Theory of electromagnetic induction in the Earth

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outline

inductive v. galvanic sources
quasistatic approximation
periodic-driven diffusion
boundary conditions
inductance
percolation
heterogeneity
energy flow
separability
similitude

acknowledgments

Nigel Edwards Steve Constable Bob Parker
Adam Schultz Alan Chave Chet Weiss +many others!

preliminary thought

difficult concepts are sometimes better understood when **learned or explained** in different ways

in words

"*an object at rest stays at rest until acted upon by a force*"'

in equations

$$m\frac{d^2x}{dt^2} = F_o\delta(t - t_0)$$

$$x(t) = 0;\ t < t_0$$

in pictures

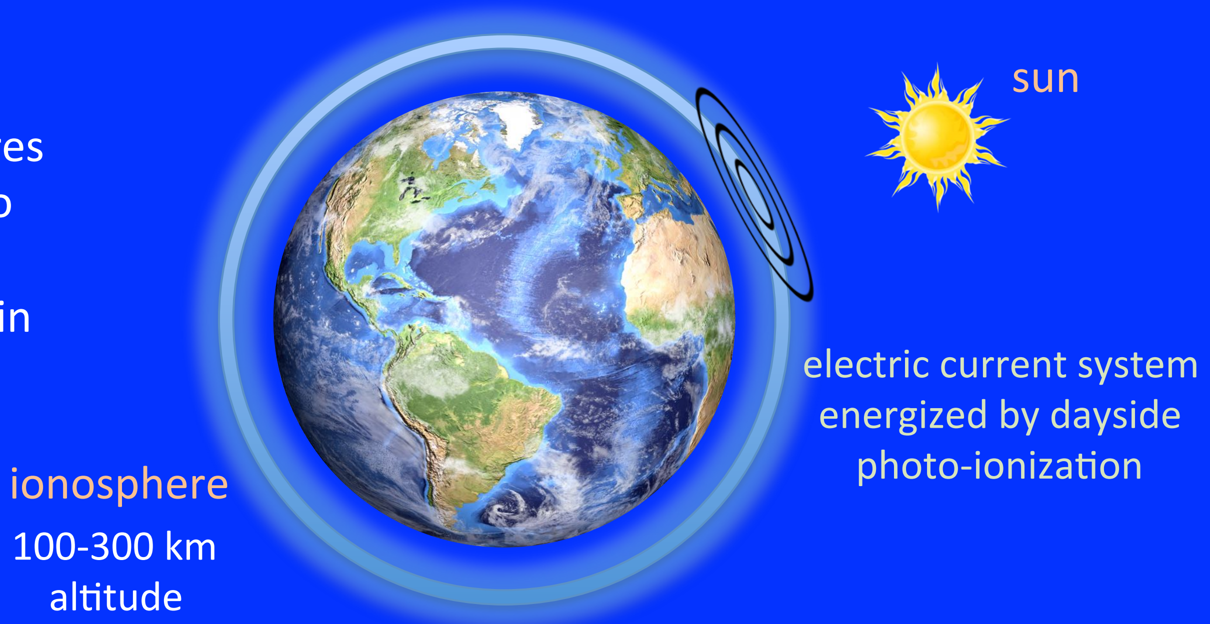
no one of these techniques is inherently superior in all respects

the electromagnetic induction method

time-variations in an external magnetic field
induce electric currents in the Earth

the external magnetic field can be natural or man-made:
passive vs. active-source methods

magnetotellurics measures the response of Earth to natural fluctuations of electric current systems in the ionosphere

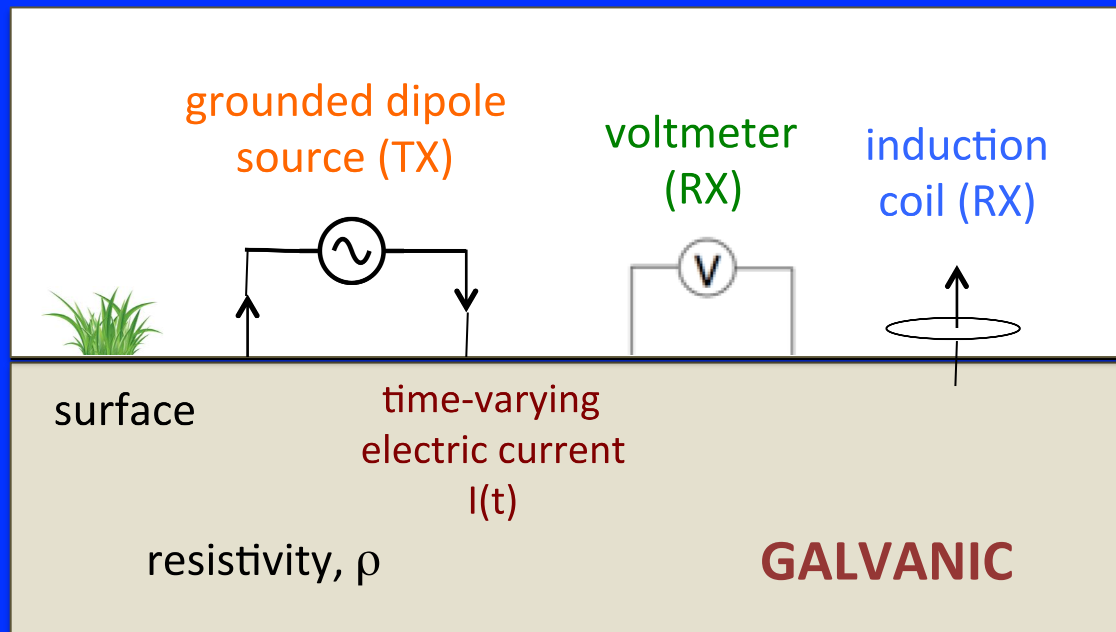


magnetotelluric data probes depth range of 10's to 100's km
(mid-lower crust to upper mantle; tectonics)

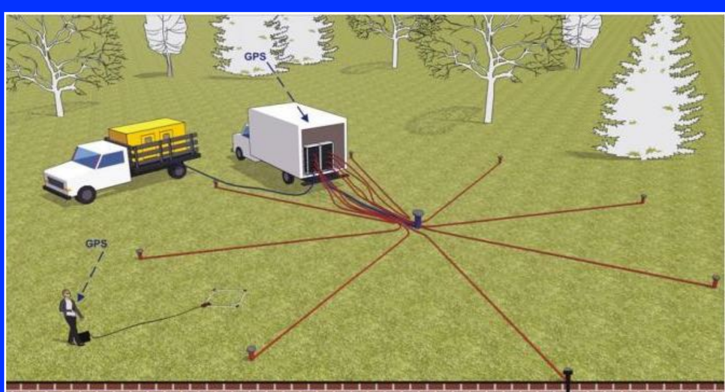
active or man-made sources

the two main types of active source
are galvanic and inductive

the grounded dipole source makes a direct electrical connection to the ground (as in ERT)



emtek.as



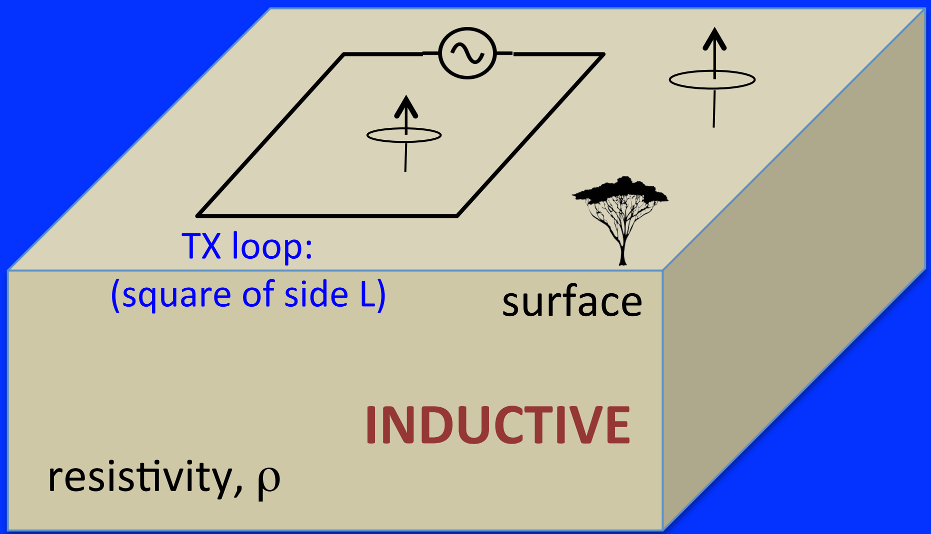
voltmeter measures voltage [V/m]
induction coil measures dB/dt [nT/s]

key difference between galvanic CSEM and ERT:
a time-varying electric current is used

inductive sources (1)

the inductive (loop) sources are
magnetically flux-linked to the ground

induction coil RX
can be placed inside
or outside the loop

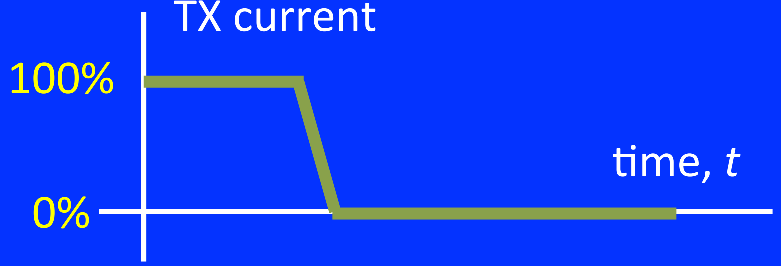


Aaron Micallef

TDEM exploration for groundwater in Malta with 10 m x 10 m TX loop and centrally-located induction coil RX

TDEM= time-domain electromagnetic method

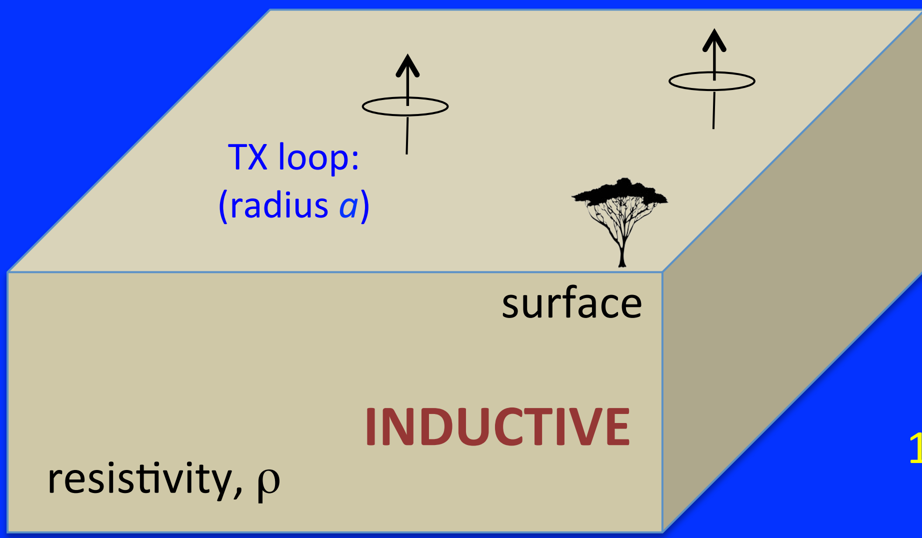
time-varying current can be an **abrupt linear ramp-off**

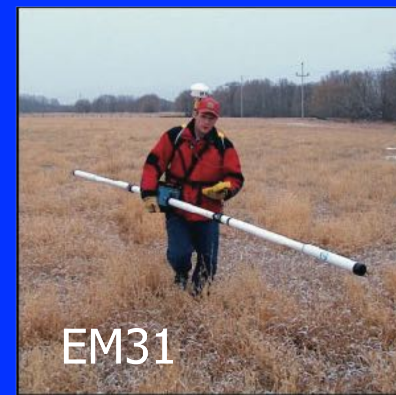


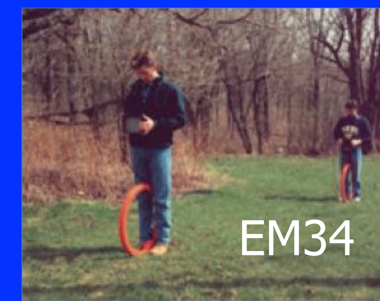
inductive sources (2)

portable loop-loop systems operating in the frequency domain

induction coil RX
an be placed inside
or outside the TX loop

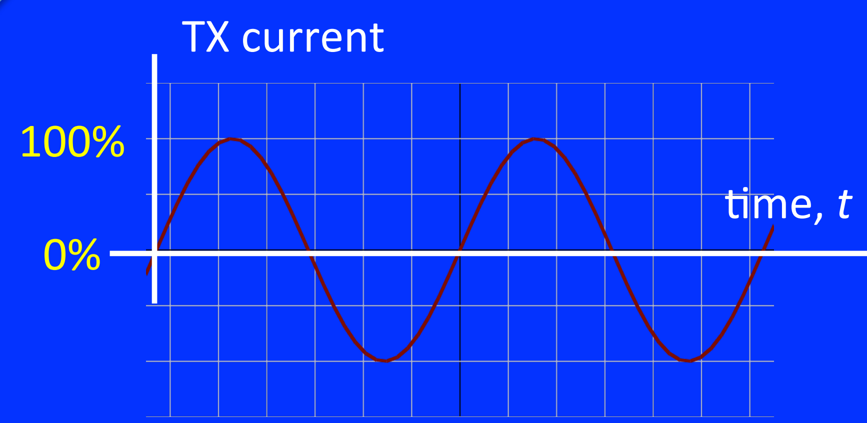






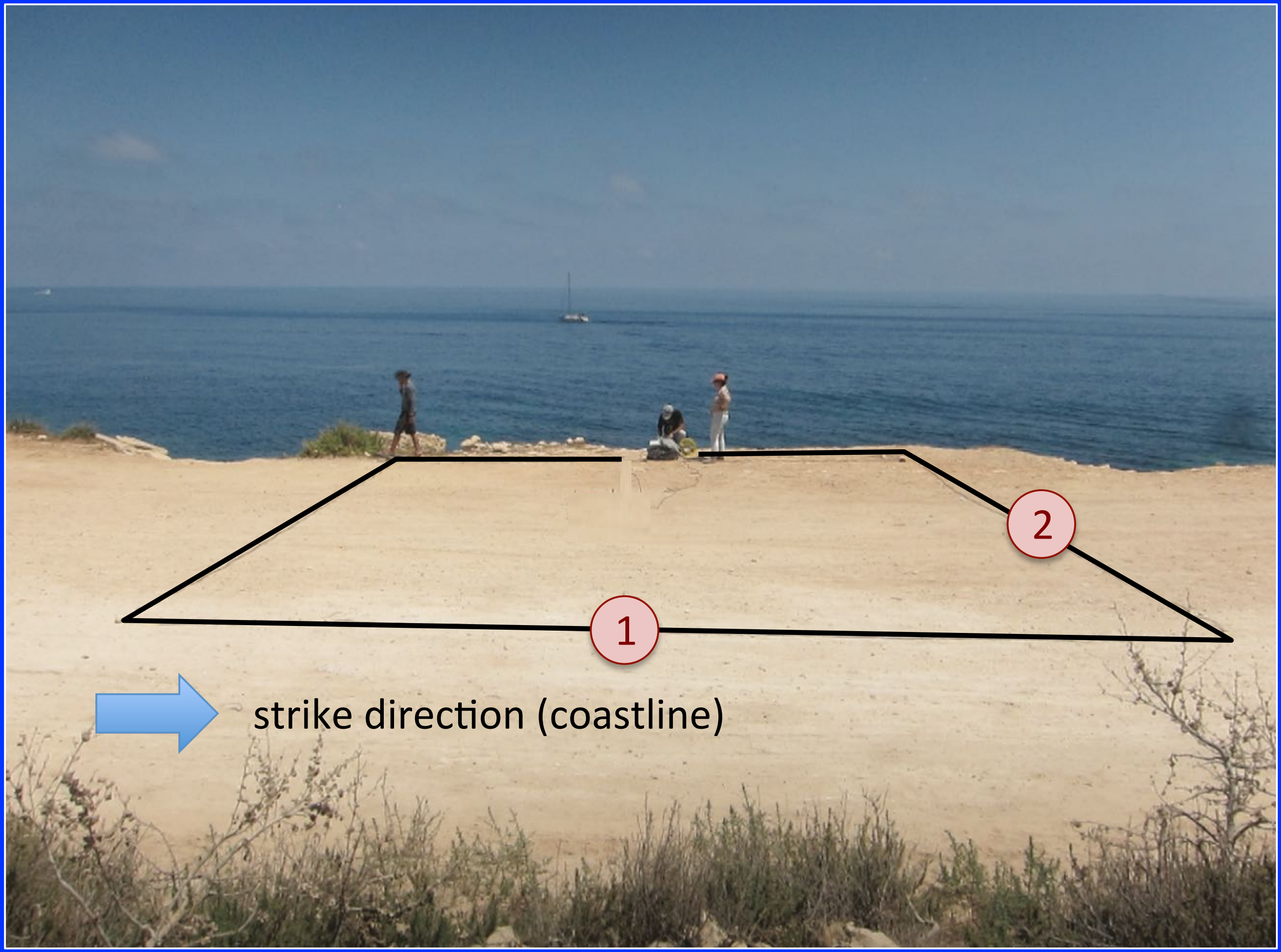
Geonics terrain conductivity meters

EMI= frequency-domain electromagnetic method



time-varying current is a **sinusoid of frequency ~10 kHz**

magnetotellurics is a controlled-source EM method
with a very large loop carrying a fluctuating current



Brad Weymer

ant-1 is making TE measurements

ant-2 is making TM measurements

tiny ants are working under the wire

magnetoquasistatic (MQS) approximation

quasistatic regime: the physical length scale is small compared to the EM wavelength λ at frequency *f*

f=1 Hz   λ=0.75 Earth-moon distance
f=1 kHz   λ=300 km
f=1 MHz   λ=300 m

EM wave velocity in water is roughly one-ninth that of light in vacuo, so that a wave propagates across a 10 km wide survey area in 300 µs

MQS approximation: velocity of EM wave is infinite

$$c \rightarrow \infty$$

EM wave at every location within the survey area has essentially the same phase, which is that of the transmitter
(entire Earth "breathes" along with the source)

magnetoquasistatic (MQS) approximation

consequences of infinite EM wave velocity

$$c = \frac{1}{\sqrt{\mu\varepsilon}} \quad \text{if} \quad c \to \infty \quad \text{then} \quad \varepsilon \to 0$$

electrical permittivity ε vanishes in the quasistatic formulation with the implications:

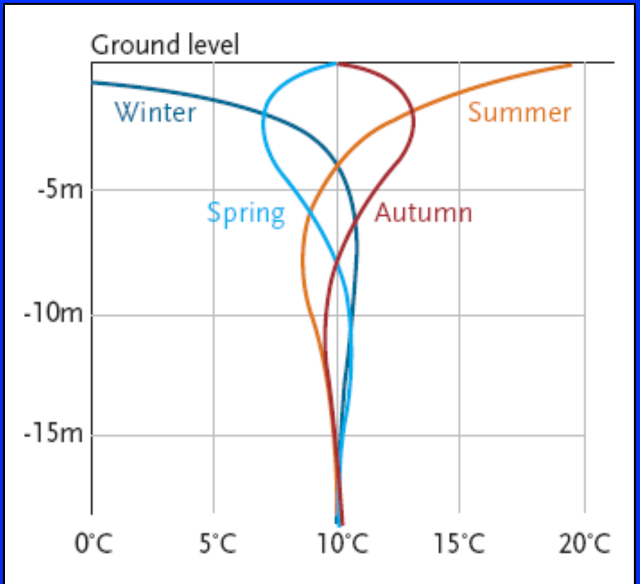
1. permittivity governs charge polarization, i.e. response of bound charges to an applied electric field, so that bound charge response is negligible
2. permittivity sets the scale for Coulomb force between charges, thus Coulomb's law is neglected
3. displacement current scales with permittivity, thus displacement curent (and hence EM wave) is neglected

capacitive effects are ignored
(present in reality but not in MQS approximation)

periodic-driven diffusion

EM induction is a diffusion process regardless of the regularity of the transient disturbance that generates it

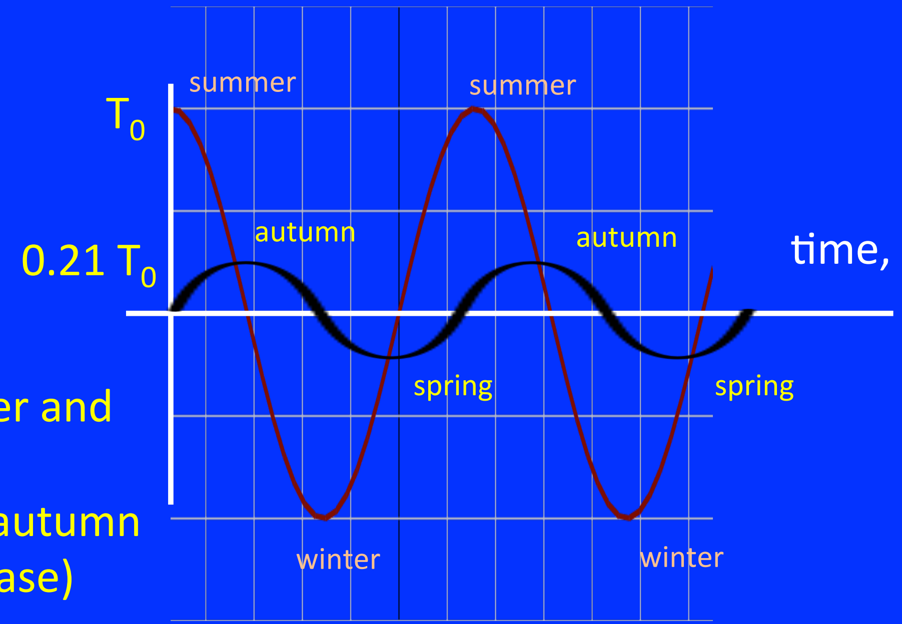
heat-conduction analogy: seasonal temperature variations in soil



UK annual soil temperatures

$$T(z,t) = T_0 \exp(-z/\delta) \exp[i(\omega t - z/\delta)]$$

soil temperature



temperature at surface is max in summer and minimum in winter

temperature at $\pi/2$ skindepths is max in autumn and minimum at spring (90^0 out of phase)

phase varies linearly with depth

no wave physics involved!

instantaneous response everywhere

step-off response of a line source in a uniform wholespace

the solution to the **damped-wave equation** including displacement current:

$$E(\rho,t) = \frac{\mu_0 I}{2\pi} \frac{\mathrm{H}(t-\sqrt{\mu_0\varepsilon_0\rho^2})}{\sqrt{t^2-\mu_0\varepsilon_0\rho^2}} \exp\left(-\frac{\sigma t}{2\varepsilon_0}\right) \cosh\left[\frac{\sigma}{2\varepsilon_0}\sqrt{t^2-\mu_0\varepsilon_0\rho^2}\right]$$

the appearance of the Heaviside function H indicates that the solution does not begin at t=0, but only after the speed-of-light propagation time

the solution to the **diffusion equation** without displacement current:

$$E(\rho,t) = \frac{\mu_0 I}{4\pi t} exp\left(-\frac{\mu_0\sigma\rho^2}{4t}\right)$$

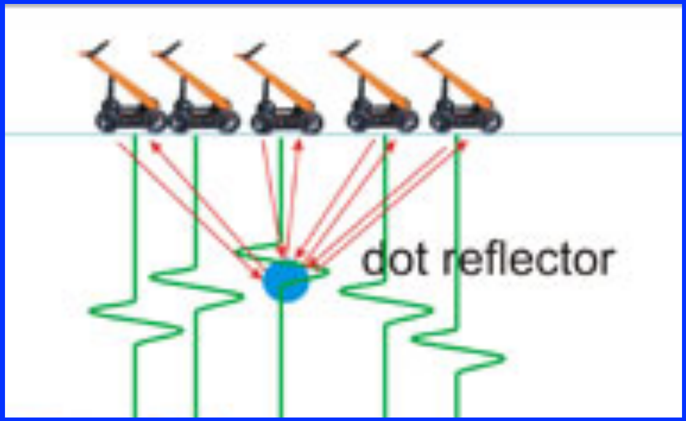
the diffusive response at all offsets begins immediately at t=0

$$t \gg \sqrt{\mu_0\varepsilon_0\rho^2}$$

displacement current can be safely neglected if all measurements are made at the times indicated above

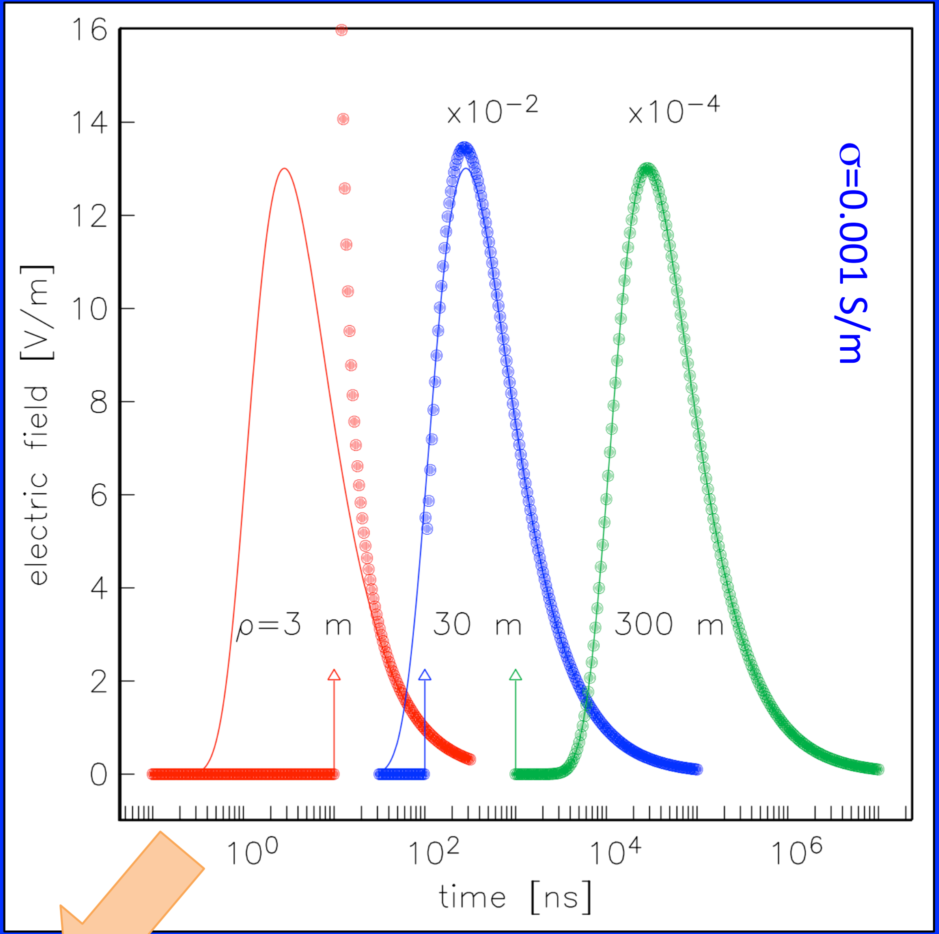
instantaneous response everywhere

electromagnetic damped wave propagation into the ground (GPR)



signals begin a finite time after TX generation

lines: diffusion
symbols: damped-wave

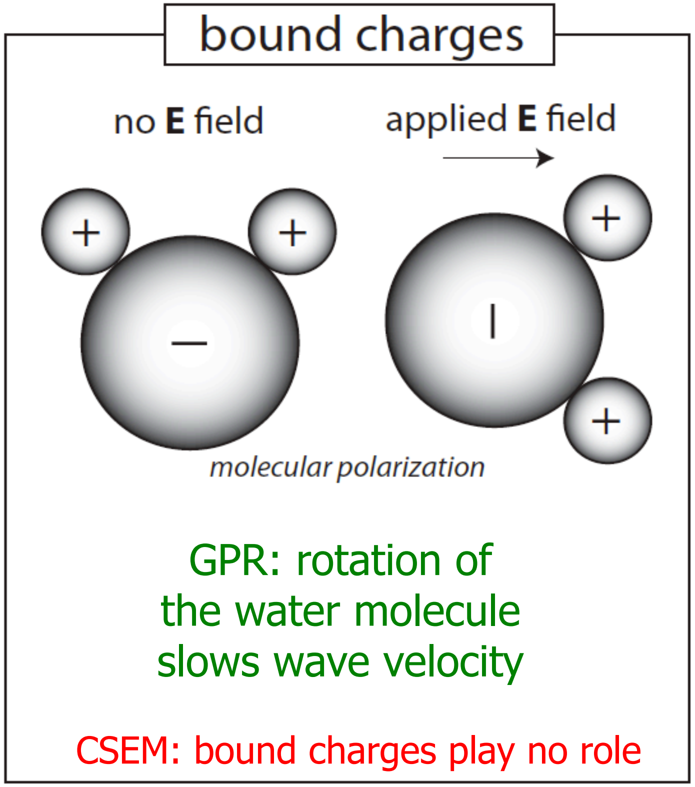


after Oristaglio and Hohmann 1989

arrows indicate "first break"

MQS approximation is excellent if measurements are made at times during which the two curves coincide some charge carriers are free or quasi-free to migrate, or drift, from place to place within the geological medium; other charges are bound to lattice atoms or material interfaces

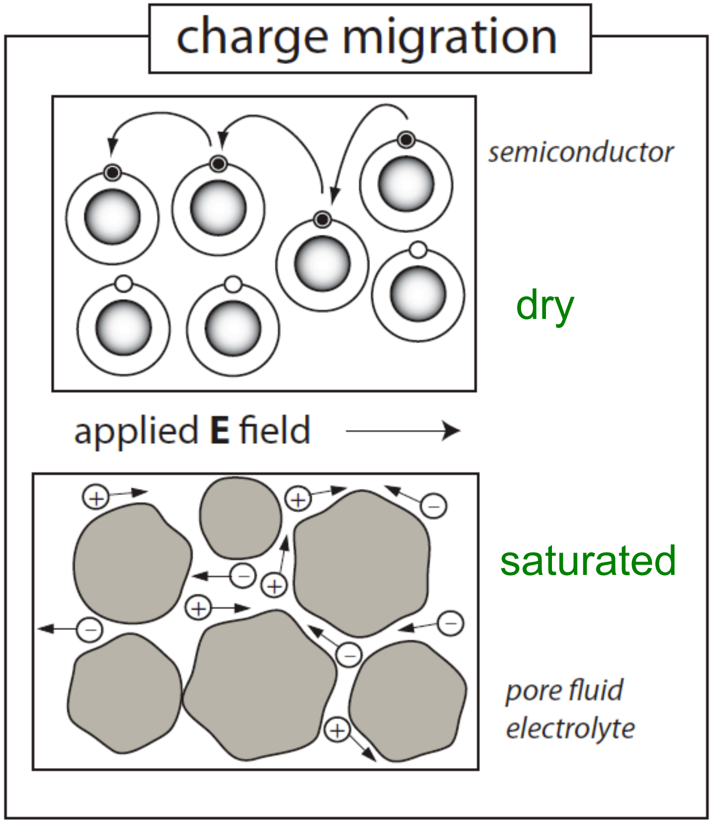
GPR only:



GPR: rotation of the water molecule slows wave velocity

CSEM: bound charges play no role

CSEM and GPR:



EM induction: all ionic charge carriers are presumed to have the same mobility

otherwise, self-potentials and streaming potentials, etc. are generated

"the world of EM-induction is a world in which all charges are mobile"

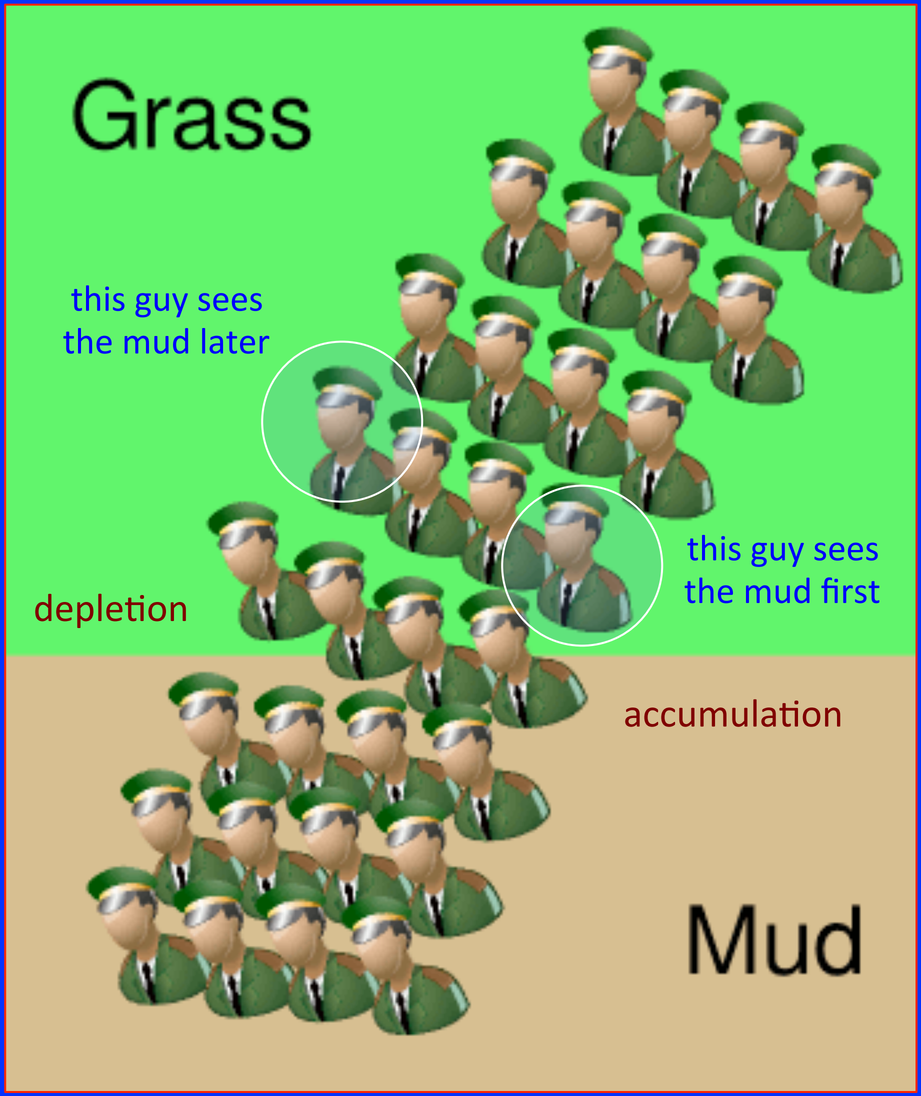
boundary conditions

wave refraction and reflection
at material boundaries obey
Fermat's least time principle but
in MQS waves propagate
with infinite velocity in each medium

at conductivity boundaries there is
a change in the mobility of ionic
charge carriers

diffusion across an interface generates
an accumulation of charge carriers
on one side and a depletion
on the other side

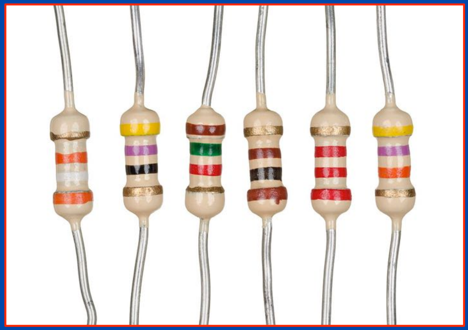
EM induction cannot be
explained by wave physics



line of marching cadets refracting
at a material discontinuity

RLC circuits

R=resistor

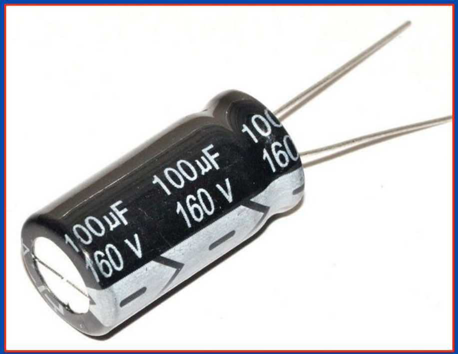


resistors are designed to reduce electric current flow in a circuit

L=inductor

inductors are designed to prevent rapid changes of electric current flow in a circuit

C=capacitor



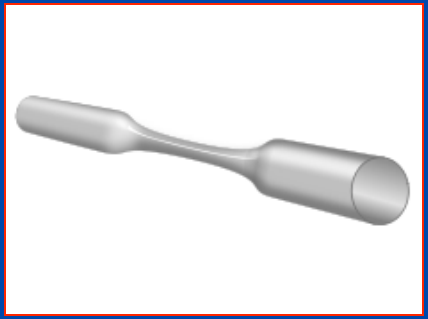
capacitors are designed to filter out slow changes of electric current flow in a circuit

Earth can act as a resistor, inductor and capacitor

hydraulic analogies of RLC circuit elements

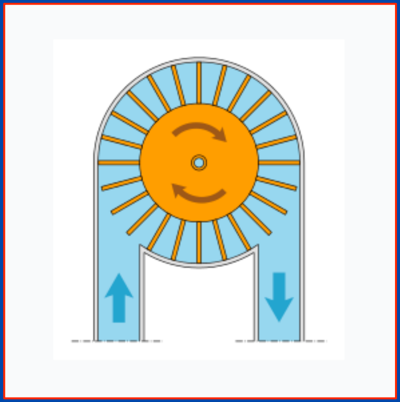
how is electric current flow in a circuit similar to water flow in a pipe?

R=resistor



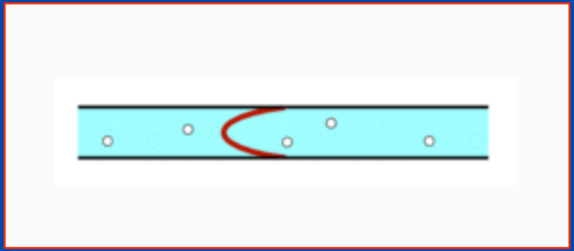
a resistor is equivalent to a constriction in a pipe: more pressure is required to pass the same amount of water as an unconstricted pipe

L=inductor



an inductor is equivalent to a paddle wheel: the heavy blades restrict the water's ability to rapidly change its rate of flow due to the effects of inertia

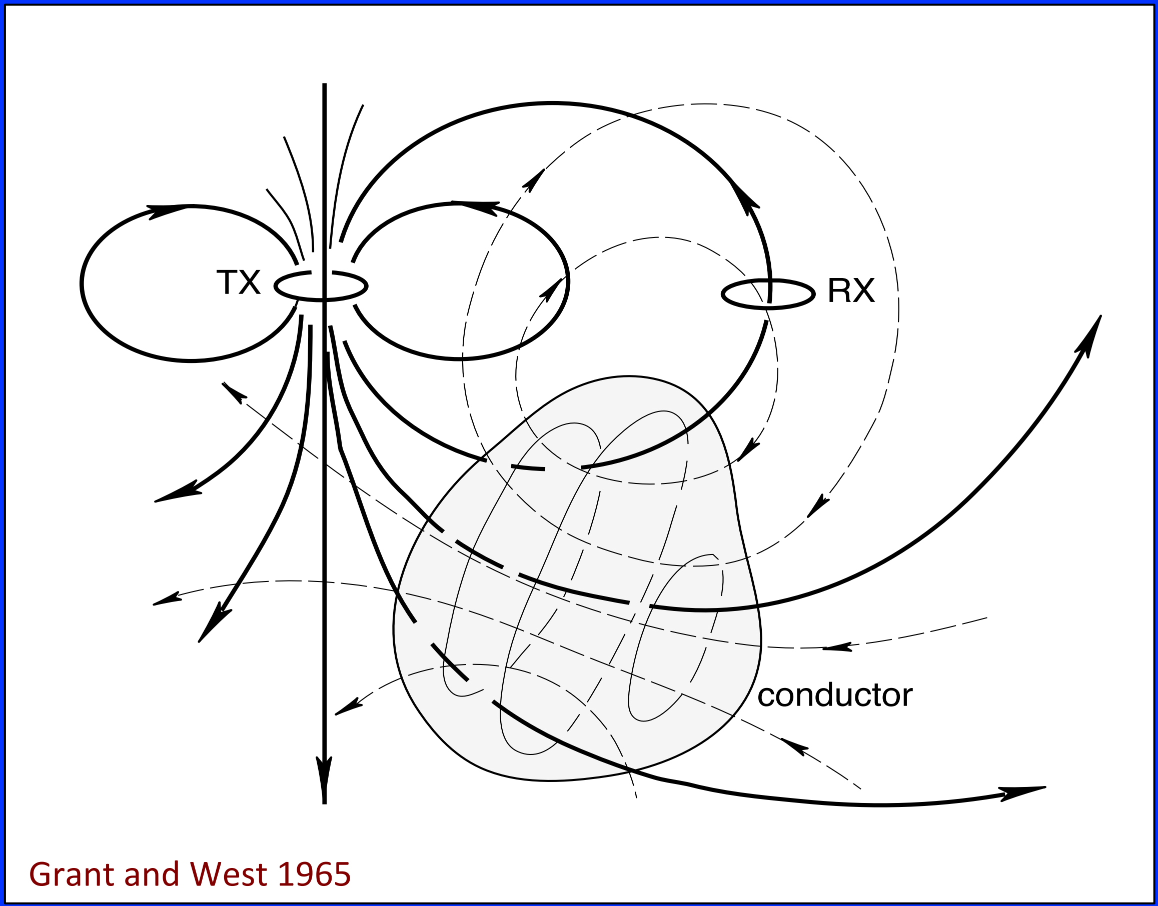
C=capacitor



a capacitor is equivalent to a rubber sheet sealed inside a pipe: water pressure can stretch the sheet but no water penetrates it

wikipedia

basic physics of electromagnetic induction



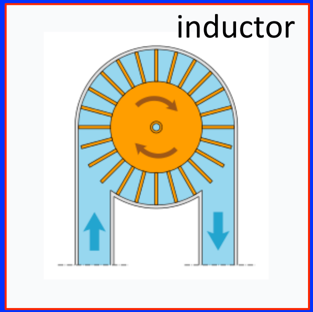
airport metal detector

keys

self-inductance

self-inductance measures the tendency of a conductive body
to oppose a sudden change in electric current flowing through it

inertia is the resistance of any physical object to any change in its velocity



a heavy paddle wheel has inertia:
it tends to resist a sudden change in fluid flow past it

$$F = m\frac{dv}{dt}$$

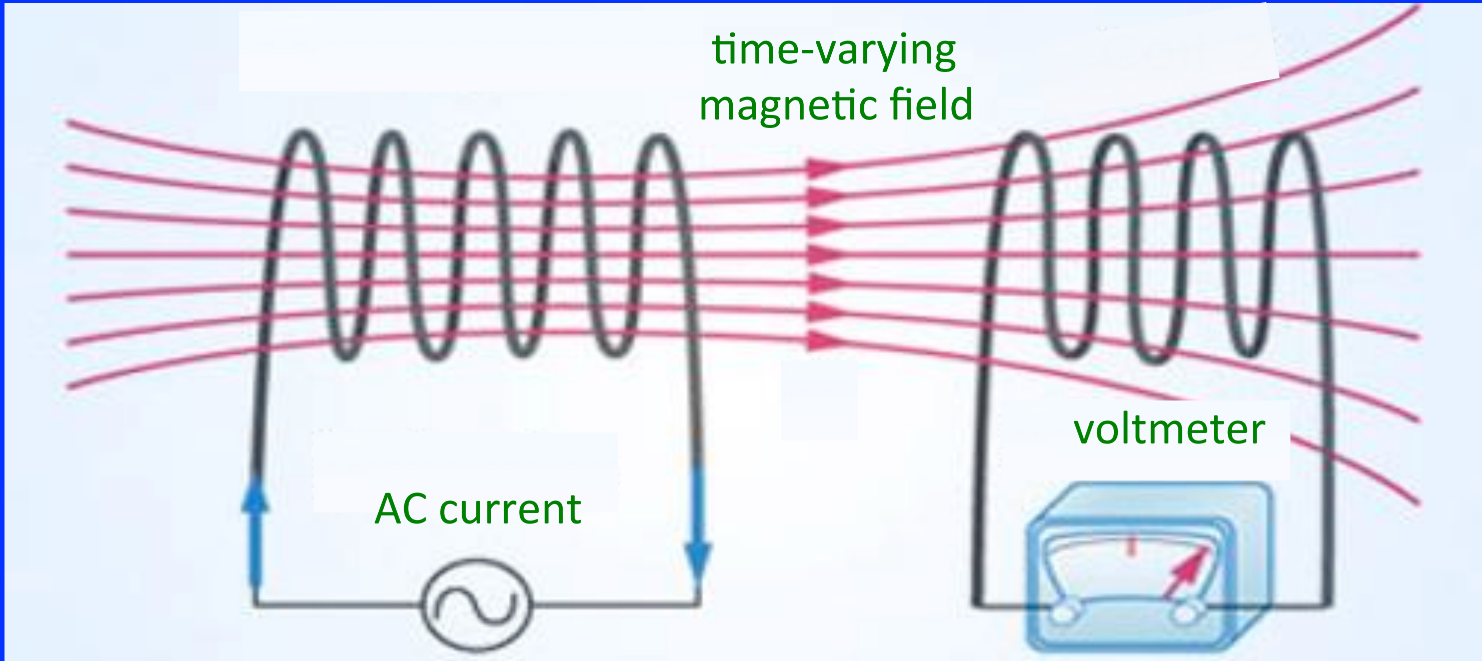
Newton's law: the mass carries inertia such that a force is required to achieve a change in velocity

$$V = L\frac{dI}{dt}$$

Faraday's law: the inductor carries inertia such that a voltage is required to achieve a change in current

mutual inductance

mutual inductance describes the
magnetic coupling between two conductors

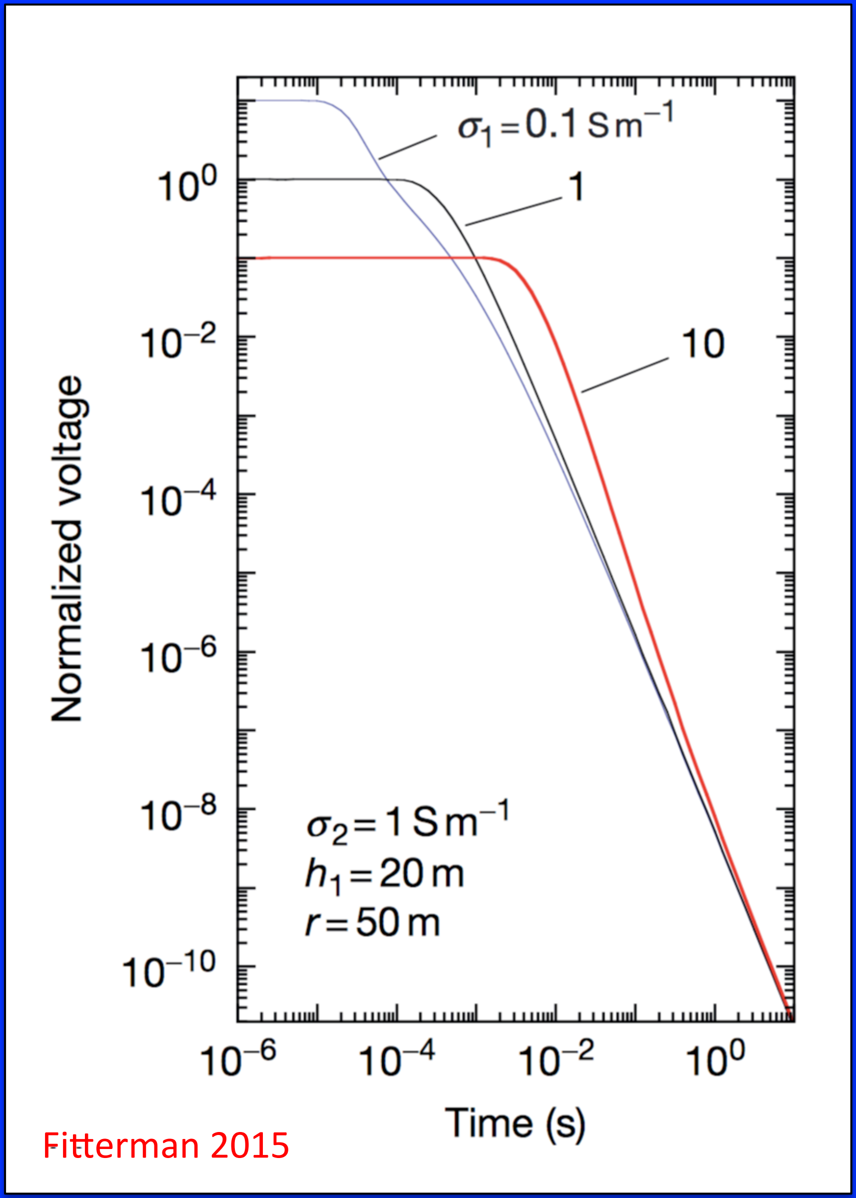


changing electric current in loop 1 induces a voltage in loop 2

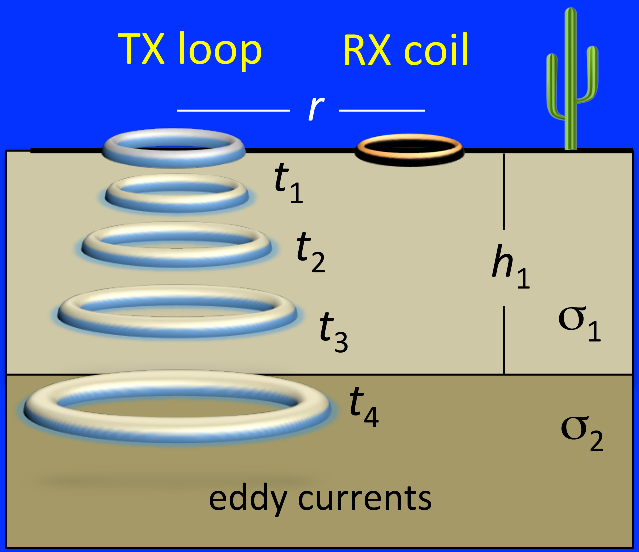
$$M_{21} = \frac{\Phi_{21}}{I_1}$$

mutual inductance between loops 1 and 2 equals the magnetic flux through loop 2 due to time-varying current flowing in loop 1

how does this apply to geophysics?



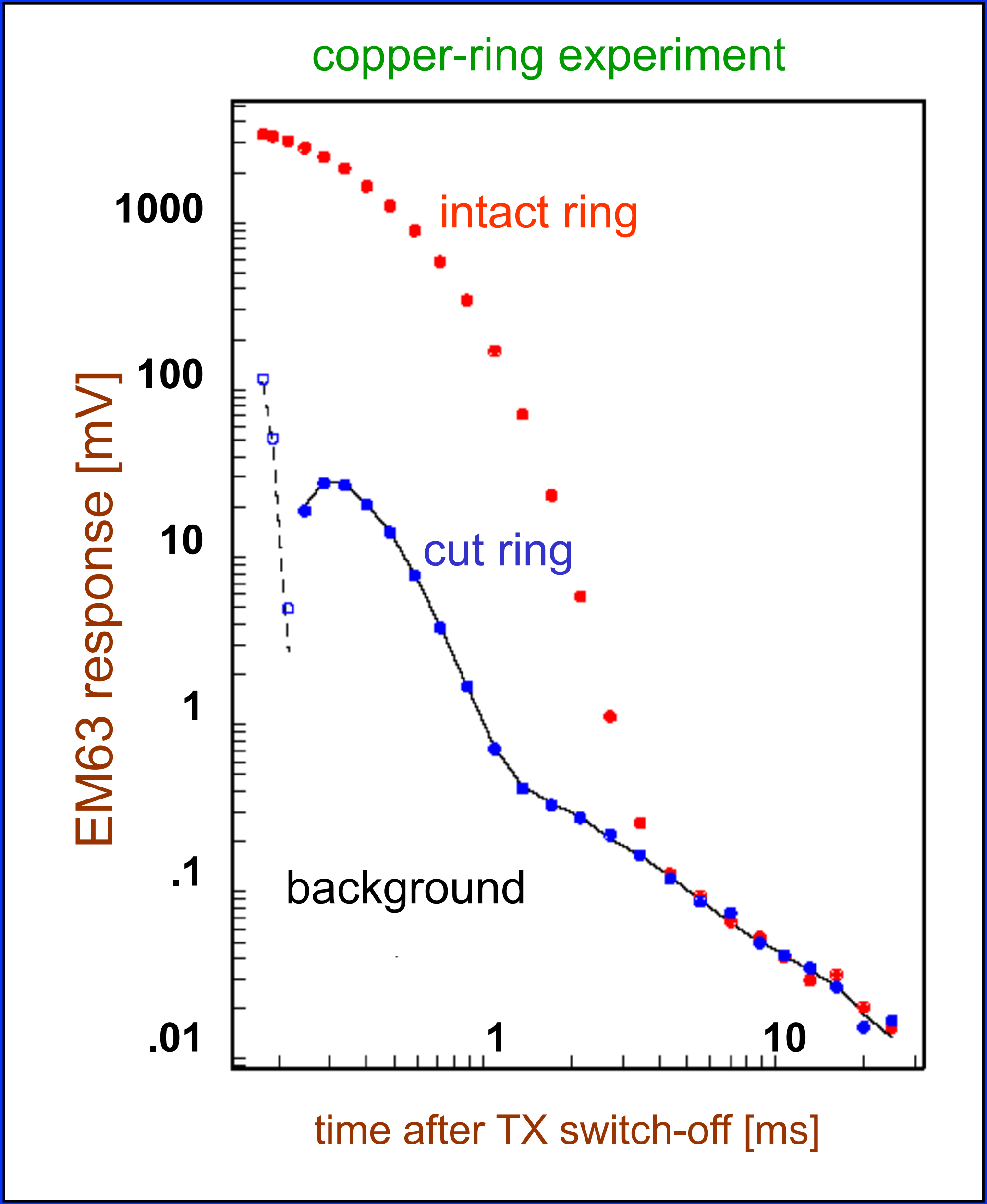
mutual inductance between TX loop and Earth causes eddy currents to flow



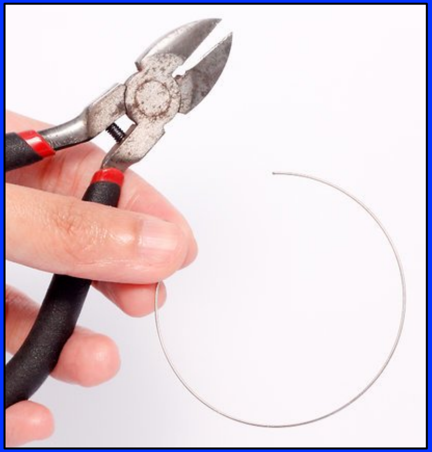
the RX voltage decays with time after TX shutoff as eddy currents dissipate

the shape of the RX-voltage curve is diagnostic of Earth's electrical conductivity

EM induction responses measure inductance, not just electrical conductivity



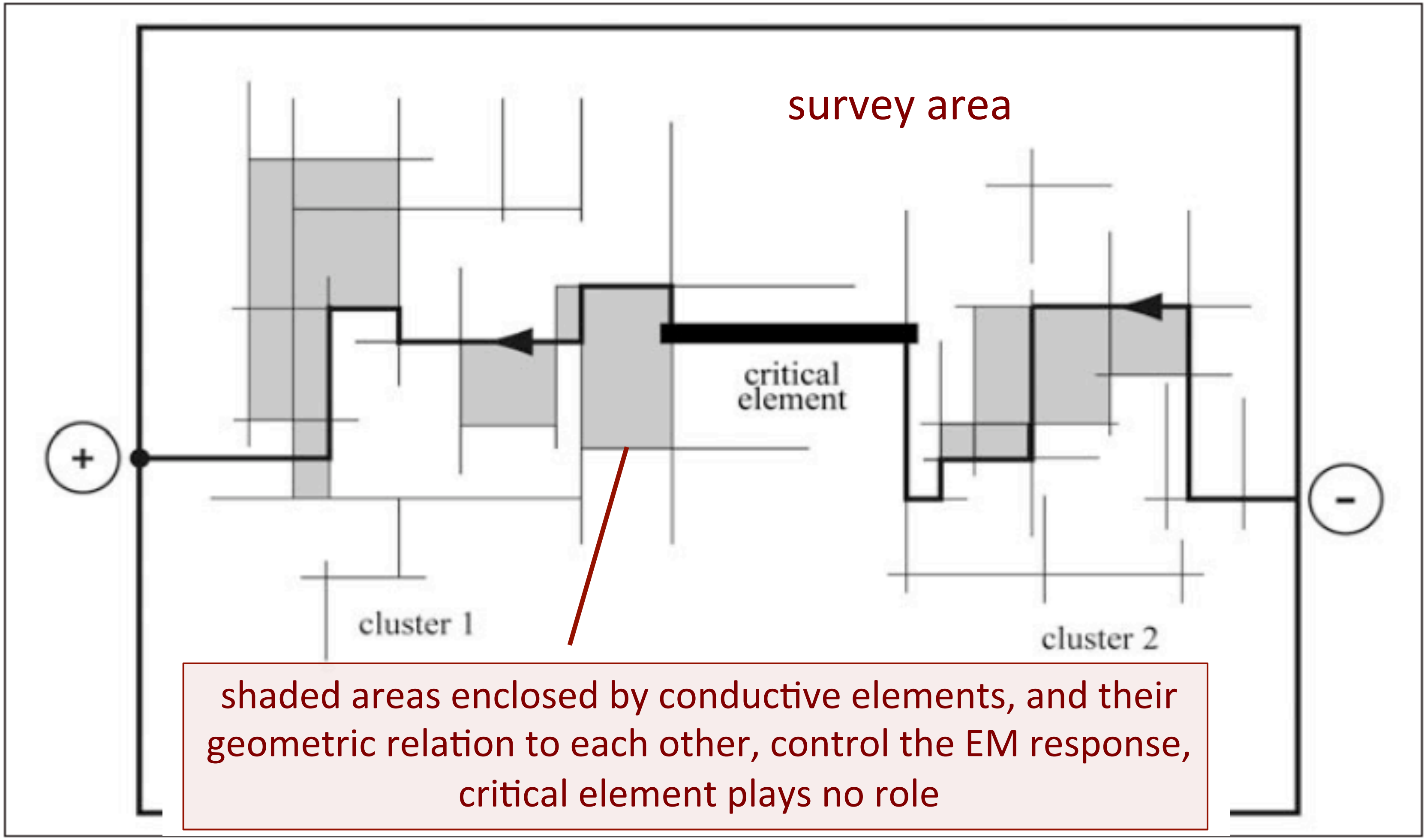
Geonics EM63 time-domain metal detector



cut ring

percolation theory has often been advanced to explain electromagnetic responses

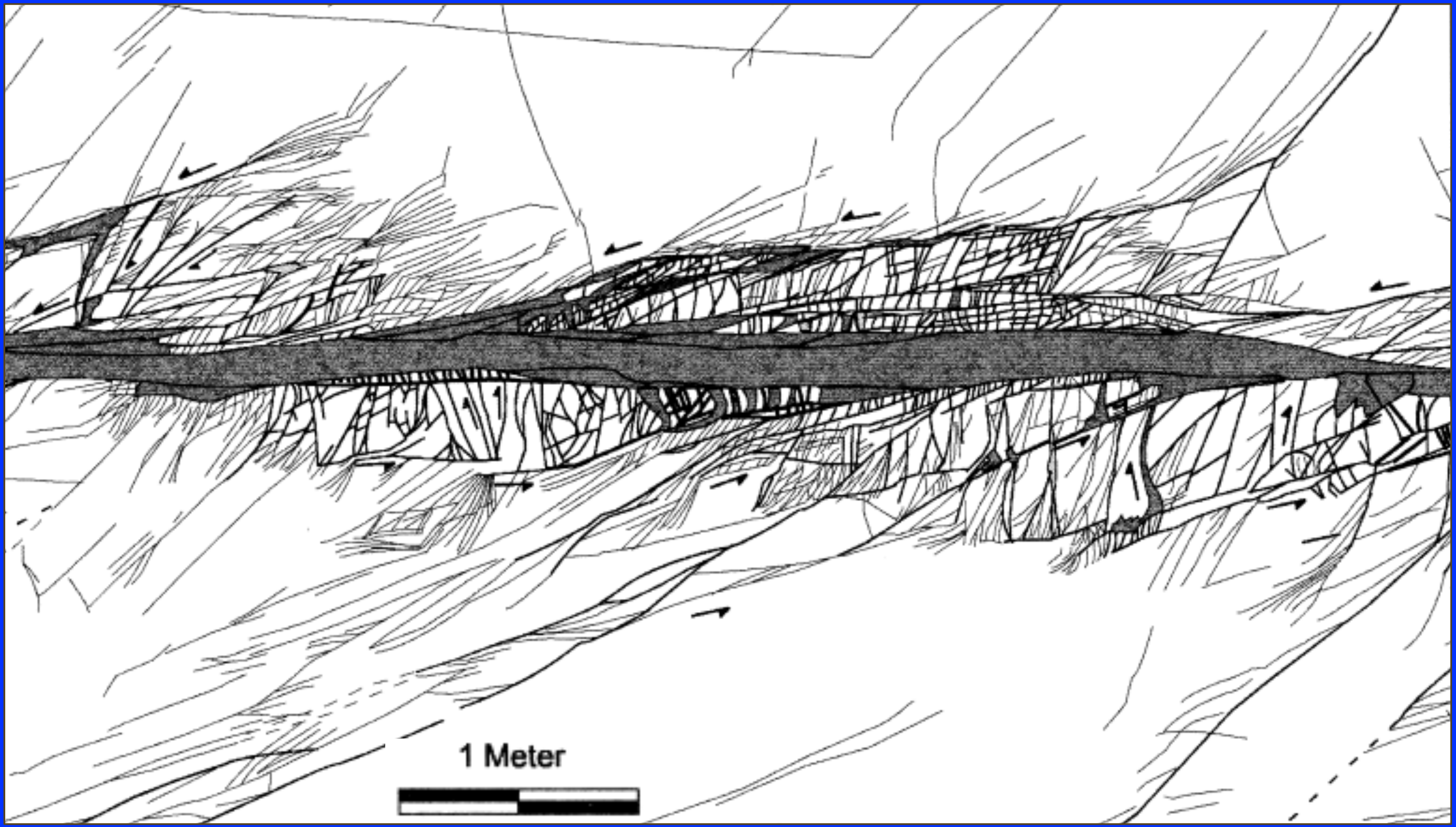
Everett 2005



shaded areas enclosed by conductive elements, and their geometric relation to each other, control the EM response, critical element plays no role

a "through-going" current must close somewhere outside the survey area

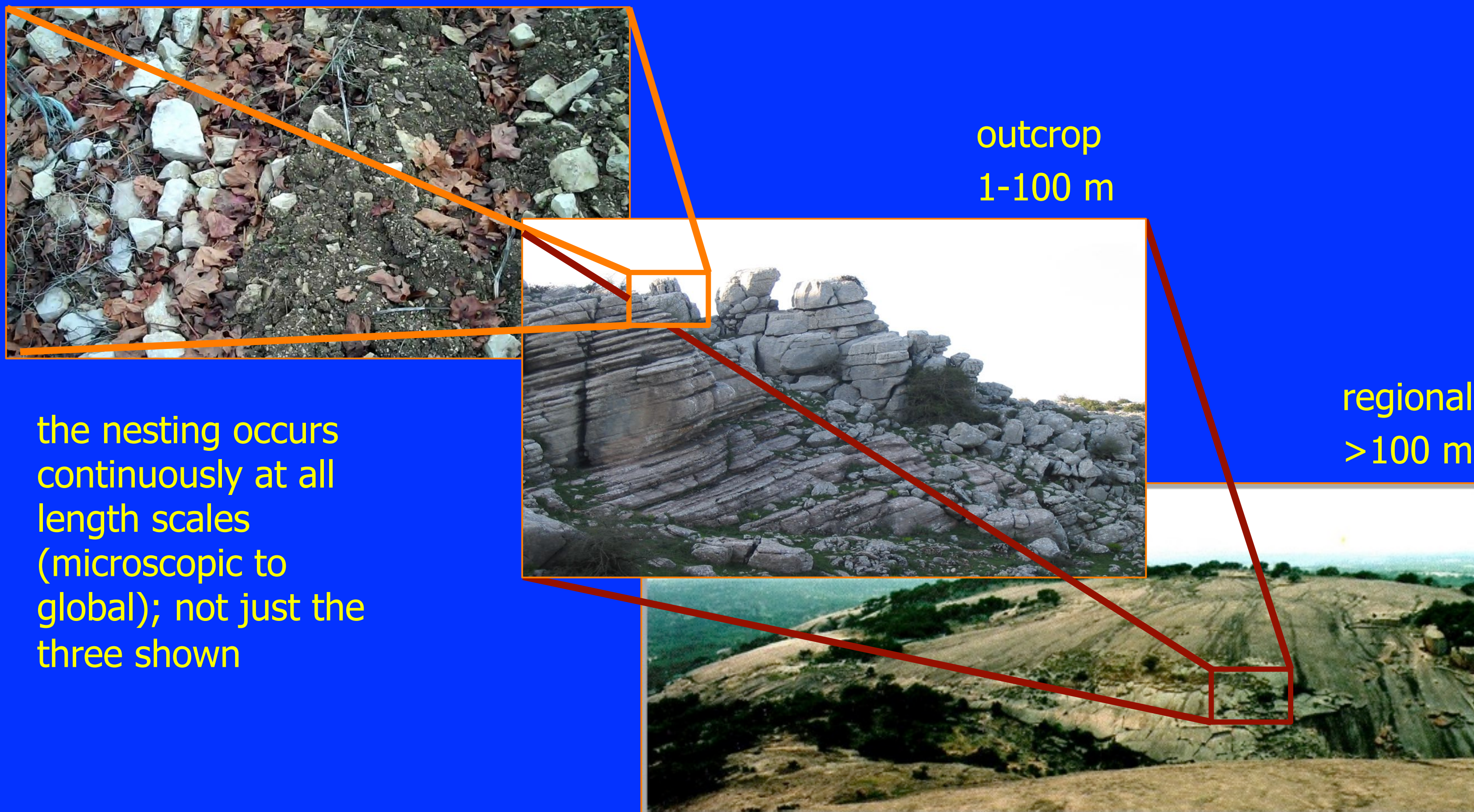
Earth is also a complex system



sketch of a 1-m-wide fault core surrounded by a 2-m-wide damage zone, sandstone formation, SE Nevada

Aydin 2000

nested levels of heterogeneity

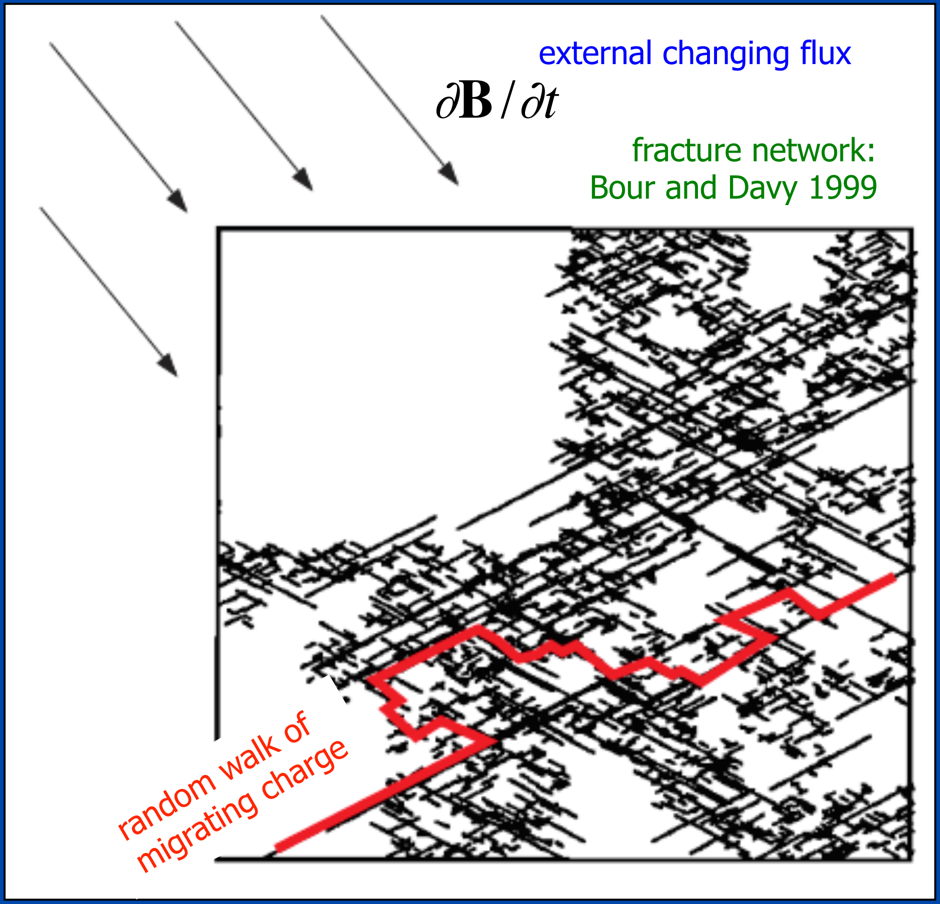


the nesting occurs continuously at all length scales (microscopic to global); not just the three shown

different 3-D spatial patterns continuously appear and disappear as you "zoom in" or "zoom out" (non-fractal)

EM induction in presence of length-scale dependent heterogeneity

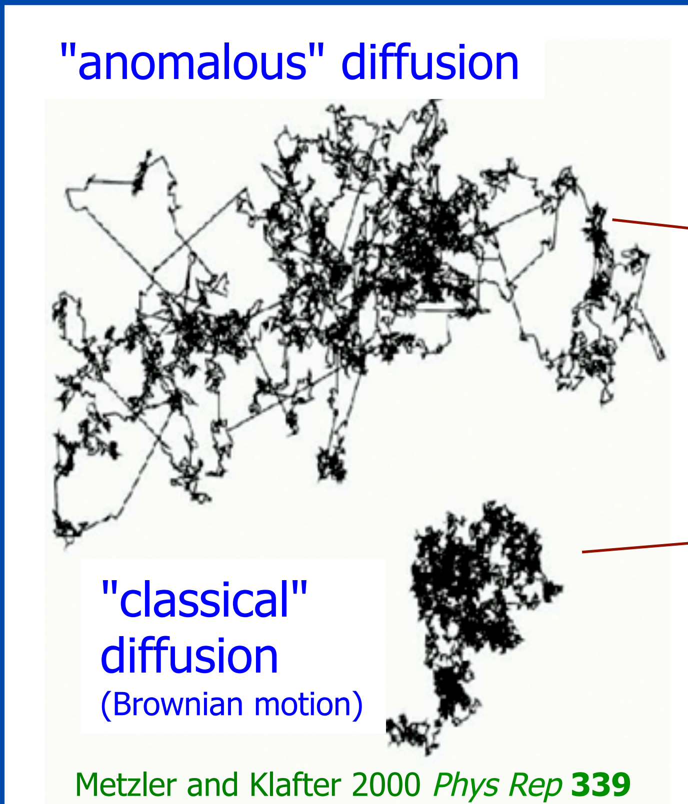
can we develop a theoretical understanding?



this is not a classical diffusion path if the jump lengths are not Gaussian distributed

porosity << 70%

CSEM responses should be diagnostic of the geometry of induced currents



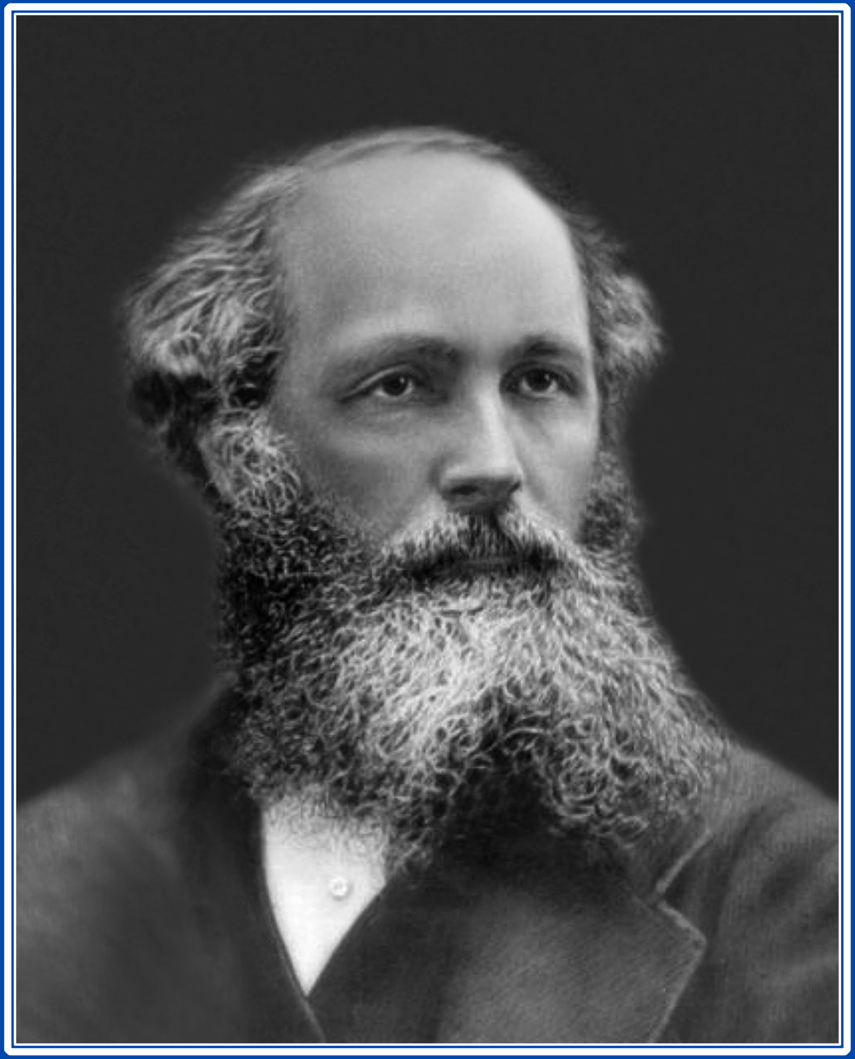
mean-squared displacement

subdiffusion ⟷ superdiffusion

$$\langle x^2(t) \rangle \sim t^{\beta}, \quad 0 < \beta < 2$$

$$\langle x^2(t) \rangle \sim t$$

anomalous diffusion may be generated by random walk through a confined, tortuous geometry



James Clerk Maxwell (1831-1879)

(a classical physicist who did not consider materials exhibiting length-scale dependent heterogeneity)

conductivity as a rough PDE coefficient

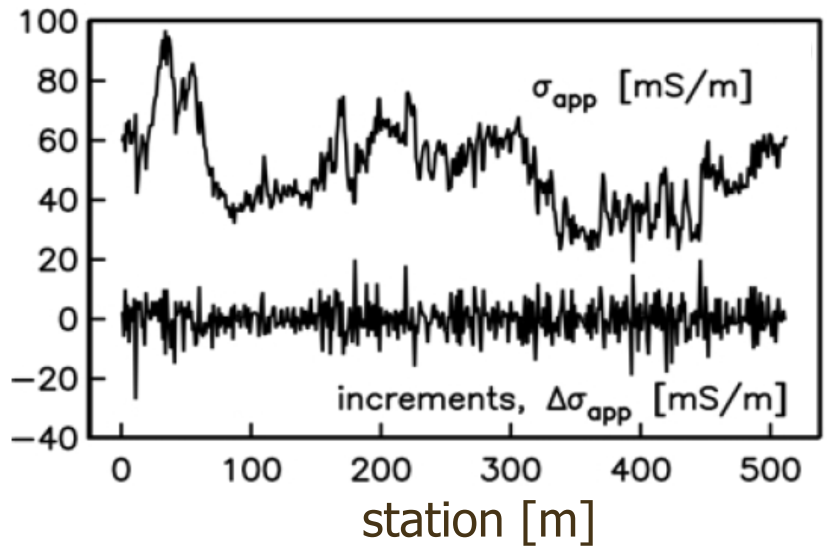
$$\nabla \times \nabla \times \mathbf{E} + \mu_0 \sigma(\mathbf{r}) \frac{\partial \mathbf{E}}{\partial t} = 0$$

CSEM responds to a "spatial average" of the subsurface conductivity

solutions to elliptic PDE's are "one degree smoother" than their coefficients

conductivity appears to be "very rough" so we expect CSEM response to be "quite rough"

EM34 data from Brazos River (TX) floodplain



fractional Maxwell equation to describe anomalous diffusion of induced currents

$$\nabla \times \nabla \times \mathbf{E}(t) = -\mu_0 \sigma_\beta \, {}_0D_t^{1-\beta} \mathbf{E}(t) - \mu_0 \frac{\partial \mathbf{J}_S}{\partial t}$$

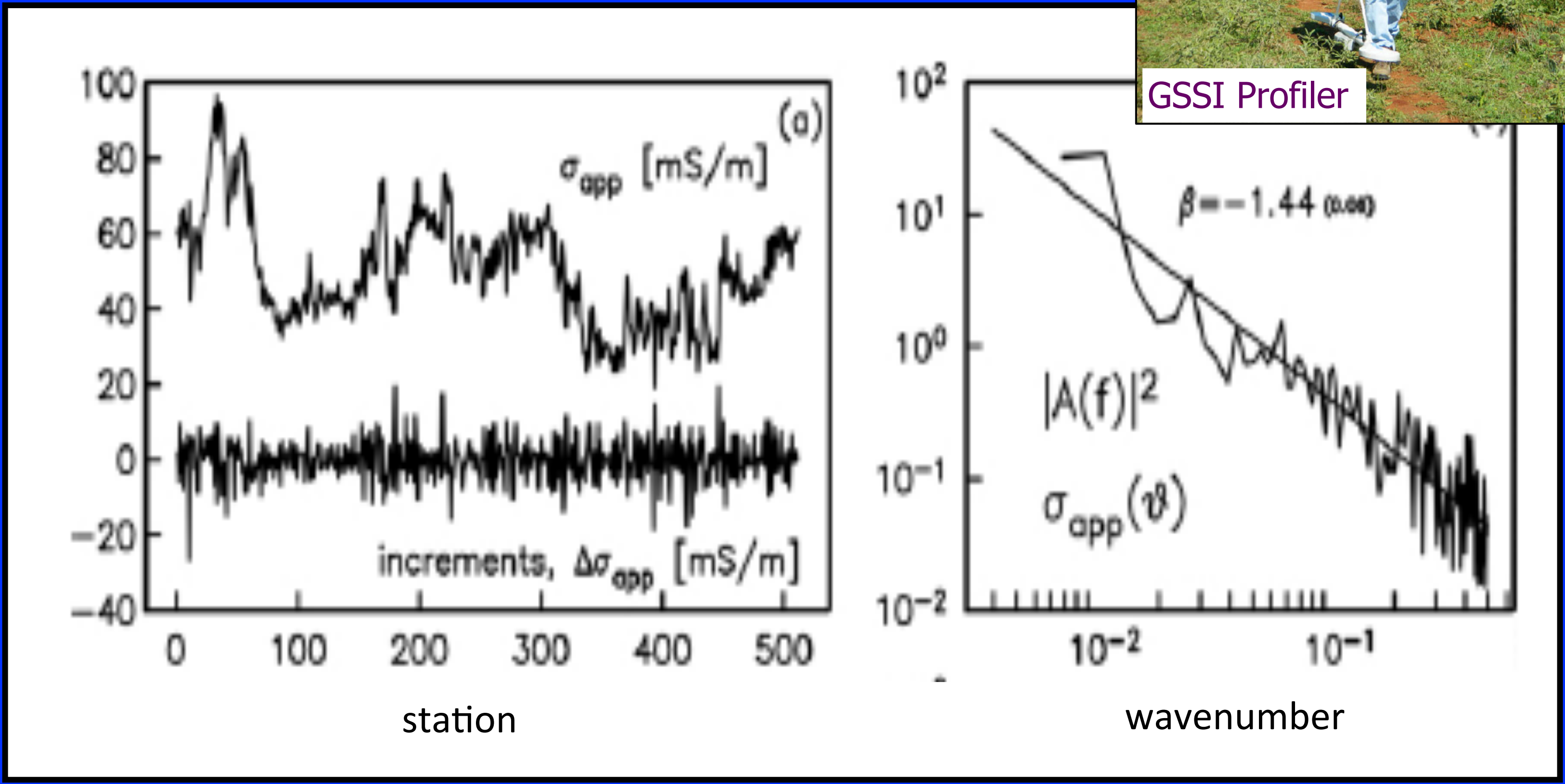
fractional derivative

Weiss and Everett (2007)

$${}_0D_t^{1-\beta} \mathbf{E}(t) = \frac{1}{\Gamma(\beta)} \frac{\partial}{\partial t} \int_0^t \frac{\mathbf{E}(t')dt'}{(t-t')^{1-\beta}}$$

Michele Caputo proposed similar ideas in elasticity

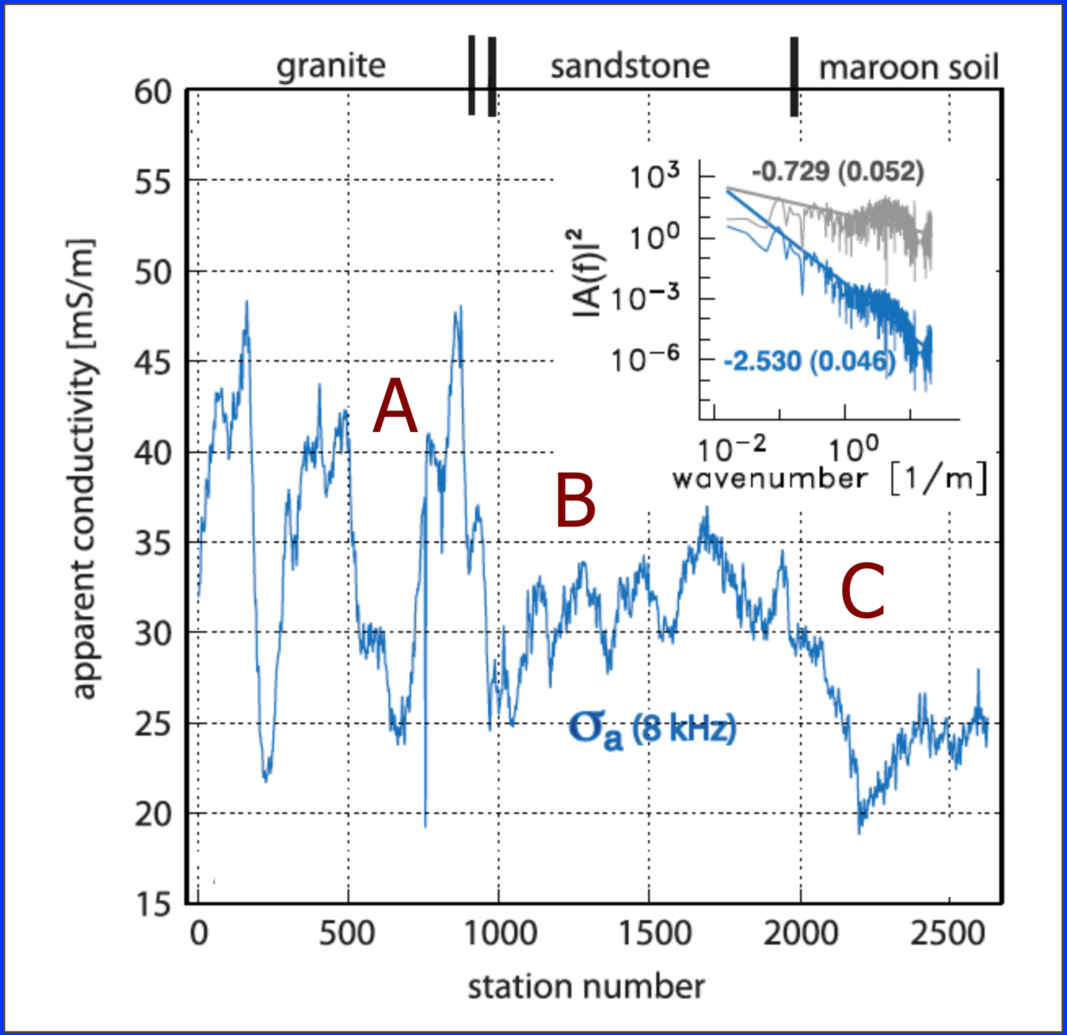
geological media exhibit
length-scale dependent heterogeneity with
persistent (long-range) correlations…



Everett and Weiss 2002

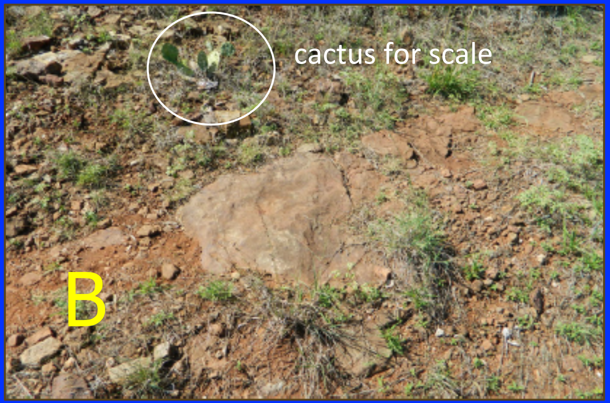
$1/f^{\alpha}$ wavenumber spectra of frequency-domain EM responses

CSEM responses of different lithologies



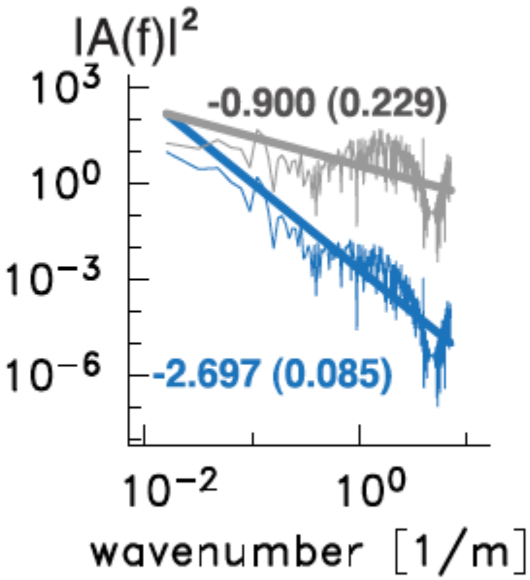
Beskardes, Weiss and Everett 2017

Llano Uplift, TX

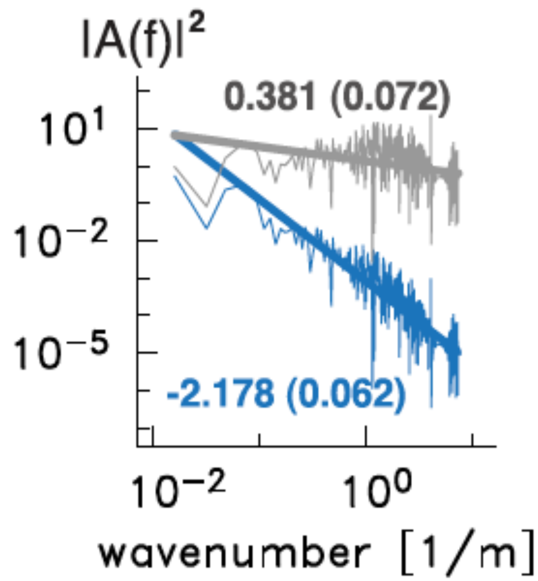


different fBm signals in the different lithologies

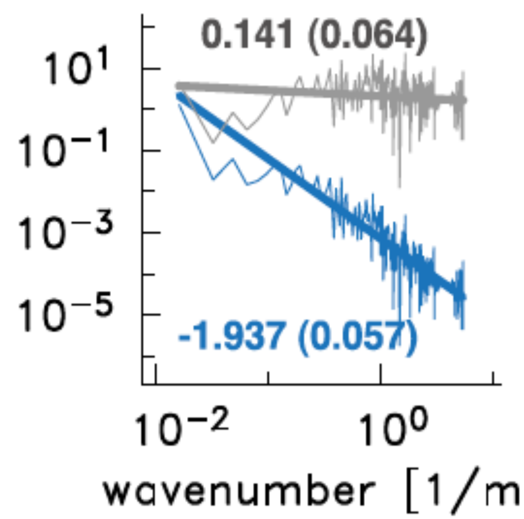
granite



sandstone



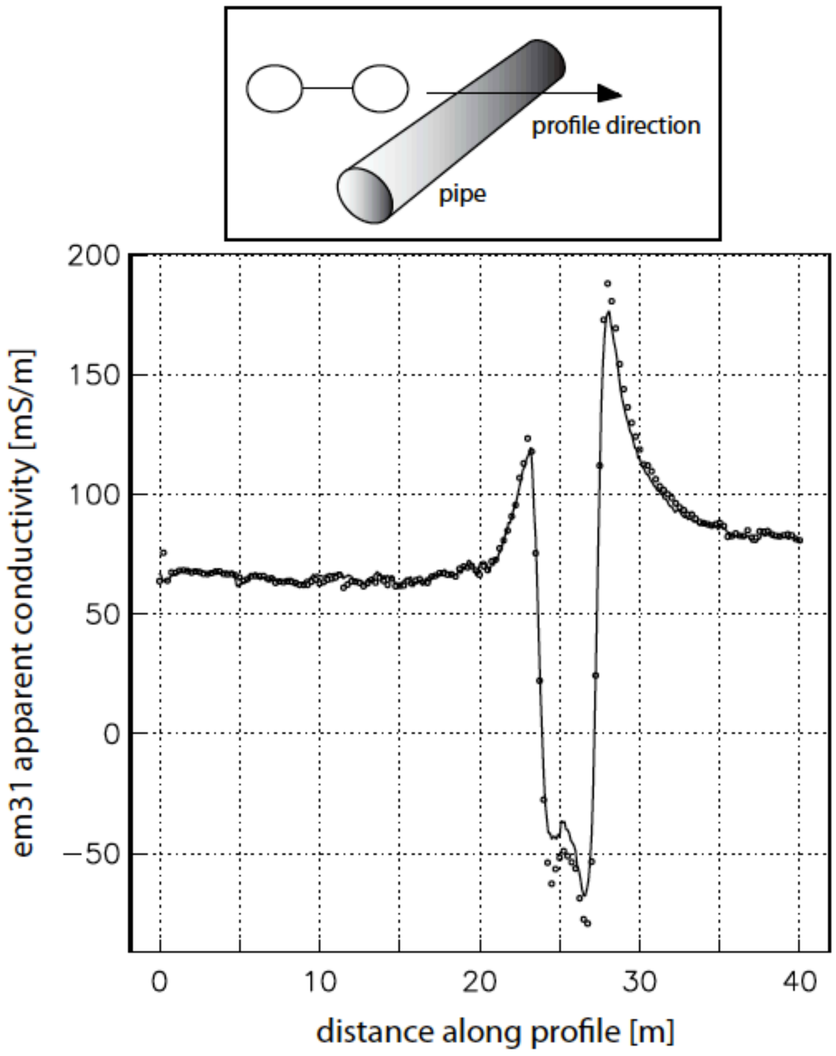
maroon soil



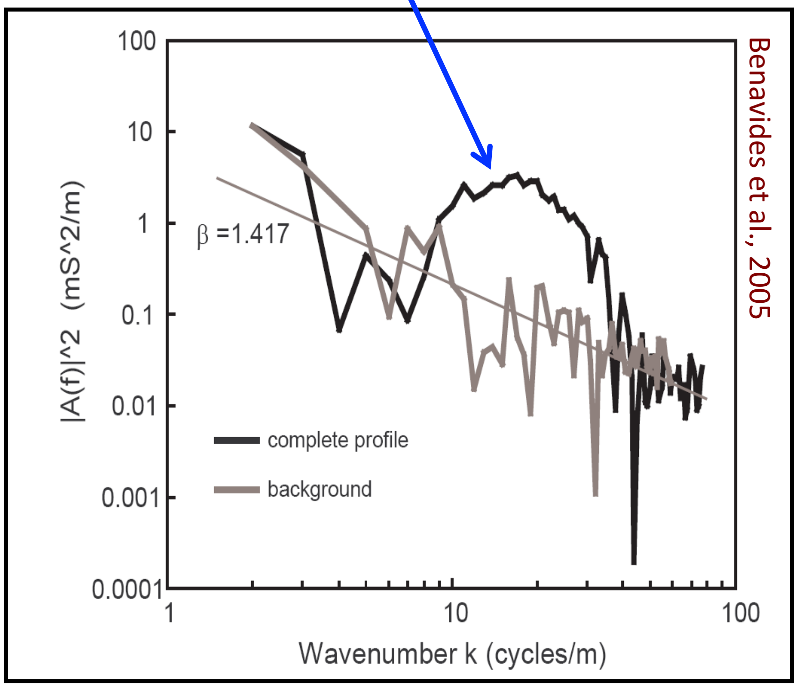
spectral density of σ_a data

spectral density of $\Delta\sigma_a$ increments

Beskardes, Weiss and Everett 2017



distortion of natural wavenumber spectrum due to man-made object



analogous to ship detection in radar sea clutter

EM31 signatures of natural gas pipeline at Texas A&M campus.

Poynting's theorem describes the flow of electromagnetic energy

$$-\partial_t \int_V \left[\varepsilon \mathbf{E}^2/2 + \mathbf{B}^2/\left(2\mu_o\right)\right] dV = \int_V \sigma \mathbf{E}^2\, dV + \oint_S \left(\mathbf{E}\times\mathbf{B}/\mu_o\right)\bullet d\mathbf{A}$$

loss of electromagnetic energy in volume V

energy loss in volume V due to Joule heating (Ohmic dissipation)

power flux through bounding surface S

differential form in the MQS approximation

$$-\partial_t u_M = \sigma \mathbf{E}\bullet\mathbf{E} + \nabla\bullet\mathbf{S}$$

loss of magnetic energy at a given point

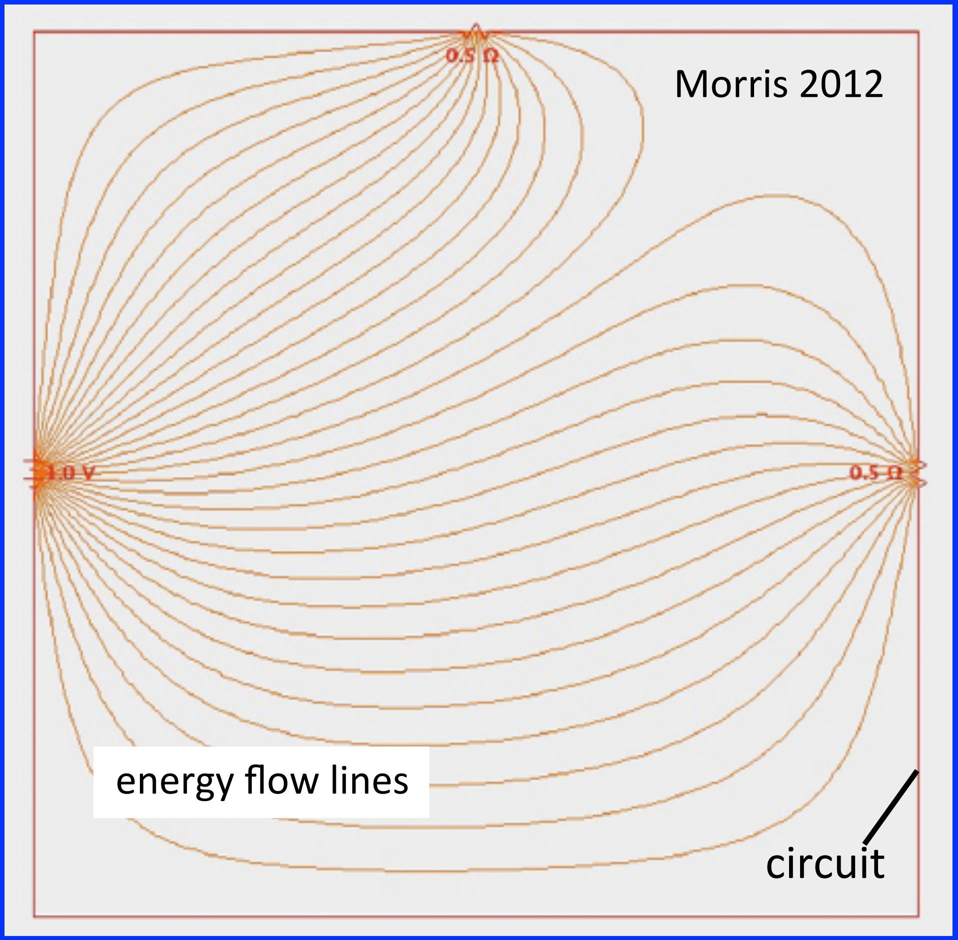
amount of heat produced at the point

divergence of energy away from the point

Poynting vector streamlines describe energy flow in circuits

resistor (sink)

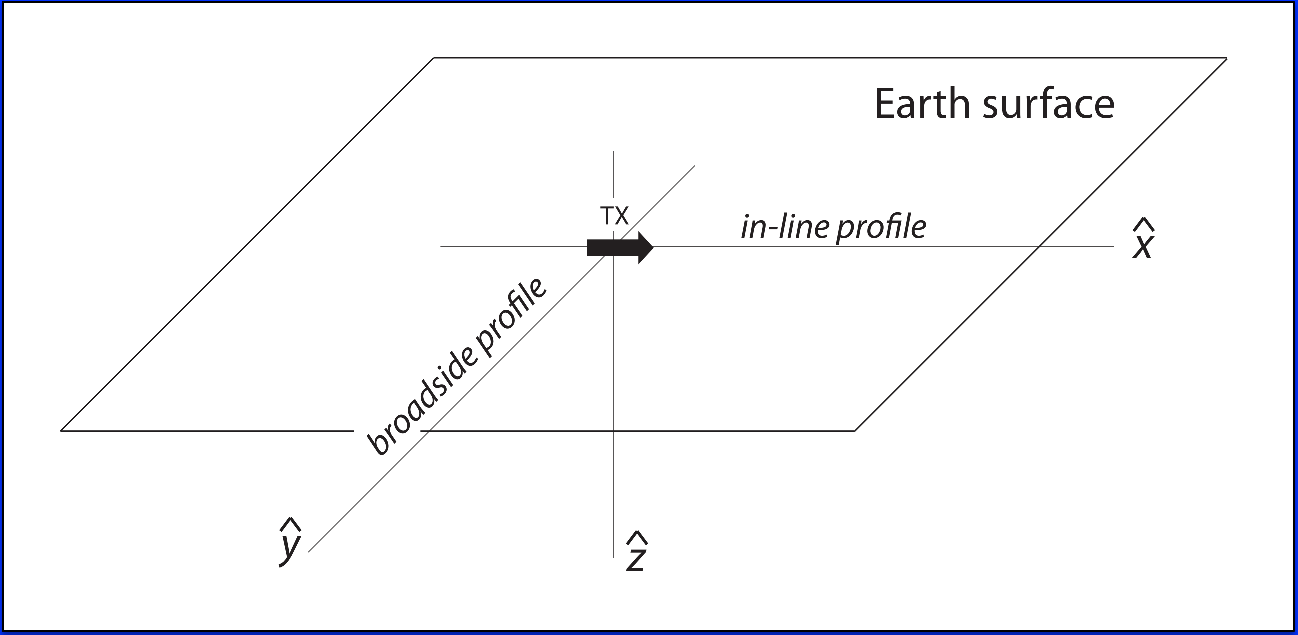
battery
(seat)



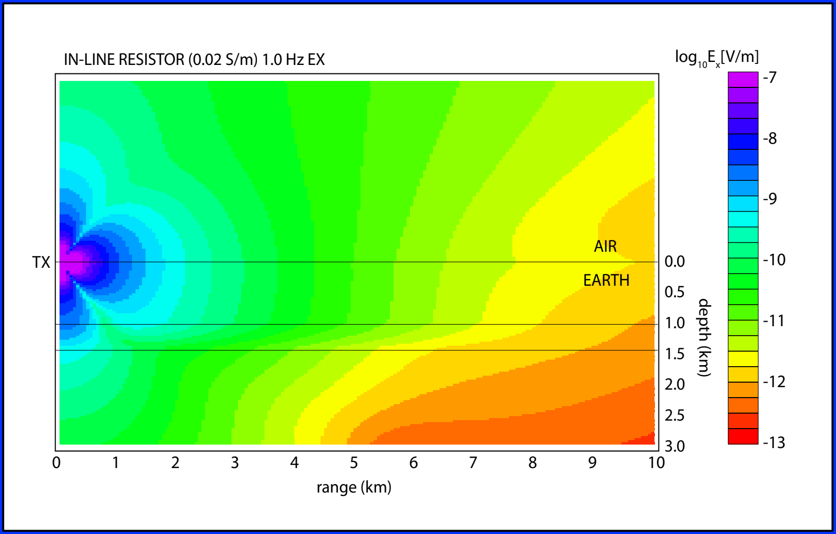
resistor
(sink)

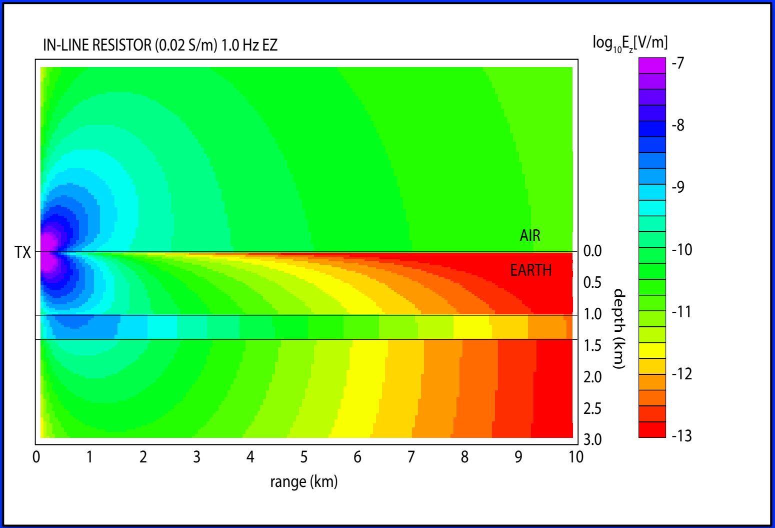
energy flows from the battery to the resistors
in the empty space between the wires, <u>not</u>
along the conductive wires

What do Poynting vector streamlines look like in a terrestrial exploration scenario?



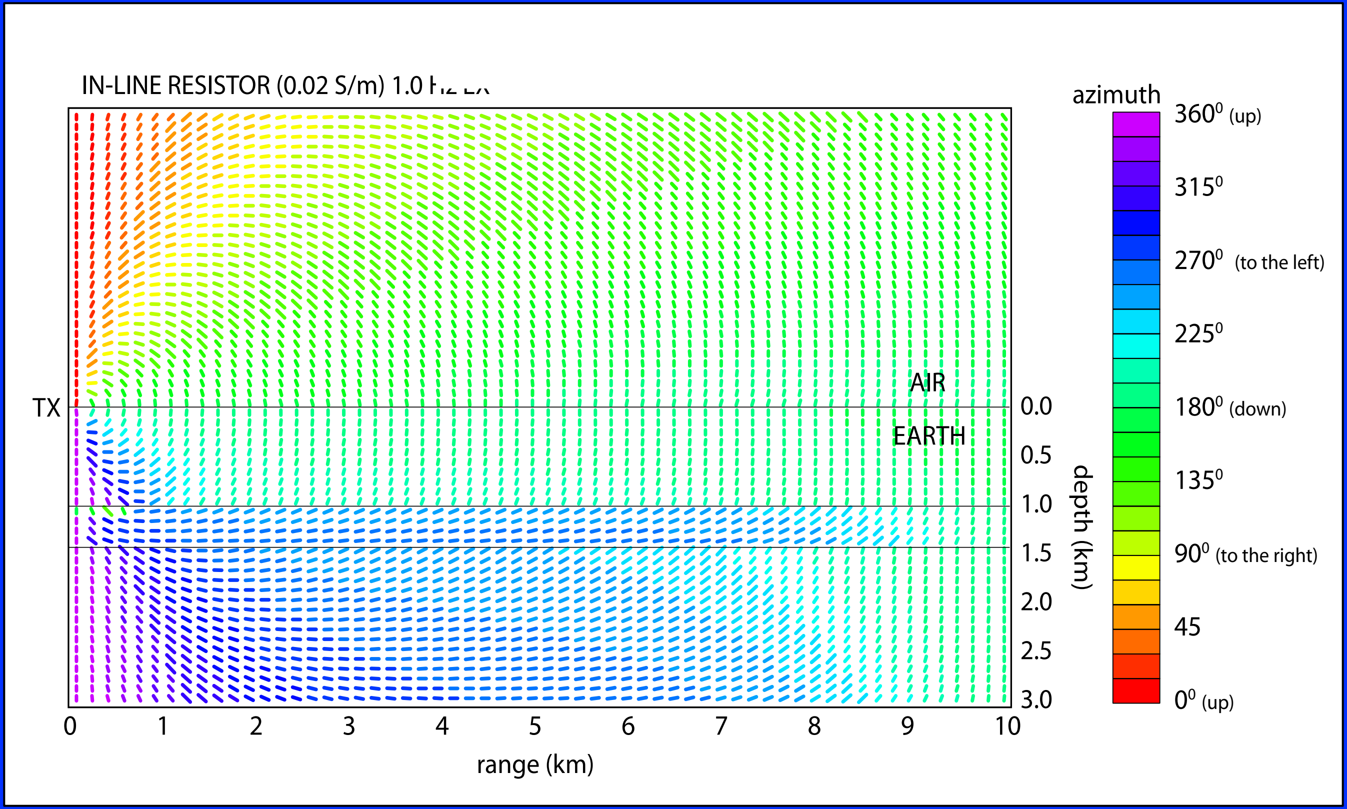
An x-directed horizontal electric dipole on a layered Earth showing in-line and broadside receiver profiles.

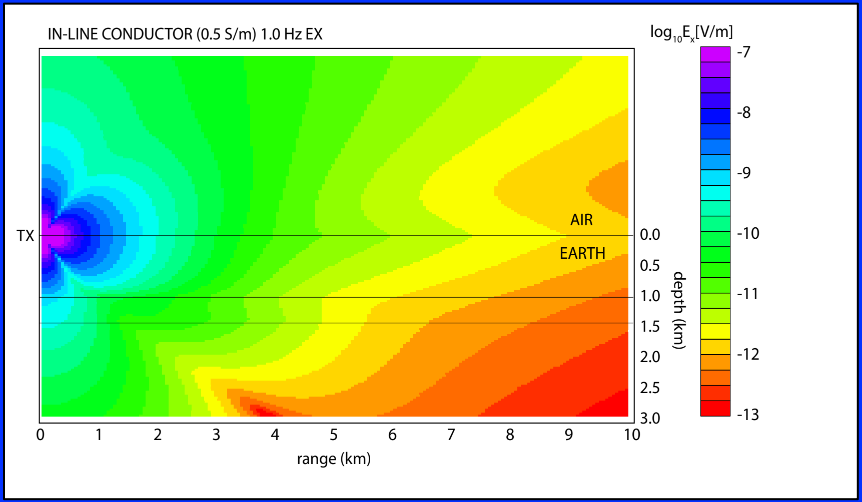


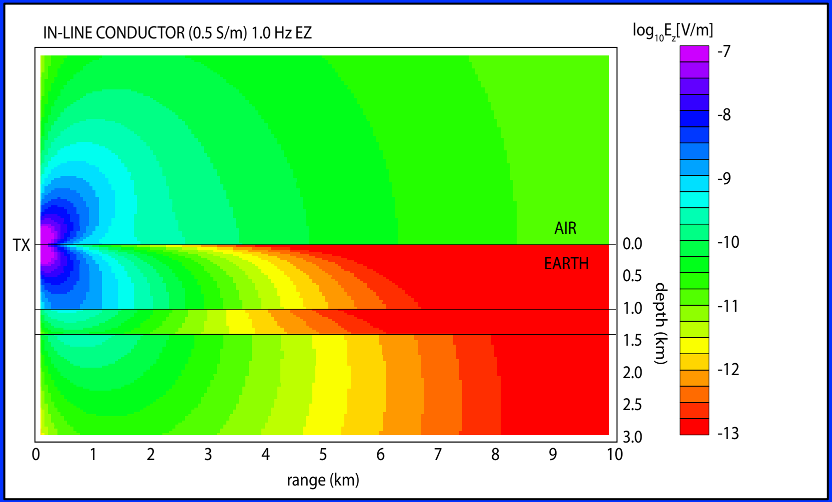


Poynting streamlines

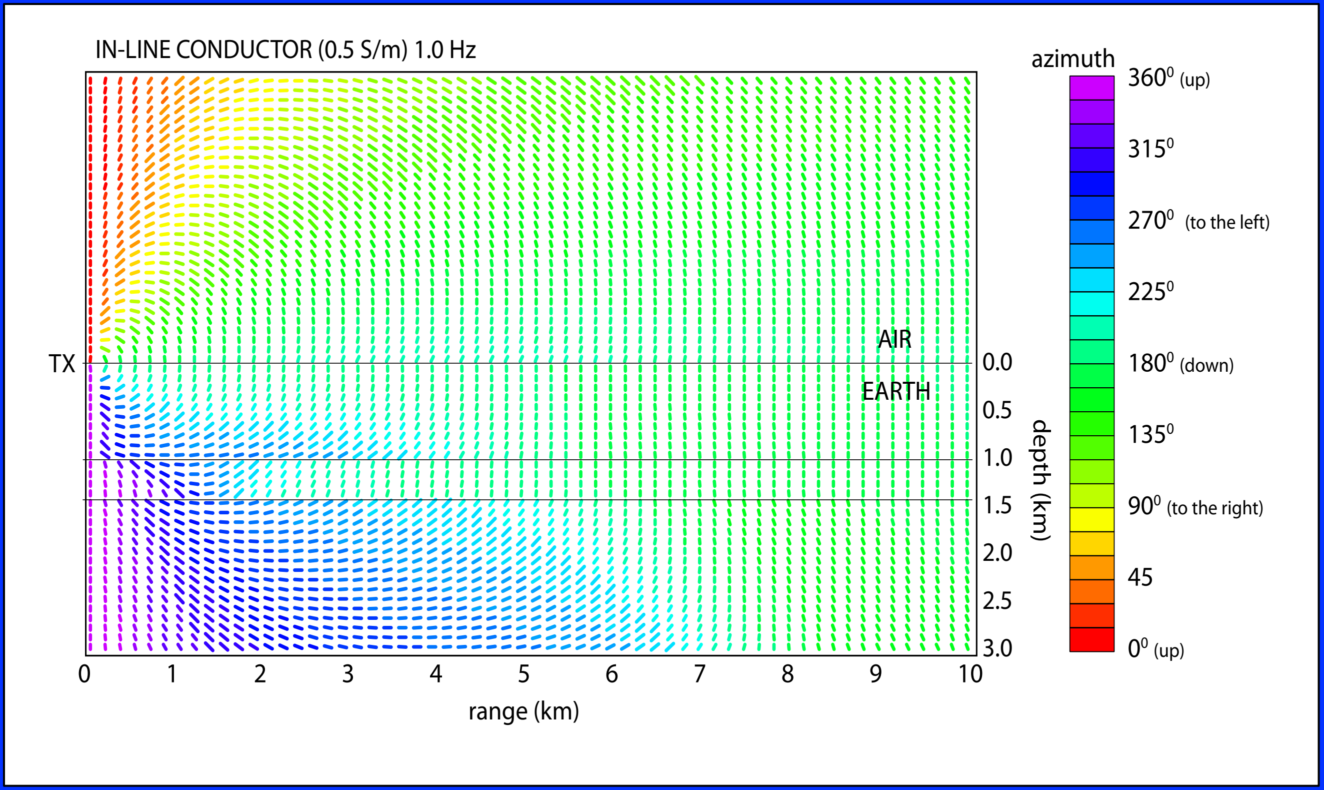
"waveguide" effect in resistive layer (active power)







Poynting streamlines



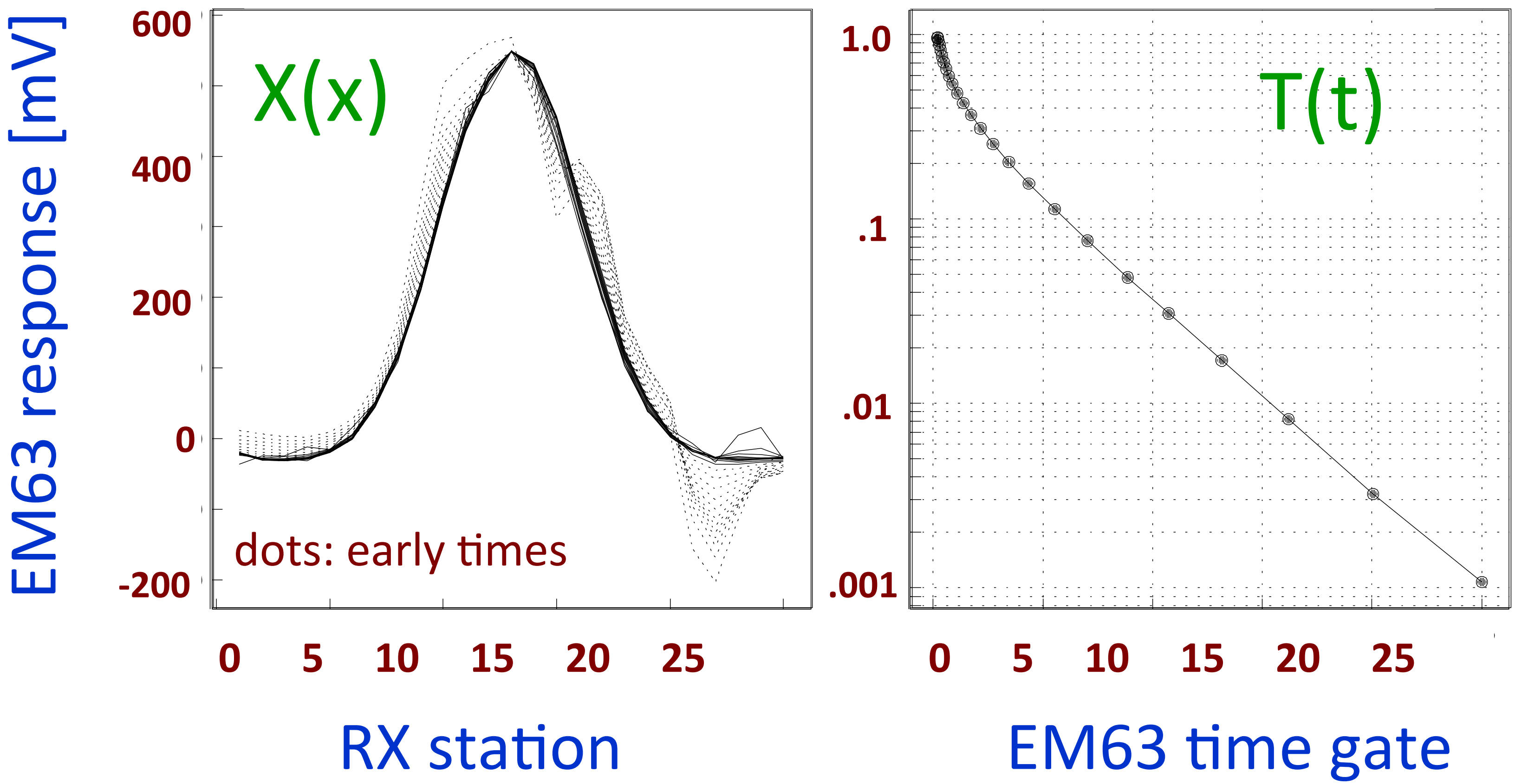
no "waveguide" effect in conductive layer (reactive power)

separability

separation of variables is widely used analytic technique to solve Helmholtz equation
is it valid for idealized systems only, or is separability found in realistic settings?

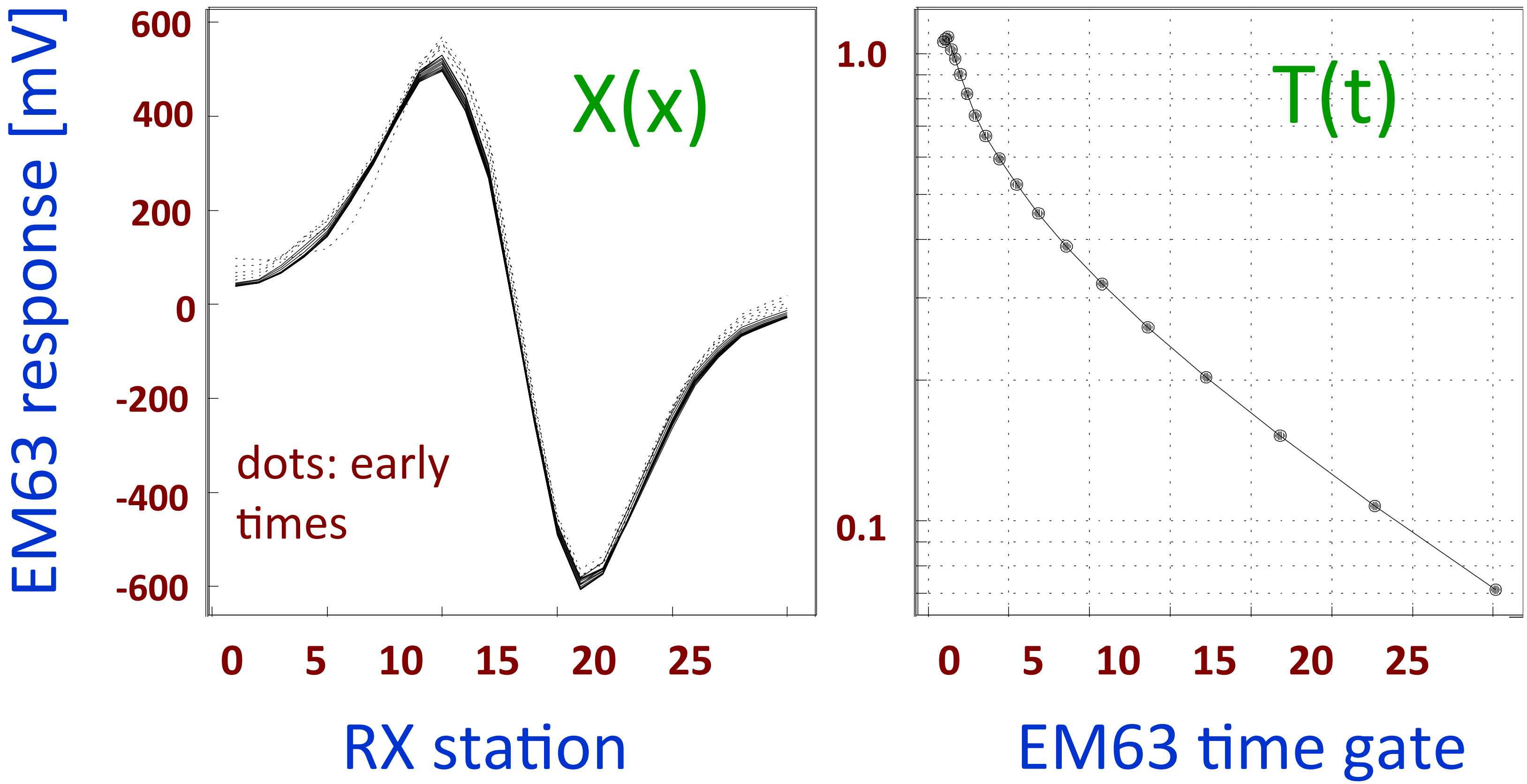
target: aluminum plates

1. horizontal aluminum plate



separable response: $R(x,t)=X(x)T(t)$

2. vertical aluminum plate



separable response: $R(x,t)=X(x)T(t)$

similitude

early analog scale modeling (Dosso 1966)

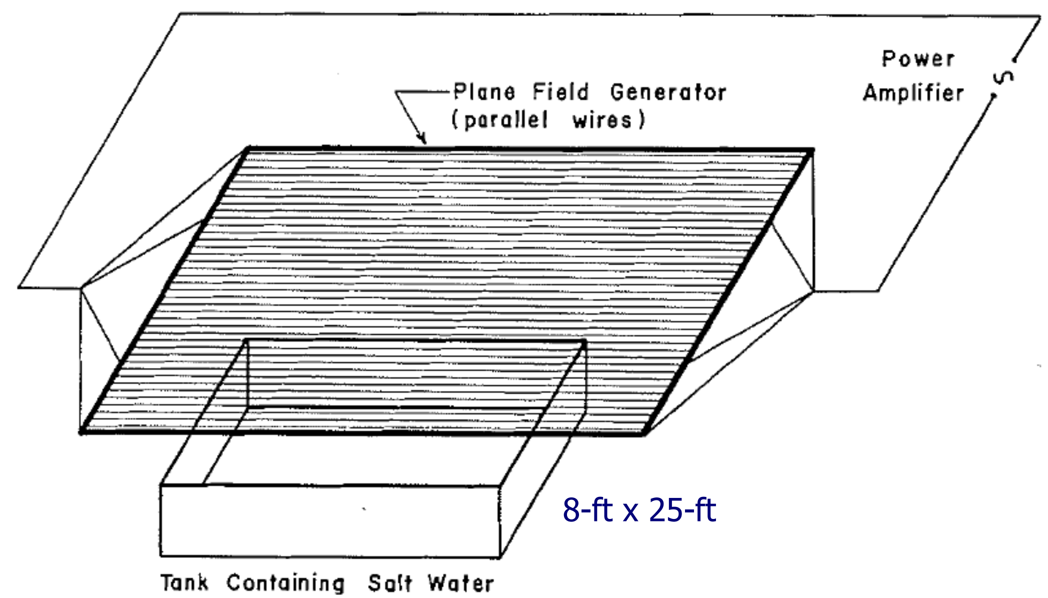


FIG. 1. Diagram of the plane-wave model.

electromagnetic **similitude**

$$\sigma f d^2 = \sigma' f' d'^2$$

lab scale geophysical scale

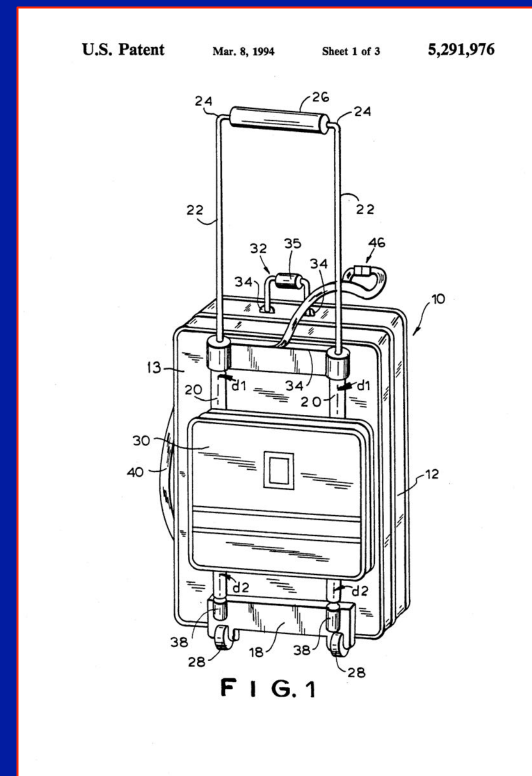
this principle is extremely useful as a debugging tool for numerical simulations (it should hold exactly)

final thought (just for fun)

the slow progress of research:
it took mankind several millenia to
realize that wheels could be attached to luggage

early wheel 5 ka

Roman satchel bag



US patent 1961

coffee-table question: why did Einstein carry his luggage?