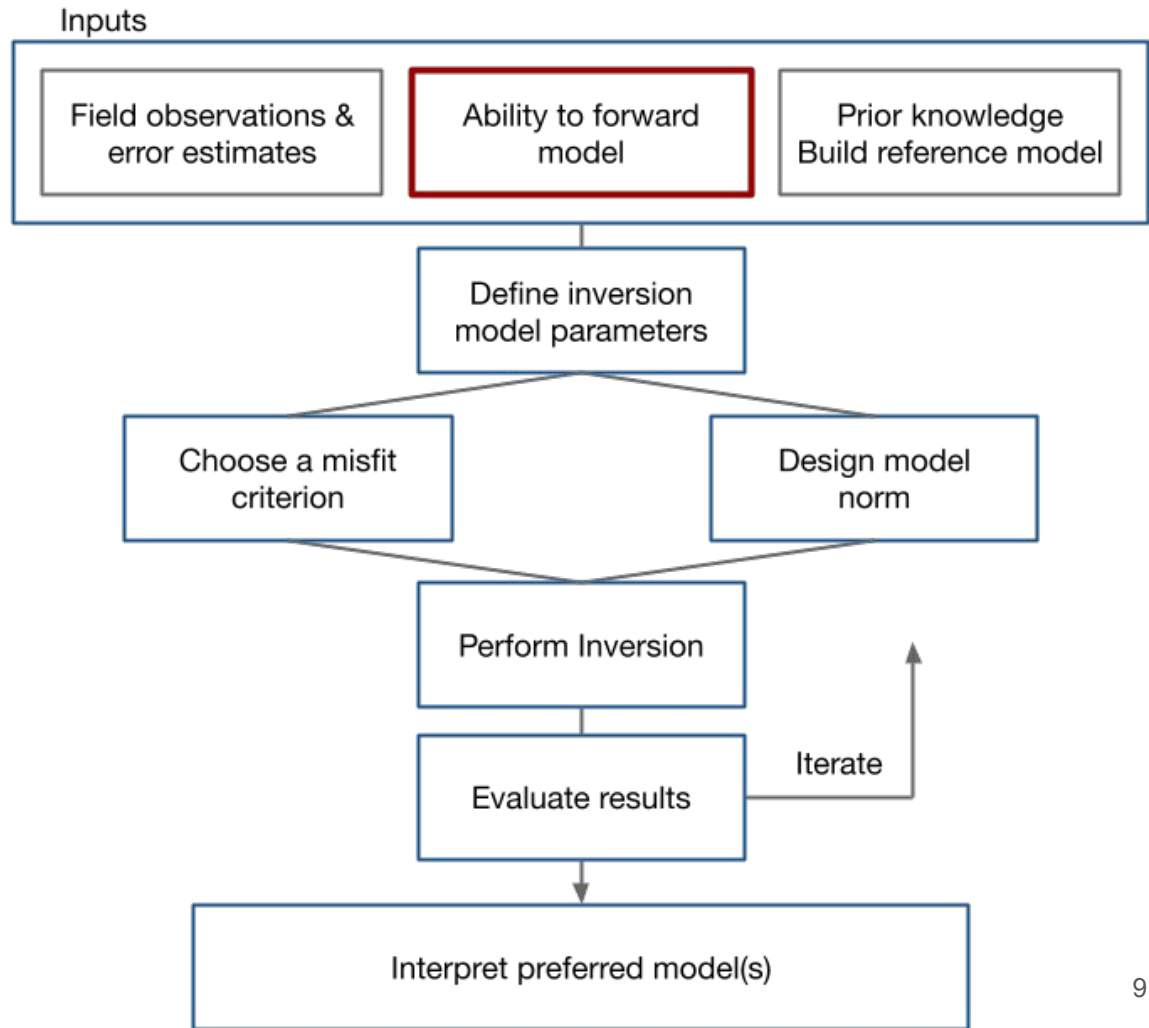
```
from SimPEG import electromagnetics
```



inversion flowchart



inversion flowchart



Electromagnetics: basic equations (quasi-static)

	Time	Frequency
Faraday's Law	$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$	$\nabla \times \vec{E} = -i\omega \vec{B} \frac{\partial \vec{b}}{\partial t}$
Ampere's Law	$\nabla \times \vec{h} = \vec{j} + \frac{\partial \vec{d}}{\partial t}$	$\nabla \times \vec{H} = \vec{J} + i\omega \vec{D} \frac{\partial \vec{b}}{\partial t}$
No Magnetic Monopoles	$\nabla \cdot \vec{b} = 0$	$\nabla \cdot \vec{B} = 0$
Constitutive Relationships (non-dispersive)	$\vec{j} = \sigma \vec{e}$ $\vec{b} = \mu \vec{h}$ $\vec{d} = \epsilon \vec{e}$	$\vec{J} = \sigma \vec{E}$ $\vec{B} = \mu \vec{H}$ $\vec{D} = \epsilon \vec{E}$

* Solve with sources and boundary conditions

Electromagnetics: frequency domain equations

Use constitutive relations, reduce to two equations, one field, one flux

$$\nabla \times \vec{E} + i\omega\vec{B} = 0$$

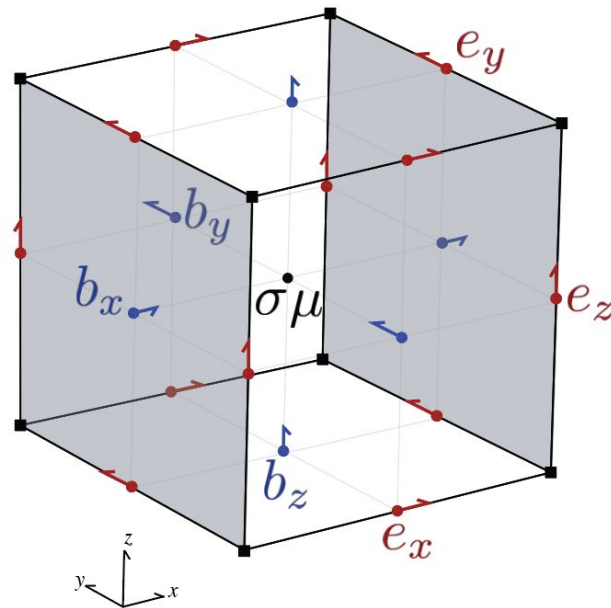
$$\nabla \times \mu^{-1}\vec{B} - \sigma\vec{E} = \vec{J}_s$$

Boundary conditions

$$\hat{n} \times \vec{B}|_{\partial\Omega} = 0$$

Staggered grid discretization

- Physical properties: cell centers
- Fields: edges
- Fluxes: faces



Electromagnetics: frequency domain equations

Continuous equations

$$\nabla \times \vec{E} + i\omega\vec{B} = 0$$

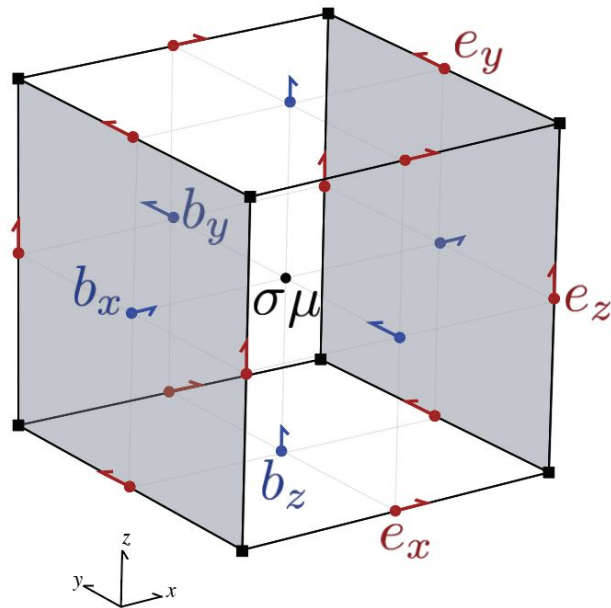
$$\nabla \times \mu^{-1}\vec{B} - \sigma\vec{E} = \vec{J}_s$$

$$\hat{n} \times \vec{B}|_{\partial\Omega} = 0$$

Finite volume discretization (see: Haber, 2014; Cockett et al., 2016)

$$\mathbf{C}\mathbf{e} + i\omega\mathbf{b} = 0$$

$$\mathbf{C}^\top \mathbf{M}_{\mu^{-1}}^f \mathbf{b} - \mathbf{M}_\sigma^e \mathbf{e} = \mathbf{M}^e \mathbf{j}_s$$



Electromagnetics: frequency domain equations

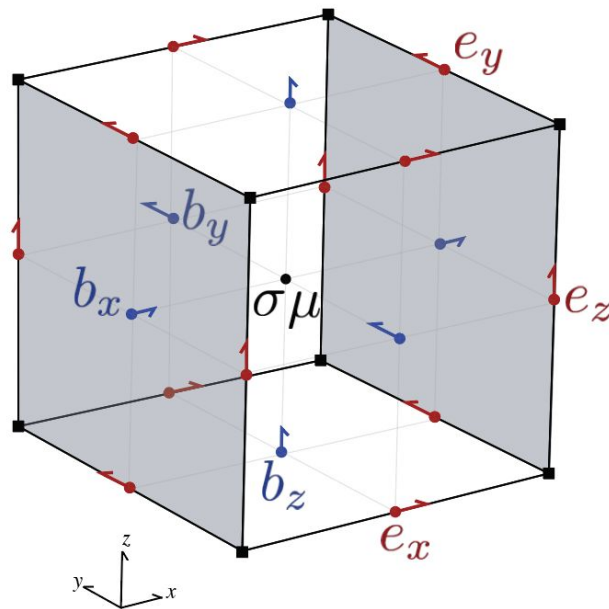
Discrete equations

$$\mathbf{C}\mathbf{e} + i\omega\mathbf{b} = 0$$

$$\mathbf{C}^T \mathbf{M}_{\mu-1}^f \mathbf{b} - \mathbf{M}_{\sigma}^e \mathbf{e} = \mathbf{M}^e \mathbf{j}_s$$

Eliminate \mathbf{b} to obtain a second-order system in \mathbf{e}

$$\underbrace{(\mathbf{C}^T \mathbf{M}_{\mu-1}^f \mathbf{C} + i\omega \mathbf{M}_{\sigma}^e)}_{\mathbf{A}(\sigma, \omega)} \underbrace{\mathbf{e}}_{\mathbf{u}} = \underbrace{-i\omega \mathbf{M}^e \mathbf{j}_s}_{\mathbf{q}(\omega)}$$



Solving an FDEM problem

$$\underbrace{(\mathbf{C}^\top \mathbf{M}_{\mu-1}^f \mathbf{C} + i\omega \mathbf{M}_\sigma^e)}_{\mathbf{A}(\sigma, \omega)} \underbrace{\mathbf{e}}_{\mathbf{u}} = \underbrace{-i\omega \mathbf{M}^e \mathbf{j}_s}_{\mathbf{q}(\omega)}$$

$\mathbf{A}(\sigma, \omega)$

- Complex
- Symmetric
- Factor once for each frequency (solve for multiple sources)
- Need to refactor on each model update
- Separable over frequencies

Solving an FDEM problem

$$\underbrace{(\mathbf{C}^\top \mathbf{M}_{\mu^{-1}}^f \mathbf{C} + i\omega \mathbf{M}_\sigma^e)}_{\mathbf{A}(\sigma, \omega)} \underbrace{\mathbf{e}}_{\mathbf{u}} = \underbrace{-i\omega \mathbf{M}^e \mathbf{j}_s}_{\mathbf{q}(\omega)}$$

```
 $\omega = 2 * \text{pi} * \text{frequency}$ 
```

```
 $\mathbf{C} = \text{mesh.edge\_curl}$ 
```

```
 $\mathbf{M}_{\mu i} = \text{mesh.get\_face\_inner\_product}(1/\mu_0)$ 
```

```
 $\mathbf{M}_\sigma = \text{mesh.get\_edge\_inner\_product}(\text{sigma})$ 
```

```
 $\mathbf{A} = \mathbf{C.T} * \mathbf{M}_{\mu i} * \mathbf{C} + i * \omega * \mathbf{M}_\sigma$ 
```

```
 $\mathbf{Ainv} = \text{Solver}(\mathbf{A})$  # acts like A inverse
```

```
 $\mathbf{M}_e = \text{mesh.get\_edge\_inner\_product}()$ 
```

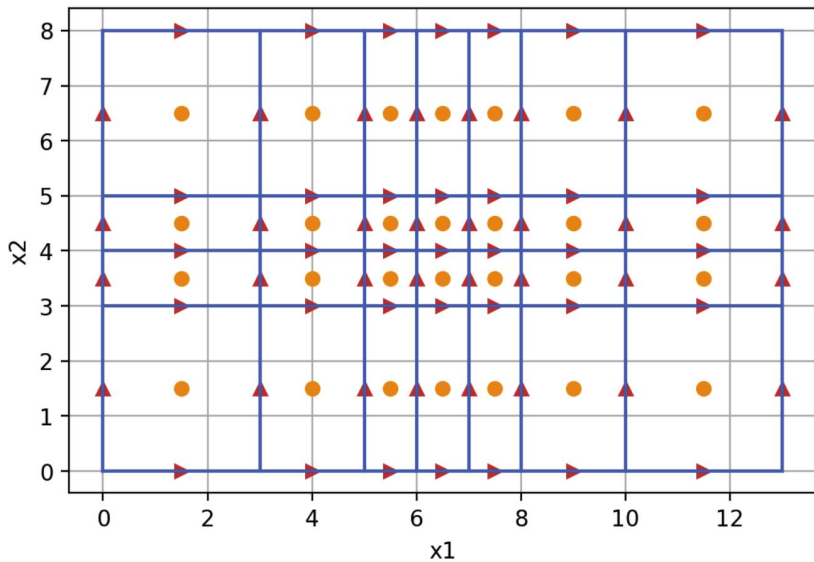
```
 $\mathbf{q} = -i * \omega * \mathbf{M}_e * \mathbf{j}_s$ 
```

```
 $\mathbf{u} = \mathbf{Ainv} * \mathbf{q}$ 
```

Create a mesh: the discretize package

```
import discretize
```

```
hx = [3, 2, 1, 1, 1, 2, 3]  
hy = [3, 1, 1, 3]  
mesh = discretize.TensorMesh([hx, hy])  
mesh.plot_grid(edges=True, centers=True);
```



Properties or Methods

dim, origin

n_cells, n_nodes, n_faces, n_edges

cell_volumes, face_areas, edge_lengths

cell_centers, nodes, edges, faces

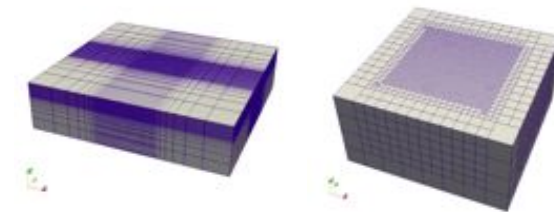
nodal_gradient, face_divergence, edge_curl

average_edge_to_cell,
average_node_to_cell, ...

get_edge_inner_product()

get_interpolation_matrix(location)

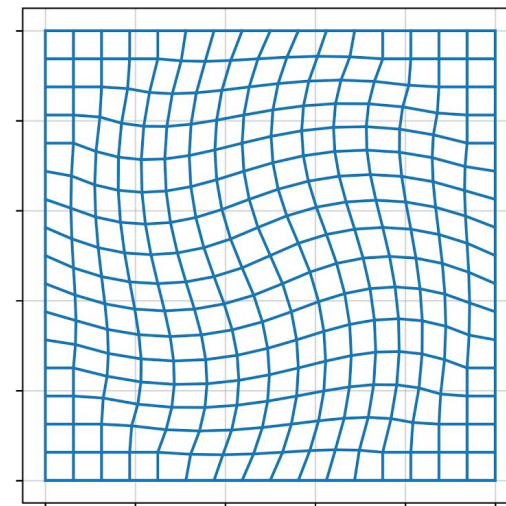
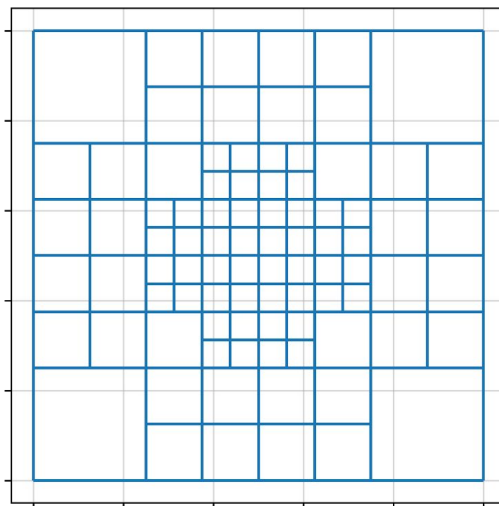
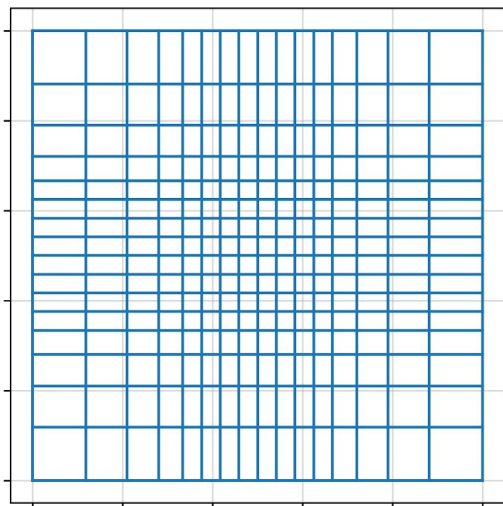
mesh types in simpeg



Tensor / Cylindrical

QuadTree / OcTree

Curvilinear



Joe
Capriotti



Survey: sources and receivers

Sources

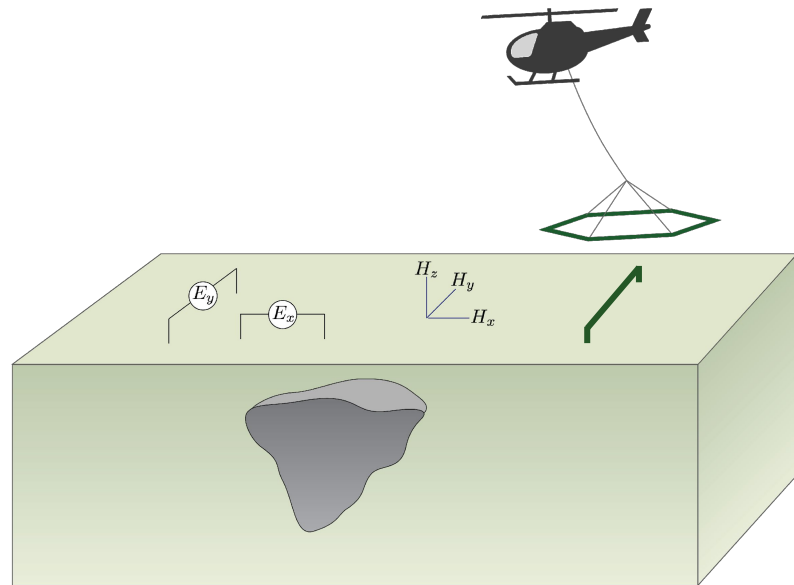
$$\underbrace{(\mathbf{C}^T \mathbf{M}_{\mu-1}^f \mathbf{C} + i\omega \mathbf{M}_{\sigma}^e)}_{\mathbf{A}(\sigma, \omega)} \underbrace{\mathbf{e}}_{\mathbf{u}} = \underbrace{-i\omega \mathbf{M}^f \mathbf{j}_s}_{\mathbf{q}(\omega)}$$

- inductive
- grounded

Receivers

- electric
- magnetic
- interpolate to locations

$$\mathbf{d}^{\text{pred}} = \mathbf{P}(\mathbf{u}, \omega)$$



```
source_list = [  
    fdem.sources.MagDipole(receiver_list, f, location)  
    for f in frequencies  
]  
survey = fdem.survey(source_list)
```

Bring it all together: simulation

```
from SimPEG.electromagnetics import frequency_domain as fdem
```

```
simulation = fdem.Simulation3DElectricField(  
    mesh=mesh, survey=survey, solver=Pardiso,  
    sigmaMap=maps.IdentityMap()  
)
```

For each frequency, solve

$$\mathbf{A}(\sigma, \omega)\mathbf{u} = \mathbf{q}(\omega)$$

```
u = simulation.fields(model)
```

Project to receiver locations

$$\mathbf{d}^{\text{pred}} = \mathbf{P}(\mathbf{u}, \omega)$$

```
dpred = simulation.dpred(model, fields=u)
```

Sensitivities (we will need them later!)

For inverse problem, also need sensitivities (and adjoint)

$$\begin{aligned}\mathbf{J} &= \frac{\partial \mathbf{d}^{\text{pred}}}{\partial \mathbf{m}} \\ &= \frac{\partial \mathbf{P}(\mathbf{u}, \omega)}{\partial \mathbf{u}} \frac{\partial \mathbf{u}}{\partial \mathbf{m}}\end{aligned}$$

where the derivative of the fields (\mathbf{u}) is computed implicitly (requires a solve)

$$\frac{\partial \mathbf{A}(\sigma, \omega) \mathbf{u}^{\text{fixed}}}{\partial \mathbf{m}} + \mathbf{A}(\sigma, \omega) \frac{\partial \mathbf{u}}{\partial \mathbf{m}} = 0$$

\mathbf{J} is a large, dense matrix \rightarrow compute products with a vector if memory-limited

Demo

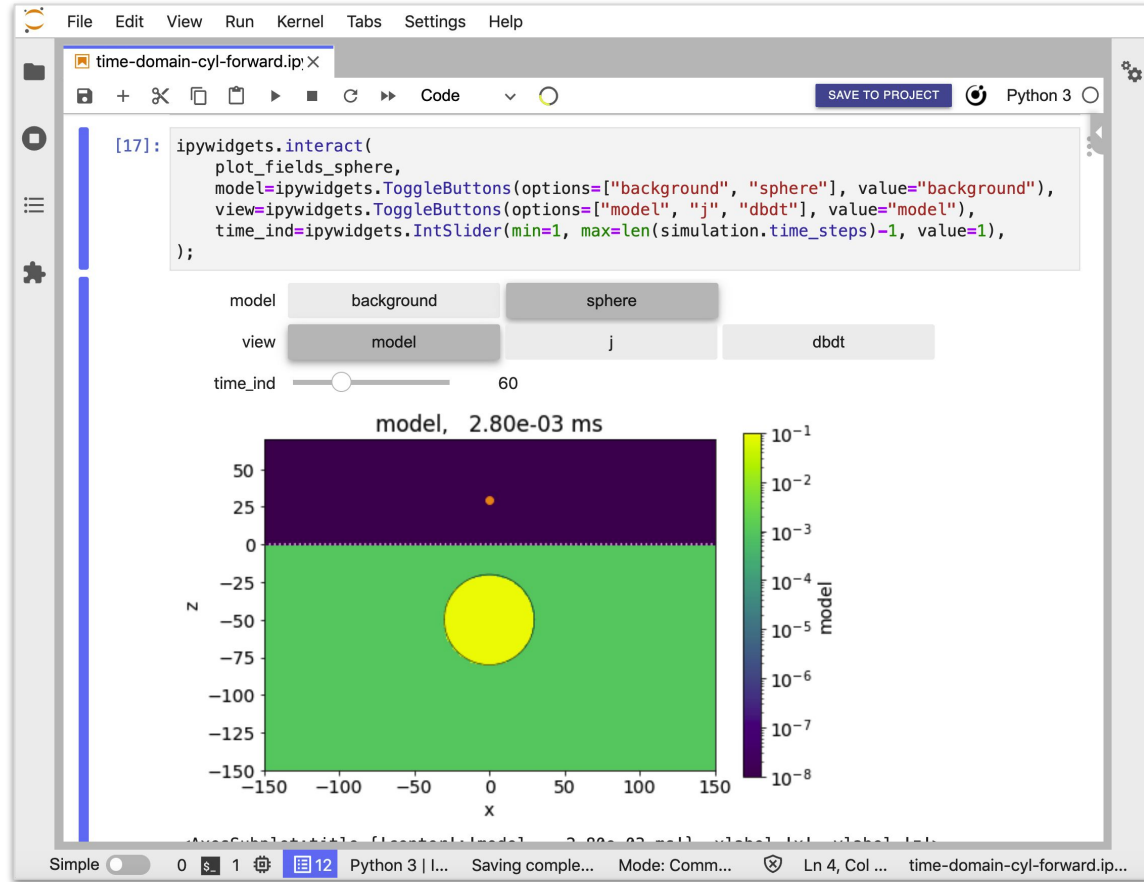
conductive sphere in a
halfspace

cylindrical mesh

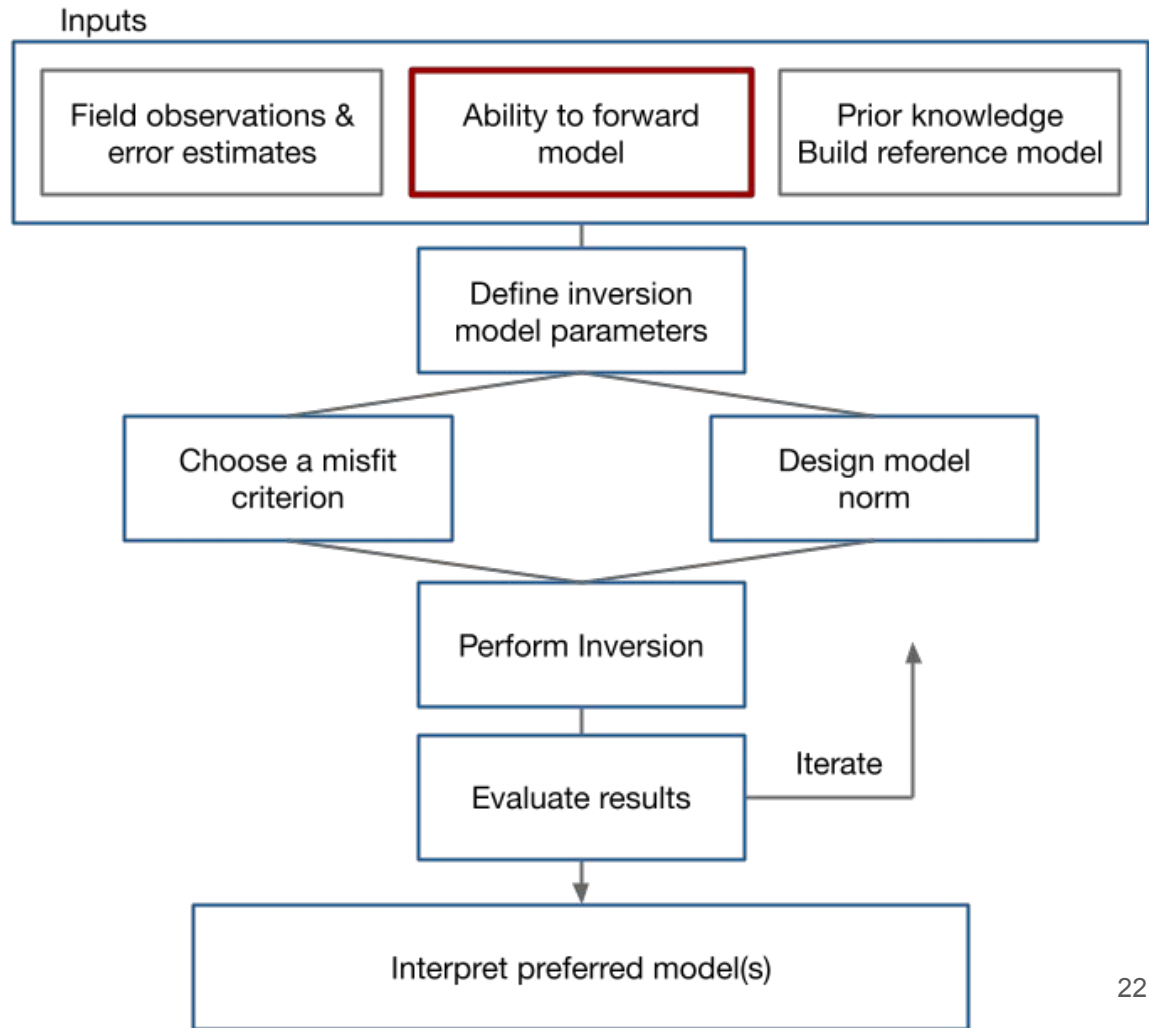
time-domain

$$\nabla \times \vec{e} = -\frac{\partial \vec{b}}{\partial t}$$

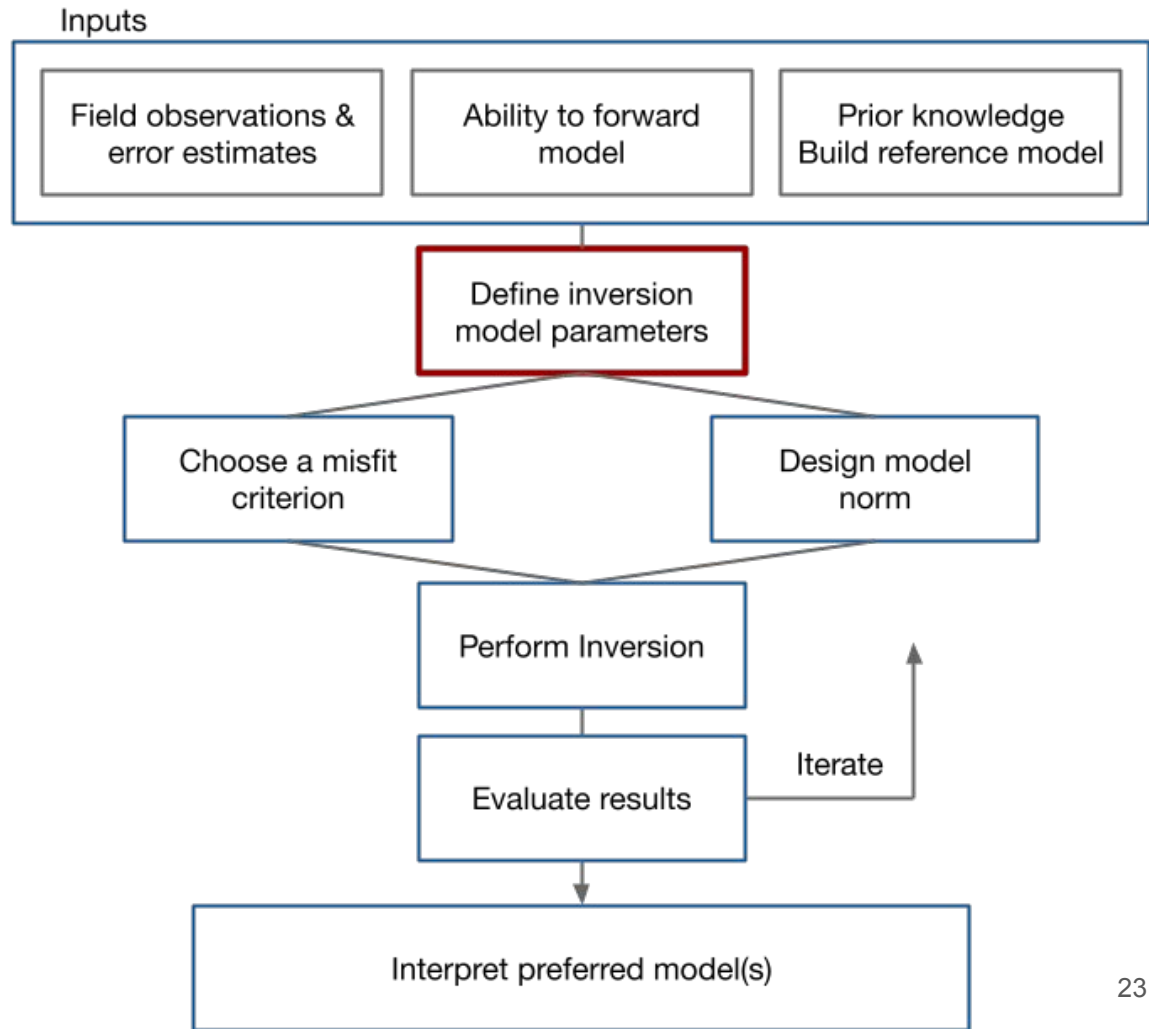
$$\nabla \times \vec{h} = \vec{j}$$



inversion flowchart

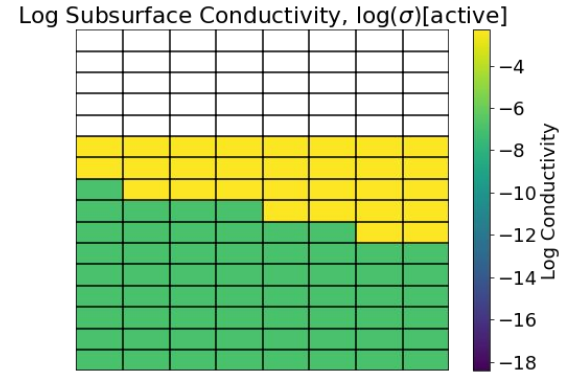
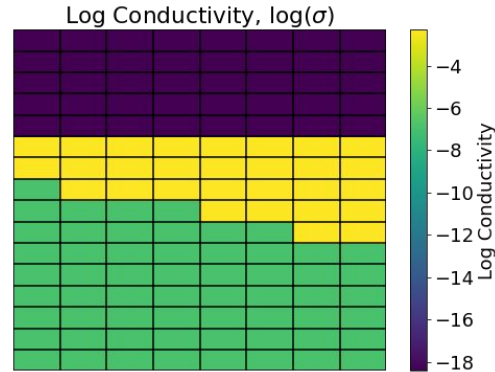
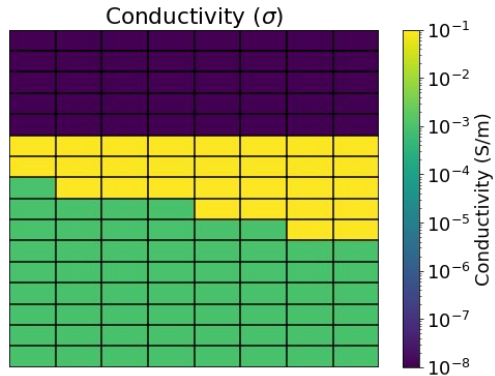


inversion flowchart



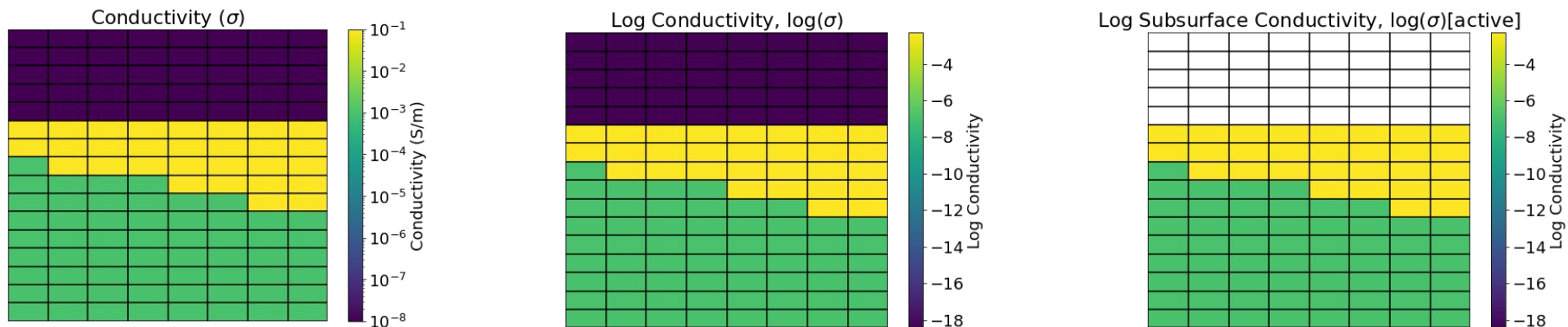
Inversion model parameters & mappings

What parameters are we inverting for?



Inversion model parameters & mappings

What parameters are we inverting for?



σ

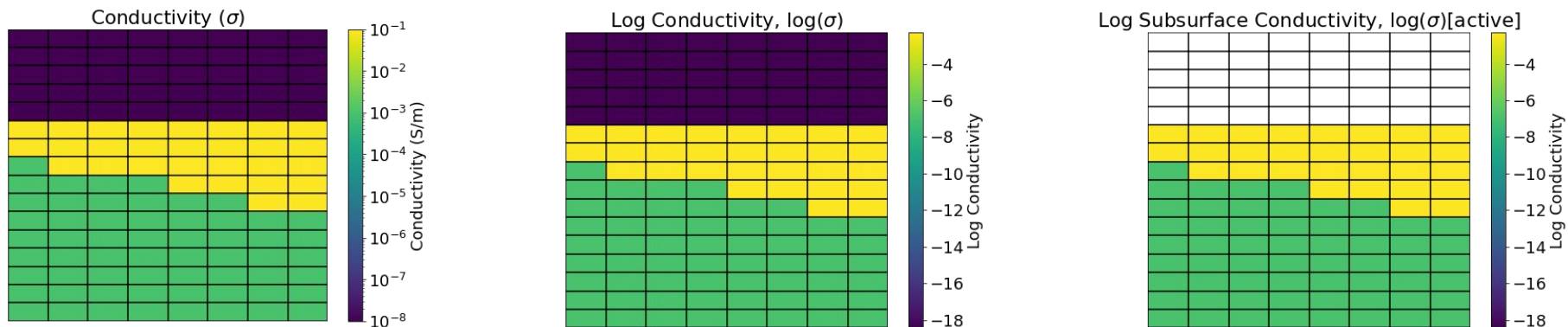
\mathbf{m}

a mapping translates model parameters to
physical properties on simulation mesh

$$\sigma = \mathcal{M}(\mathbf{m})$$

Inversion model parameters & mappings

What parameters are we inverting for?

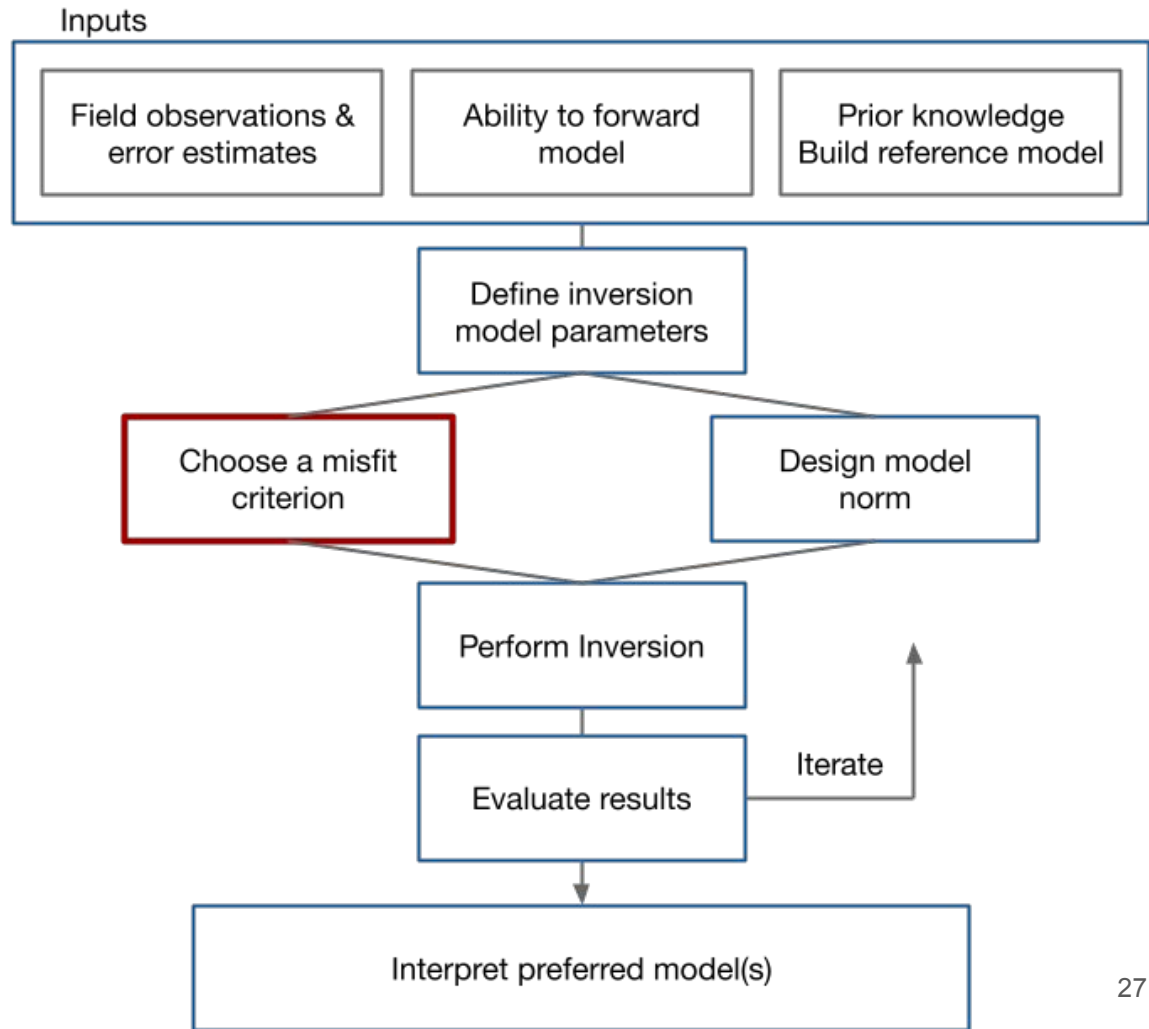


σ ← `maps.ExpMap` ————— `maps.InjectActiveCells` — \mathbf{m}

$$\sigma = \mathcal{M}(\mathbf{m})$$

- Mappings can be composed
- Includes parametric models
- Keep track of derivatives (for sensitivities)

inversion flowchart



observed data, uncertainties, and data misfit

Data misfit term

$$\phi_d = \|\mathbf{W}_d(\mathcal{F}(\mathbf{m}) - \mathbf{d}^{\text{obs}})\|^2$$

uncertainties captured in W matrix

$$\mathbf{W}_d = \text{diag} \left(\frac{1}{\boldsymbol{\epsilon}} \right)$$

$$\epsilon_j = \%|d_j^{\text{obs}}| + \text{floor}$$

observed data, uncertainties, and data misfit

Data misfit term

$$\phi_d = \|\mathbf{W}_d(\mathcal{F}(\mathbf{m}) - \mathbf{d}^{\text{obs}})\|^2$$

uncertainties captured in W matrix

$$\mathbf{W}_d = \text{diag} \left(\frac{1}{\epsilon} \right)$$

$$\epsilon_j = \%|d_j^{\text{obs}}| + \text{floor}$$

Data class: survey geometry, observed data, assigned uncertainties

```
from SimPEG import Data
data = Data(survey, dobs, relative_error, floor)
```

Data misfit instantiated with

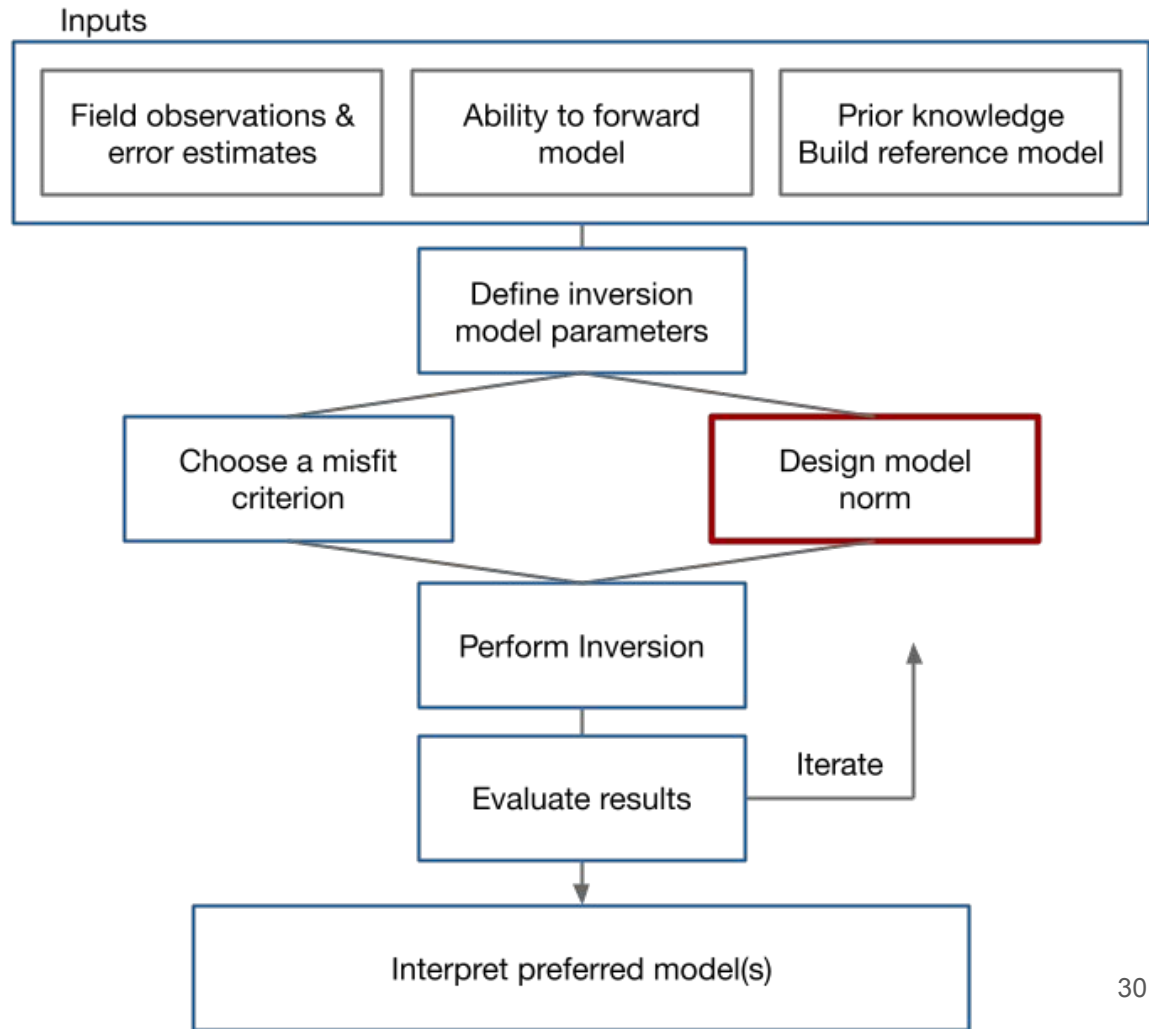
- simulation: to compute $\mathcal{F}(\mathbf{m})$
- data: defines $\mathbf{W}_d, \mathbf{d}^{\text{obs}}$

```
from SimPEG import data_misfit
phi_d = data_misfit.L2DataMisfit(data, simulation)
```

can now evaluate data misfit + derivatives

```
phi_d(m), phi_d.deriv(m), phi_d.deriv2(m, v)
```

inversion flowchart



Designing a model norm: regularization class

Basic Tikhonov regularization

$$\phi_m = \underbrace{\alpha_s \int_V w_s (m - m_{\text{ref}})^2 dV}_{\text{smallness}} + \underbrace{\alpha_x \int_V w_x \frac{d(m - m_{\text{ref}})^2}{dx} dV}_{\text{smoothness}}$$

discretize

$$\phi_m = \alpha_s \|\mathbf{W}_s(\mathbf{m} - \mathbf{m}_{\text{ref}})\|^2 + \alpha_x \|\mathbf{W}_x(\mathbf{m} - \mathbf{m}_{\text{ref}})\|^2$$

Choices:

- α - parameter values
- reference model
- mref in the smoothness terms
- norm applied on each term

Designing a model norm: regularization class

Basic Tikhonov regularization

$$\phi_m = \underbrace{\alpha_s \int_V w_s (m - m_{\text{ref}})^2 dV}_{\text{smallness}} + \underbrace{\alpha_x \int_V w_x \frac{d(m - m_{\text{ref}})^2}{dx} dV}_{\text{smoothness}}$$

discretize

$$\phi_m = \alpha_s \|\mathbf{W}_s(\mathbf{m} - \mathbf{m}_{\text{ref}})\|^2 + \alpha_x \|\mathbf{W}_x(\mathbf{m} - \mathbf{m}_{\text{ref}})\|^2$$

Choices:

- α - parameter values
- reference model
- mref in the smoothness terms
- norm applied on each term

Regularization instantiated with

- mesh: to evaluate spatial derivs
- alphas, mref have default values, can be replaced with user values

```
from SimPEG import regularization
```

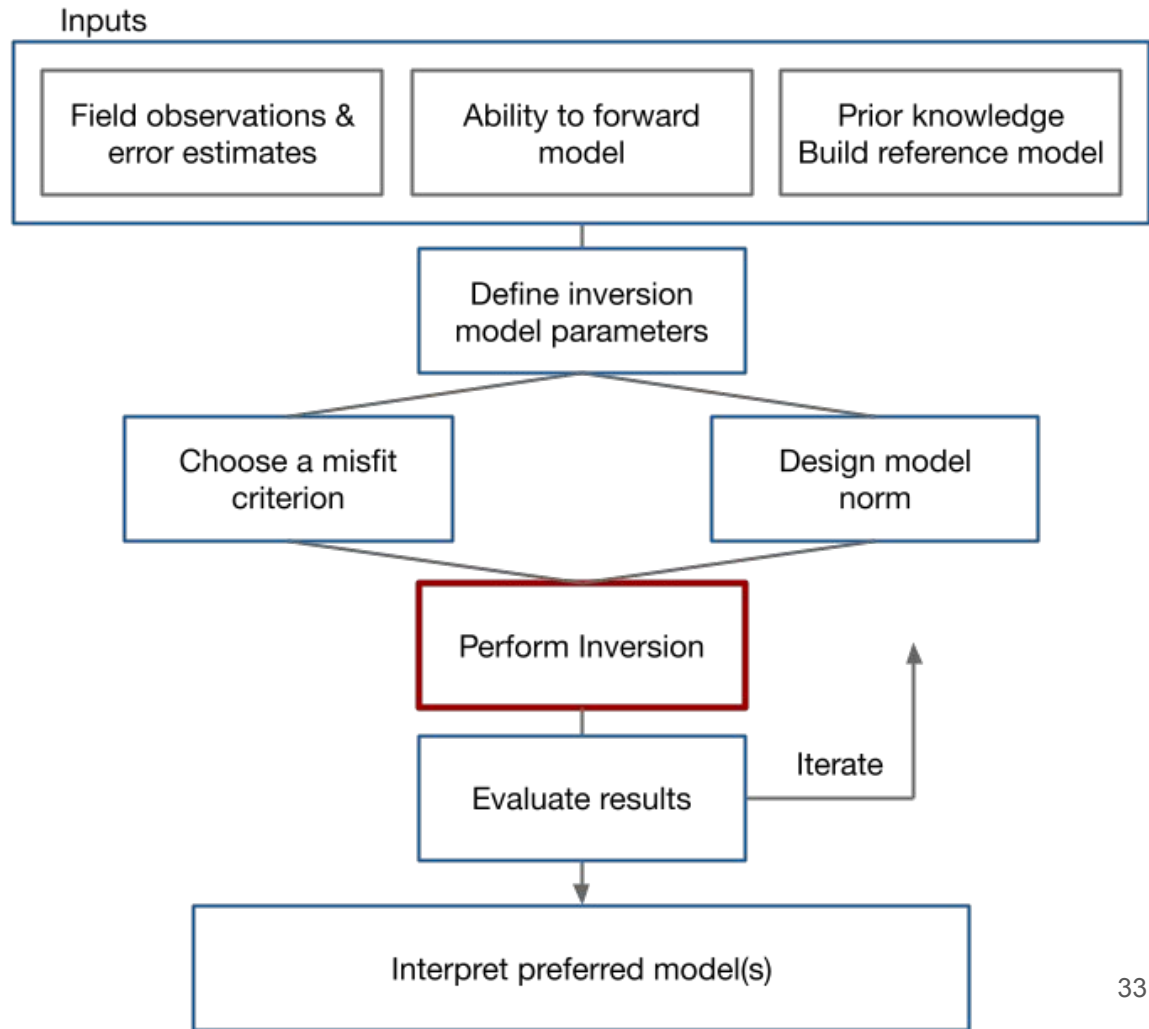
```
phi_m = regularization.Tikhonov(  
    mesh, mref=mref, alpha_s=alpha_s, alpha_x=alpha_x  
)
```

```
phi_m_sparse = regularization.Sparse(  
    mesh, mref=mref, norms=[1, 1]  
)
```

can now evaluate phi_m + derivatives

```
phi_m(m), phi_m.deriv(m), phi_m.deriv2(m, v)
```


inversion flowchart



Perform the inversion: stating the objective function

Inversion as an optimization problem (deterministic approach)

$$\begin{aligned} \min_{\mathbf{m}} \phi(\mathbf{m}) &= \phi_d(\mathbf{m}) + \beta \phi_m(\mathbf{m}) \\ \text{s.t. } \phi_d &\leq \phi_d^* \quad \mathbf{m}_L \leq \mathbf{m} \leq \mathbf{m}_U \end{aligned}$$

At this stage, we have specified

- parameters we are inverting for
- data misfit
- model norm

Perform the inversion: stating the objective function

Inversion as an optimization problem (deterministic approach)

$$\begin{aligned} \min_{\mathbf{m}} \phi(\mathbf{m}) &= \phi_d(\mathbf{m}) + \beta \phi_m(\mathbf{m}) \\ \text{s.t. } \phi_d &\leq \phi_d^* \quad \mathbf{m}_L \leq \mathbf{m} \leq \mathbf{m}_U \end{aligned}$$

Still to define

- optimization method
- upper and lower bounds
- choice of initial beta
- choice of beta-cooling schedule
- target misfit and stopping the inversion

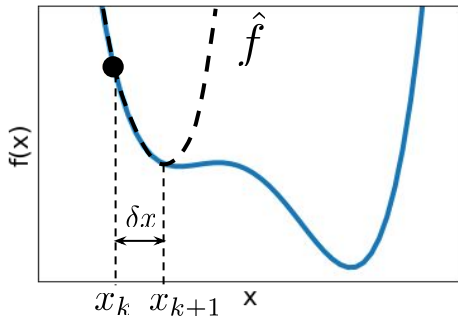
Perform the inversion: optimization approach

Inversion as an optimization problem (deterministic approach)

$$\underline{\min}_{\mathbf{m}} \phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m})$$

$$\text{s.t. } \phi_d \leq \phi_d^* \quad \underline{\mathbf{m}}_L \leq \mathbf{m} \leq \underline{\mathbf{m}}_U$$

Second-order methods



```
from SimPEG import optimization
```

```
opt = optimization.InexactGaussNewton(maxIter=20, maxIterCG=20)
```

```
opt_with_bounds = optimization.ProjectedGNCG(lower=0)
```

Perform the inversion: inversion directives

Inversion as an optimization problem (deterministic approach)

$$\begin{aligned} \min_{\mathbf{m}} \phi(\mathbf{m}) &= \phi_d(\mathbf{m}) + \beta \phi_m(\mathbf{m}) \\ \text{s.t. } \phi_d &\leq \underline{\phi_d^*} \quad \mathbf{m}_L \leq \mathbf{m} \leq \mathbf{m}_U \end{aligned}$$

We use **directives** to make parameter updates and orchestrate the inversion, e.g.

- estimating initial beta
- defining a beta-cooling schedule
- stopping the inversion when target misfit reached

Perform the inversion: inversion directives

Inversion as an optimization problem (deterministic approach)

$$\begin{aligned} \min_{\mathbf{m}} \phi(\mathbf{m}) &= \phi_d(\mathbf{m}) + \beta \phi_m(\mathbf{m}) \\ \text{s.t. } \phi_d &\leq \phi_d^* \quad \mathbf{m}_L \leq \mathbf{m} \leq \mathbf{m}_U \end{aligned}$$

Initial beta

- estimate “size” of data misfit and model norm by approximating eigenvalues of

$$\mathbf{J}^T \mathbf{W}_d^T \mathbf{W}_d \mathbf{J}, \quad \mathbf{W}_m^T \mathbf{W}_m$$

- take ratio, weight by a parameter controlling relative importance of each

```
from SimPEG import directives
beta0 = directives.BetaEstimate_ByEig(beta0_ratio=100)
```

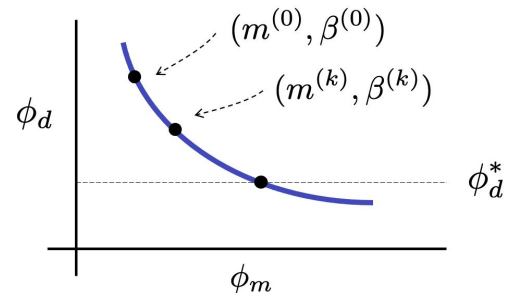
Perform the inversion: inversion directives

Inversion as an optimization problem (deterministic approach)

$$\begin{aligned} \min_{\mathbf{m}} \phi(\mathbf{m}) &= \phi_d(\mathbf{m}) + \beta \phi_m(\mathbf{m}) \\ \text{s.t. } \phi_d &\leq \phi_d^* \quad \mathbf{m}_L \leq \mathbf{m} \leq \mathbf{m}_U \end{aligned}$$

Beta-cooling

- Define how often beta is reduced (every N iterations)
- Define how much beta is reduced by



```
beta_cooling = directives.BetaSchedule(coolingRate=2, coolingFactor=4)
```

Perform the inversion: inversion directives

Inversion as an optimization problem (deterministic approach)

$$\begin{aligned} \min_{\mathbf{m}} \phi(\mathbf{m}) &= \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m}) \\ \text{s.t. } \phi_d &\leq \underline{\phi_d^*} \quad \mathbf{m}_L \leq \mathbf{m} \leq \mathbf{m}_U \end{aligned}$$

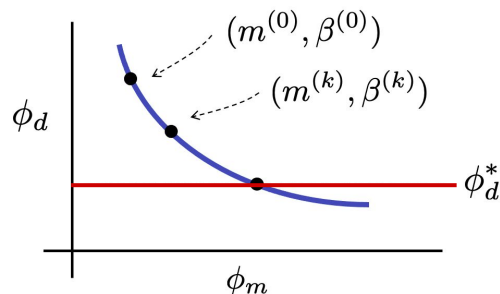
Target misfit

- Expected value of data misfit

$$E[\phi_d] \simeq N$$

- Define target misfit as (default $\chi=1$)

$$\phi_d^* = \chi N$$



```
target_misfit = directives.TargetMisfit(chifact=1)
```


Perform the inversion: inversion directives

Inversion as an optimization problem (deterministic approach)

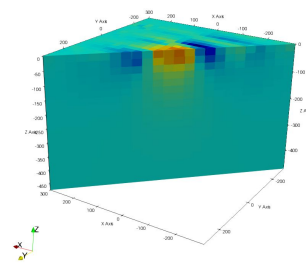
$$\min_{\mathbf{m}} \phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m})$$

$$\text{s.t. } \phi_d \leq \phi_d^* \quad \mathbf{m}_L \leq \mathbf{m} \leq \mathbf{m}_U$$

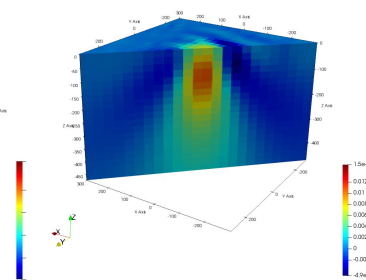
Other uses for directives

- saving inversion model at each iteration
- saving inversion progress (beta, data misfit, ...)
- including / updating sensitivity weighting
- updating values for norms (L2 \rightarrow Lp)

Recovered susceptibility



Sensitivity weighted susceptibility



Perform the inversion: bringing it all together

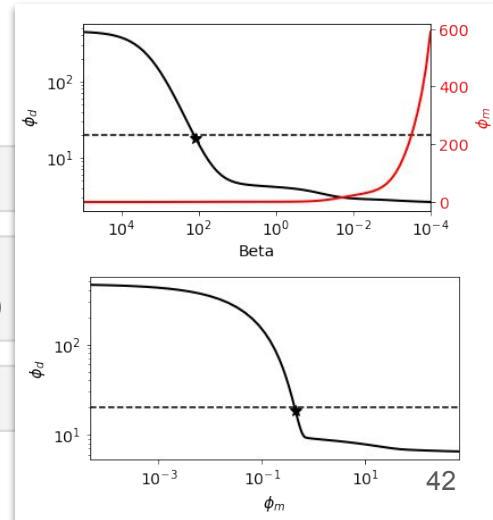
Inversion as an optimization problem (deterministic approach)

$$\begin{aligned} \min_{\mathbf{m}} \phi(\mathbf{m}) &= \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m}) \\ \text{s.t. } \phi_d &\leq \phi_d^* \quad \mathbf{m}_L \leq \mathbf{m} \leq \mathbf{m}_U \end{aligned}$$

```
from SimPEG import inverse_problem, inversion
```

```
inv_prob = inverse_problem.BaseInvProblem(phi_d, phi_m, opt)  
inv = inversion.BaseInversion(inv_prob, [beta0, beta_cooling, target_misfit])
```

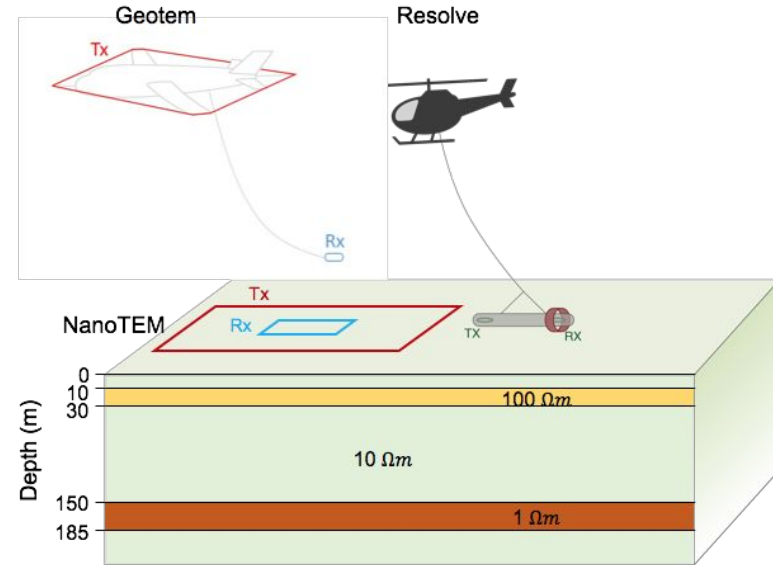
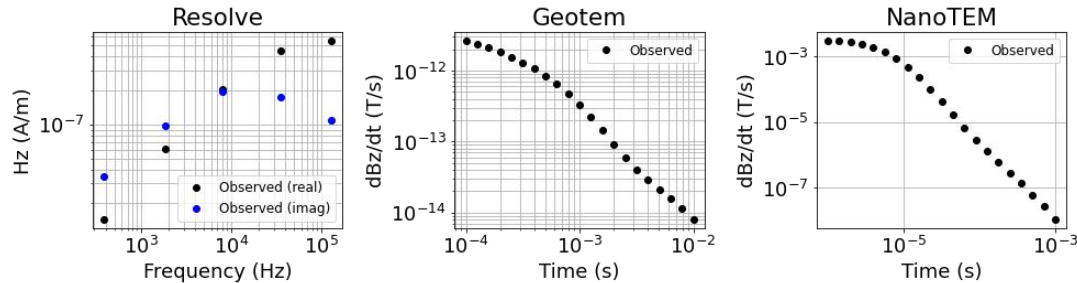
```
inv.run(m0)
```



An example: 1D inversions

Layered earth, 3 different EM systems

- Resolve (airborne, frequency)
- Geotem (airborne, time-domain)
- NanoTEM (ground, time-domain)



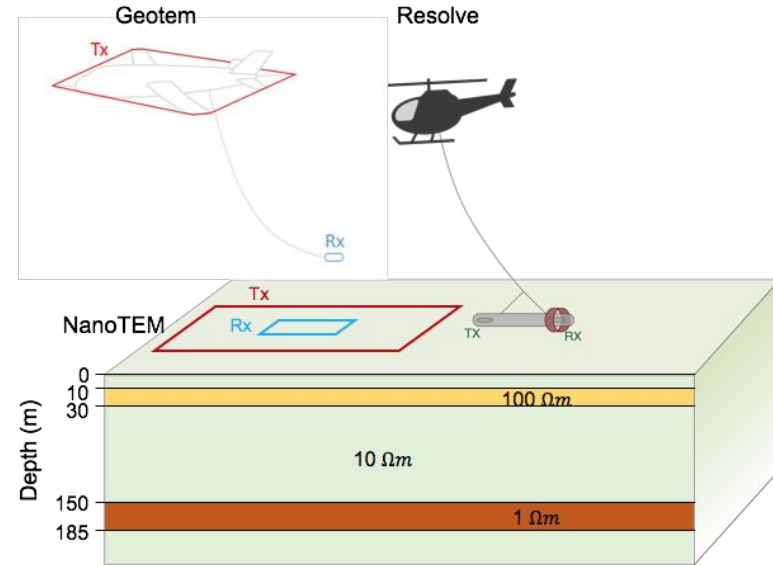
(Oldenburg et al, 2020)

SimPEG EM1D



Seogi Kang

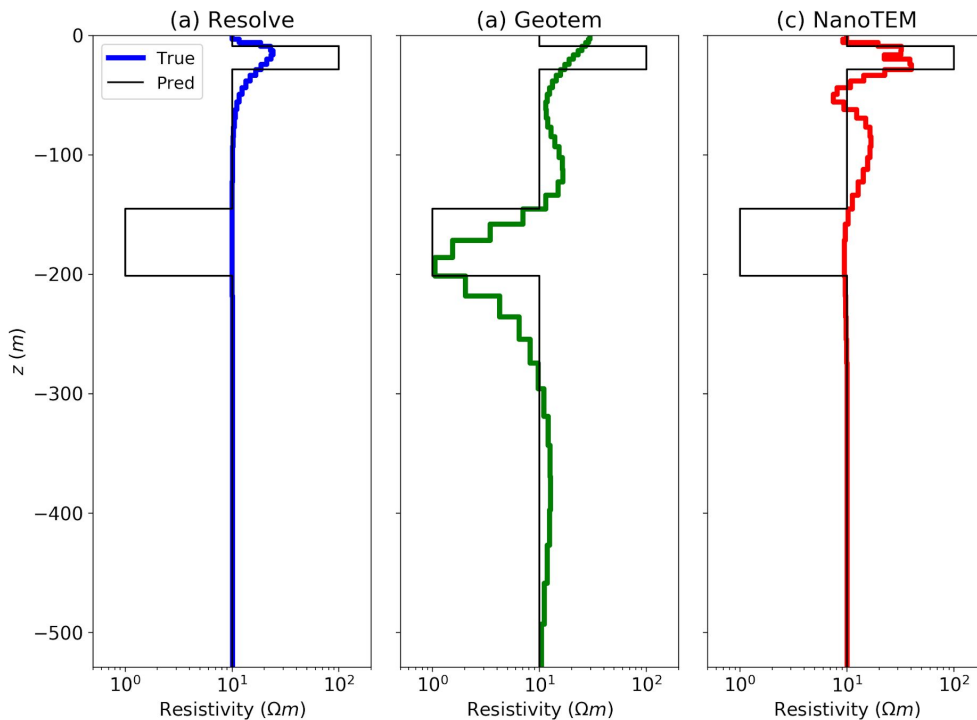
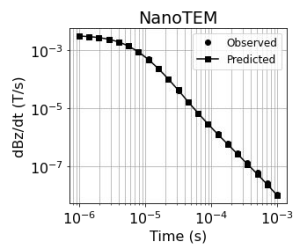
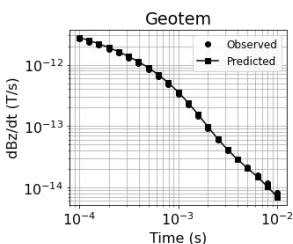
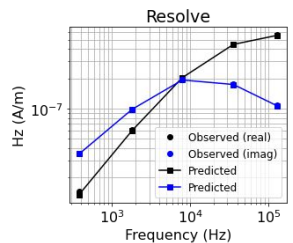
- Efficient forward simulation, sensitivity calculation using digital filters
 - relies on empymod (Werthmüller, 2017)
- Parallelized over soundings
- Common FDEM, TDEM system parameters implemented



(Kang et al, 2018)

Individual inversions

L2 regularization

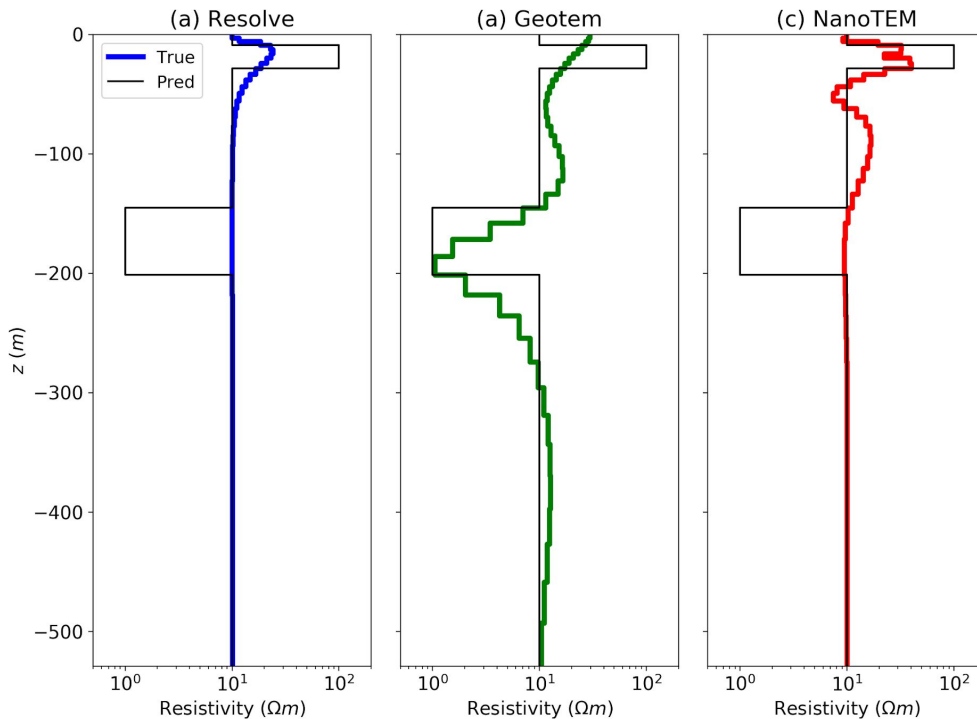
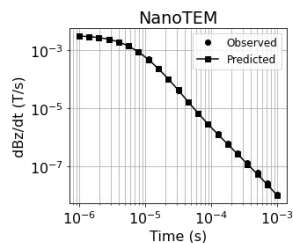
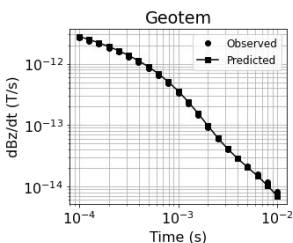
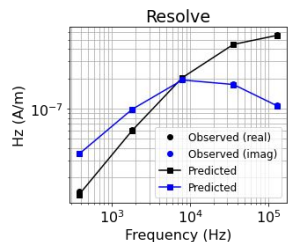


Joint Inversion

$$\phi(\mathbf{m}) = \underbrace{\phi_d^{\text{Resolve}} + \phi_d^{\text{Geotem}} + \phi_d^{\text{NanoTEM}}}_{\phi_d(\mathbf{m})} + \beta\phi_m(\mathbf{m})$$

L2 regularization

$$\text{phi_d} = \text{phi_d_resolve} + \text{phi_d_geotem} + \text{phi_d_nanotem}$$

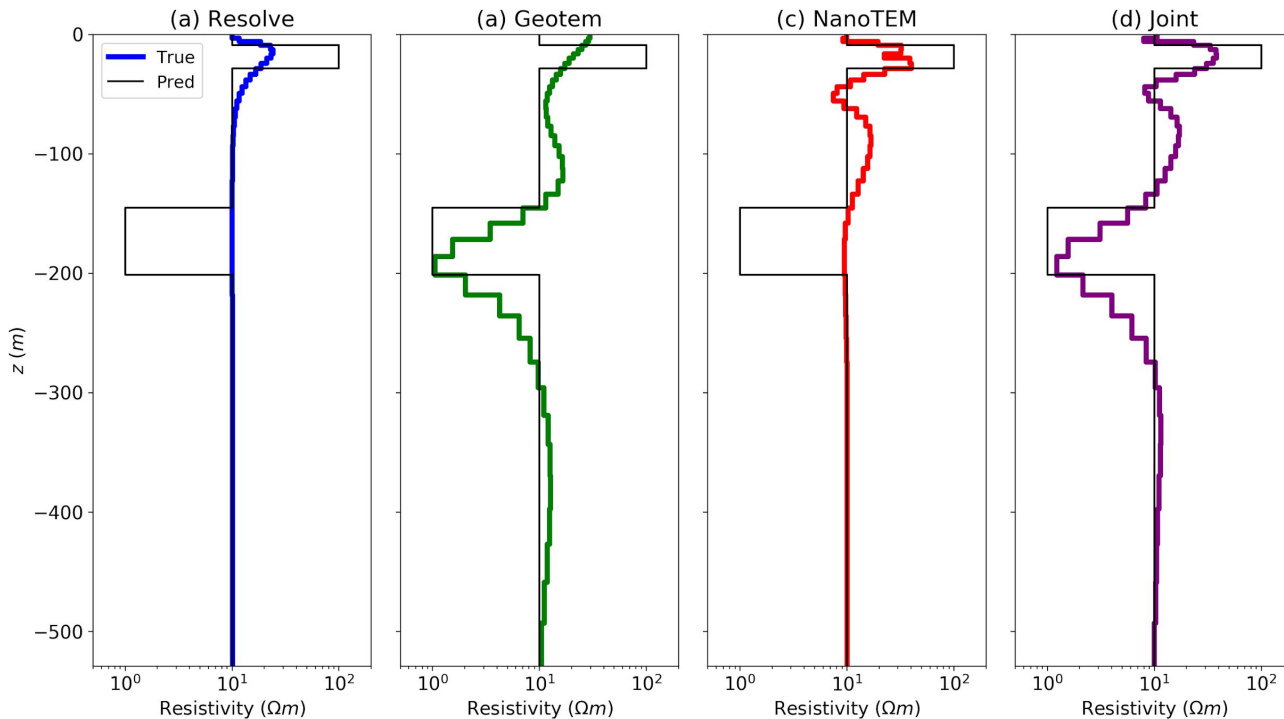
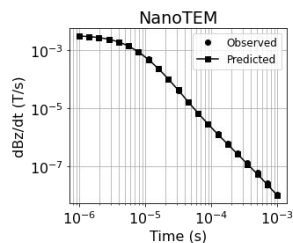
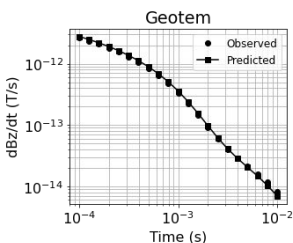
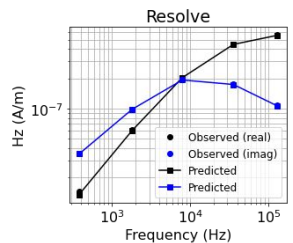


Joint Inversion

$$\phi(\mathbf{m}) = \underbrace{\phi_d^{\text{Resolve}} + \phi_d^{\text{Geotem}} + \phi_d^{\text{NanoTEM}}}_{\phi_d(\mathbf{m})} + \beta\phi_m(\mathbf{m})$$

L2 regularization

$$\text{phi_d} = \text{phi_d_resolve} + \text{phi_d_geotem} + \text{phi_d_nanotem}$$

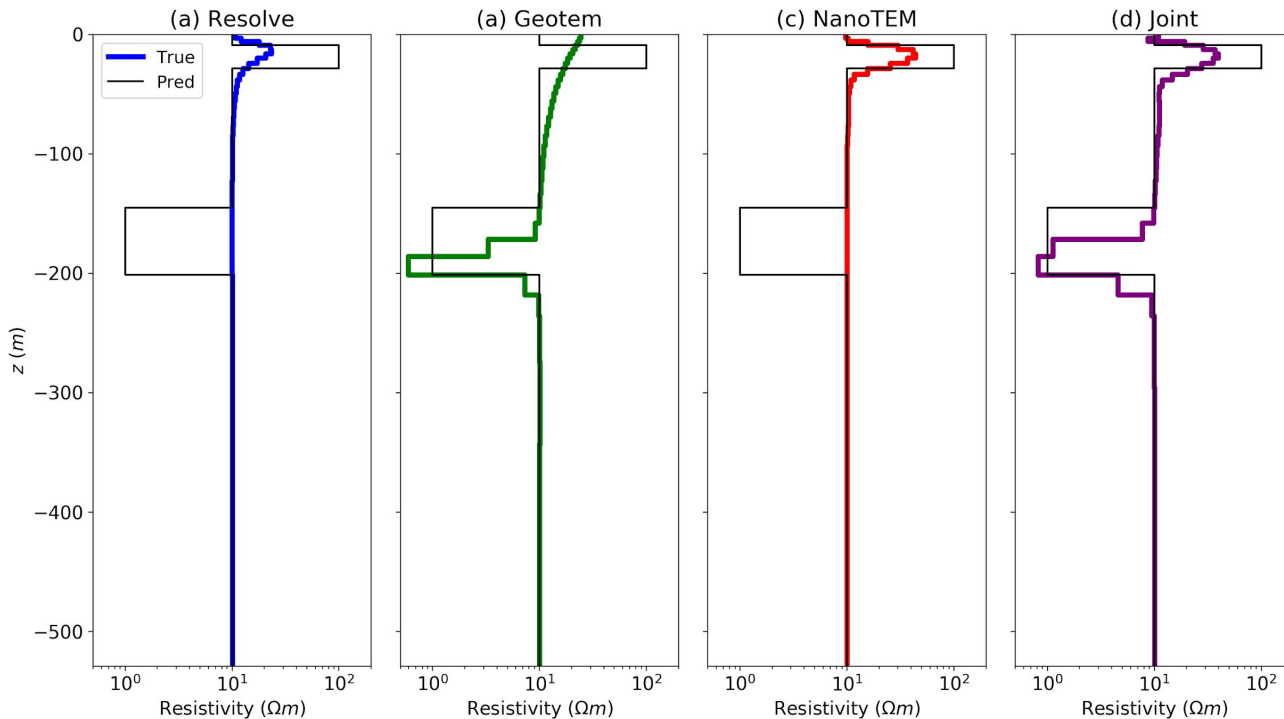
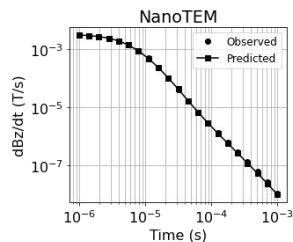
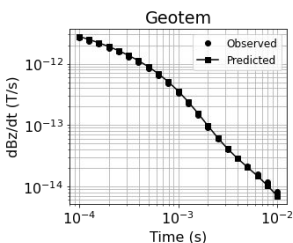
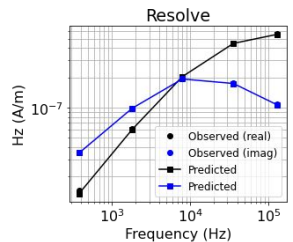


Joint Inversion

$$\phi(\mathbf{m}) = \underbrace{\phi_d^{\text{Resolve}} + \phi_d^{\text{Geotem}} + \phi_d^{\text{NanoTEM}}}_{\phi_d(\mathbf{m})} + \beta\phi_m(\mathbf{m})$$

L0 regularization

$$\text{phi_d} = \text{phi_d_resolve} + \text{phi_d_geotem} + \text{phi_d_nanotem}$$

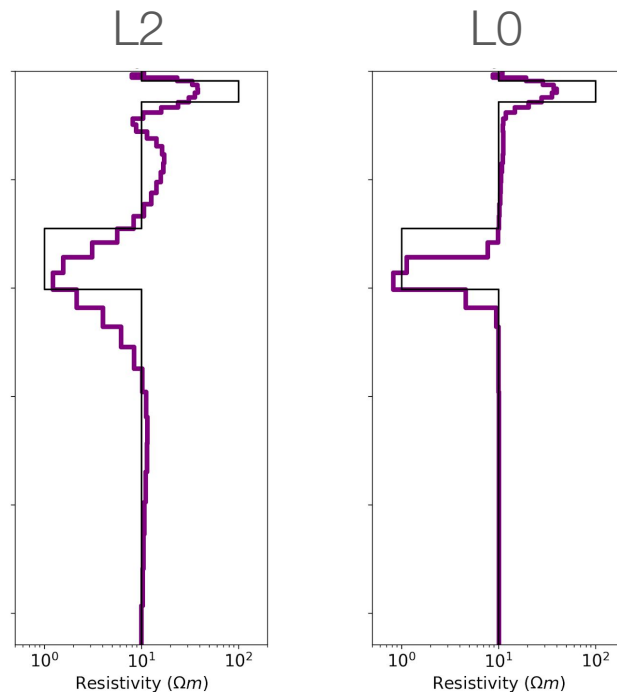


Joint Inversion

Flexibility to handle:

- multiple surveys / physics
- different model parameterizations
- different simulation mesh for each datum
- separate forward simulation and inversion meshes

$$\text{phi_d} = \text{phi_d_resolve} + \text{phi_d_geotem} + \text{phi_d_nanotem}$$



Example: Bookpurnong

Murray River Floodplain

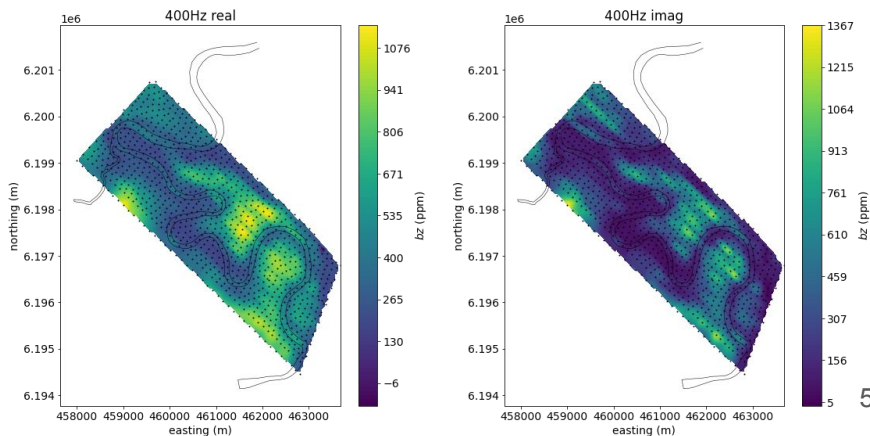
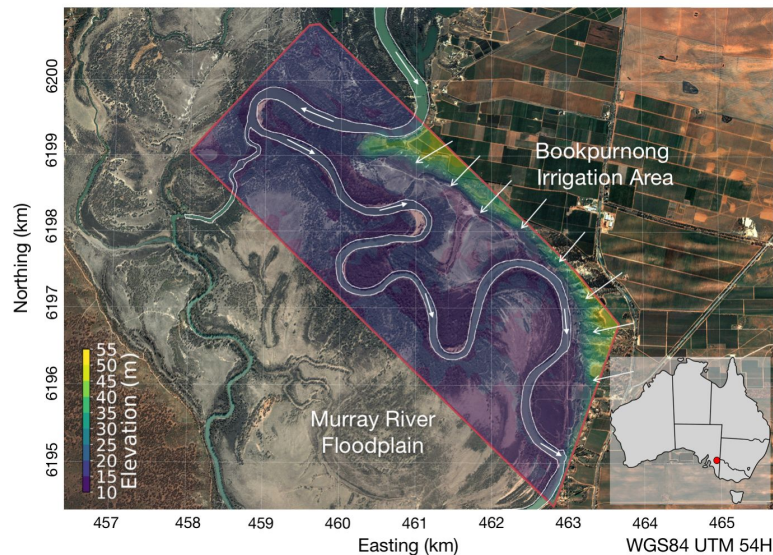
- over-irrigation and drought
- saline water recharges river
- floodplain salinization

Data

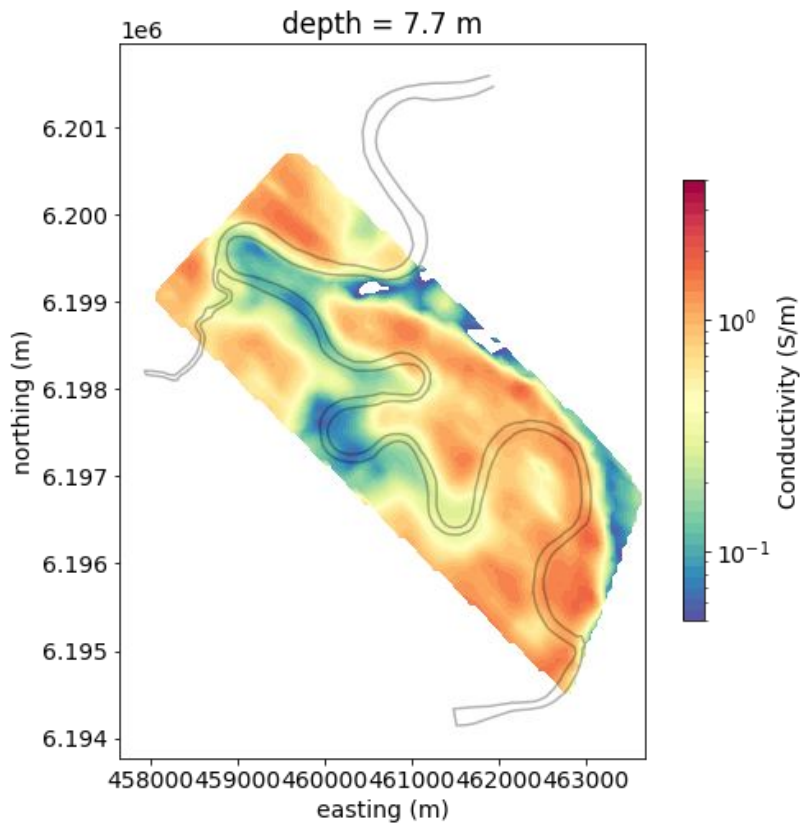
- 2006: SkyTEM (time-domain)
- 2008: RESOLVE (frequency-domain)

Inversion

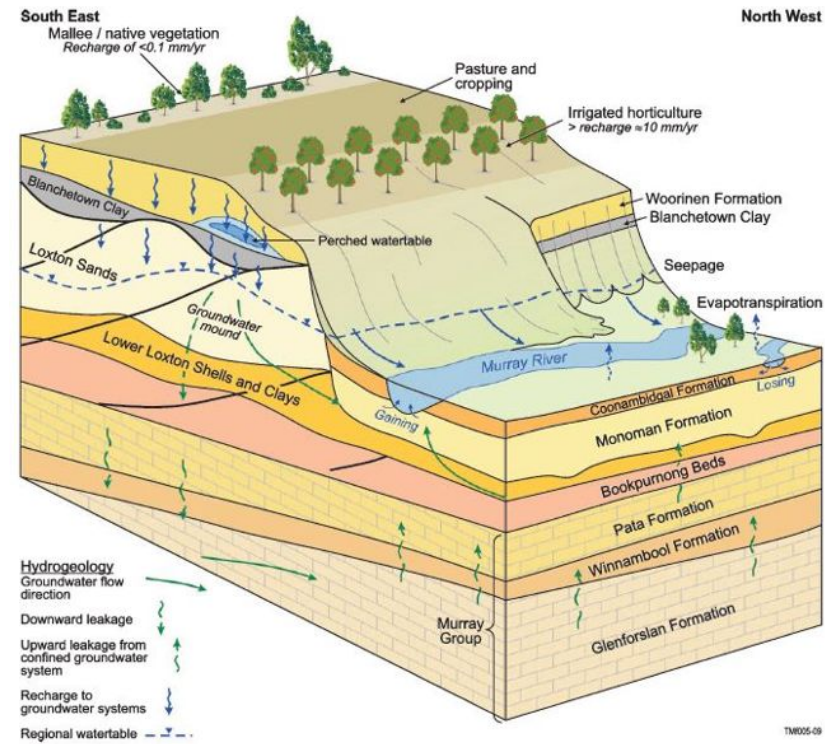
- Spatially constrained 1D



Example: Bookpurnong

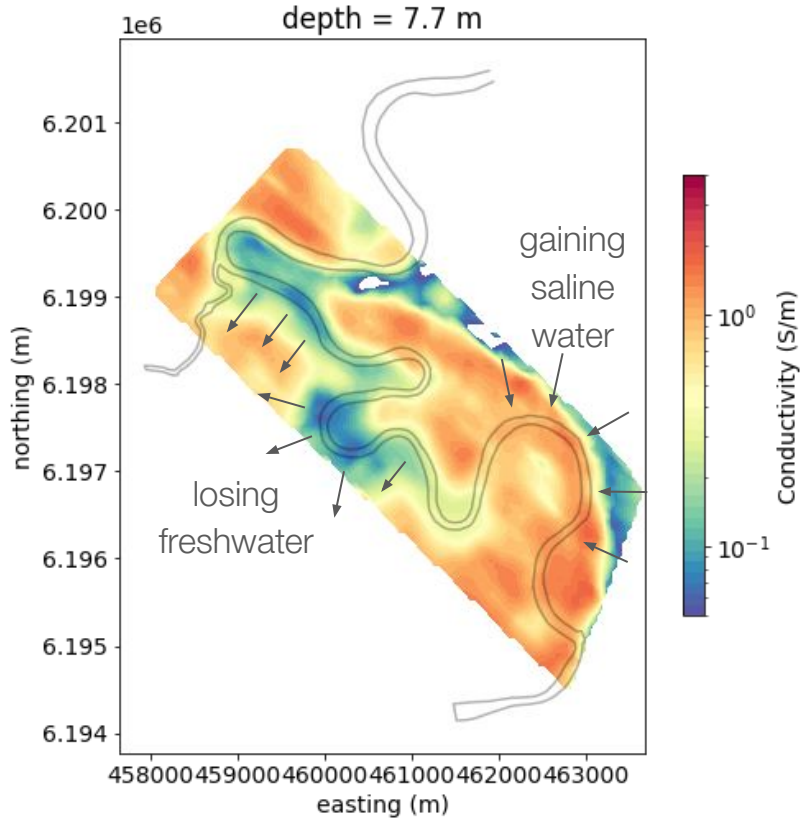


coming to the [docs](#) soon!

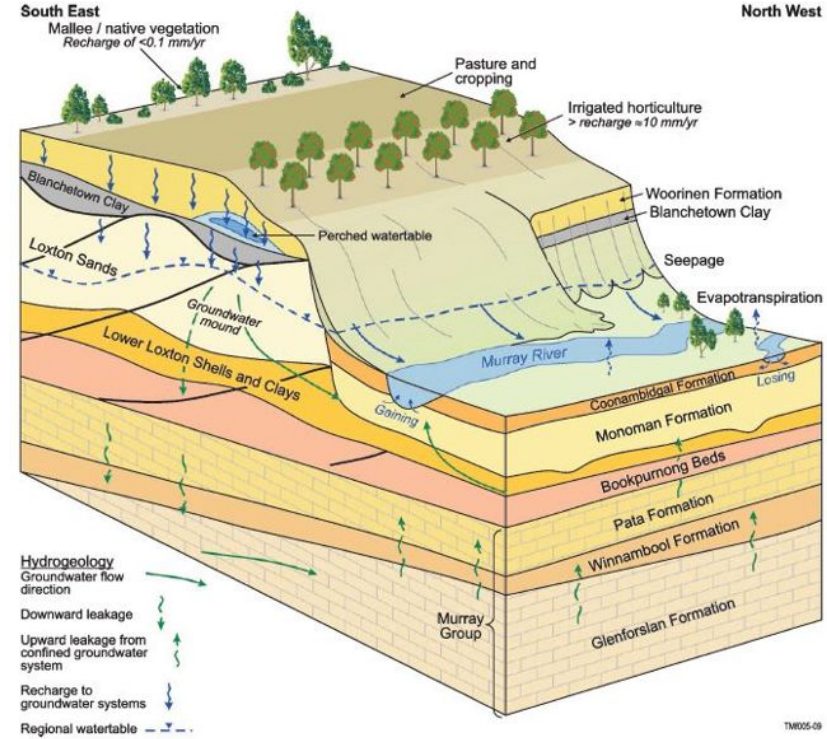


(Viezzoli et al., 2009)

Example: Bookpurnong



coming to the [docs](#) soon!

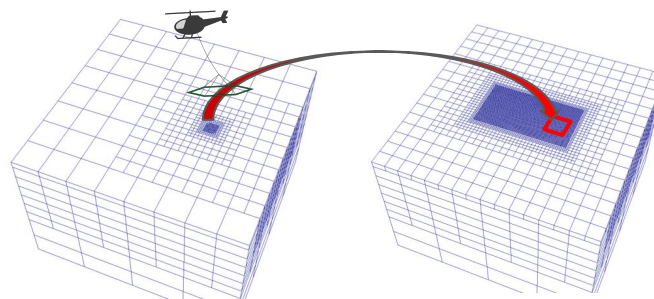
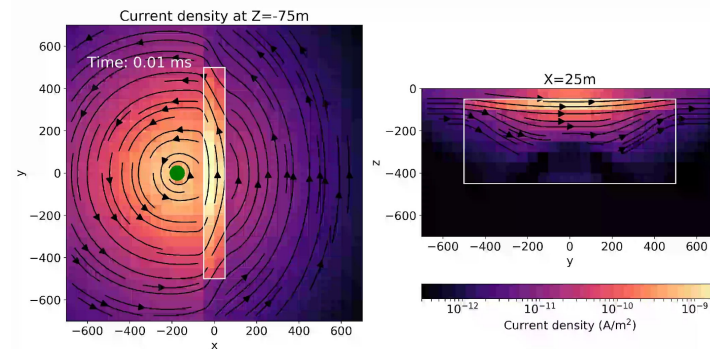


(Viezzoli et al., 2009)

geophysical methods in SimPEG



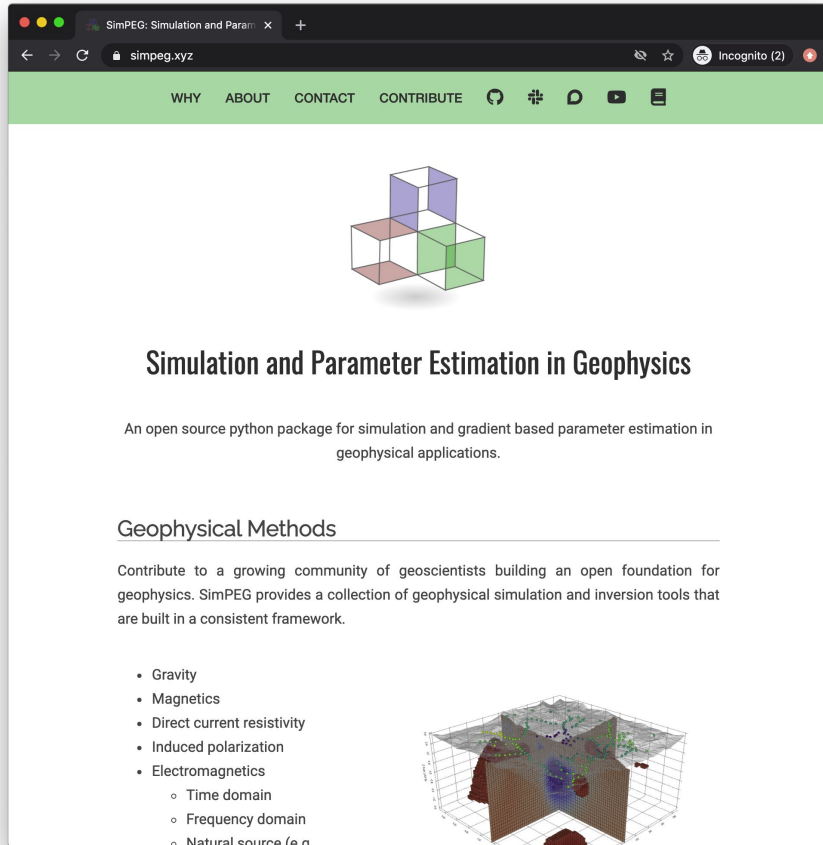
- Gravity
- Magnetics
- Direct current resistivity
- Induced polarization
- Electromagnetics
 - Frequency Domain
 - Time Domain
 - Controlled + natural sources
- Fluid Flow
 - Richards Equation



Dom Fournier

Tiled, parallelized inversions in progress

what is simpeg?



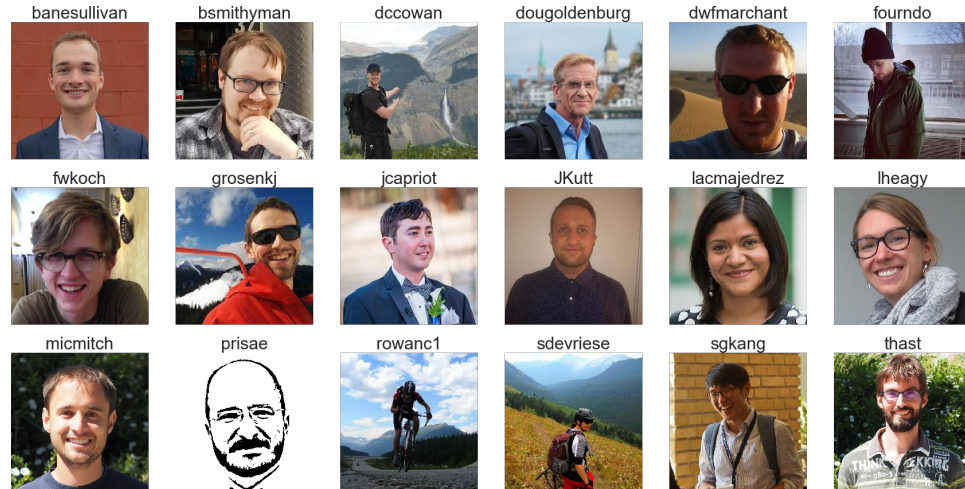
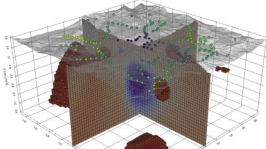
The screenshot shows the website for SimPEG: Simulation and Parameter Estimation in Geophysics. The page features the SimPEG logo, the title "Simulation and Parameter Estimation in Geophysics", and a description: "An open source python package for simulation and gradient based parameter estimation in geophysical applications." Below this, there is a section for "Geophysical Methods" with a list of methods and a 3D visualization of a geological model.

Simulation and Parameter Estimation in Geophysics

An open source python package for simulation and gradient based parameter estimation in geophysical applications.

Geophysical Methods

- Gravity
- Magnetics
- Direct current resistivity
- Induced polarization
- Electromagnetics
 - Time domain
 - Frequency domain
 - Natural source (e.g. lightning)



<https://simpeg.xyz>

code + community

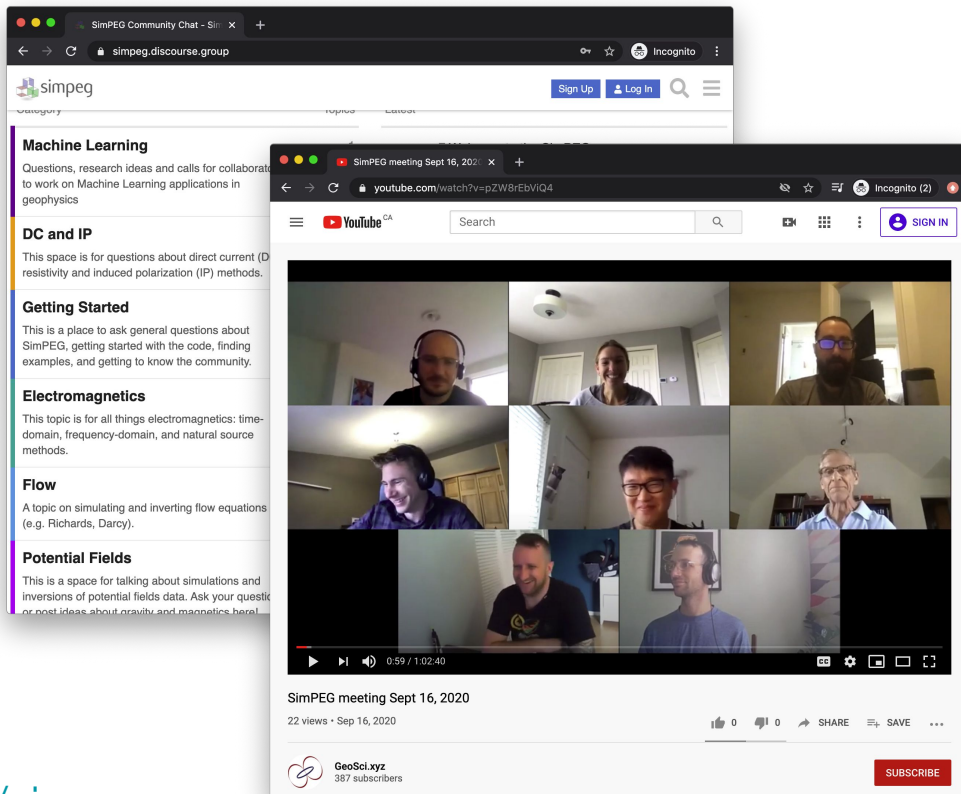


Software practices

- Versioning code
- Tracking issues
- Testing code
- Suggesting changes
- Peer-reviewing changes

Communication

- Weekly meetings (recorded)
- Discourse forum for Q&A
- Chat with slack



<https://simpeg.xyz>

testing

mathematical
properties

analytic
solutions

code
comparisons

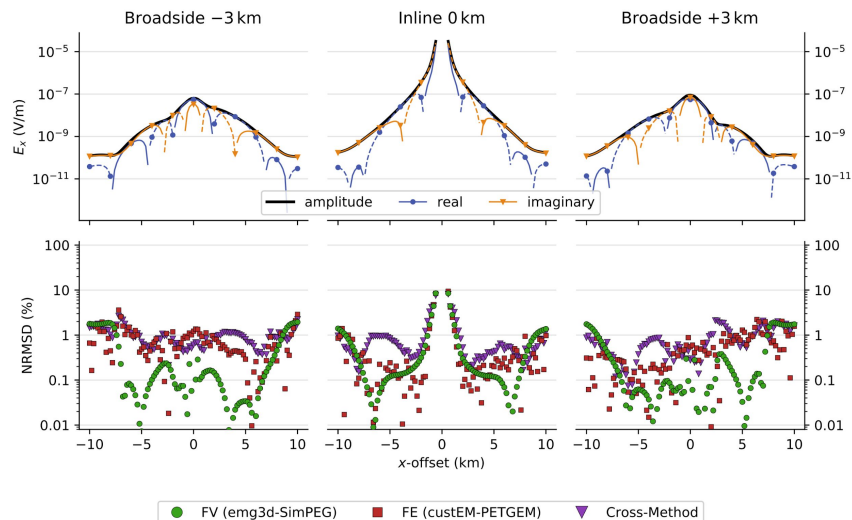
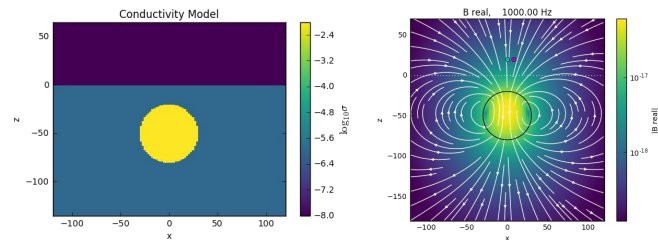
?

confidence

vector identity: $\nabla \cdot \nabla \times \vec{v} = 0$

```
[2]: v = np.random.rand(mesh.nE)  
np.all(mesh.faceDiv * mesh.edgeCurl * v == 0)
```

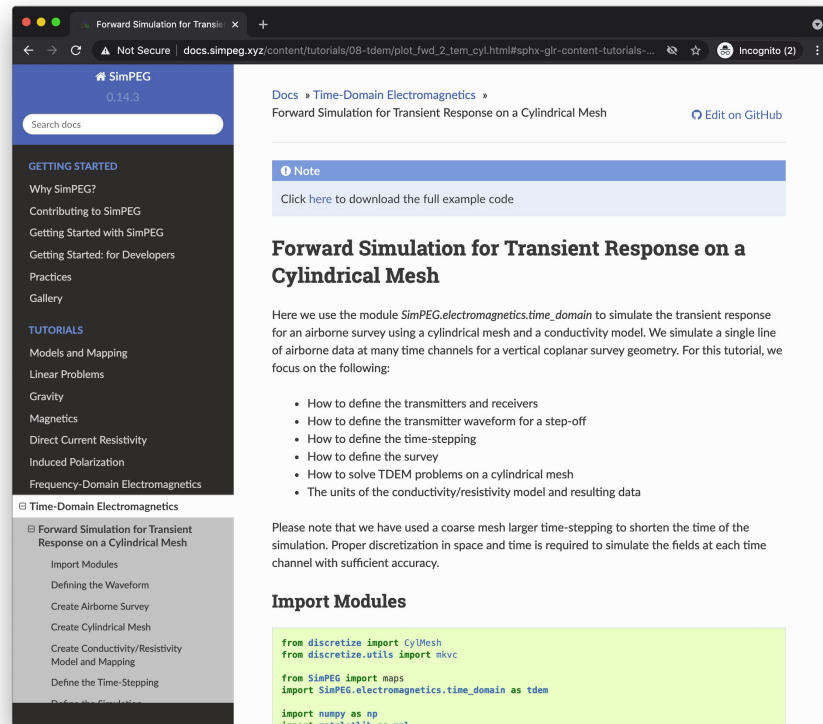
[2]: True



(Werthmüller et al., 2020)

community: connecting + resources

- documentation:
docs.simpeg.xyz
- community forum:
simpeg.discourse.group
- chat:
slack.simpeg.xyz
- meeting notes + recordings:
curvenote.com/@simpeg/meeting-notes



The screenshot shows a web browser displaying the SimPEG documentation page for a tutorial titled "Forward Simulation for Transient Response on a Cylindrical Mesh". The page includes a navigation sidebar on the left with sections like "GETTING STARTED", "TUTORIALS", and "Time-Domain Electromagnetics". The main content area features a "Note" box with a download link, the tutorial title, an introductory paragraph, a bulleted list of topics, and a code block for "Import Modules".

Forward Simulation for Transient Response on a Cylindrical Mesh

Note

Click here to download the full example code

Forward Simulation for Transient Response on a Cylindrical Mesh

Here we use the module `SimPEG.electromagnetics.time_domain` to simulate the transient response for an airborne survey using a cylindrical mesh and a conductivity model. We simulate a single line of airborne data at many time channels for a vertical coplanar survey geometry. For this tutorial, we focus on the following:

- How to define the transmitters and receivers
- How to define the transmitter waveform for a step-off
- How to define the time-stepping
- How to define the survey
- How to solve TDEM problems on a cylindrical mesh
- The units of the conductivity/resistivity model and resulting data

Please note that we have used a coarse mesh larger time-stepping to shorten the time of the simulation. Proper discretization in space and time is required to simulate the fields at each time channel with sufficient accuracy.

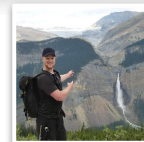
Import Modules

```
from discretize import CyMesh
from discretize.utils import mkvc

from SimPEG import maps
import SimPEG.electromagnetics.time_domain as tdem

import numpy as np
import matplotlib as mpl
```

Devin
Cowan

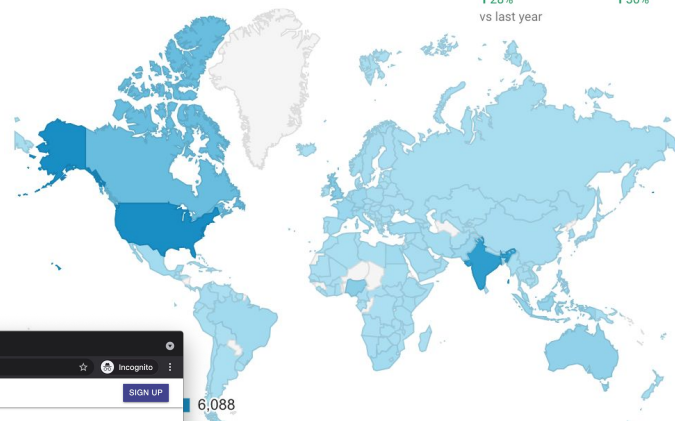
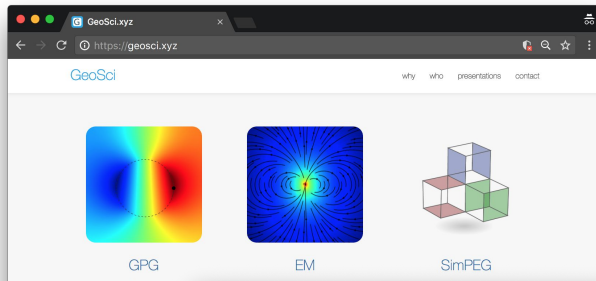


GeoSci.xyz

<https://geosci.xyz>

Users
30K
↑ 28%
vs last year

Sessions
48K
↑ 30%



DISC 2017
Geophysical Electromagnetics: Fundamentals & Applications

Linear Tikhonov Inversion

AUTHOR: Douglas Oldenburg DATE: Jan 18, 2021

In this chapter we present the basic elements for how an inverse problem can be formulated and solved using optimization theory. The quantity to be minimized is a weighted sum of misfit and regularization terms with their relative importance controlled by an adjustable Tikhonov parameter.

The inverse problem has many elements and a solution is best achieved by adhering to the workflow shown below. Throughout this chapter we investigate each of these steps and illustrate the concepts with a simple linear problem. Jupyter notebooks are provided so that the concepts can be explored and all figures can be reproduced. The formative material for this chapter is extracted from the tutorial paper by Oldenburg and Li (Oldenburg & Li 2005).

Inputs

- Field observations & error estimates
- Ability to forward model
- Prior knowledge (Built reference model)

Define inversion model parameters

Choose a method

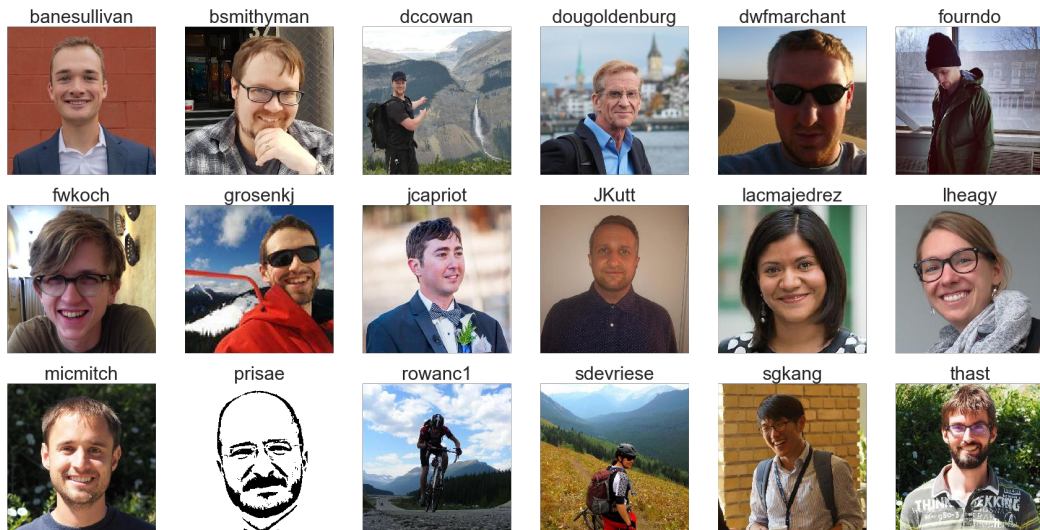
Define model



26 locations worldwide

curvenote.com/@geosci/inversion-module

thank you!



simpeg.xyz



geosci.xyz



curvenote.com/@geosci



lhagy@eoas.ubc.ca



Alan Jones



Max Moorkamp

references

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- Viezzoli, A., Munday, T., Auken, E., & Christiansen, A. V. (2010). Accurate quasi 3D versus practical full 3D inversion of AEM data the Bookpurnong case study. *Preview*, 2010(149), 23–31. <https://doi.org/10.1071/PVv2010n149> p23
- Werthmüller, D. (2017). An open-source full 3D electromagnetic modeler for 1D VTI media in Python: empymod. *GEOPHYSICS*, 82(6), WB9–WB19. <https://doi.org/10.1190/geo2016-0626.1>
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