MARE2DEM: An open-source code for 2D inversion of MT, CSEM, DC resistivity and borehole EM data

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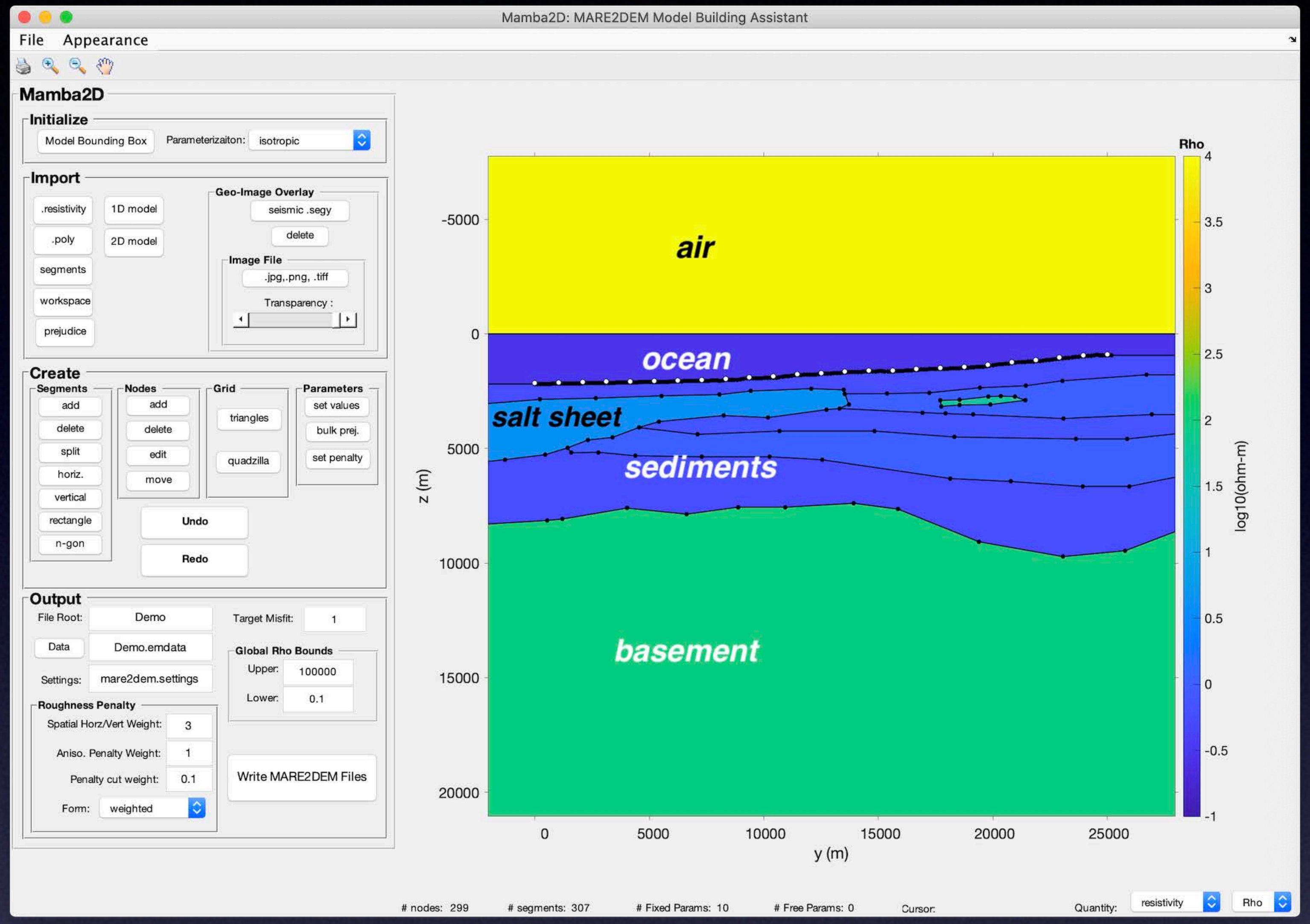
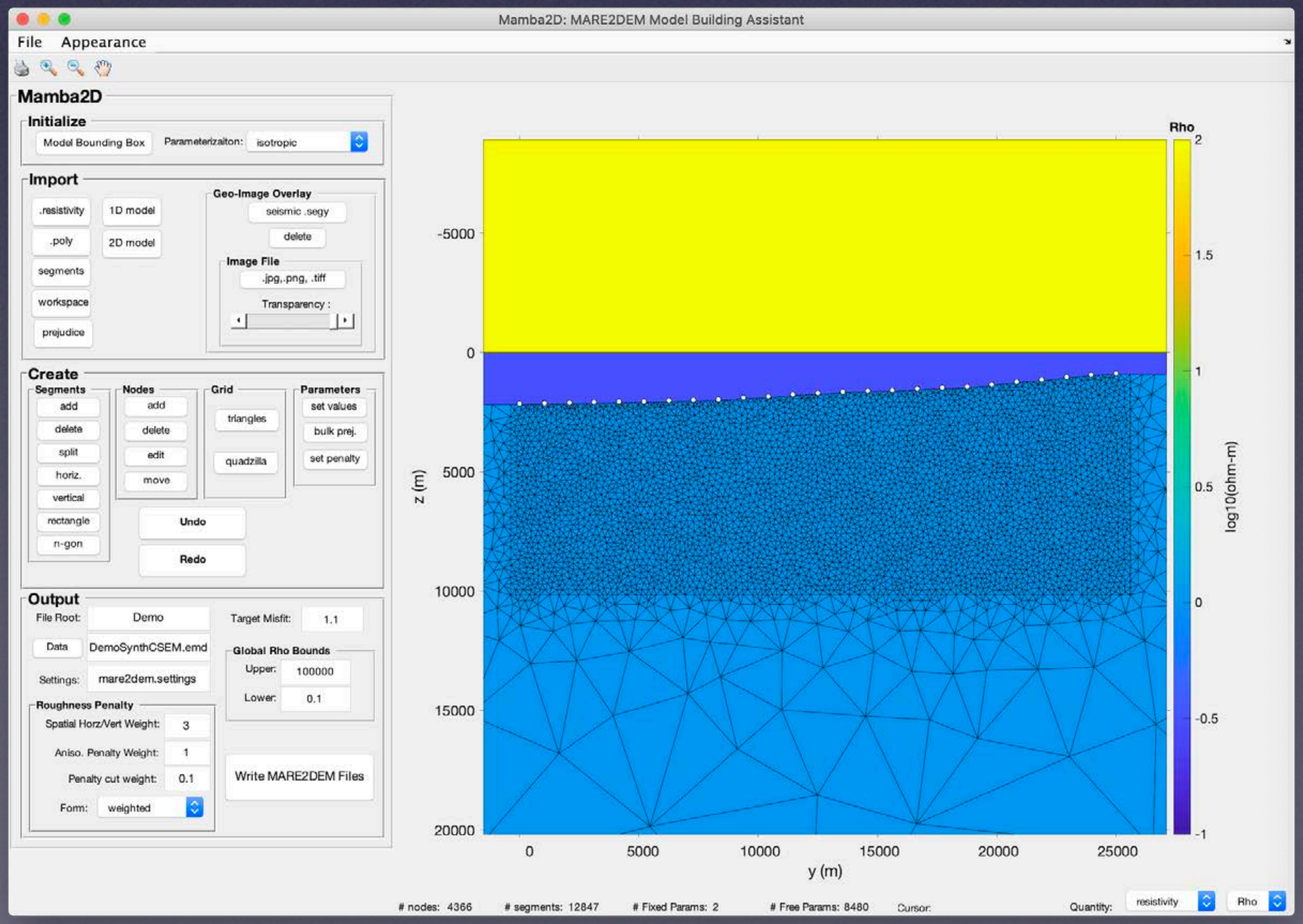
http://mare2dem.bitbucket.io

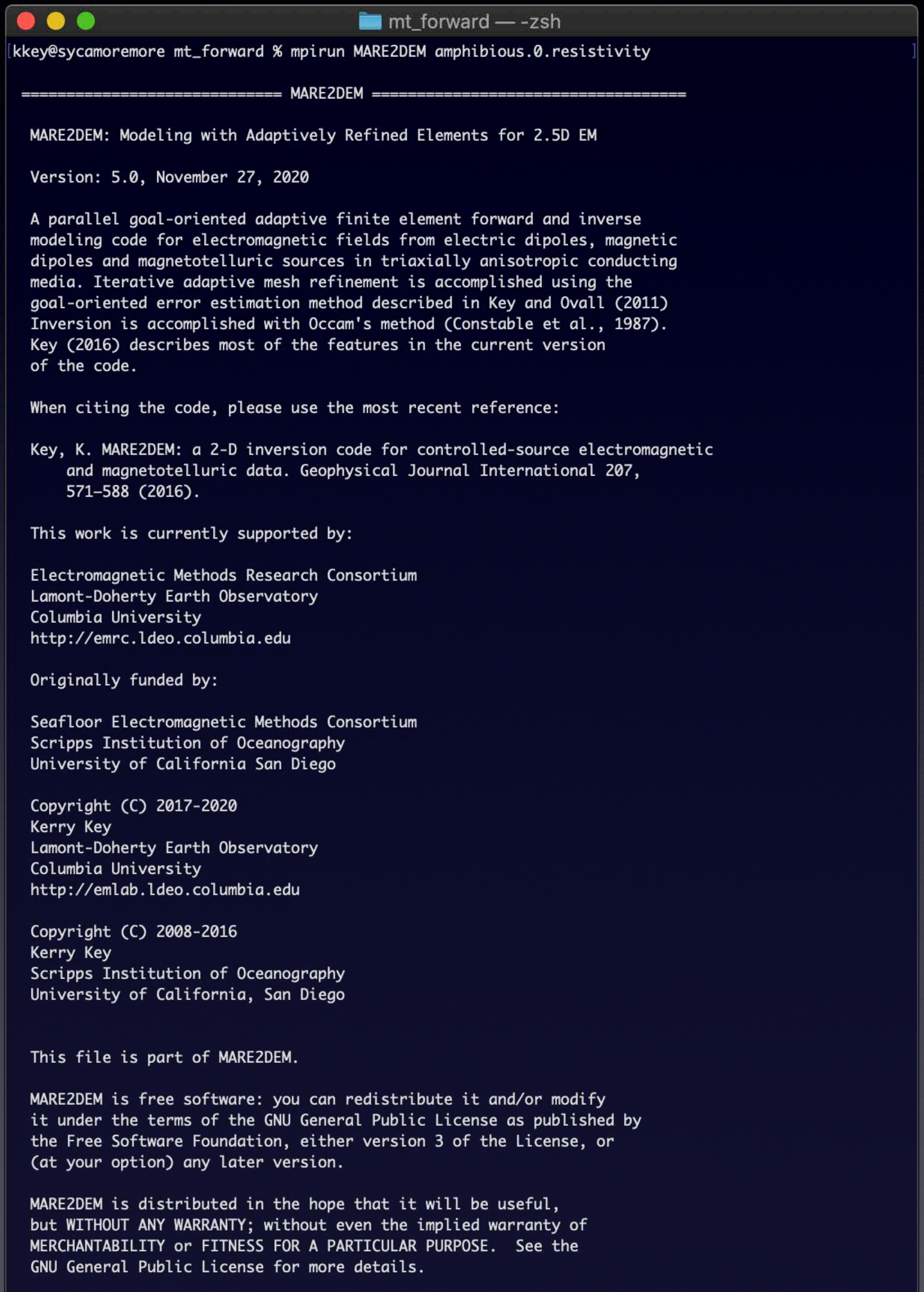
What is MARE2DEM?

- A MATLAB graphical user interface for creating forward models, inversion grids and routines for making data files

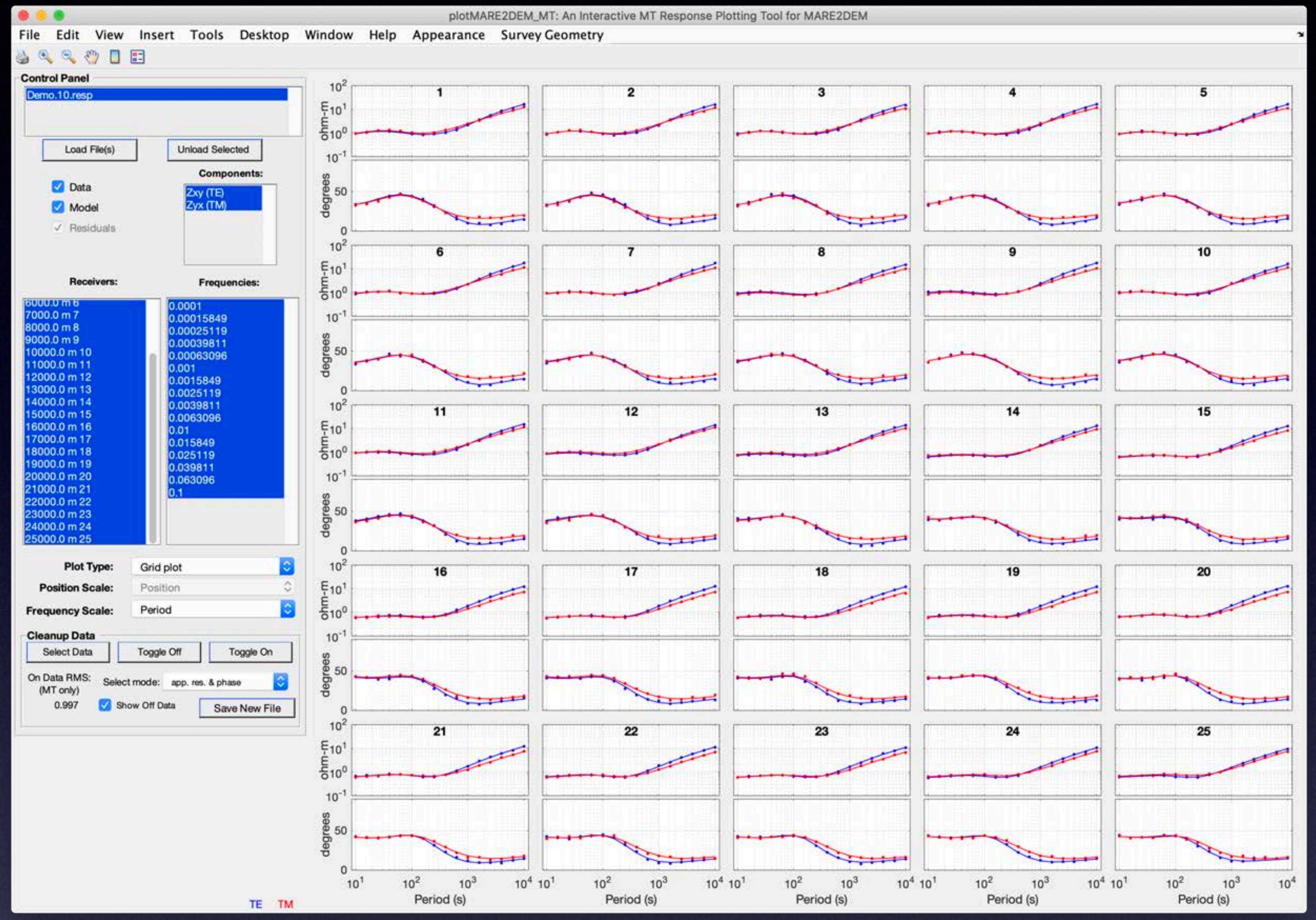


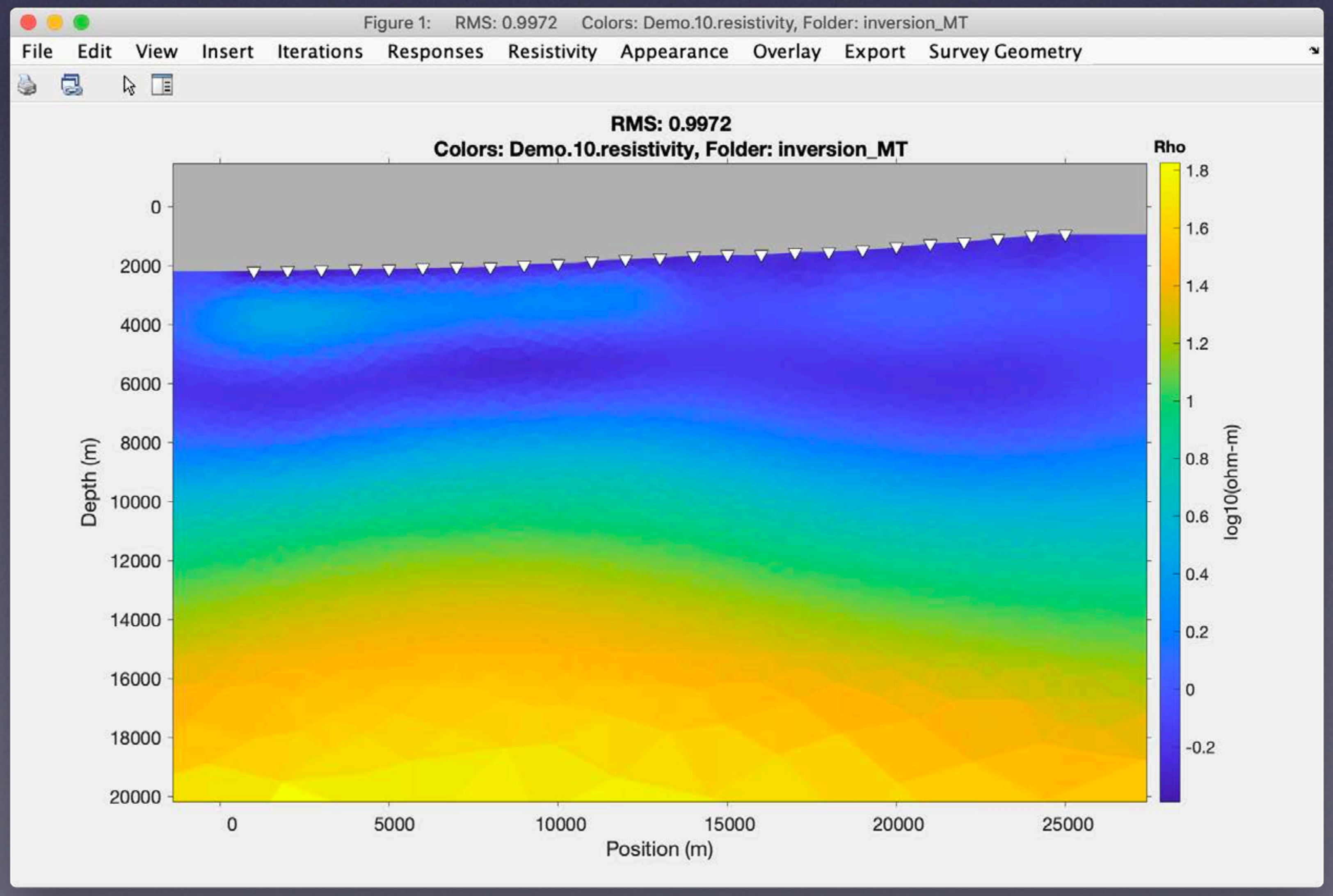


- An MPI-Fortran-C code that can be run on laptops, desktops and cluster computers for computing EM forward responses and running inversions



- A MATLAB graphical user interface for viewing responses, data fits and inversion models

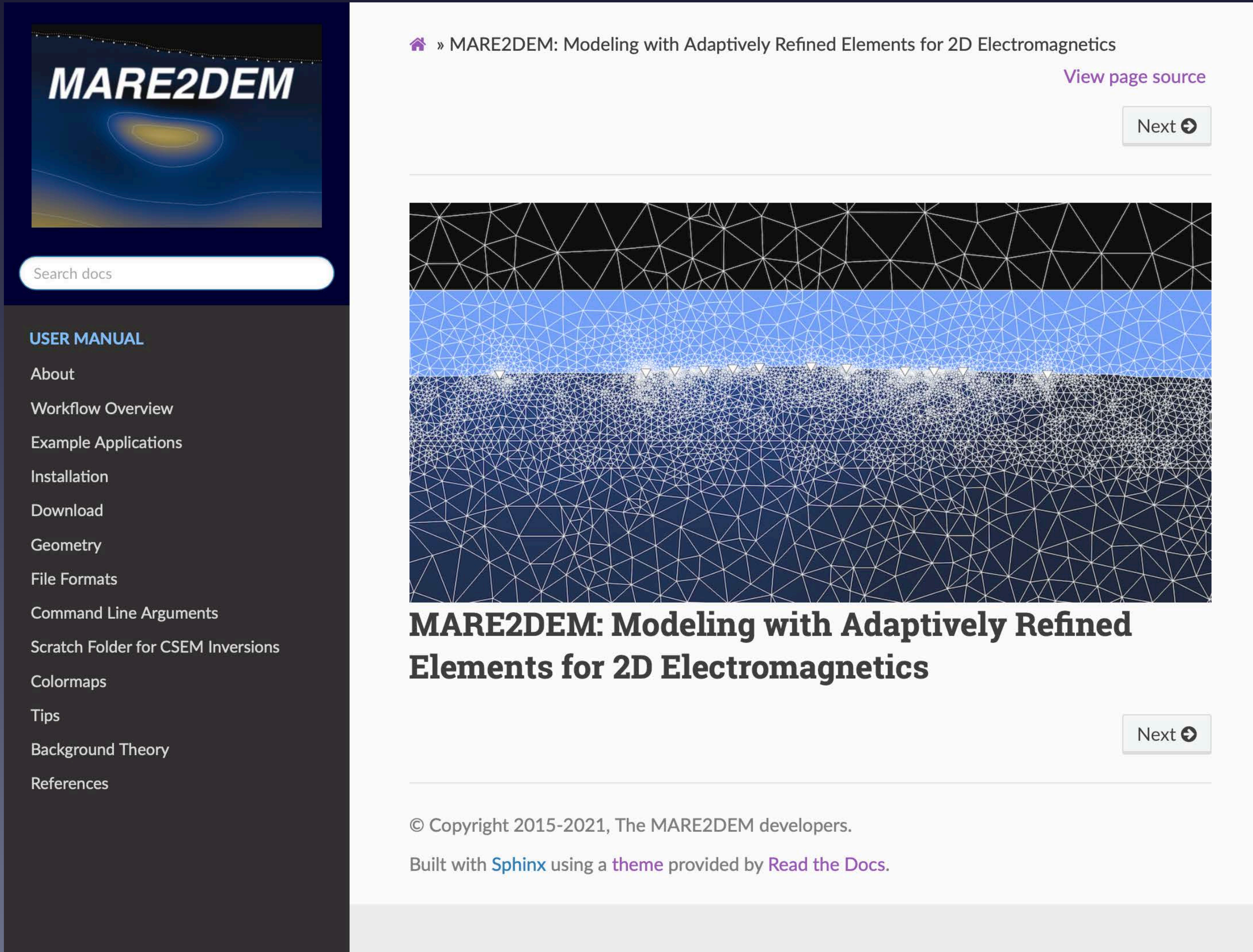




MARE2DEM: Modeling with Adaptively Refined Elements for 2D Electromagnetics

http://mare2dem.bitbucket.io

- EM Methods : CSEM, MT, DC resistivity
- Applications: land, marine, amphibious, polar, borehole, crosswell, land-air, EM physics studies
- Conductivity: isotropic, transversely isotropic (X,Y or Z), triaxial, complex, Cole-Cole IP
- System requirements:
  - User interface: MATLAB
  - MARE2DEM code: Unix based operating system with Intel Fortran & C compilers (now freely available), MPI compiler.
- Freely available under GNU GPLv3 License.
- Developed under industry sponsorship from *Electromagnetic Methods Research Consortium* at Columbia University and previously the *Seafloor Electromagnetic Methods Consortium* at Scripps Institution of Oceanography.



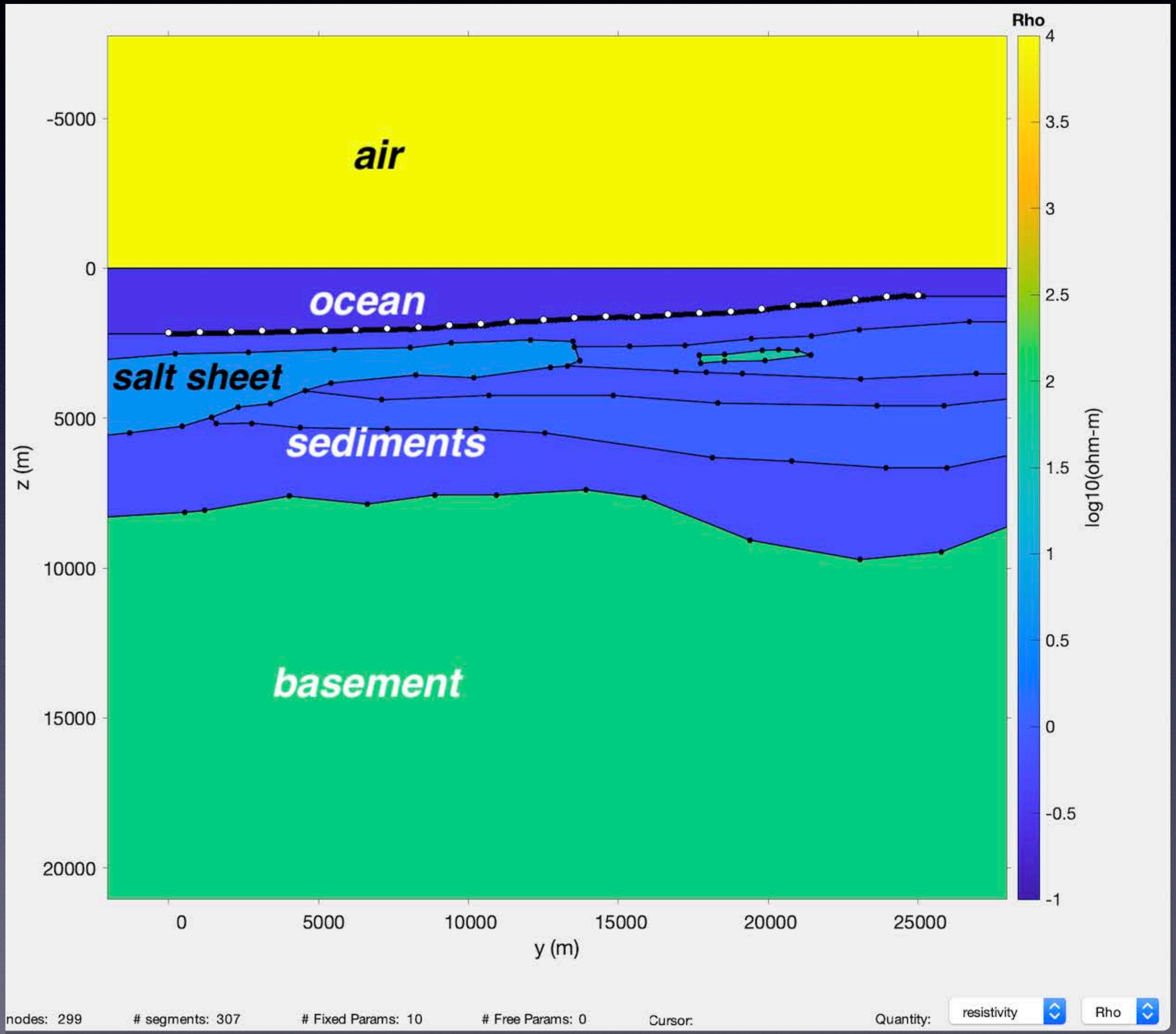
Model Parameterization

- Parameters are defined using polygons of piecewise constant conductivity

- Polygons are closed regions defined by nodes connected by segments

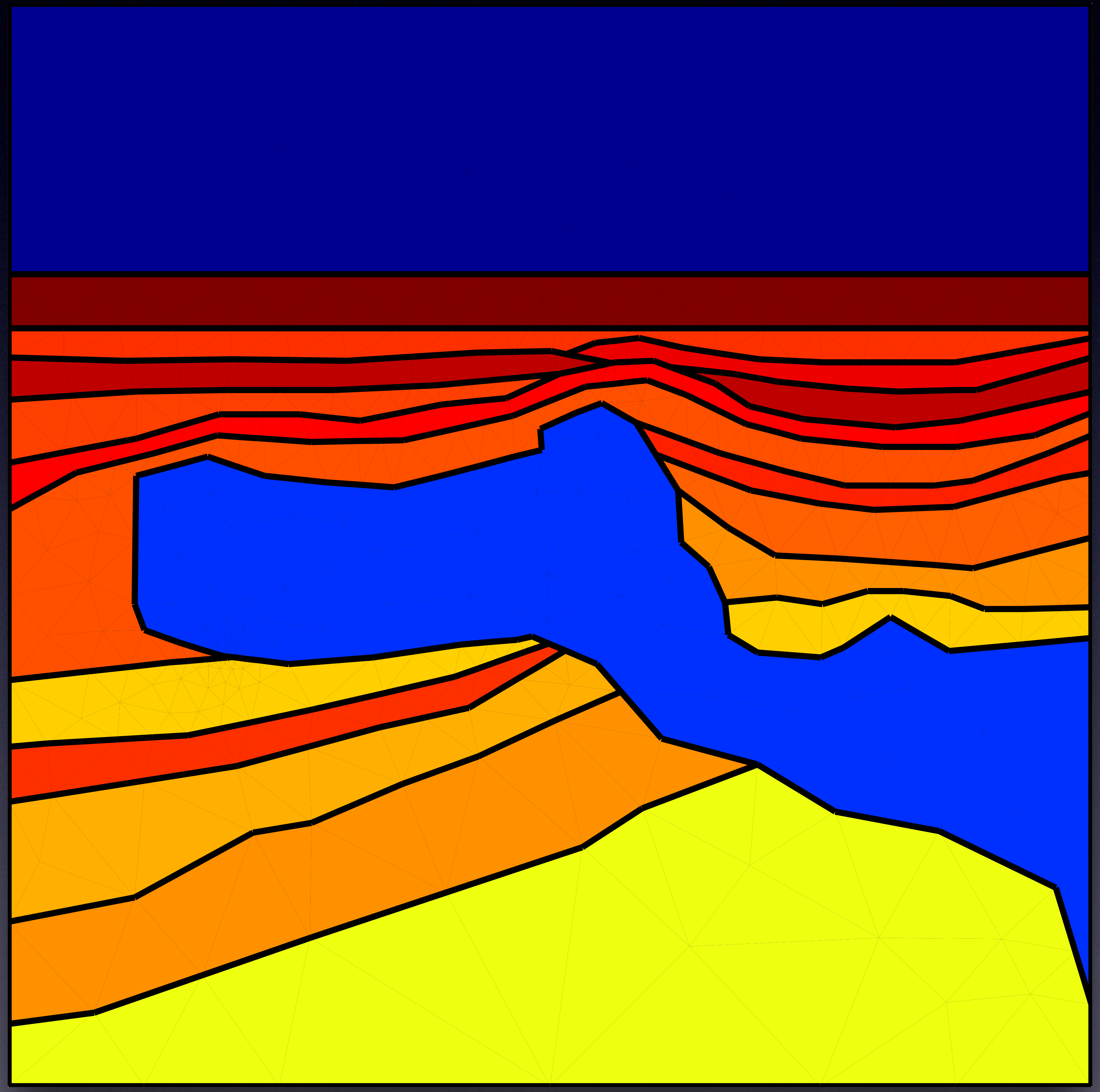
- Highly flexible for efficiently handling complicated topography and other geologic surfaces

- No need to be a meshing expert. Just create the model polygons and MARE2DEM will handle the finite element meshing internally.

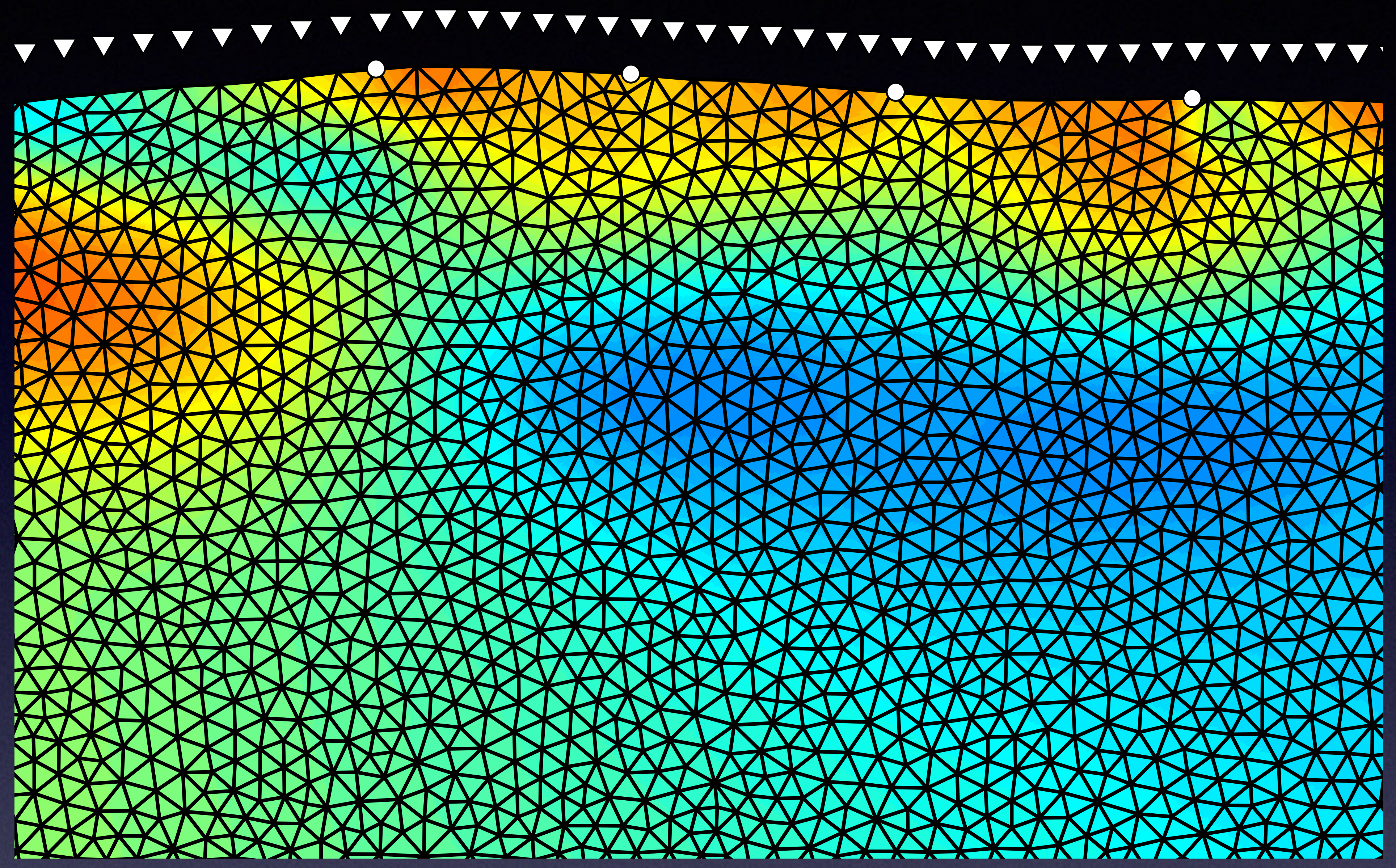


Flexible Parameterization

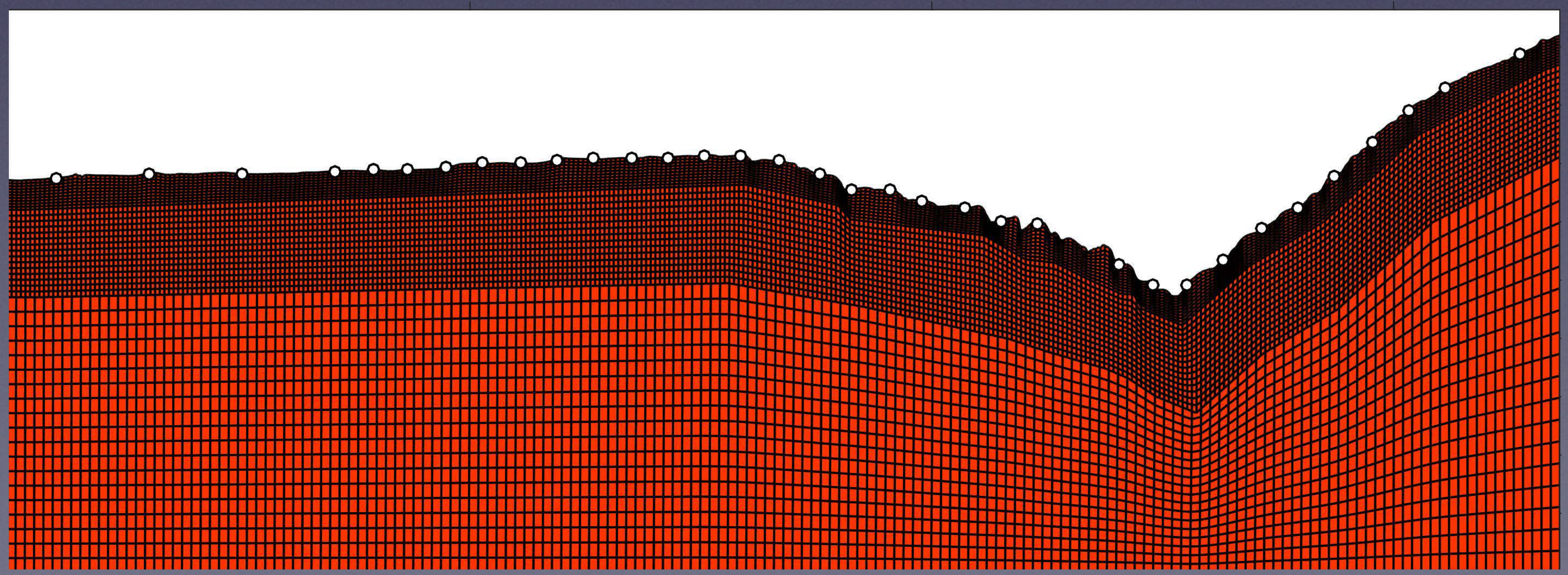
Arbitrary polygons



Triangles



Quadrilaterals



Governing Equations for MT

TM Mode: $$\nabla \sigma_t^{-1} \nabla H_x + i\omega\mu H_x = 0$$

TE Mode: $$\nabla \cdot \nabla E_x + i\omega\mu\sigma_x E_x = 0$$

$$\bar{\bar{\sigma}} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} \qquad \sigma_t = \begin{pmatrix} \sigma_y & 0 \\ 0 & \sigma_z \end{pmatrix}$$

1D boundary conditions applied to model side boundaries. Unit downward component of magnetic source field applied at top of model domain.

Auxiliary *y* and *z* electric and magnetic fields found from spatial derivatives of *x* fields via Faraday's and Ampere's equations.

MARE2DEM has option for scattered-field MT solution, but total field solution is often faster and just as accurate.

Governing Equations for 2.5D EM

(3D sources in 2D conductivity)

$$-\nabla \cdot (A \nabla \mathbf{u}) + C\mathbf{u} \quad = \quad \mathbf{f}$$

where $\quad \mathbf{u} = (\hat{E}_x, \hat{H}_x)$

The details:

$$A \quad = \quad \begin{pmatrix} \lambda\sigma_t & ik_x\lambda R \\ ik_x R\lambda & i\omega\mu\lambda' \end{pmatrix}, \qquad C = \begin{pmatrix} \sigma_x & 0 \\ 0 & i\omega\mu \end{pmatrix},$$

where

$$R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \;\; \sigma_t = \begin{pmatrix} \sigma_y & 0 \\ 0 & \sigma_z \end{pmatrix}, \;\; \lambda^{-1} = \begin{pmatrix} k_x^2 - i\omega\mu\sigma_y & 0 \\ 0 & k_x^2 - i\omega\mu\sigma_z \end{pmatrix}, \quad \lambda' = R^T \lambda R.$$

$$\mathbf{f} \quad = \quad \nabla \cdot (AQ^T \mathbf{s}_t) - \mathbf{s}_x$$

where

$$AQ^T = \begin{pmatrix} ik_x\lambda & -\sigma_t\lambda R \\ -i\omega\mu R\lambda & ik_x\lambda' \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & R \\ R & 0 \end{pmatrix}, \quad \mathbf{s}_t = (\hat{\mathbf{J}}_t^s, \hat{\mathbf{M}}_t^s), \quad \mathbf{s}_x = (\hat{J}_x^s, \hat{M}_x^s).$$

Solve for wavenumber domain E_x and H_x over a spectrum of wavenumbers (k_x) and then inverse Fourier transform to get spatial domain fields.

Adaptive Finite Element Method

Asymptotically exact solution through iterative mesh refinement:

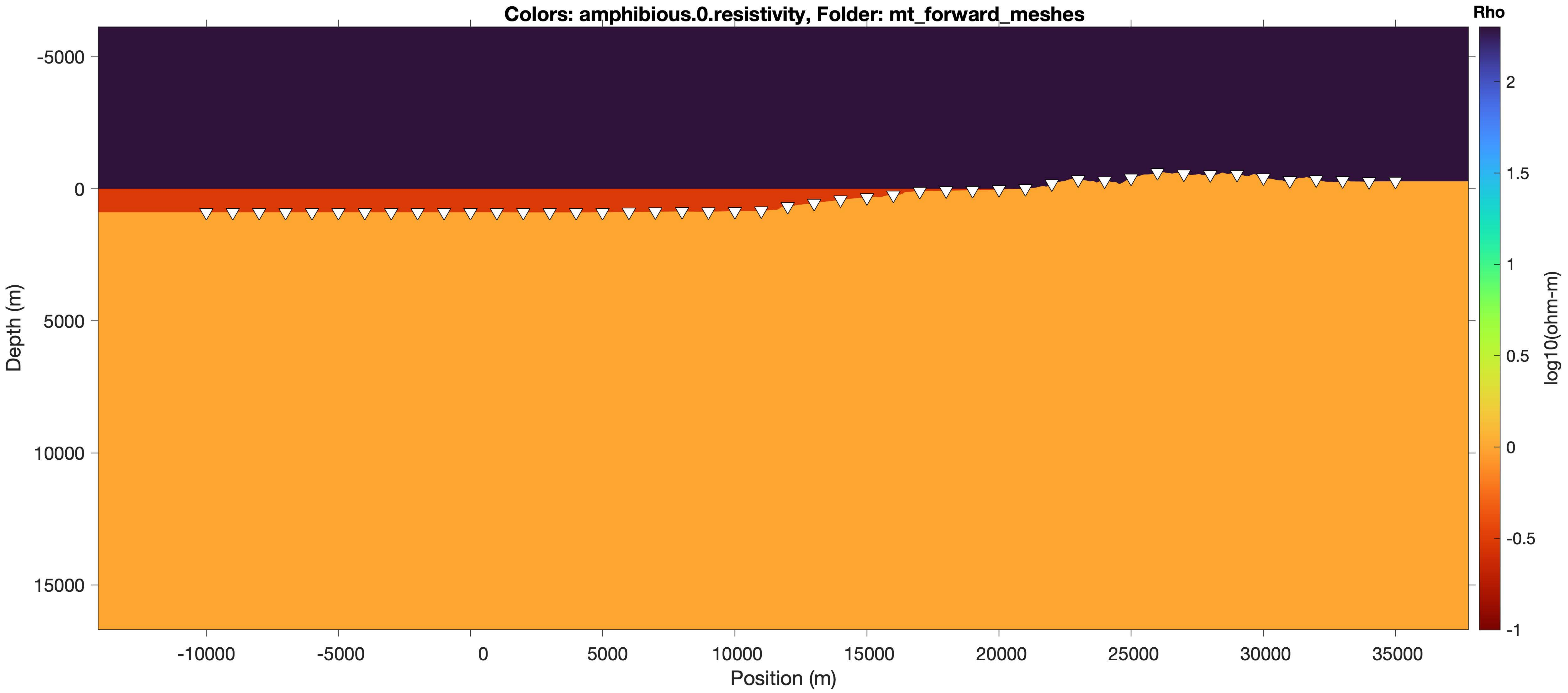
0. **Local a priori refinement** around receivers and transmitters

1. **Solve** governing PDE on finite element mesh
   - MARE2DEM uses unstructured linear triangular finite elements
2. **Estimate** error for each mesh element
   - MARE2DEM uses a goal-oriented error estimator designed to reduce *relative* error at each receiver. Requires adjoint solution.
3. **Refine** mesh
   - Select fraction of elements with large error and refine them.

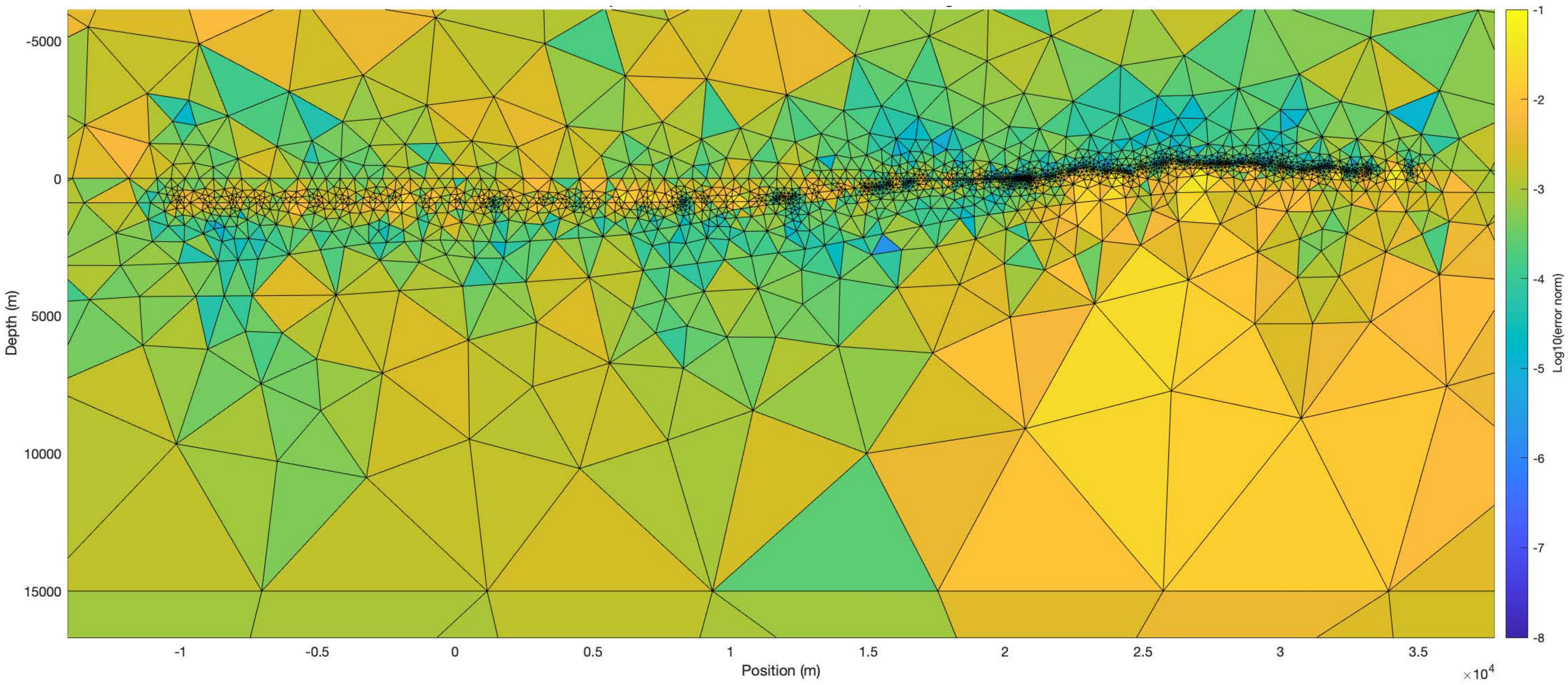
Iterate 1–3 until solution converges to user specified tolerance (usually 1%).

Details in Key and Ovall (2011), Key (2016)

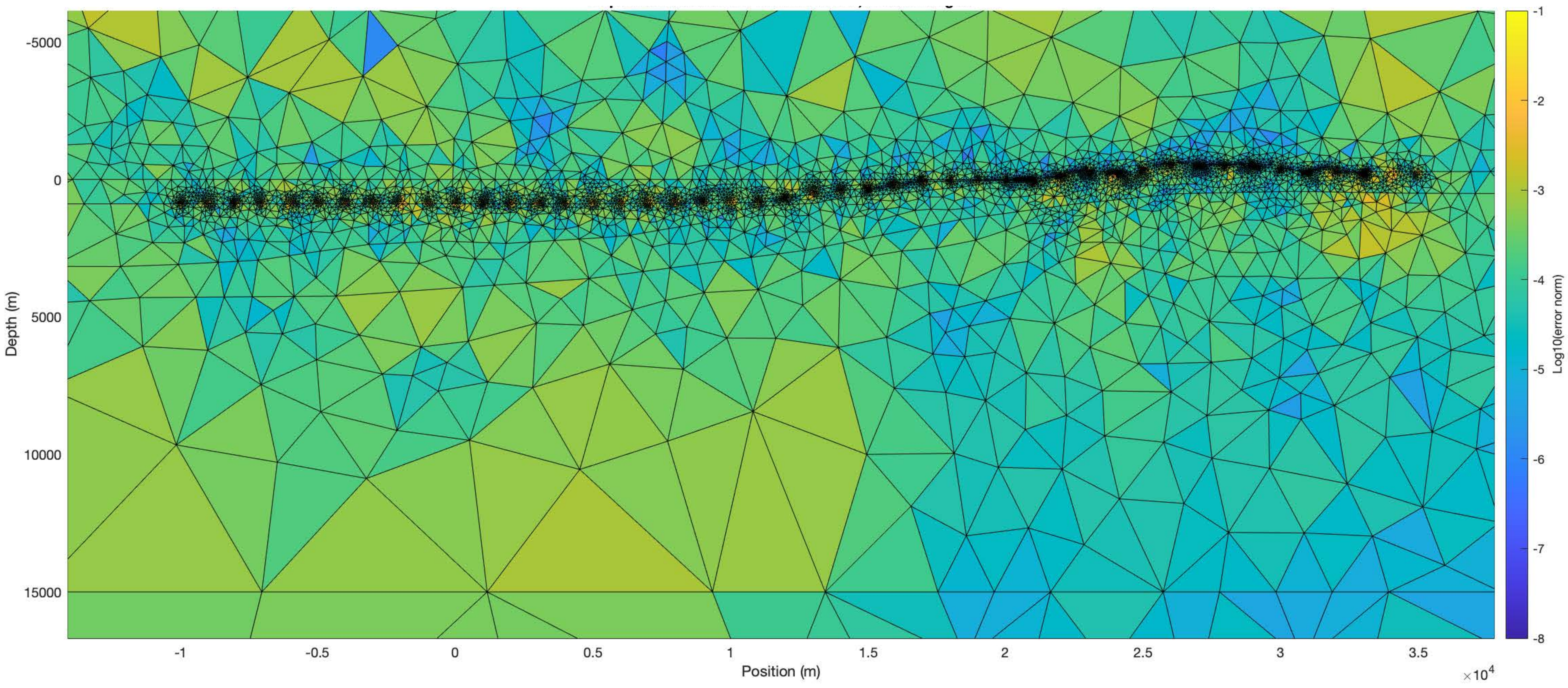
Example of Goal-Oriented Adaptive Mesh Refinement



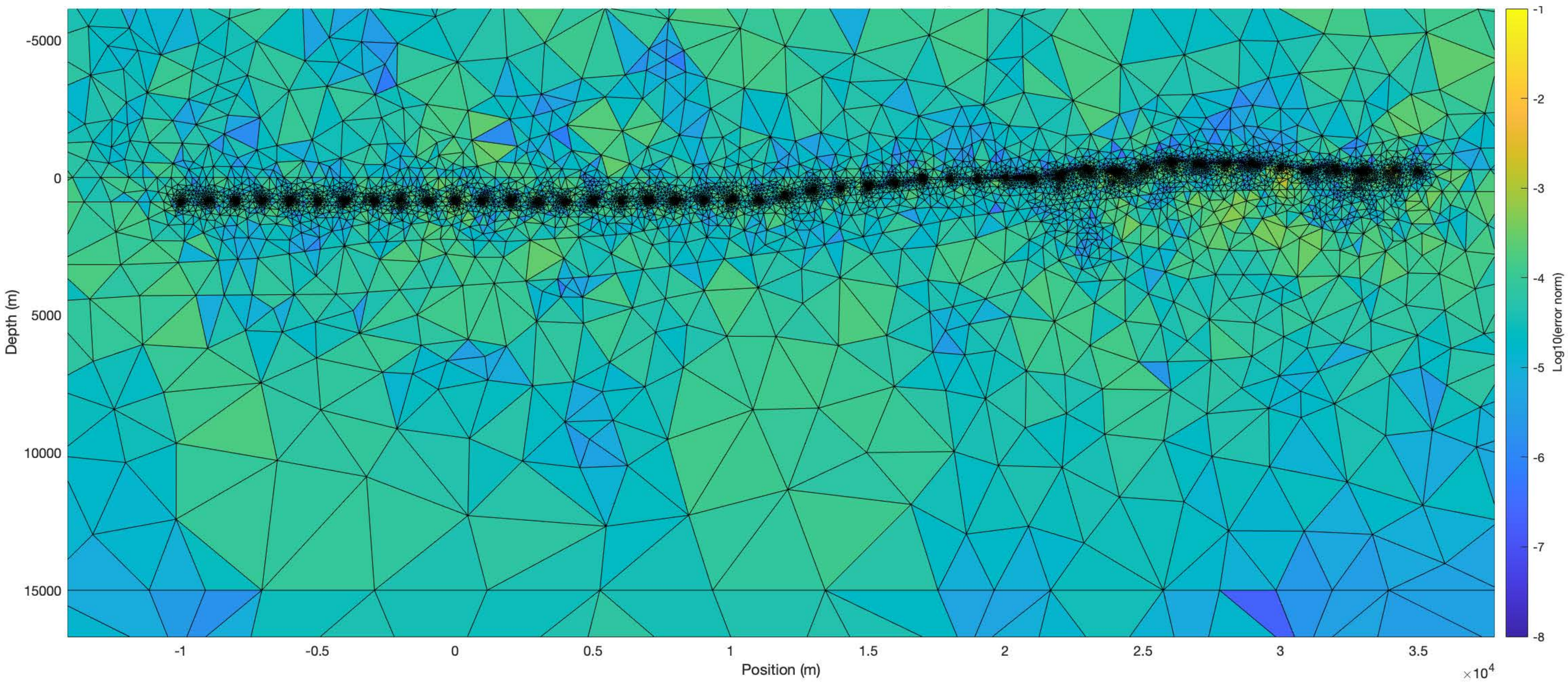
Mesh 1 and Error Estimate



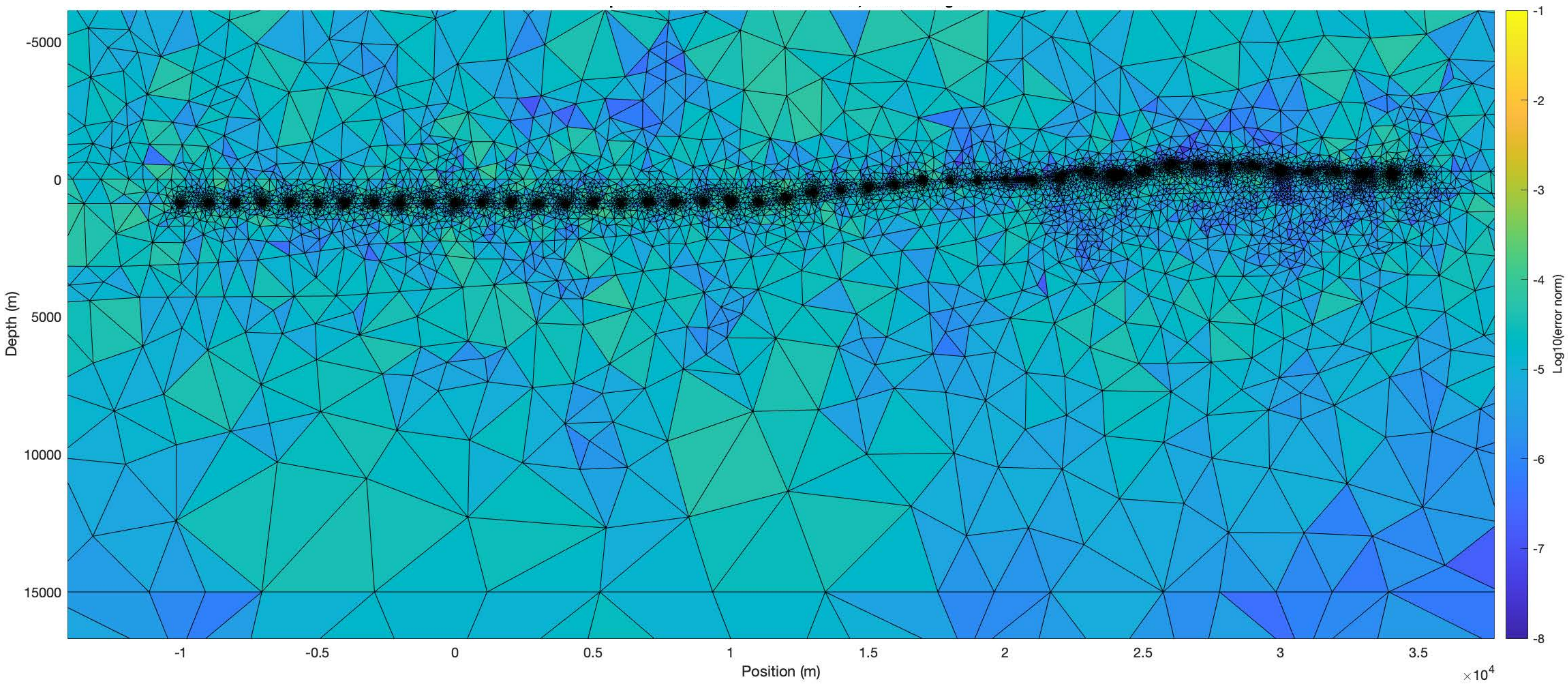
Mesh 2 and Error Estimate



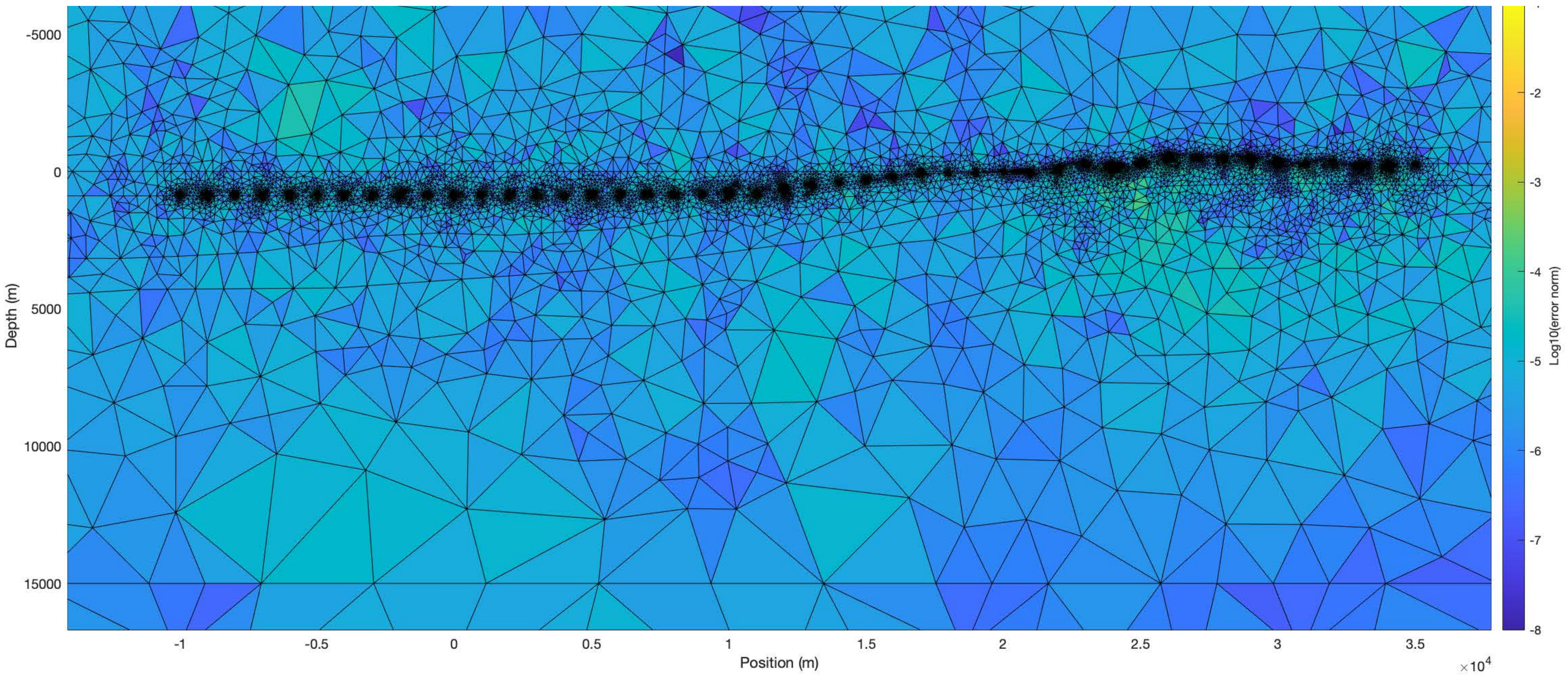
Mesh 3 and Error Estimate



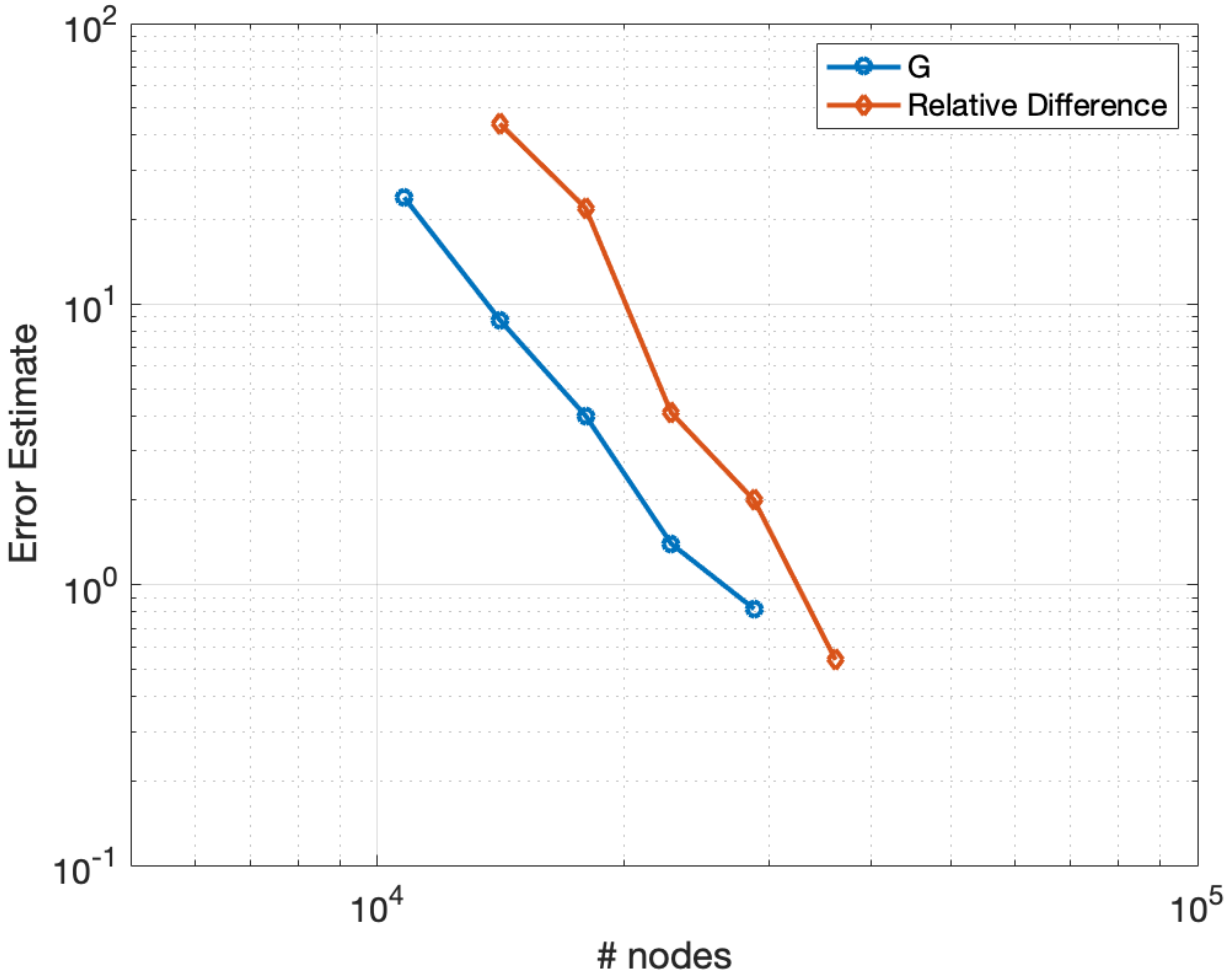
Mesh 4 and Error Estimate



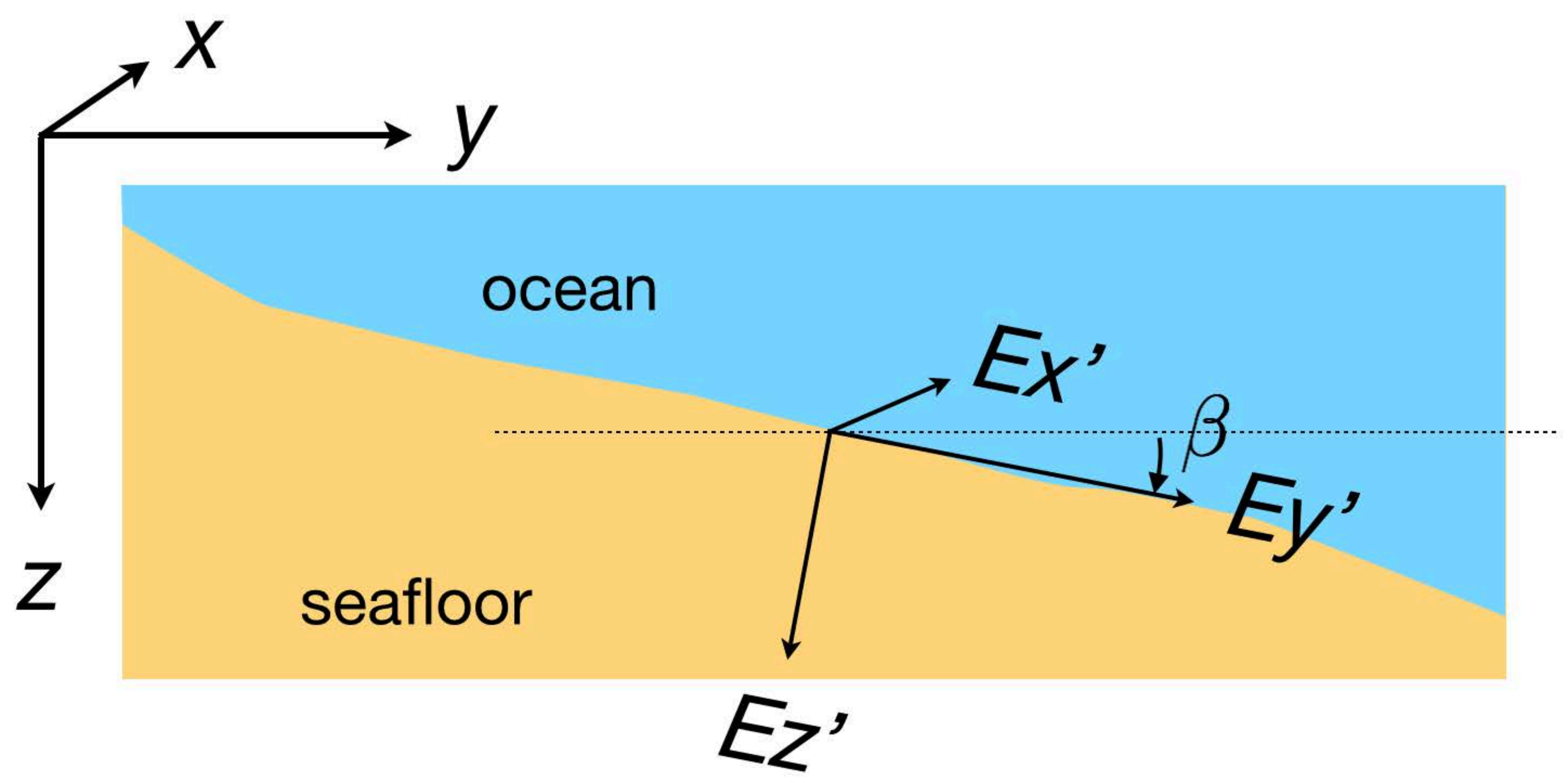
Mesh 5 and Error Estimate



Convergence of forward solution

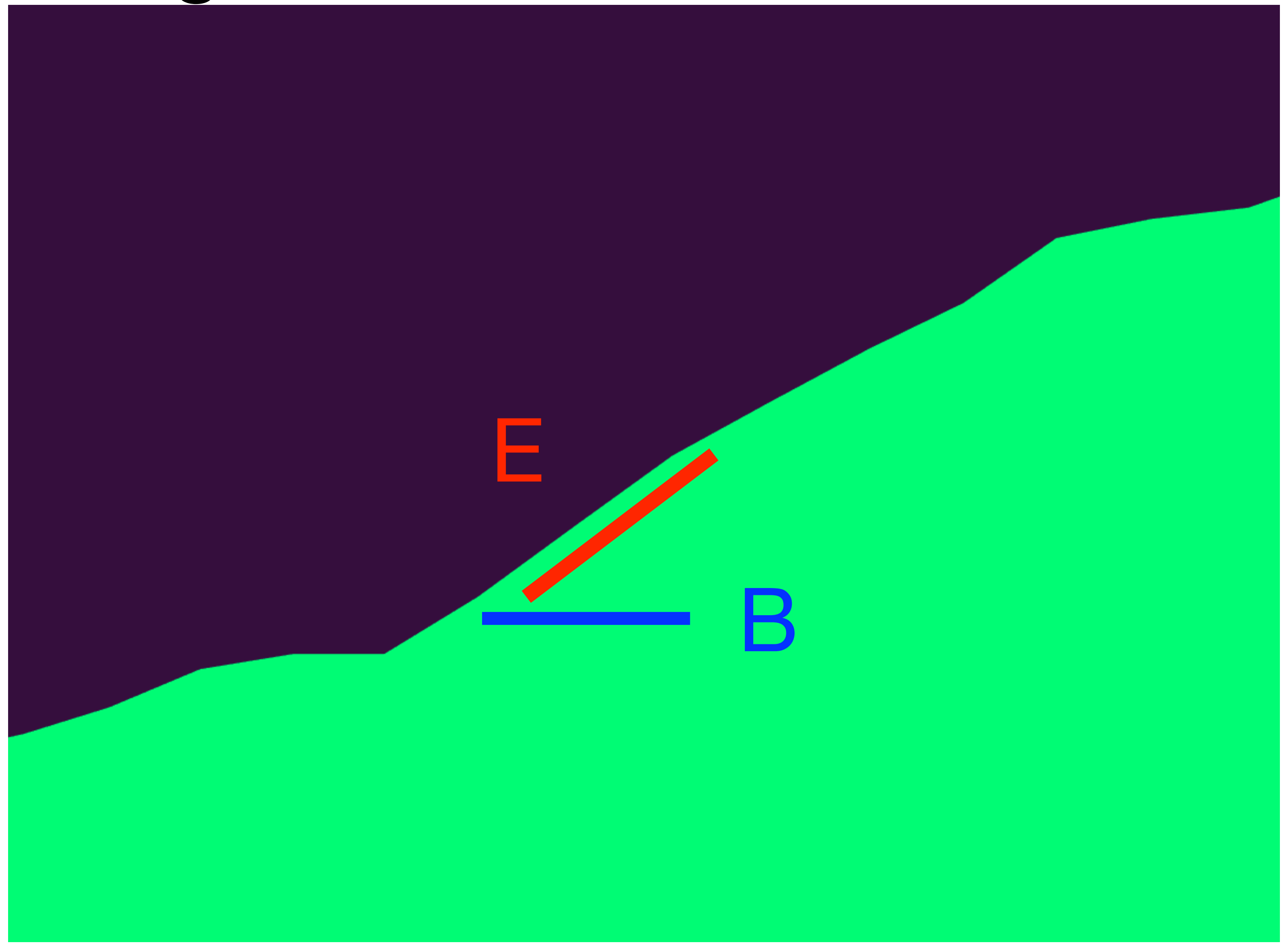


Geometry



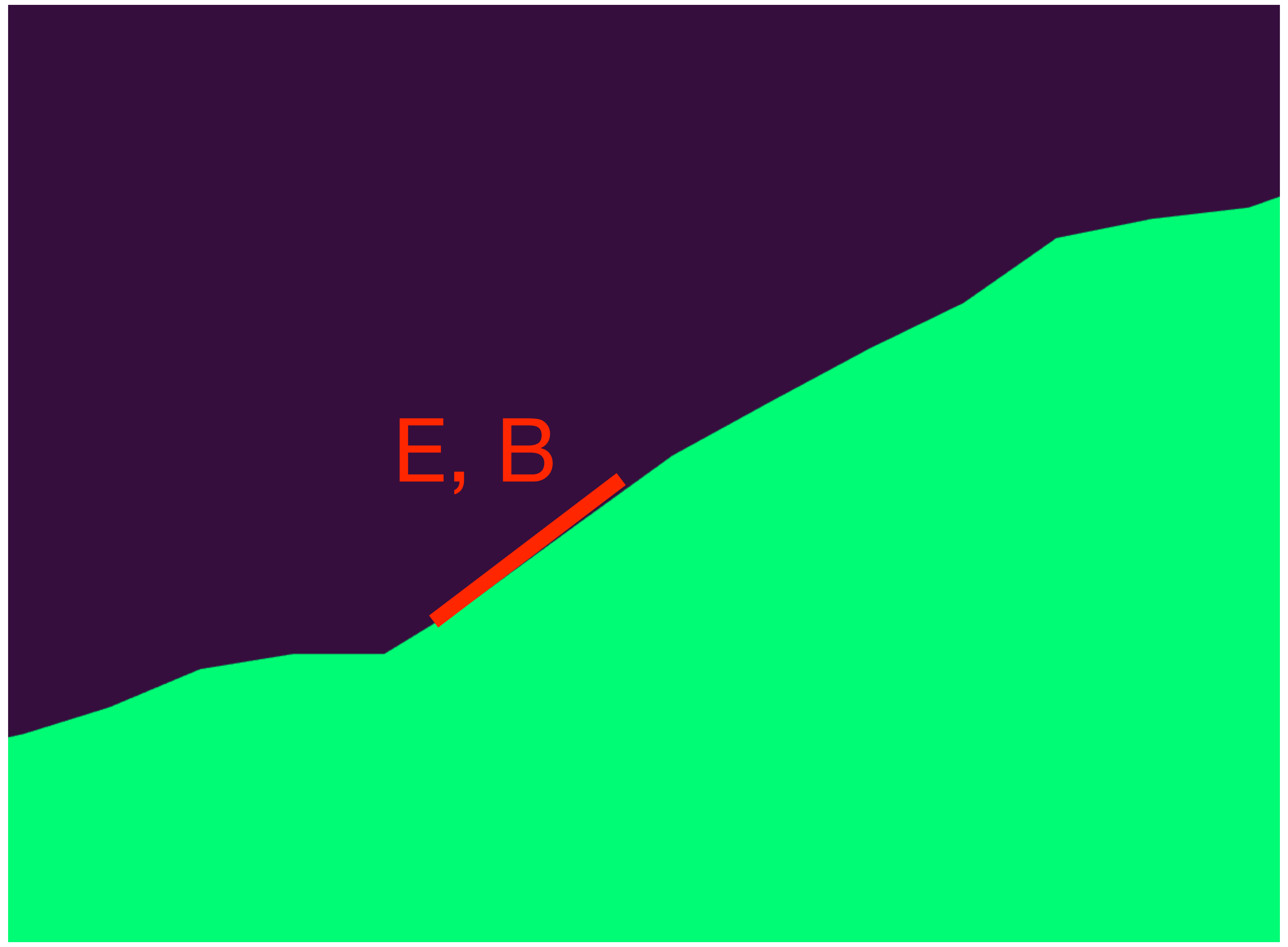
Land MT

slope parallel electric and horizontal magnetic fields



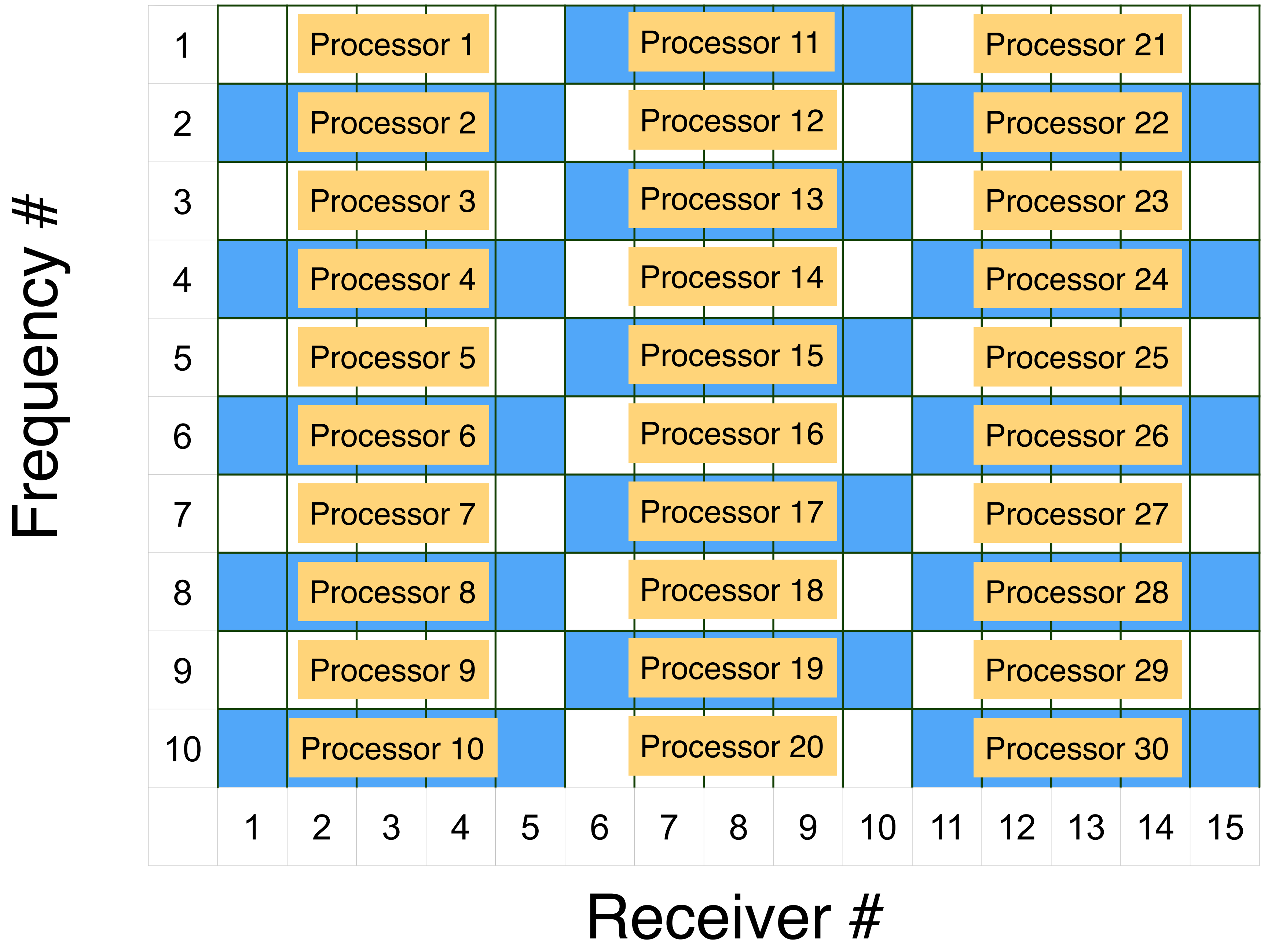
Marine MT

slope parallel electric and magnetic fields



Parallel Data Decomposition

- Forward calculations done in parallel using manager-worker model:
- **Frequencies** modeled independently
- **Receiver** subsets modeled independently
- **Transmitter** subsets modeled independently



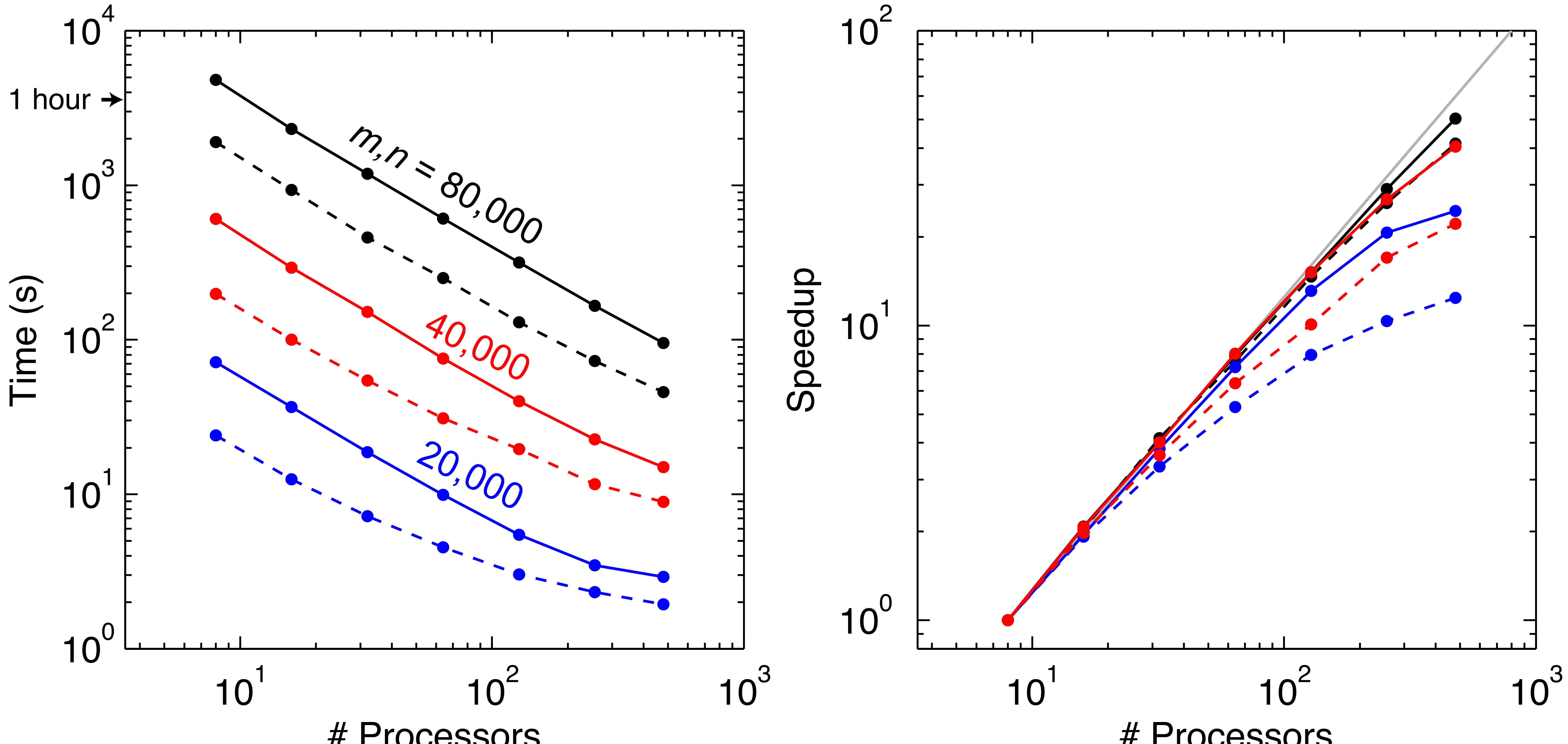
- Example parallel decomposition for a large marine CSEM problem:
- 1000 transmitters, 80 receivers, 10 frequencies:
- (1000/10 transmitters per subset) x (80/10 receivers per subset) x 10 frequencies = 8000 parallel tasks

Regularized Nonlinear EM Inversion using Occam method

Objective function: $$U = \|\mathbf{Rm}\|^2 + \mu^{-1}\|\mathbf{W}(\mathbf{d} - \mathcal{F}(\mathbf{m}))\|^2$$

- Model parameter grid can using any polygon shapes. MARE2DEM takes care of the finite element meshing internally.
- Model parameters can be bounded using non-linear transform approach

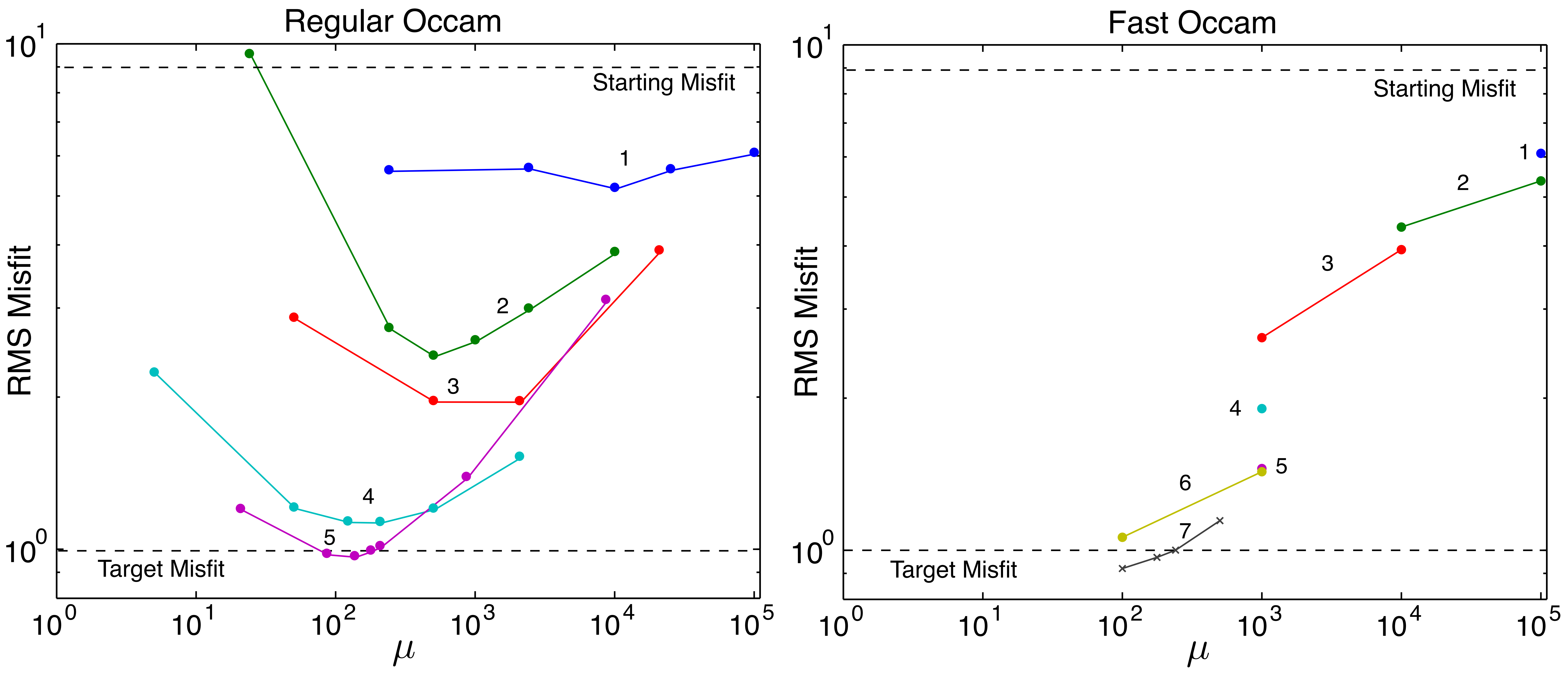
Dense matrix operations done in parallel using ScaLAPACK



Fast Occam Approach

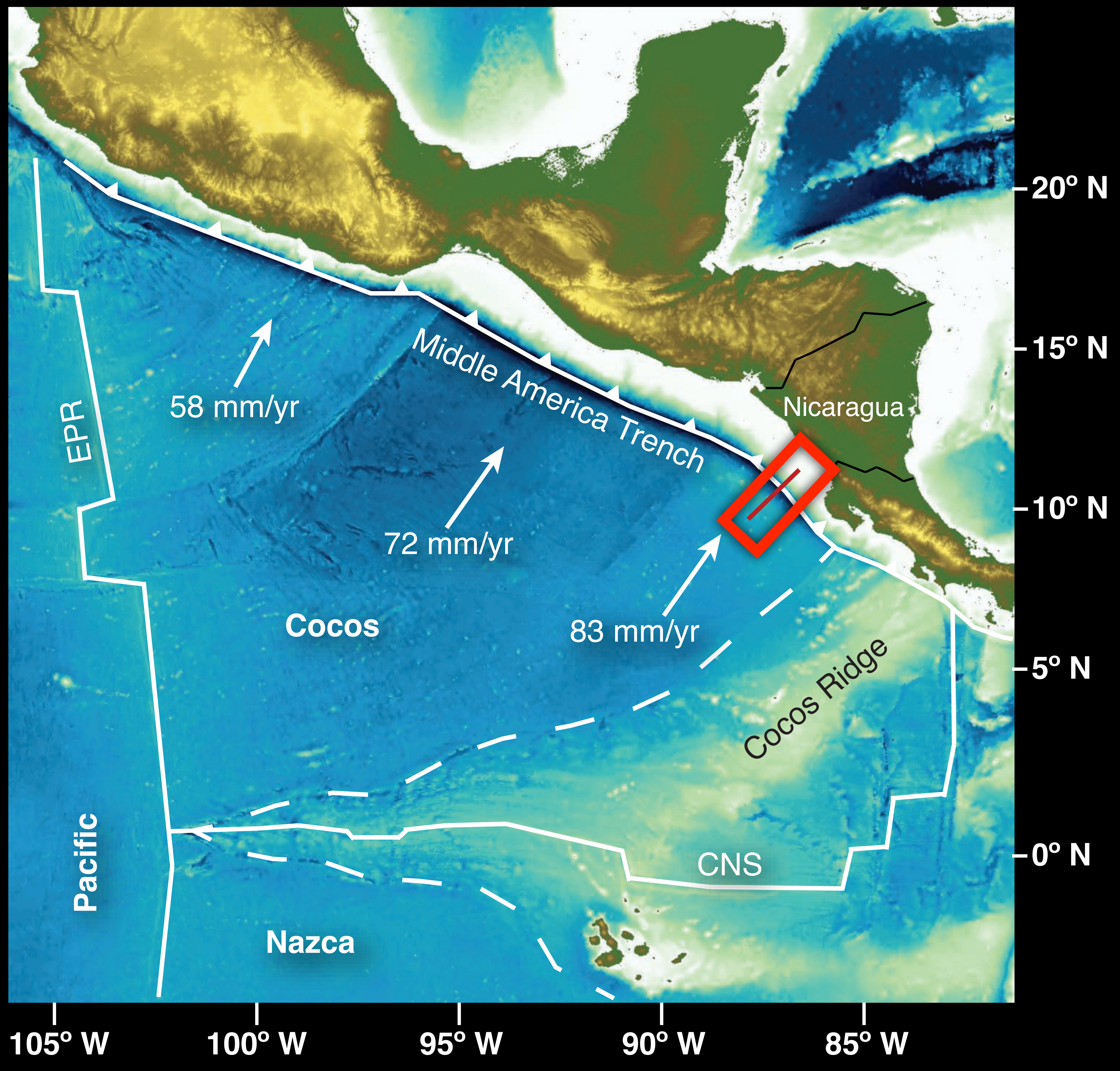
End line search for optimal mu early if “large” misfit decrease found for a test model:

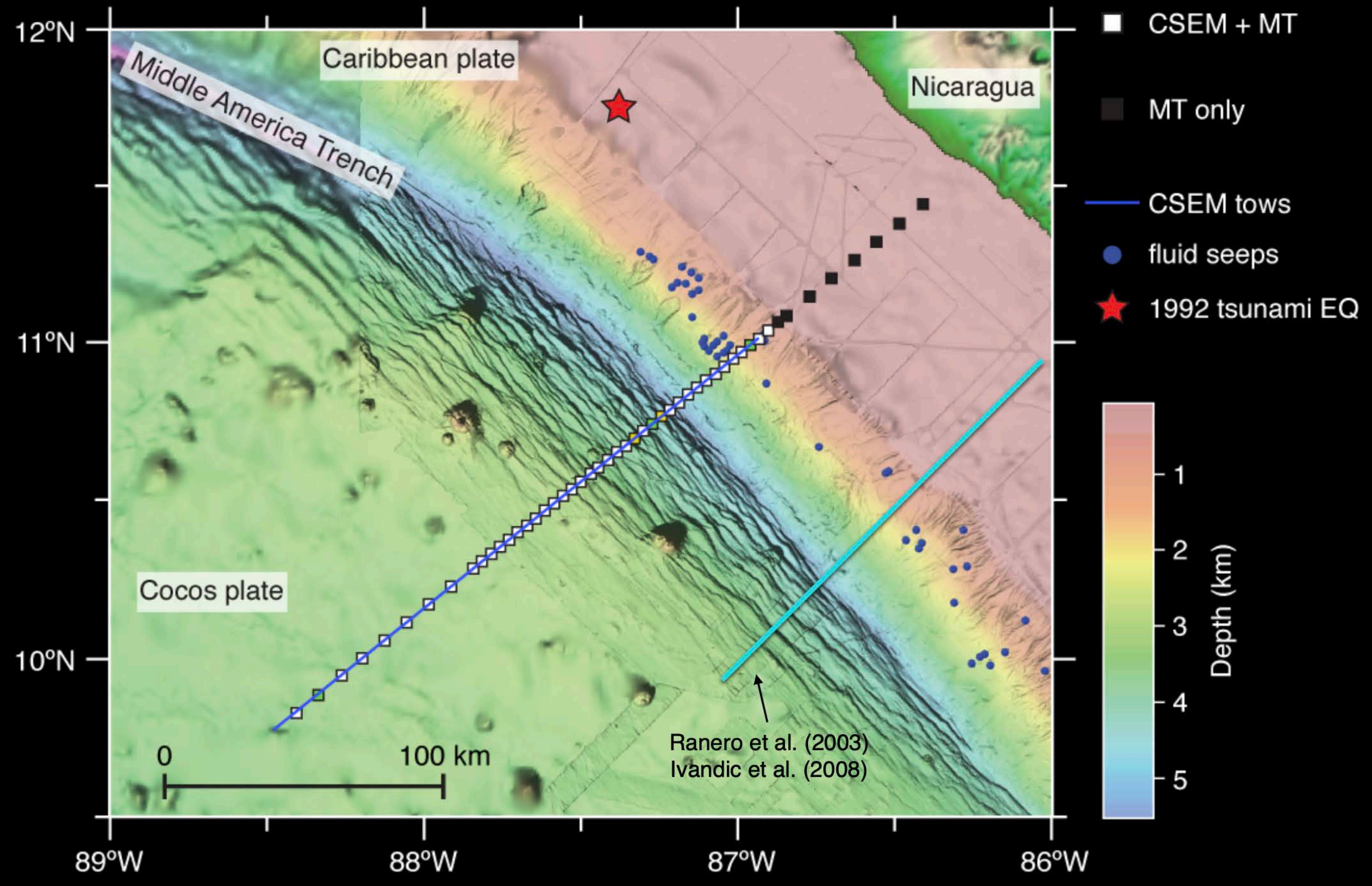
$$\mathbf{m}_{k+1} = \left[\mu\left(\mathbf{R}^T\mathbf{R}\right) + \left(\mathbf{W}\mathbf{J}_k\right)^T \mathbf{W}\mathbf{J}_k\right]^{-1}\left[\left(\mathbf{W}\mathbf{J}_k\right)^T \mathbf{W}\hat{\mathbf{d}}\right]$$



- Dense matrix operations done in parallel using ScaLAPACK

Marine EM Survey of the Middle America Trench



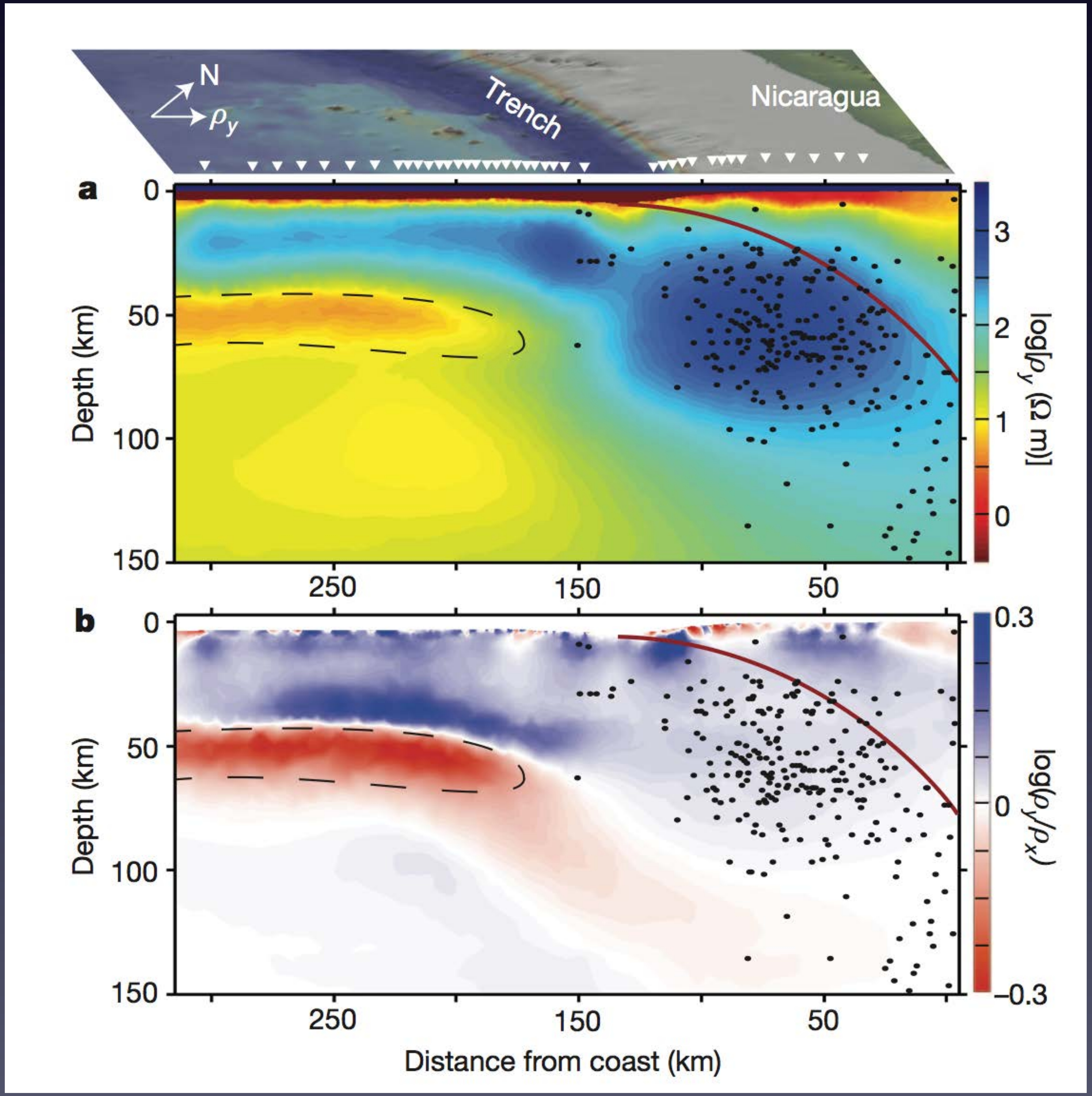


- First CSEM survey of a subduction zone

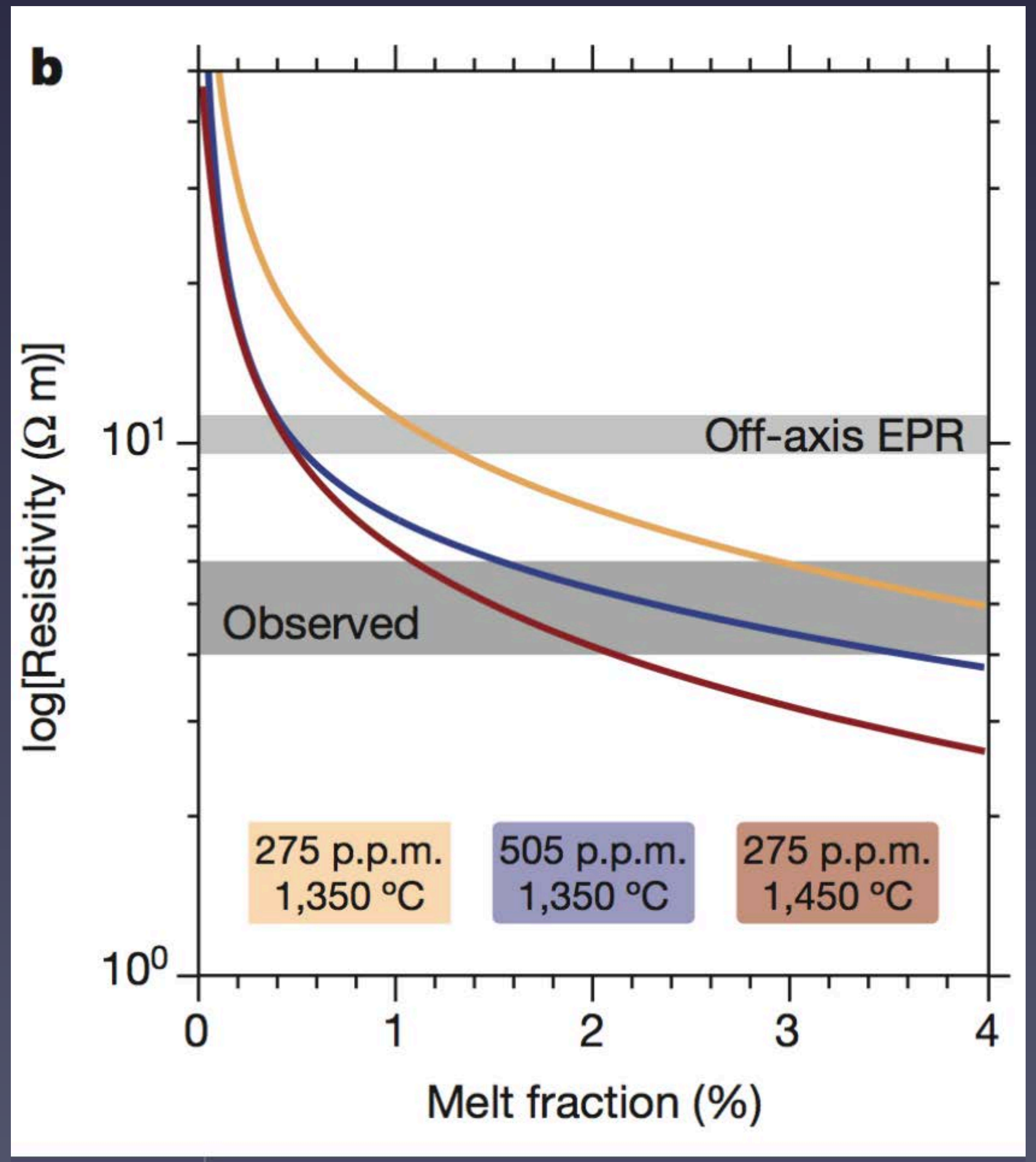
Collaborators:

Samer Naif (LDEO), Steven Constable (SIO), Rob L Evans (WHOI)

Magnetotelluric Results

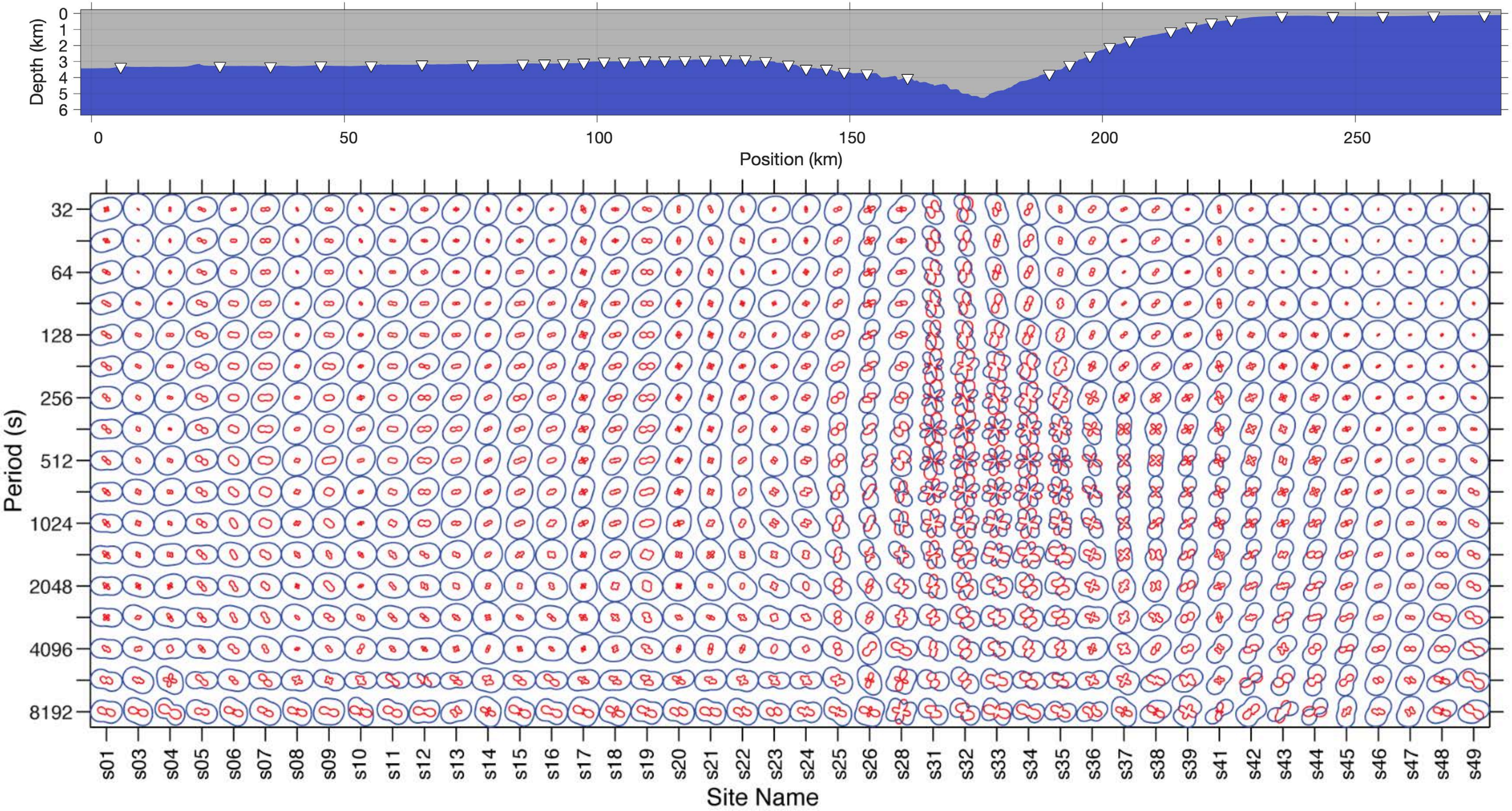


- Conductive channel along the lithosphere-asthenosphere boundary
- Anisotropic (3x)
- Implies sheared partial melt, which may act to lubricate tectonic plate motions



Naif, S., Key, K., Constable, S., & Evans, R. L. (2013). Melt-rich channel observed at the lithosphere-asthenosphere boundary. *Nature*, *495*(7441), 356–359.

Selecting 2D Compatible Data: Impedance Polar Diagrams



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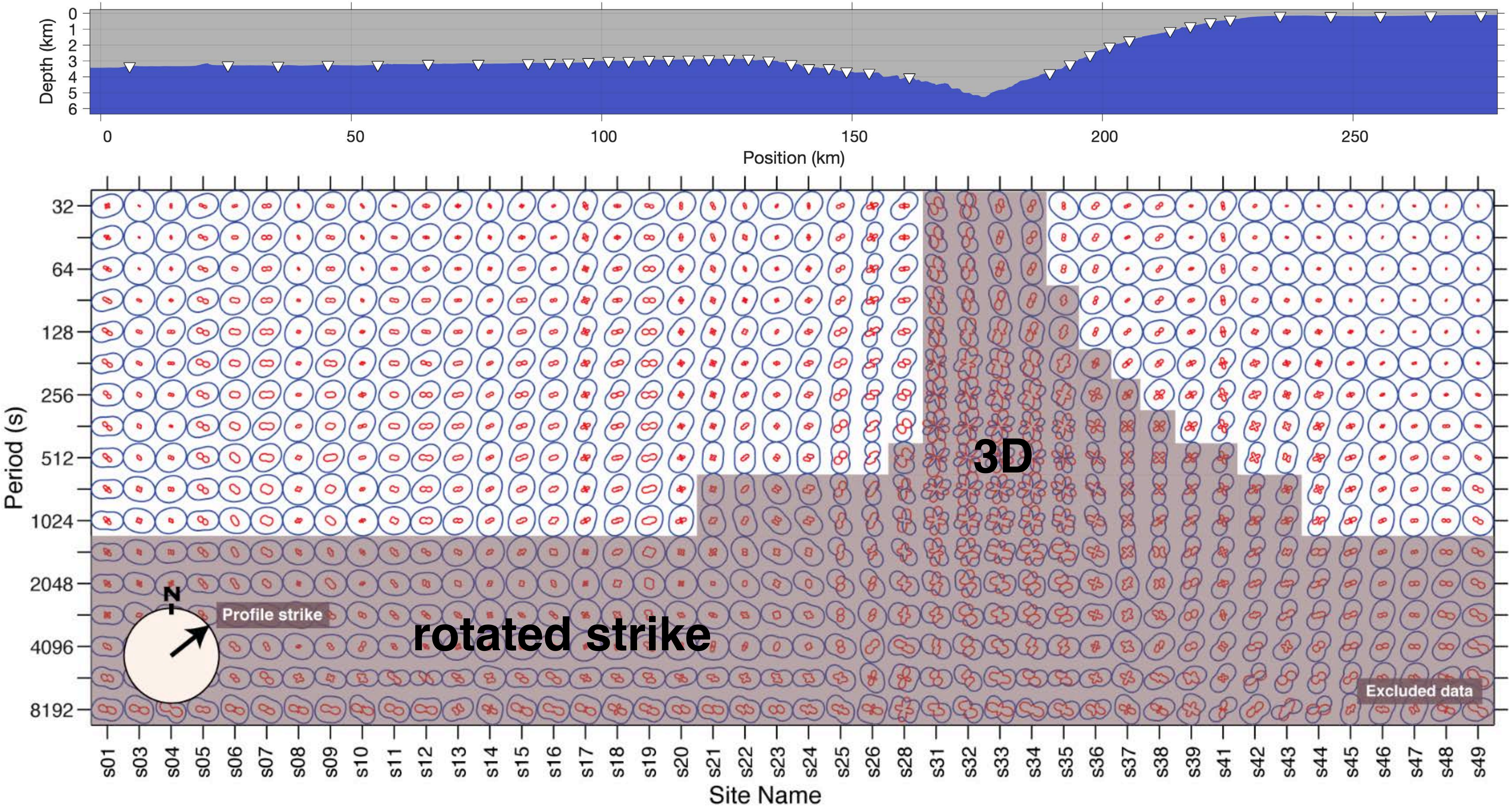
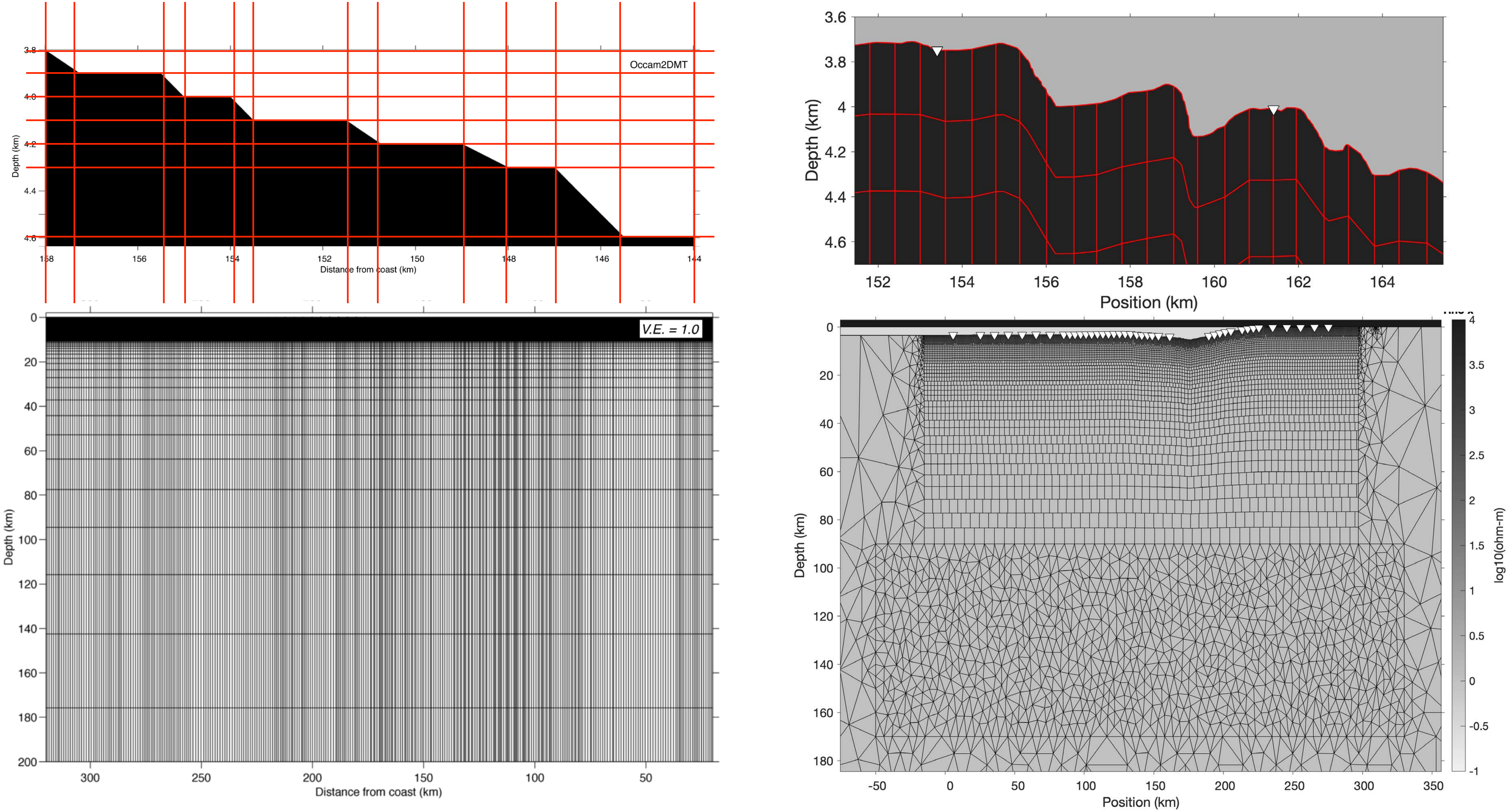
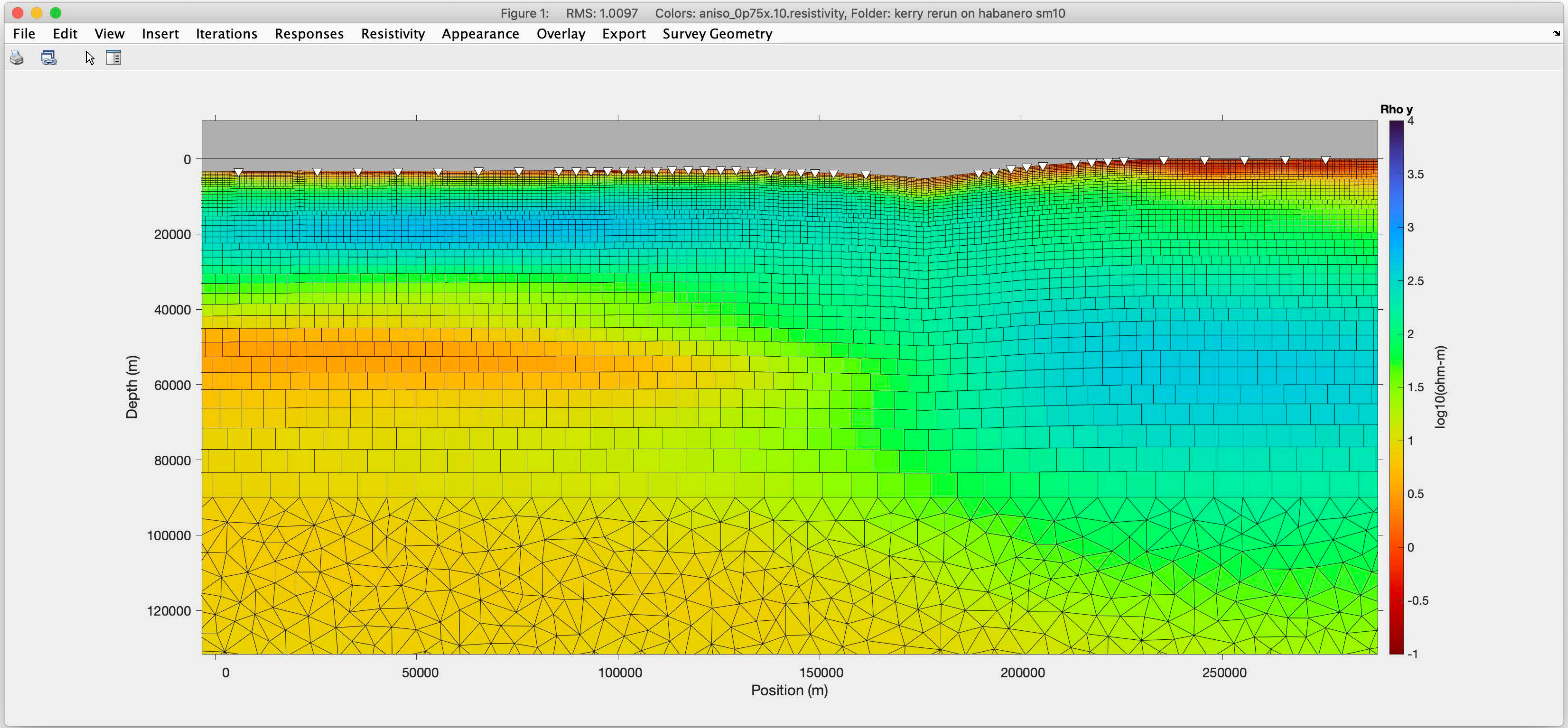


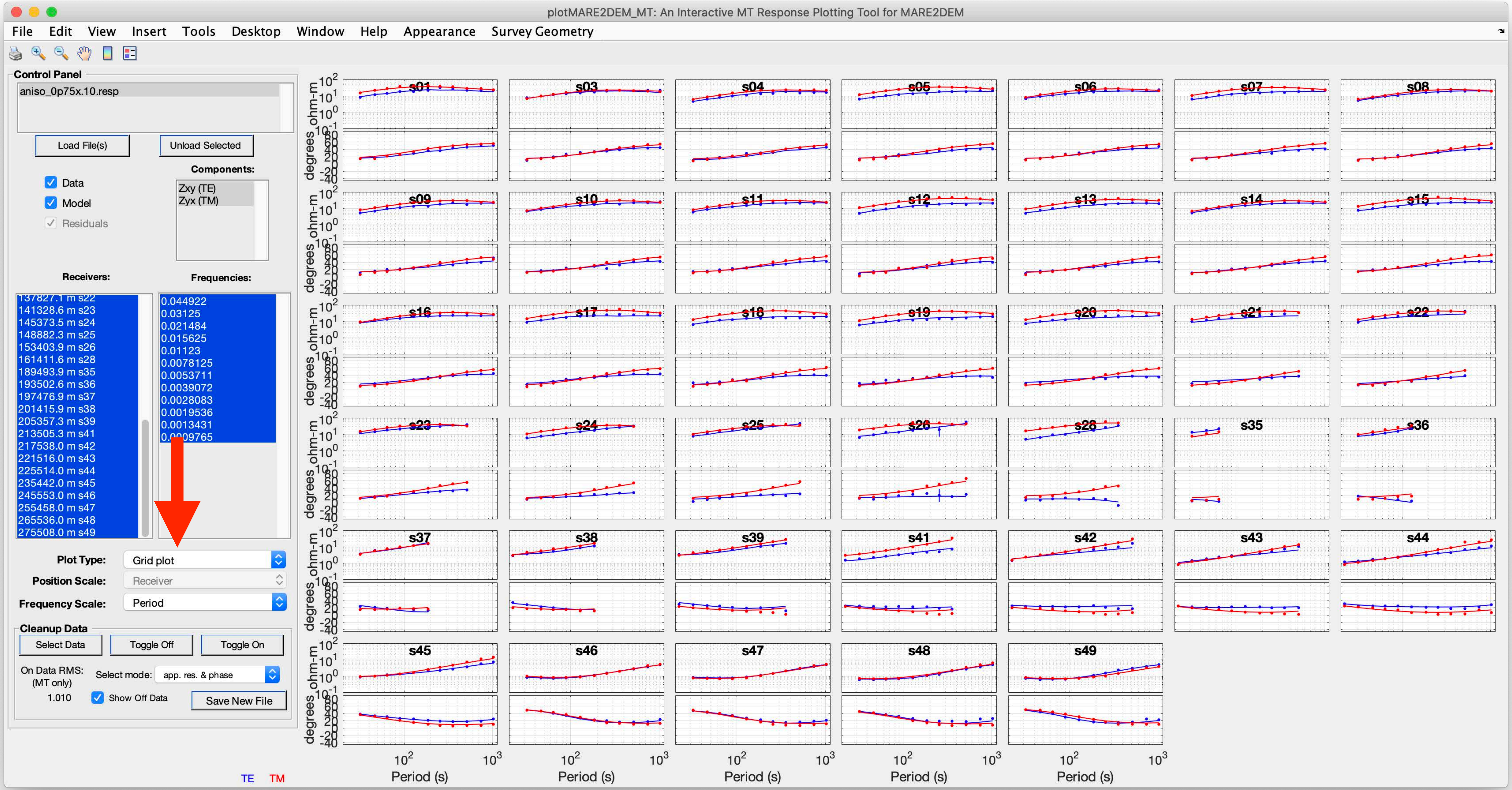
Inversion Parameter Grid: Occam2DMT versus MARE2DEM



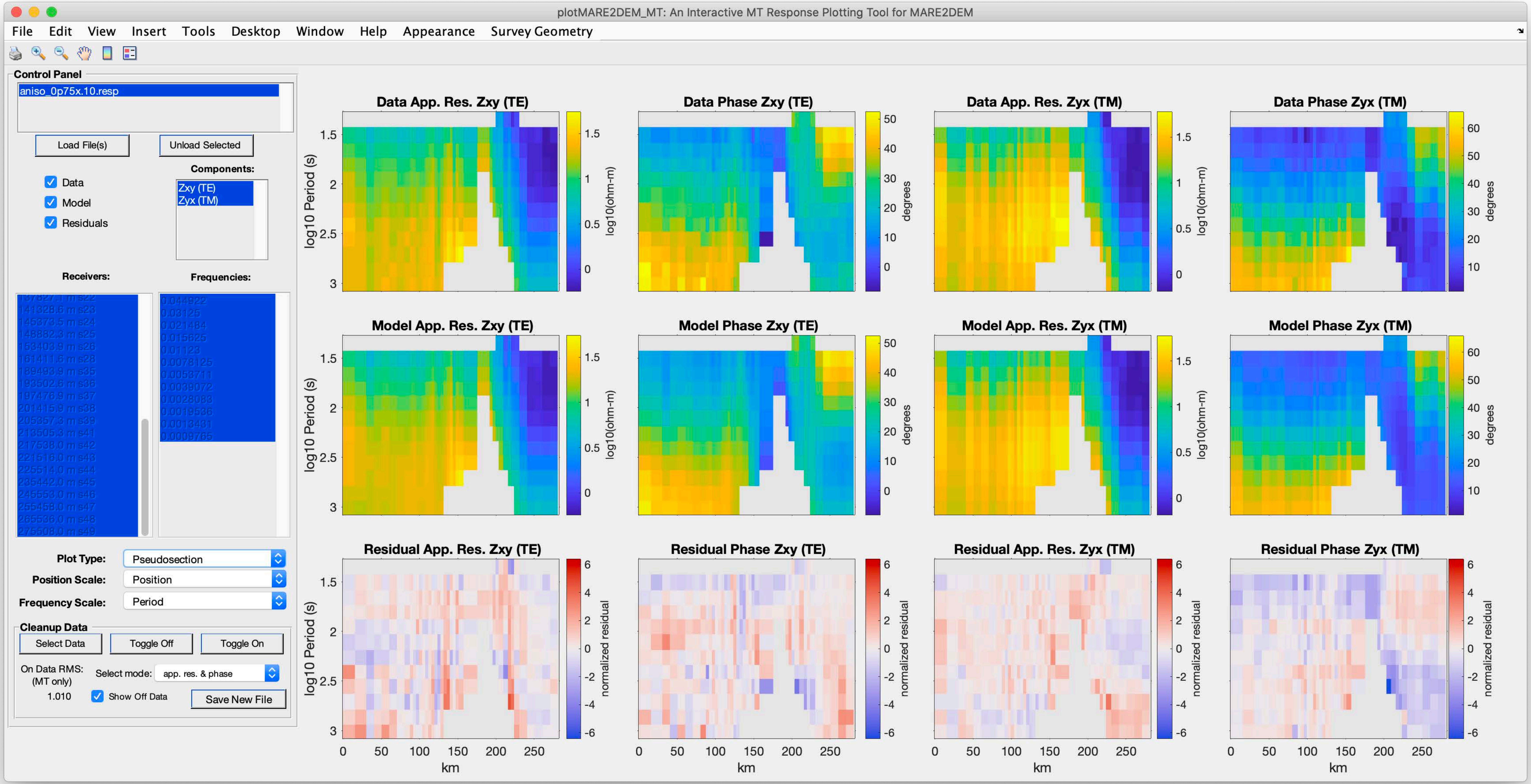
plotMARE2DEM.m: Inversion Results



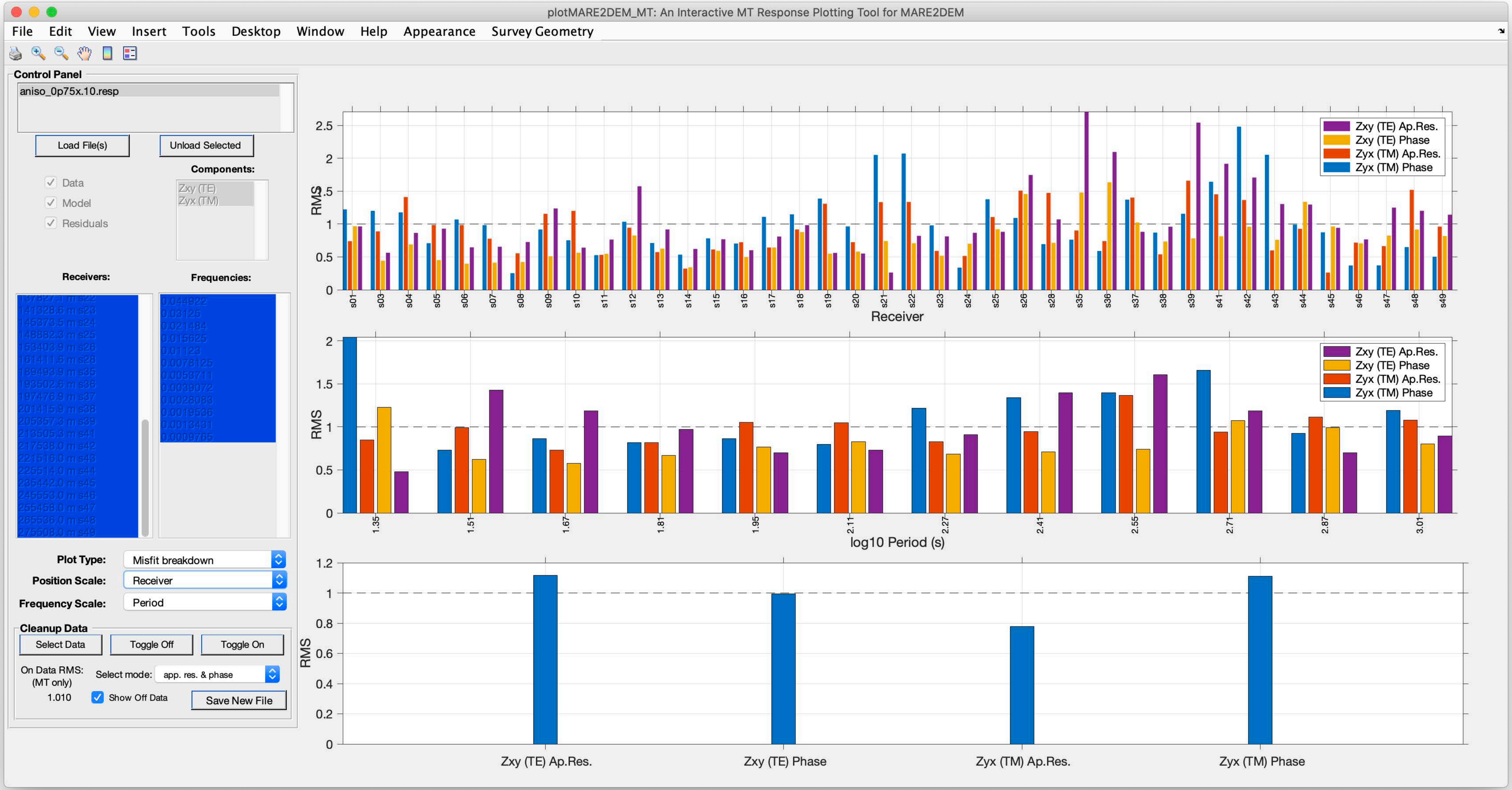
plotMARE2DEM_MT.m: Model Fit to the Data



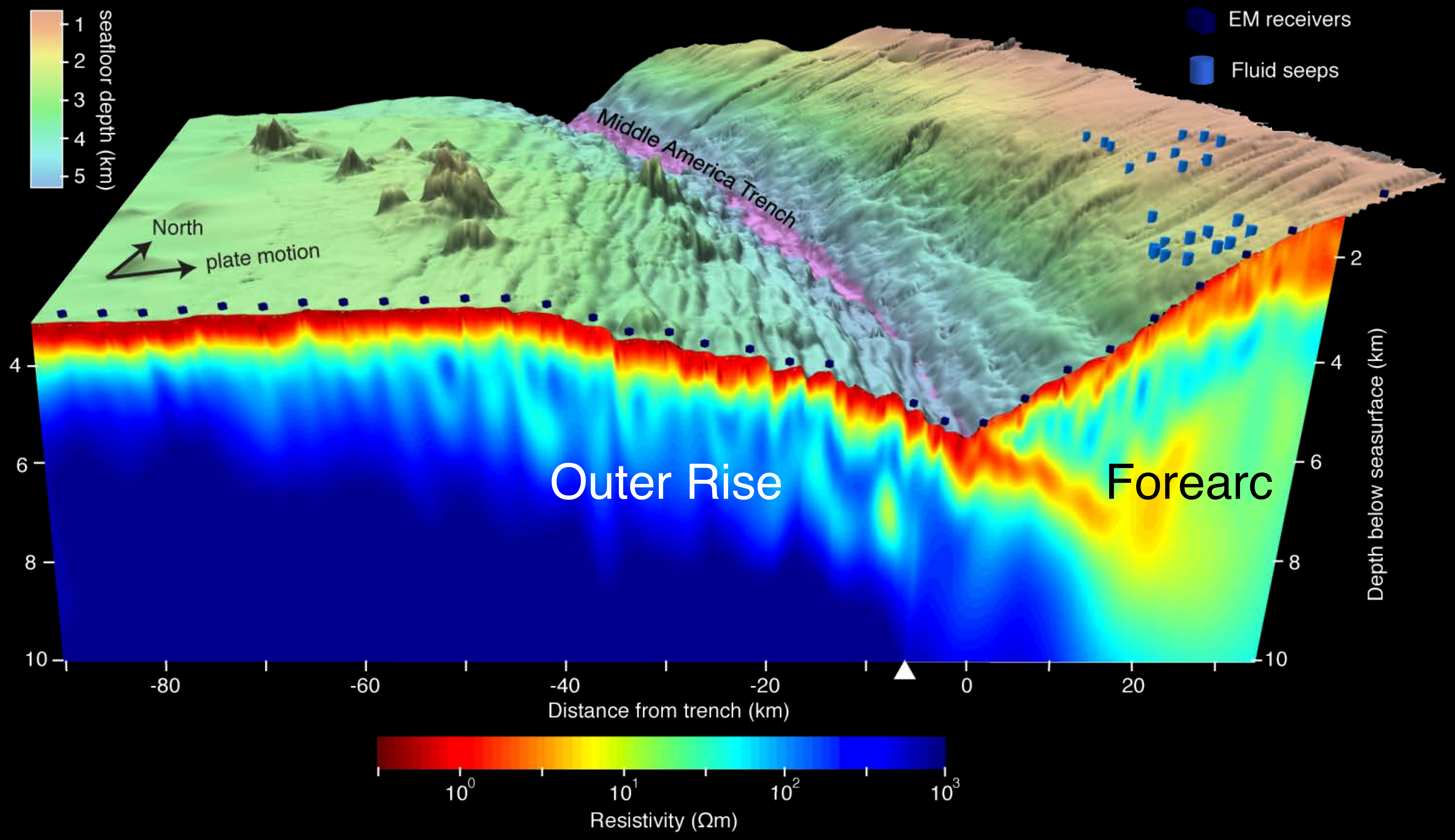
plotMARE2DEM_MT.m: Model Fit to the Data



plotMARE2DEM_MT.m: Model Fit to the Data

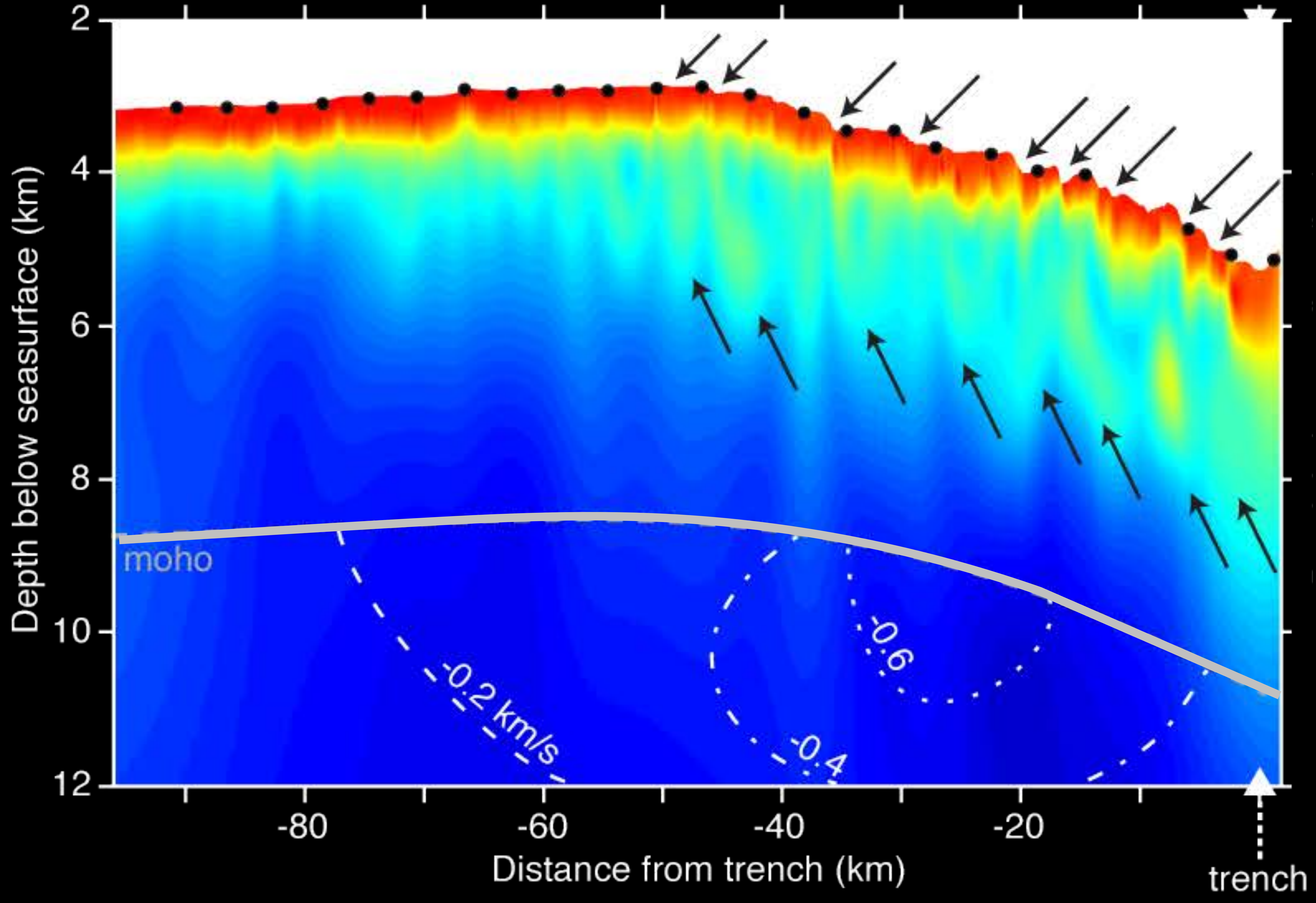


CSEM Results



Naif, S., K. Key, S. Constable, and R. L. Evans (2016), Porosity and fluid budget of a water-rich megathrust revealed with electromagnetic data at the Middle America Trench, Geochem Geophy Geosy, 17(11), 4495–4516.

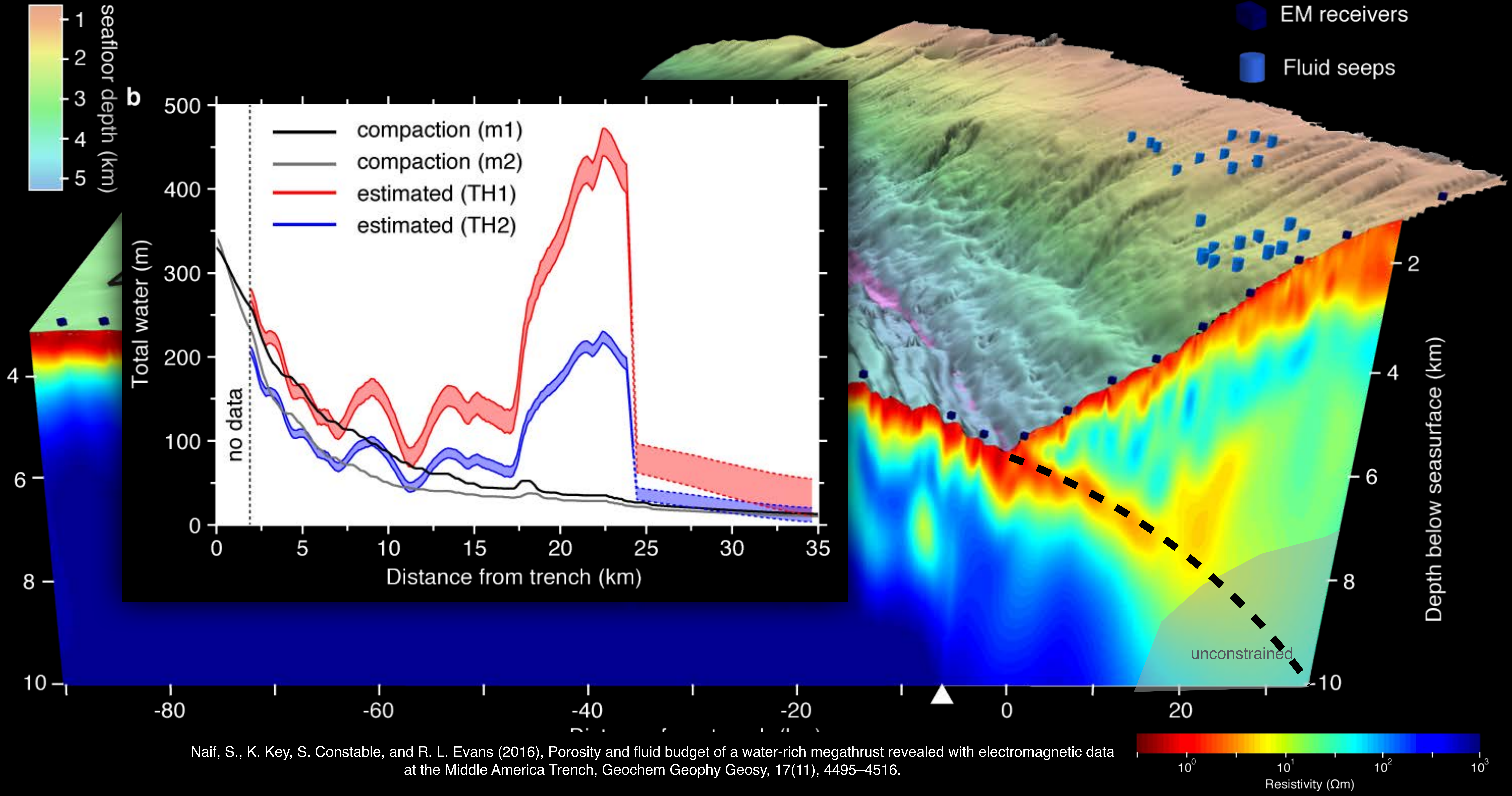
Outer Rise



- Dashed lines: P-wave velocity anomalies (Ivandic et al. 2008)
- Fault scarps correlate with steeply dipping conductive channels
- Porous channels along the fault traces drive fluids into the slab

Naif, S., K. Key, S. Constable, and R. L. Evans (2015), Water-rich bending faults at the Middle America Trench, *Geochem Geophy Geosy*, *16*(8), 2582–2597.

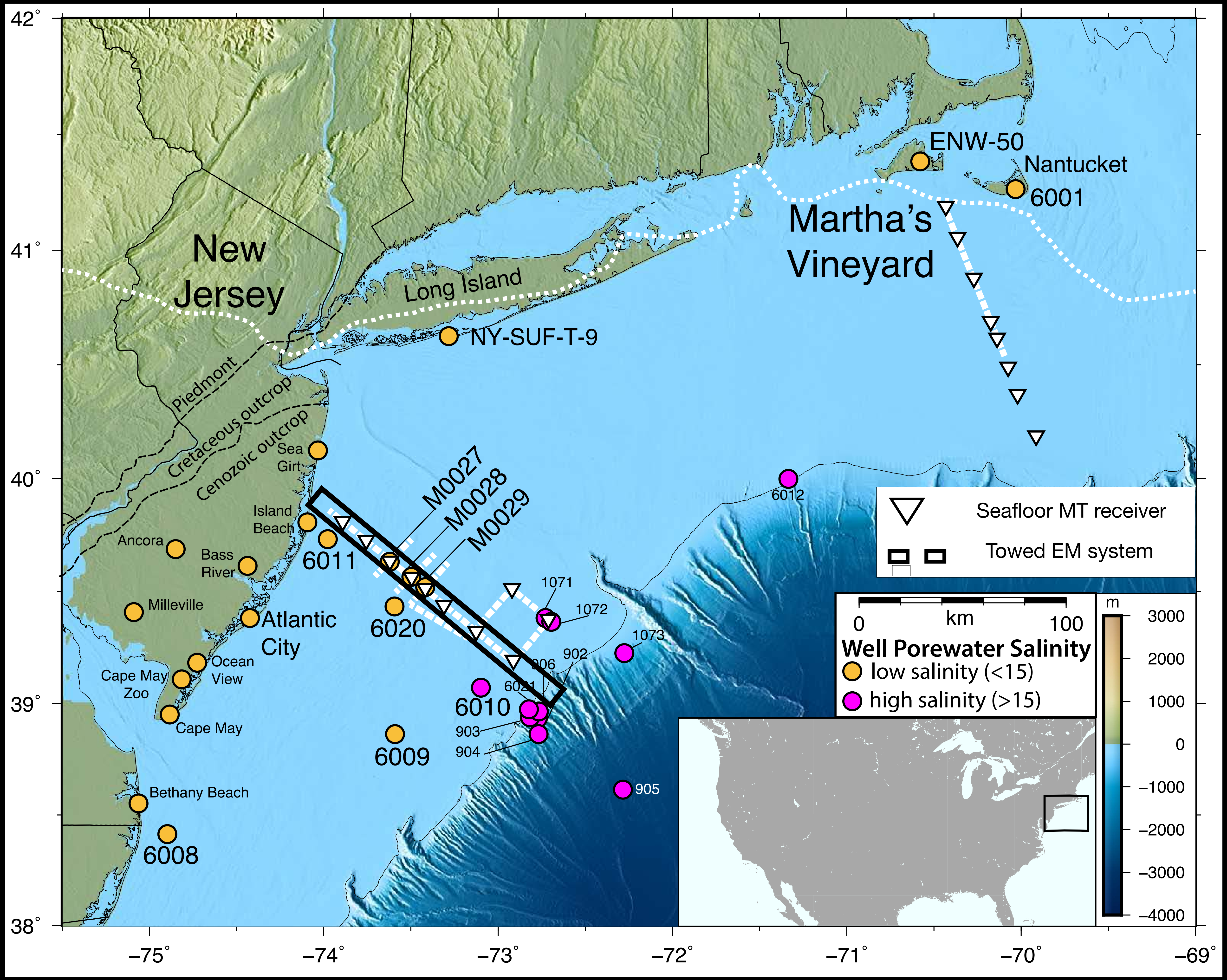
Forearc Structure



Naif, S., K. Key, S. Constable, and R. L. Evans (2016), Porosity and fluid budget of a water-rich megathrust revealed with electromagnetic data at the Middle America Trench, Geochem Geophy Geosy, 17(11), 4495–4516.

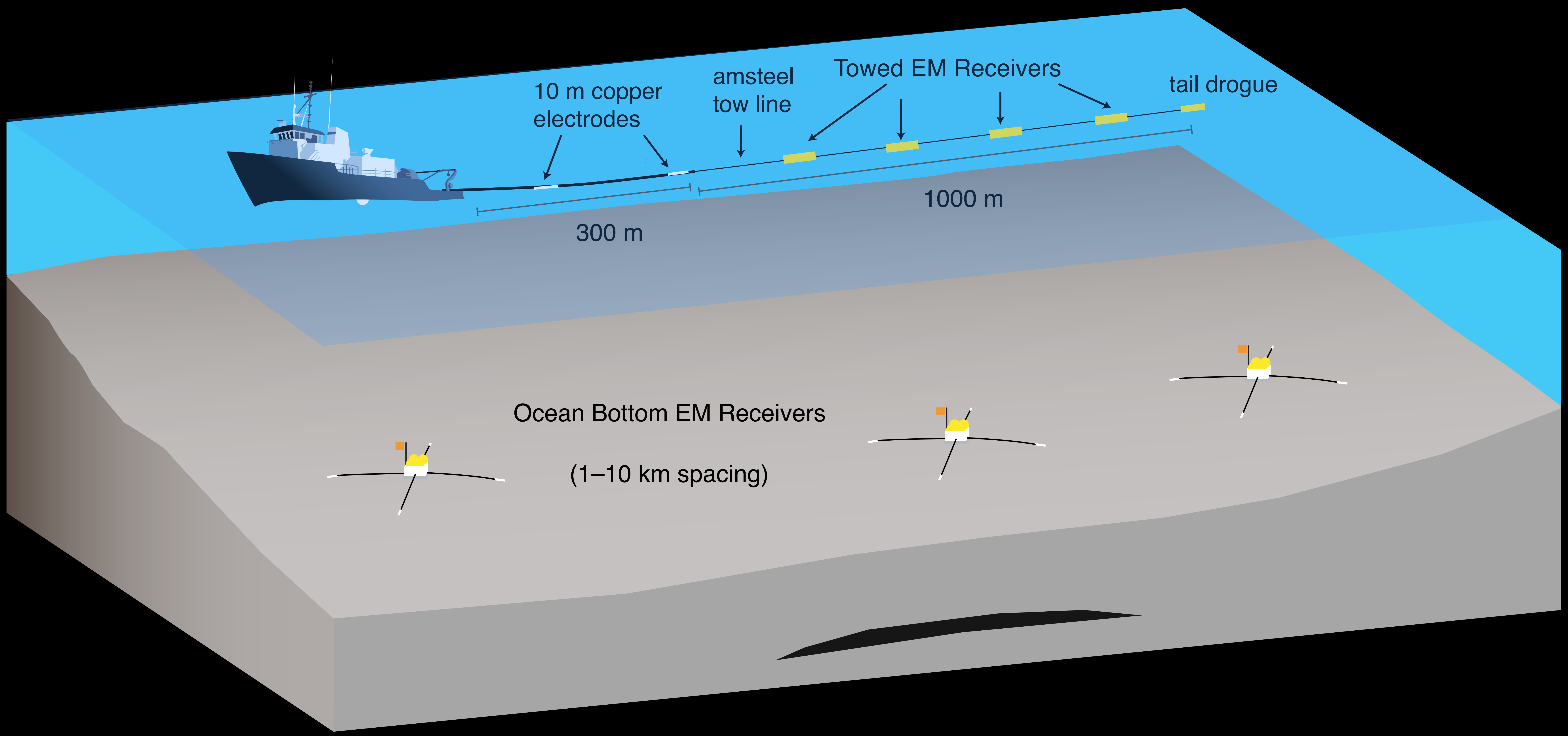
Mapping Offshore Groundwater

Collaborators: Chloe Gustafson (LDEO) and Rob Evans (WHOI)



Gustafson, C., Key, K., & Evans, R. L. (2019). Aquifer systems extending far offshore on the U.S. Atlantic margin. Scientific Reports, 9(1), 1–10.

Surface Towed CSEM and MT



- 336 m dipole transmitter, surface towed, 100 A current
- 4 towed receivers (600, 870, 1120, 1380 m) offsets
- 10 seafloor EM/MT receivers

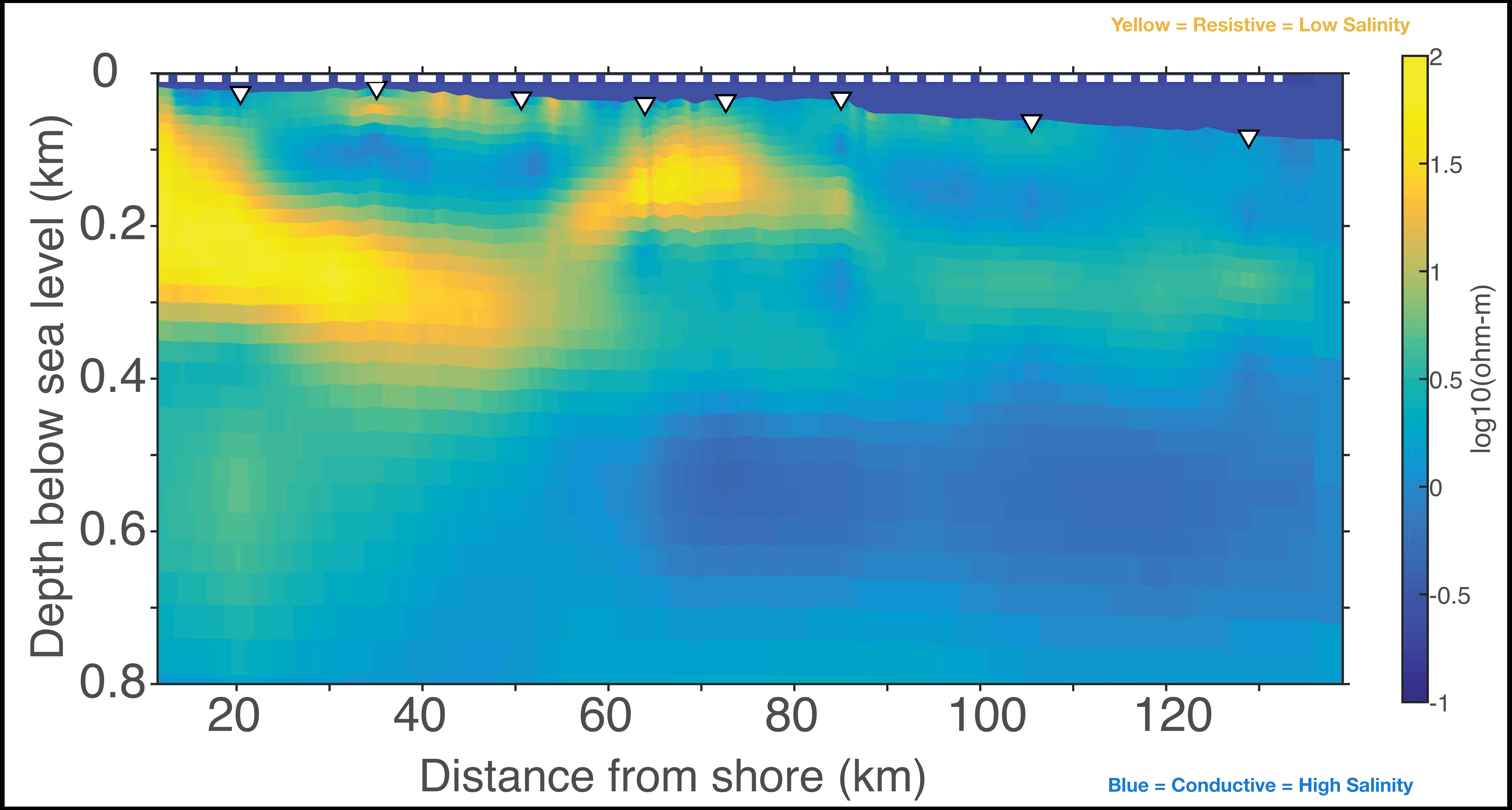
<u>Frequency range:</u>

MT: 0.0005 - 80 Hz

CSEM: 0.75, 1.75 Hz

New Jersey resistivity model

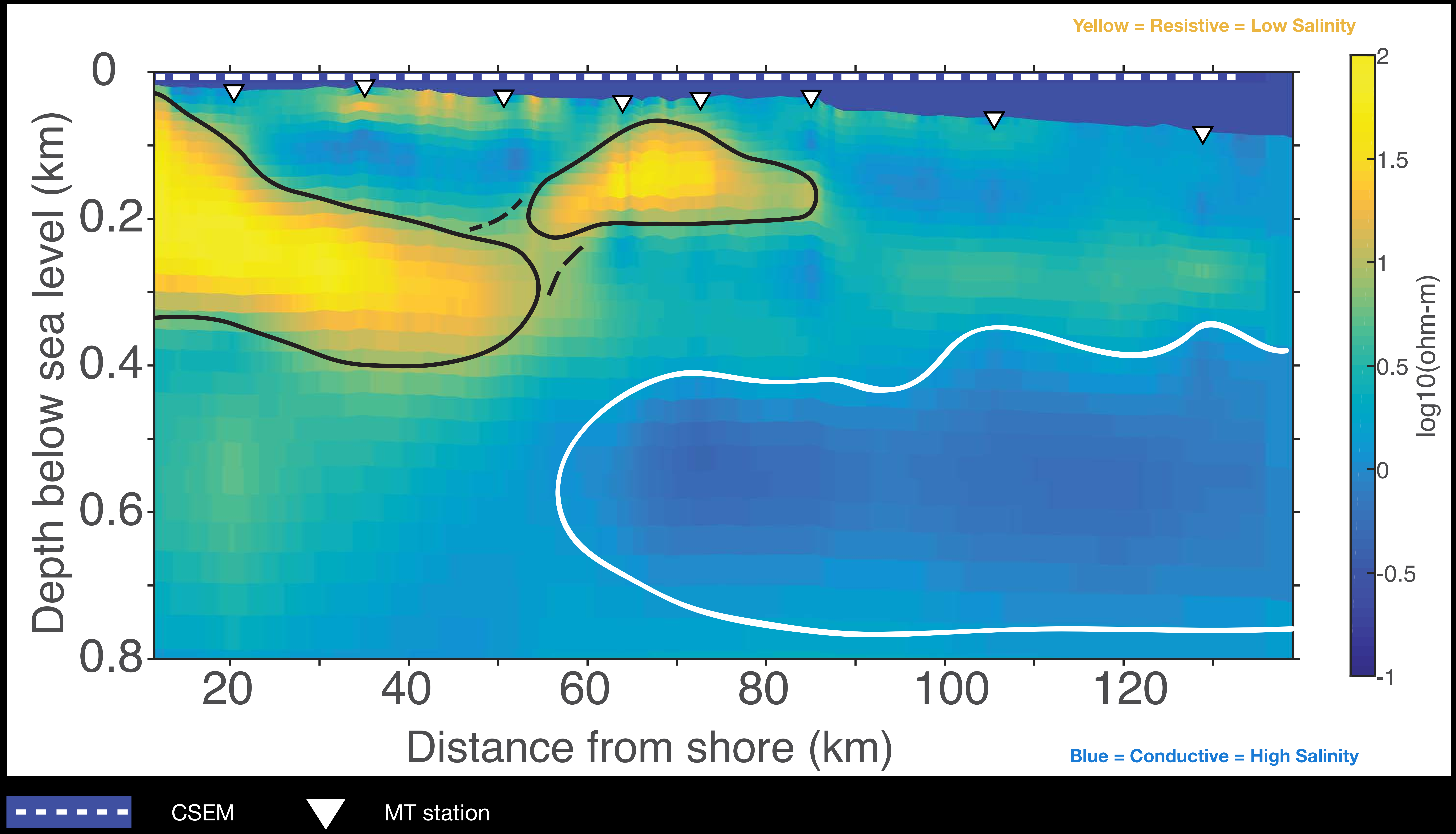
V. E. = 160x



CSEM MT station

Gustafson et al 2019, Scientific Reports

New Jersey resistivity model



Gustafson et al 2019, Scientific Reports

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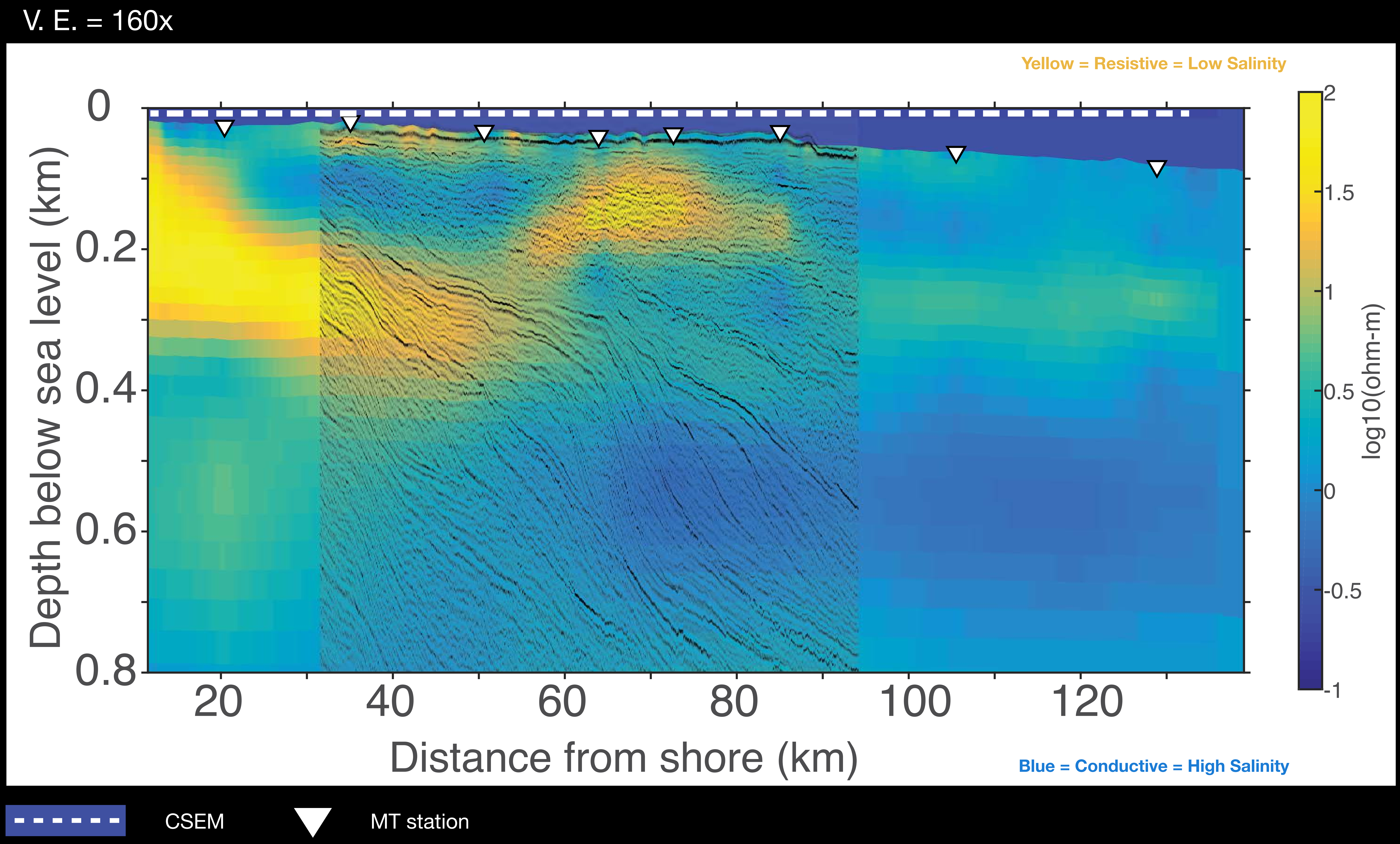

Gustafson et al 2019, Scientific Reports

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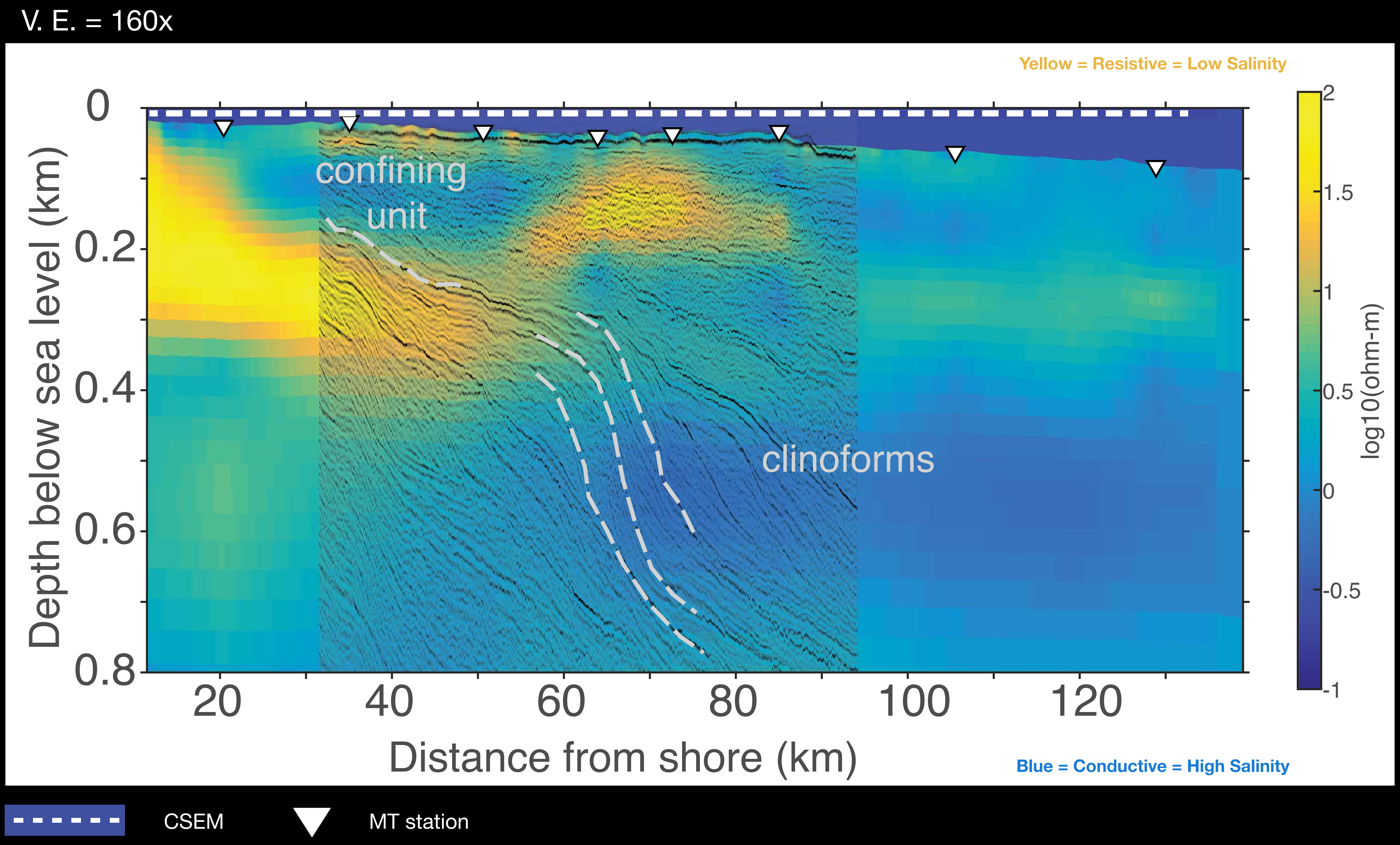

Gustafson et al 2019, Scientific Reports

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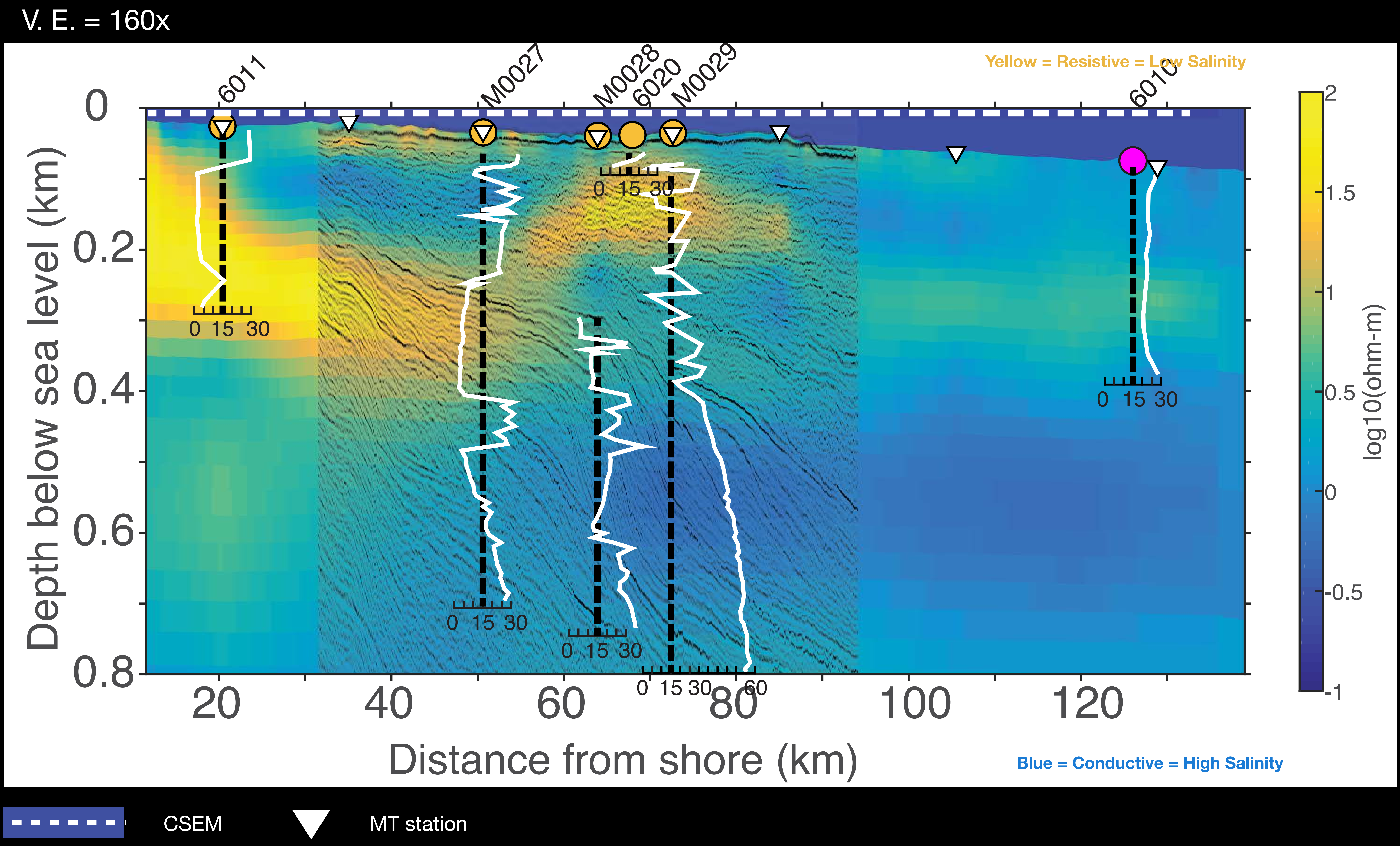
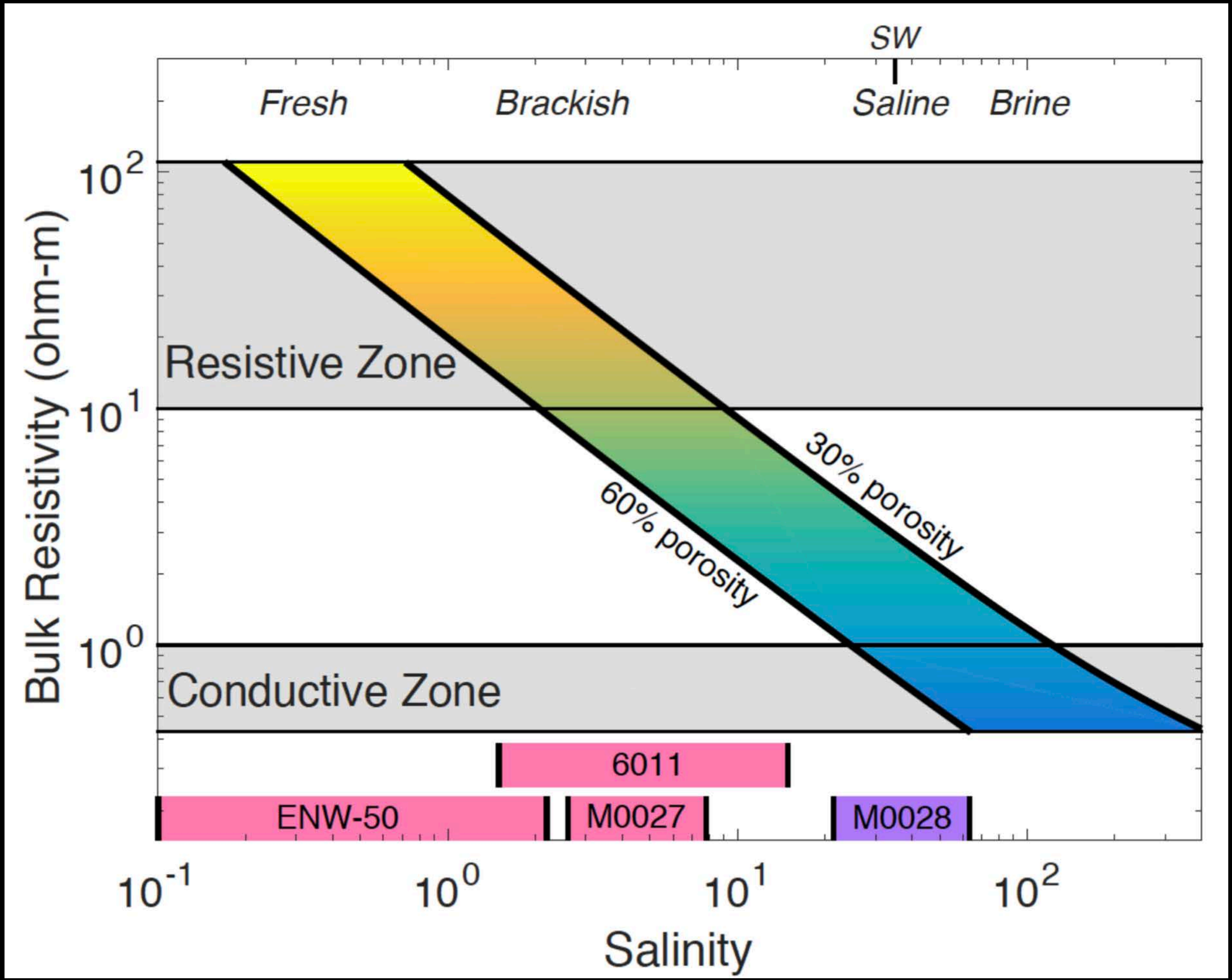


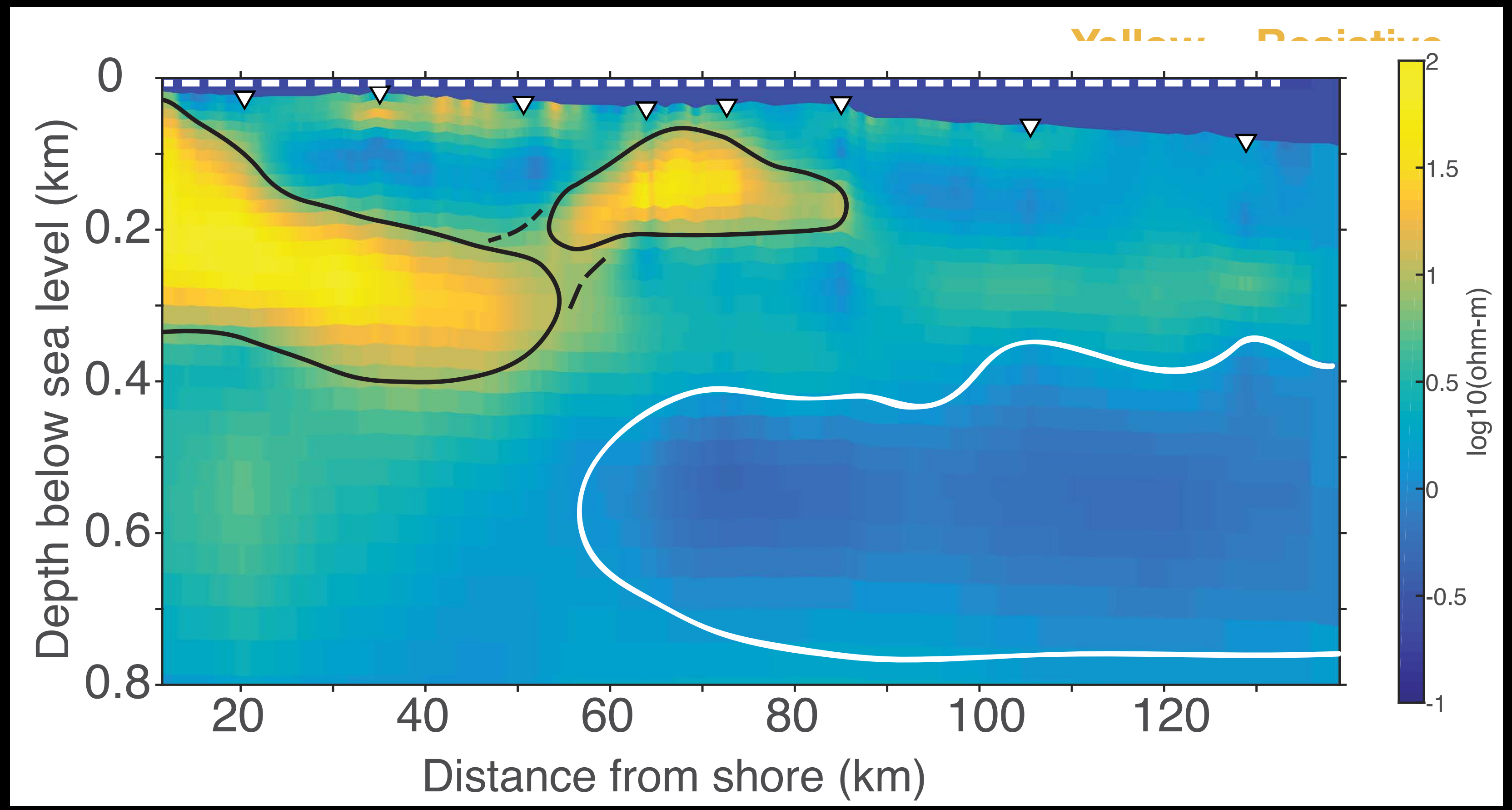
Gustafson et al 2019, Scientific Reports

Resistivity as a Function of Salinity



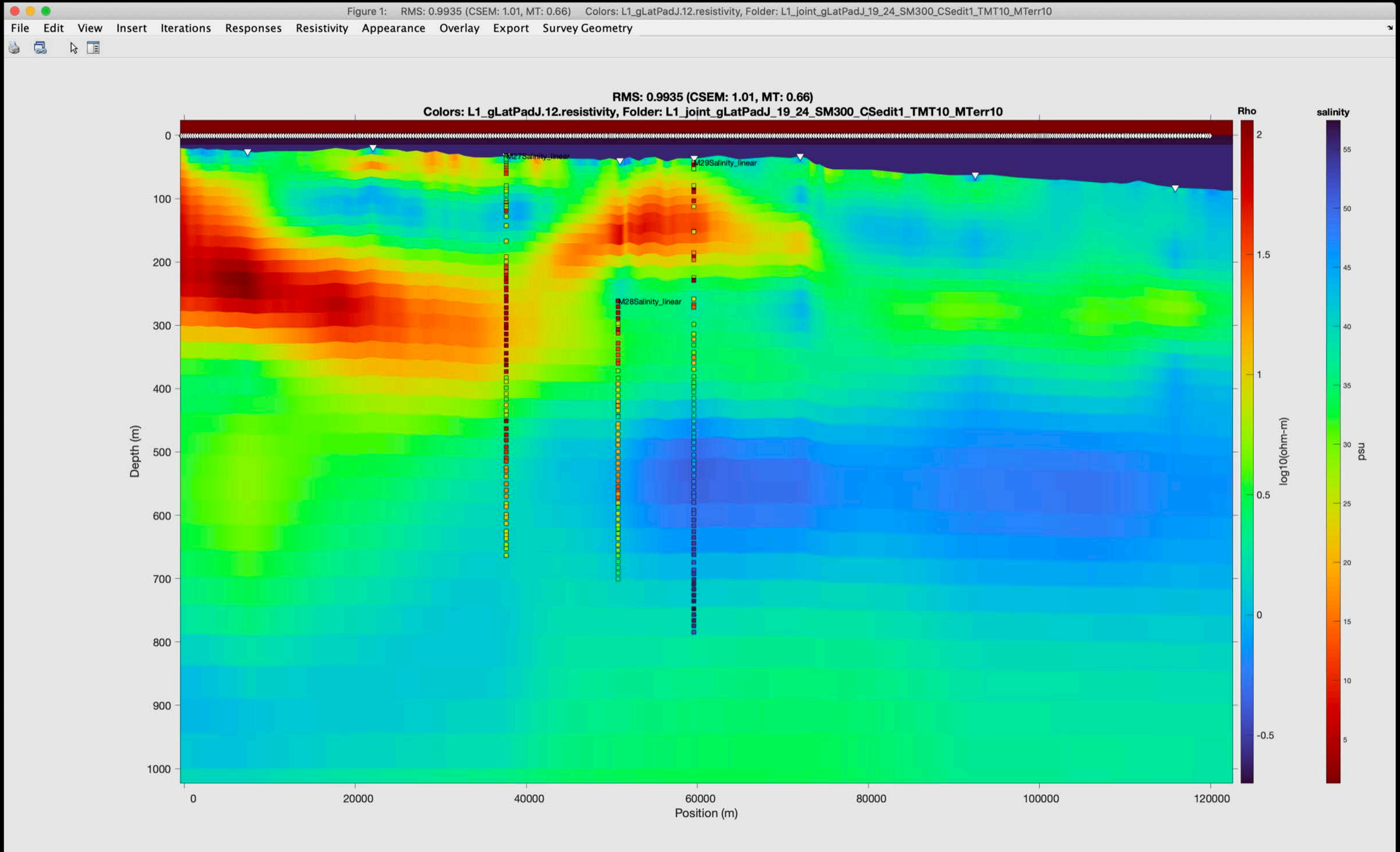
uses Archie's law

$$\rho = \rho_f \phi^{-m}$$

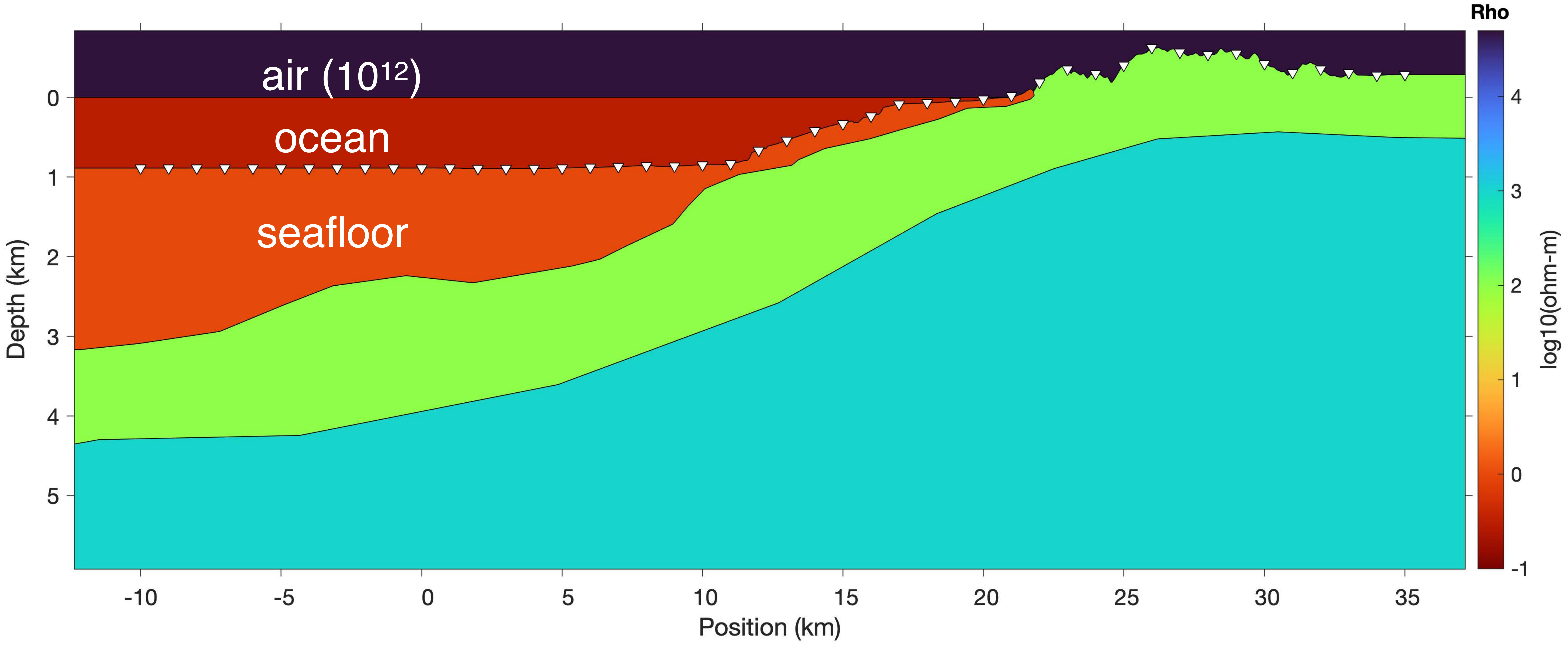


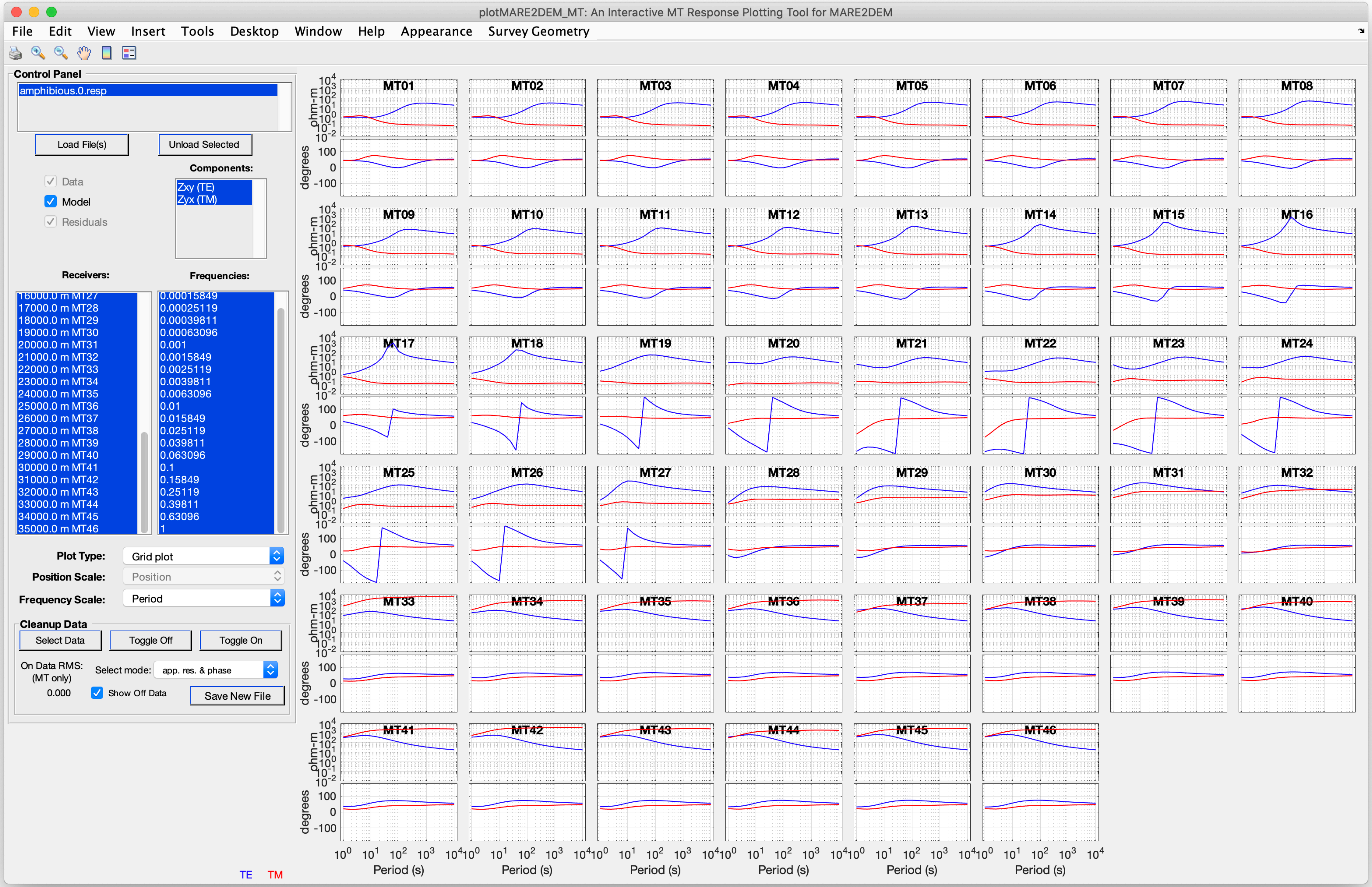
Example of overlaying well-data on top of resistivity with plotMARE2DEM.m

(see the Overlay menu)

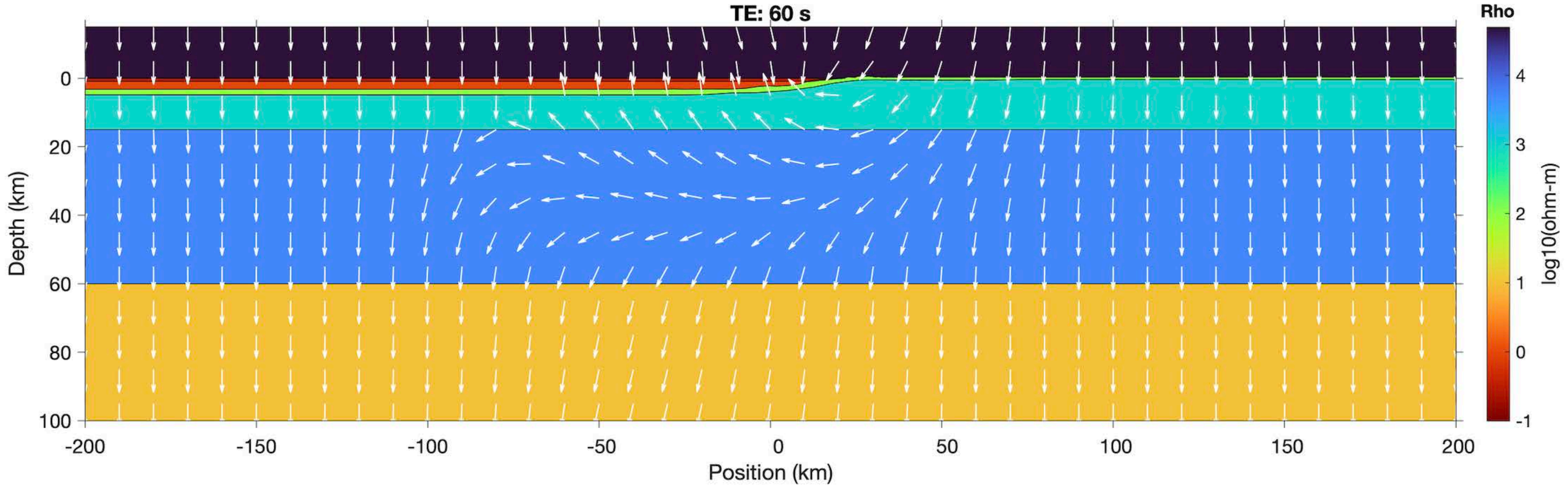


Using MARE2DEM to study MT Physics: TE mode coast effect study

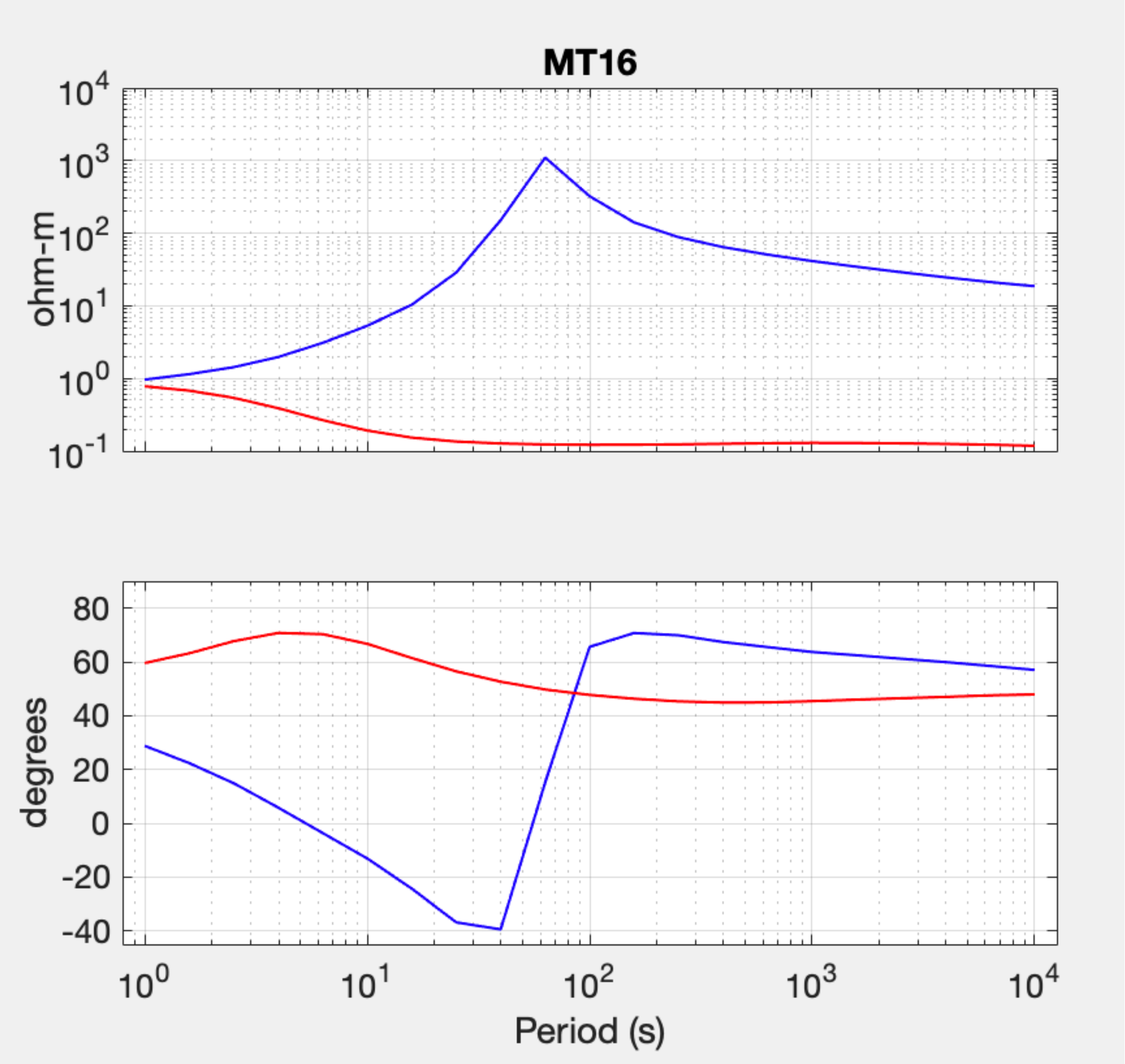



plotMARE2DEM_MT: An Interactive MT Response Plotting Tool for MARE2DEM
File Edit View Insert Tools Desktop Window Help Appearance Survey Geometry
Control Panel
amphibious.0.resp
Load File(s)
Unload Selected
Data
Model
Residuals
Components:
Zxy (TE)
Zyx (TM)
Receivers:
16000.0 m MT27
17000.0 m MT28
18000.0 m MT29
19000.0 m MT30
20000.0 m MT31
21000.0 m MT32
22000.0 m MT33
23000.0 m MT34
24000.0 m MT35
25000.0 m MT36
26000.0 m MT37
27000.0 m MT38
28000.0 m MT39
29000.0 m MT40
30000.0 m MT41
31000.0 m MT42
32000.0 m MT43
33000.0 m MT44
34000.0 m MT45
35000.0 m MT46
Frequencies:
0.00015849
0.00025119
0.00039811
0.00063096
0.001
0.0015849
0.0025119
0.0039811
0.0063096
0.01
0.015849
0.025119
0.039811
0.063096
0.1
0.15849
0.25119
0.39811
0.63096
1
Plot Type: Grid plot
Position Scale: Position
Frequency Scale: Period
Cleanup Data
Select Data
Toggle Off
Toggle On
On Data RMS: (MT only)
0.000
Select mode: app. res. & phase
Show Off Data
Save New File
TE TM
MT01 MT02 MT03 MT04 MT05 MT06 MT07 MT08
MT09 MT10 MT11 MT12 MT13 MT14 MT15 MT16
MT17 MT18 MT19 MT20 MT21 MT22 MT23 MT24
MT25 MT26 MT27 MT28 MT29 MT30 MT31 MT32
MT33 MT34 MT35 MT36 MT37 MT38 MT39 MT40
MT41 MT42 MT43 MT44 MT45 MT46
ohm-m
degrees
100
0
-100
Period (s)

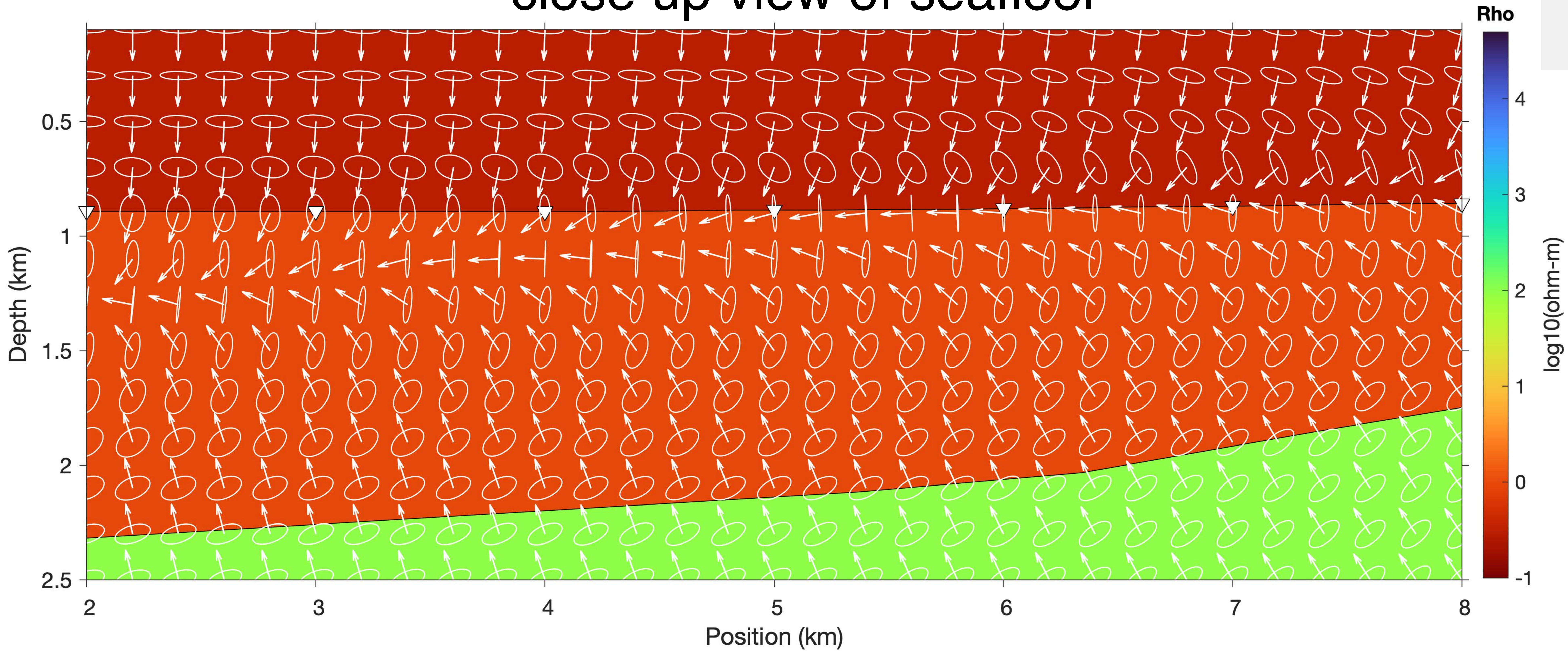
TE Poynting Vectors: $\mathbf{S} = (\mathbf{E} \times \mathbf{H}^*)/2$
60 s period



TE Poynting Vector: **S** = (**E** x **H***)/2
TE Polarization Ellipses (H_y,H_z)
60 s period



close up view of seafloor

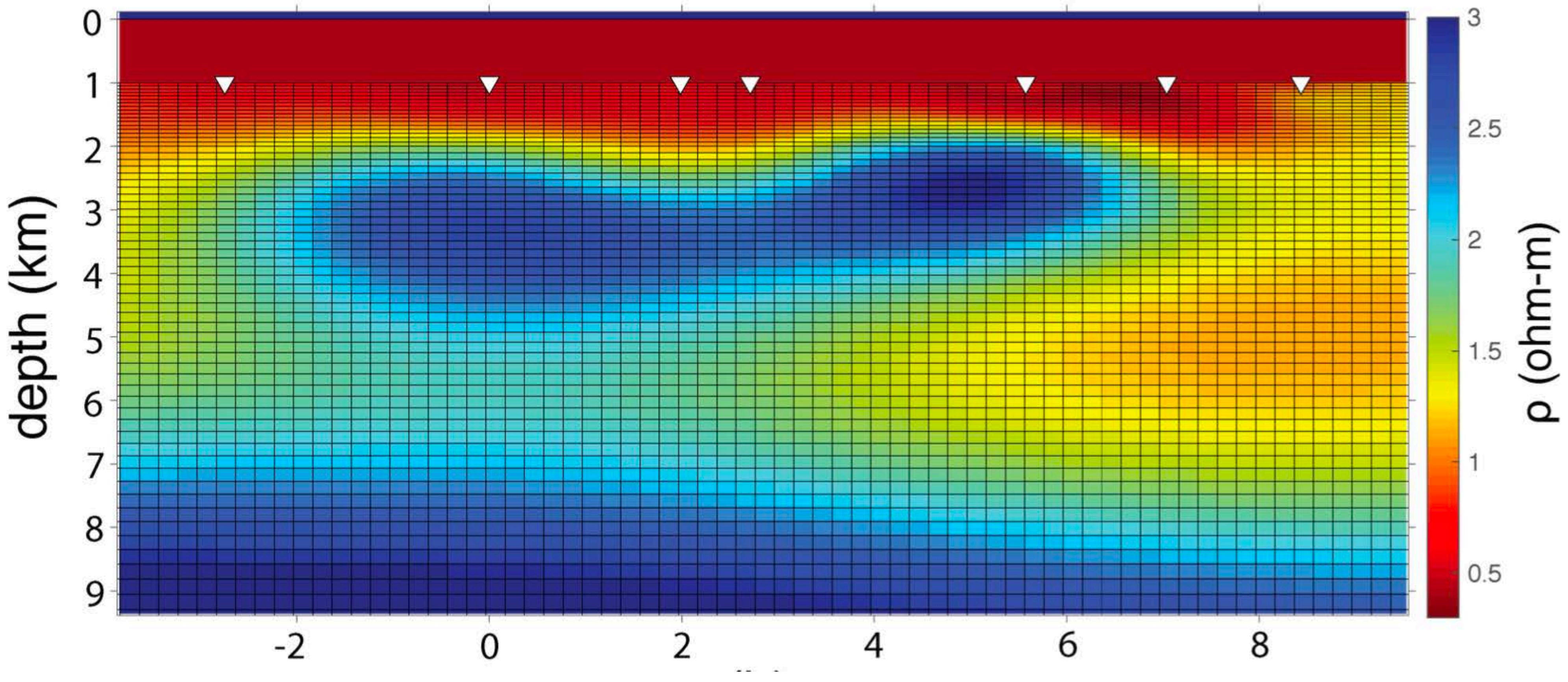


Tips:

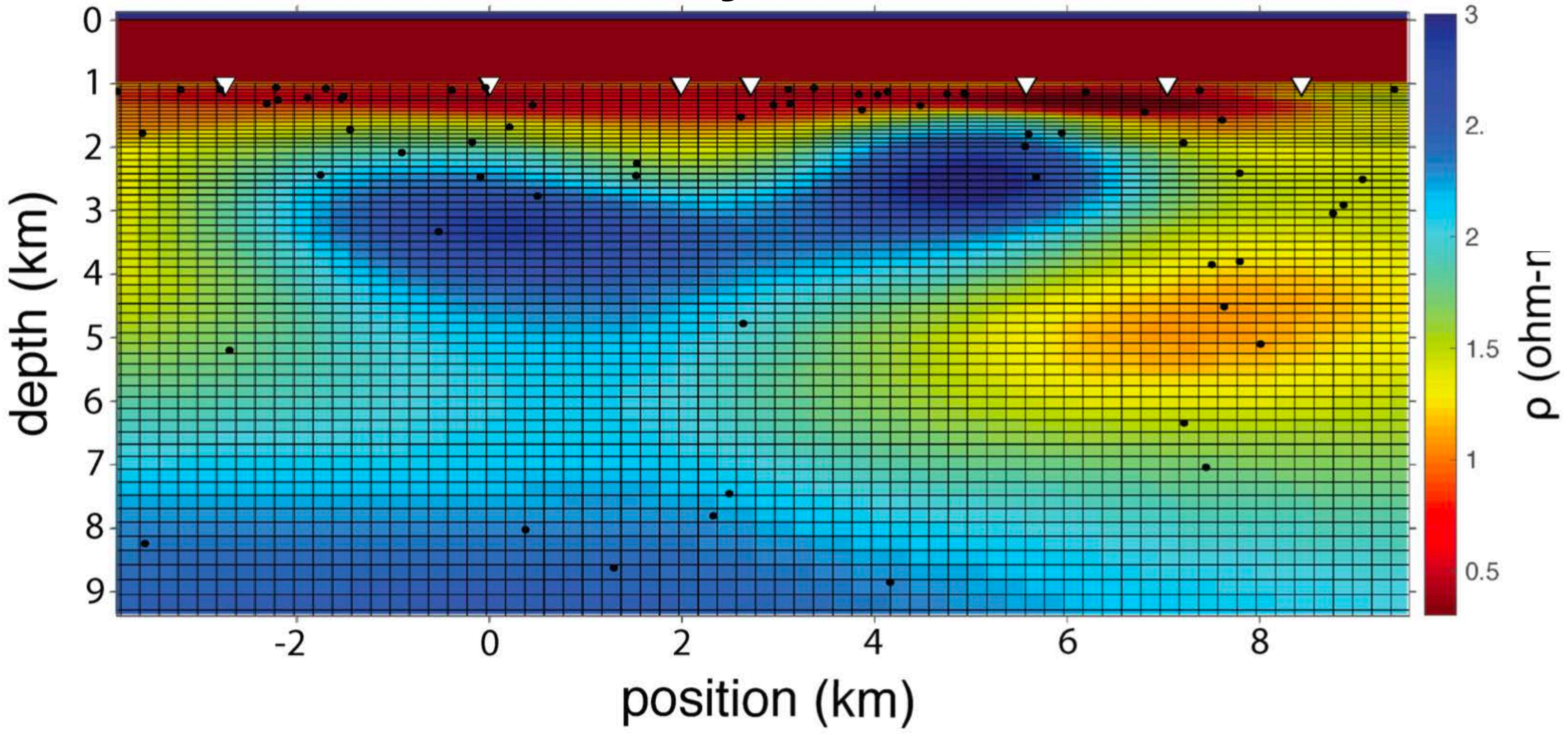
- MARE2DEM has many special features. Don’t use them!
- **Keep it simple**. Don’t use anisotropy, complex conductivity, finite dipoles, prejudice values, parameter bounds etc, unless you understand what you’re doing and have already run inversions without using these settings.
- Avoid making slivers (pinch-outs) in model segments.
- Plot the model and data files to check on the setup before submitting your job to the cluster queue.

Upcoming Add-on to MARE2DEM: Trans-dimensional Bayesian Inversion

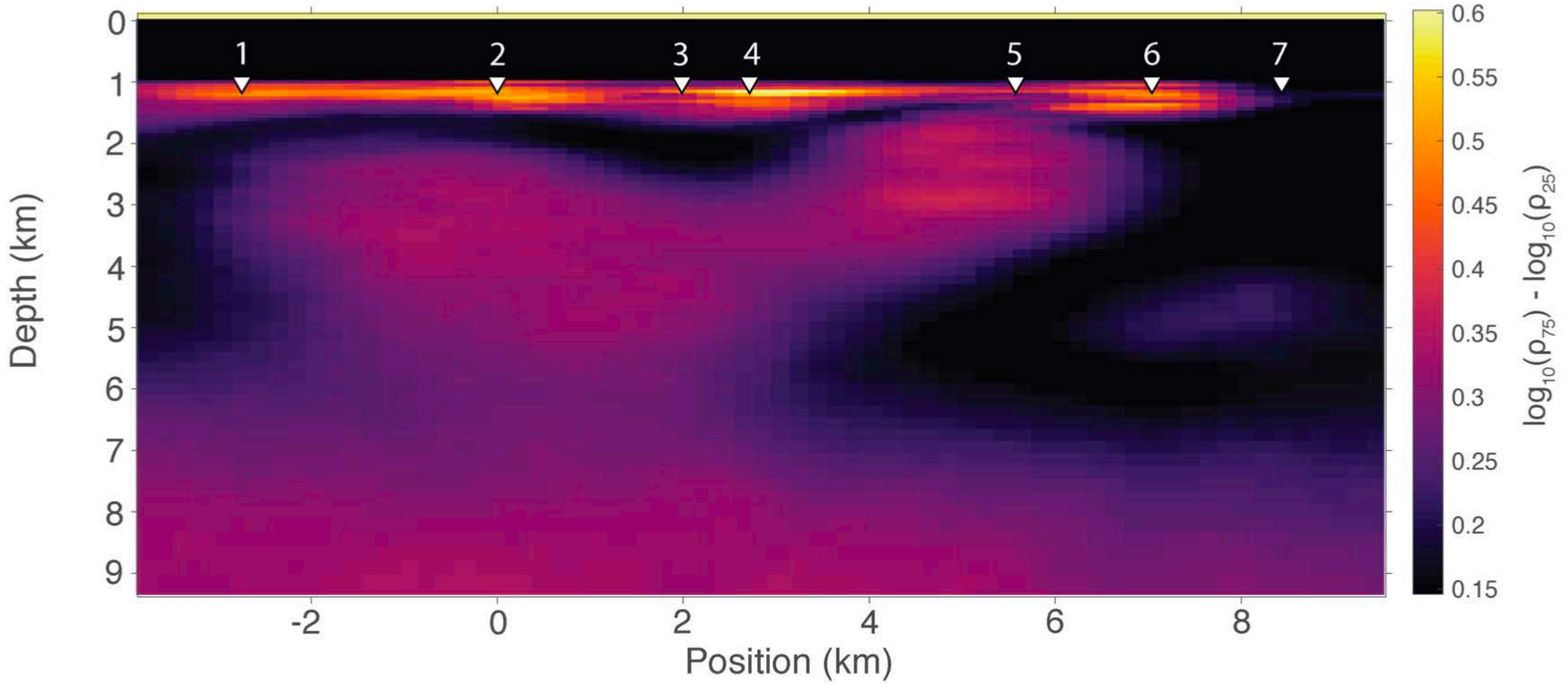
MARE2DEM inversion



Mean of Bayesian Ensemble



Interquartile range of ensemble



Daniel Blatter, Anandaroop Ray, Kerry Key, Two-dimensional Bayesian inversion of magnetotelluric data using trans-dimensional Gaussian processes, *Geophysical Journal International*, 2021;, ggab110, https://doi.org/10.1093/gji/ggab110

Work in progress and planned features

- Upgrade user interface using MATLAB's new App Designer
- Scriptable forward and inverse model construction (i.e. without UI)
- *Julia* library interface (load files, forward & inverse iterations)
- New scripts and UI for importing and reformatting MT responses into MARE2DEM format

Developer collaboration and community contributions encouraged!