EMinar:

16:00 UT, 8th December, 2021

Thibaut Astic on:

An integrative framework for geophysical inversion: merging geophysics, petrophysics and geology with machine learning

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An integrative framework for geophysical inversion:

merging geophysics, petrophysics and geology with machine learning

Thibaut Astic and the SimPEG team.
University of British Columbia, Geophysical Inversion Facility (UBC-GIF)

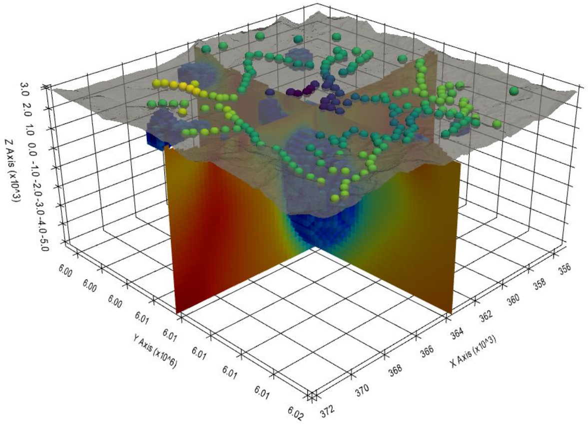
http://bit.ly/astic_EMinar21

Exploration toolkit

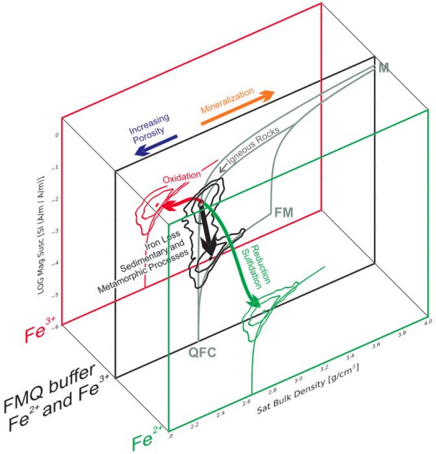
Geophysical inversion

Petrophysical characterization

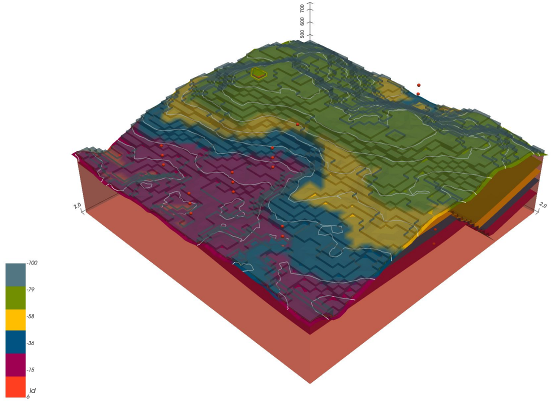
Geological modelling



Cockett *et al.* 2015



Enkin *et al.* 2020



de la Varga *et al.* 2019

Objective:

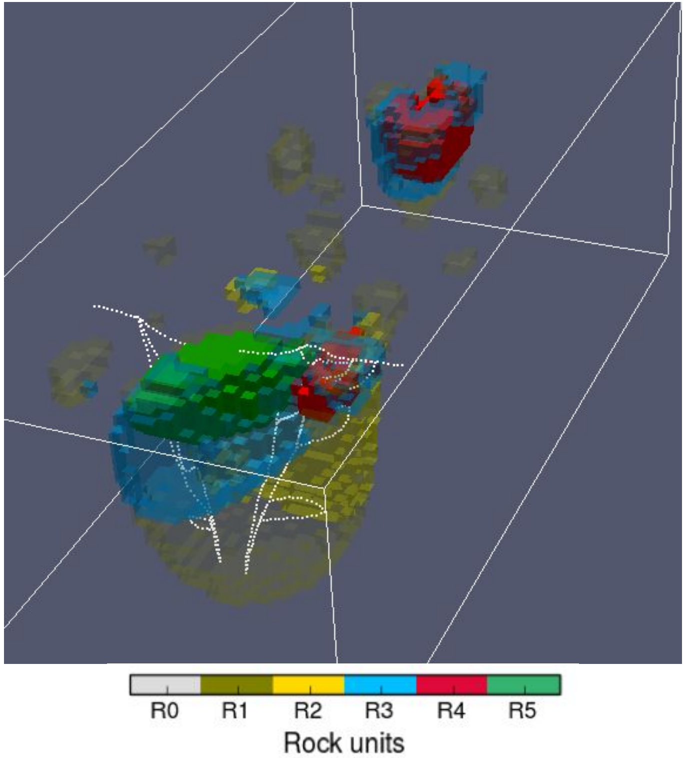
Tie geophysical, petrophysical and geological information

Towards geologic inversions

Objective:
using geophysical, petrophysical and geological information, generate a "quasi-geology model" (Li *et al.* 2019) that facilitates the answering of geologic questions.

Approach:

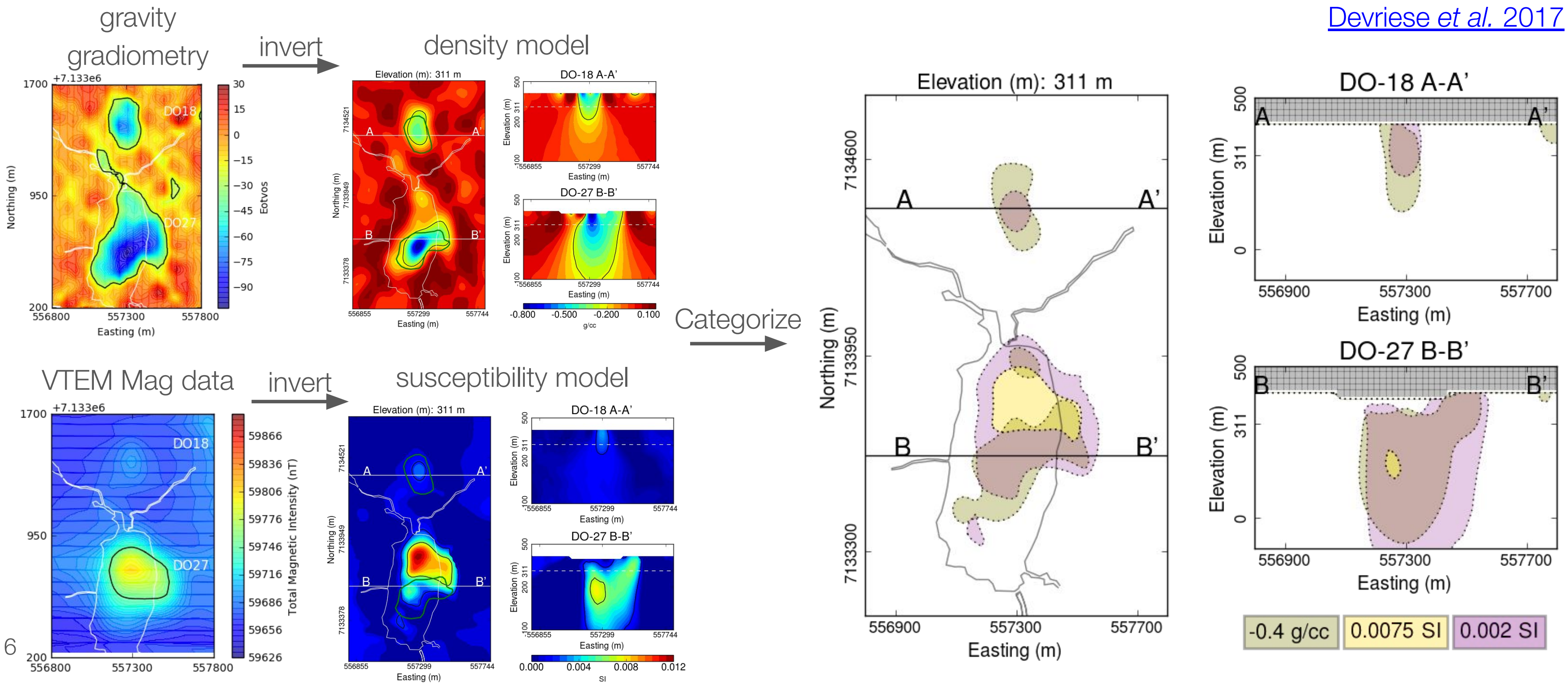
- Integrate and reproduce petrophysical and geological information in geophysical inversion.
- Jointly invert for multiple physical properties.
- **Relate the inversion to the geologic questions.**



Kang *et al.* 2017

Conventional inversions & post-classification: DO-27 & DO-18 kimberlite pipes (NWT, Canada)

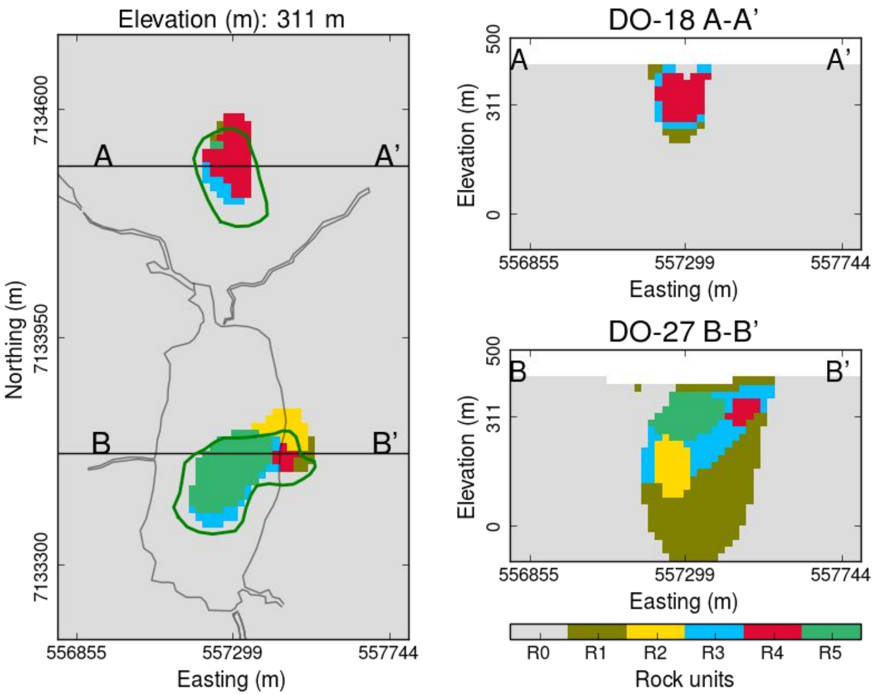
Devriese *et al.* 2017



Conventional inversions & post-classification: DO-27 & DO-18 kimberlite pipes (NWT, Canada)

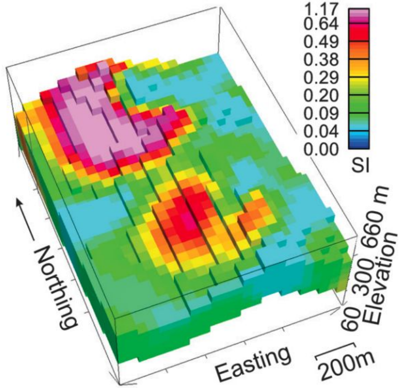
Rock type	Glacial till	Host rock	HK	VK	PK
Density	Moderate	Moderate	Low	Low	Low
Susceptibility	None	None	High	Low-moderate	Low-moderate
Conductivity	Moderate-high	Low	Low-moderate	Moderate-high	Moderate-high
Chargeability	Low	Low	?	?	?

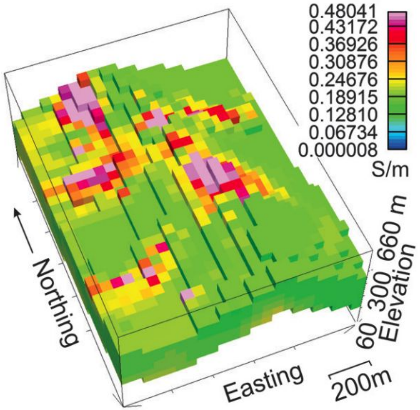
Rock Unit	ρ	κ	σ	$\tilde{\eta}_E$	$\tilde{\eta}_L$	τ	Interpretation
R0	Mod.	Low	Low	Low	Low	N/A	Host Rock
R1	Low	Low	Low	Low	Low	N/A	Kimberlite
R2	Low	High	Low	Low	Low	N/A	HK
R3	Low	Mod.	Mod.	Low	Low	N/A	PK or VK
R4	Low	Mod.	Mod.	High	Low	Small	VK
R5	Low	Mod.	Mod.	Low	High	Large	PK

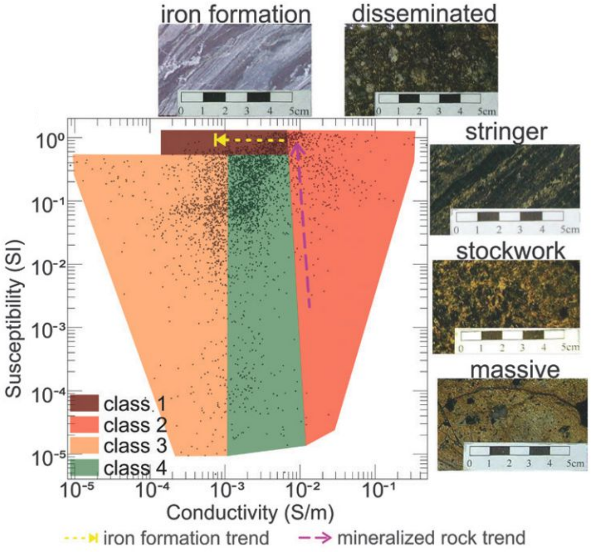


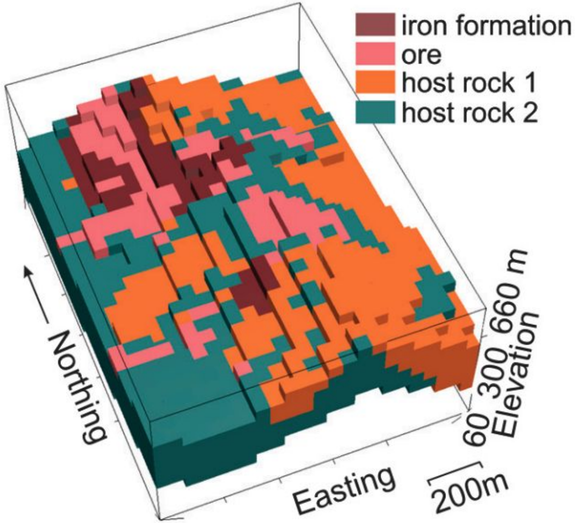
- Magnetics, Gravity: Devriese *et al.* 2017
- Electromagnetics: Fournier *et al.* 2017
- IP effects in EM data: Kang *et al.* 2017

Conventional inversions & post-classification: IOCG, Cristalino (Brazil)



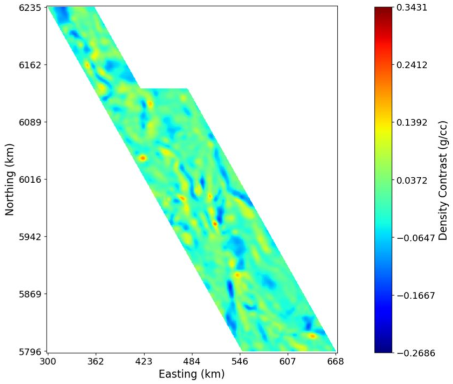


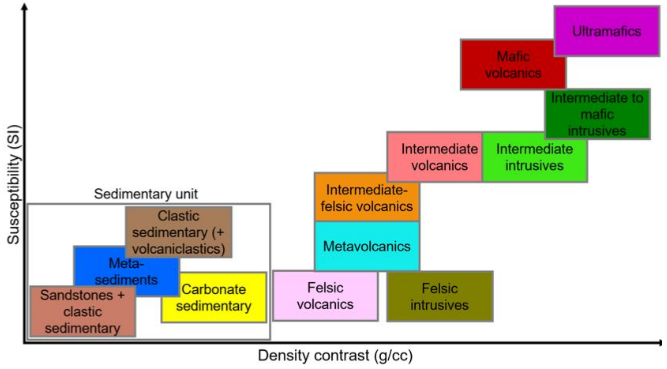




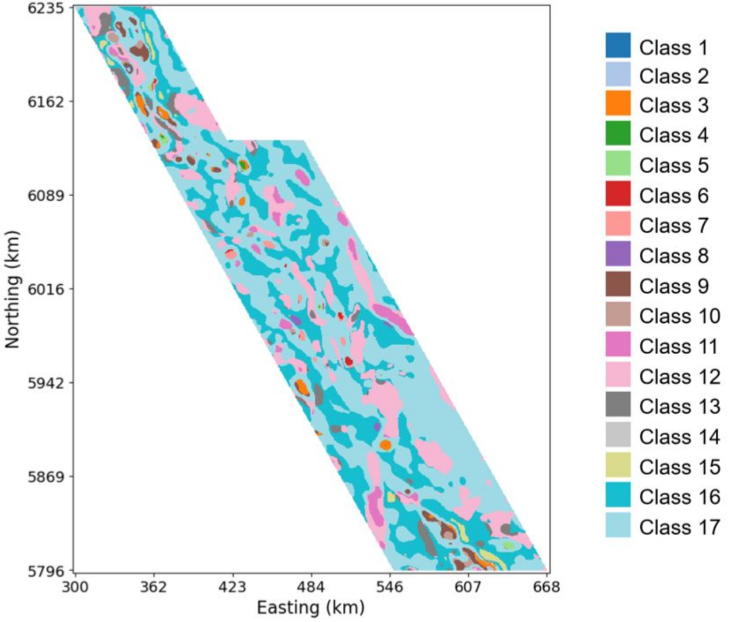
- Magnetics, Gravity: [Martinez & Li 2015]
- Magnetics, DC resistivity: [Melo *et al.* 2017]

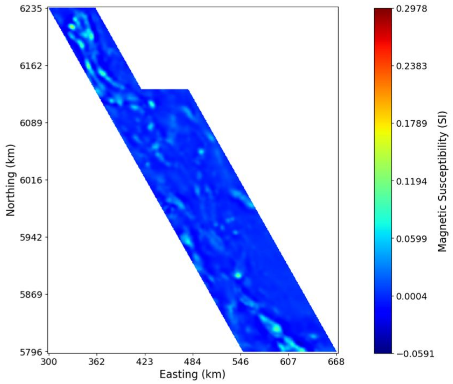
Cross-gradients & differentiation (QUEST, BC,Canada)

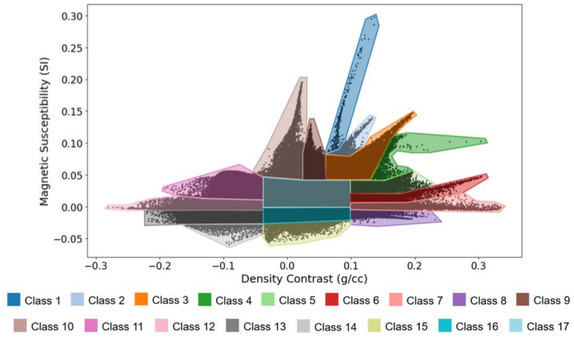




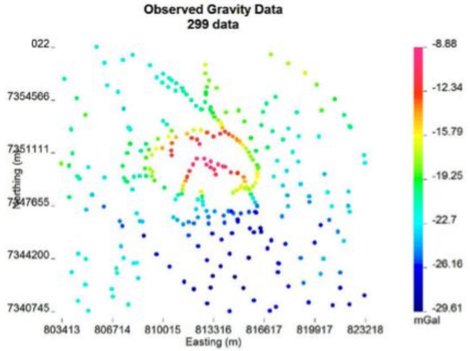
Kim *et al.* 2020

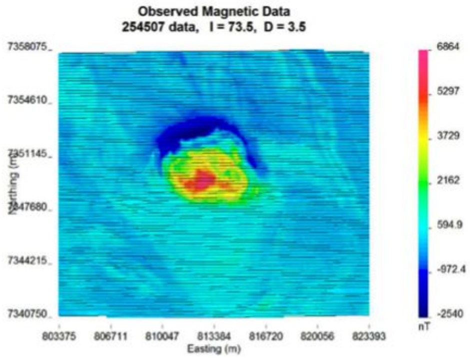


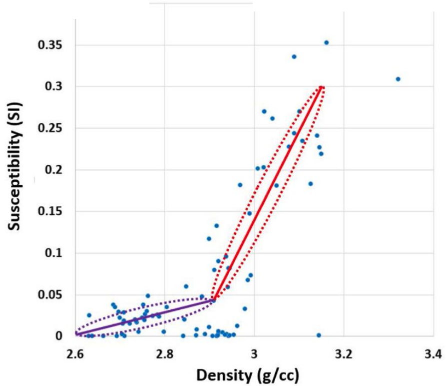




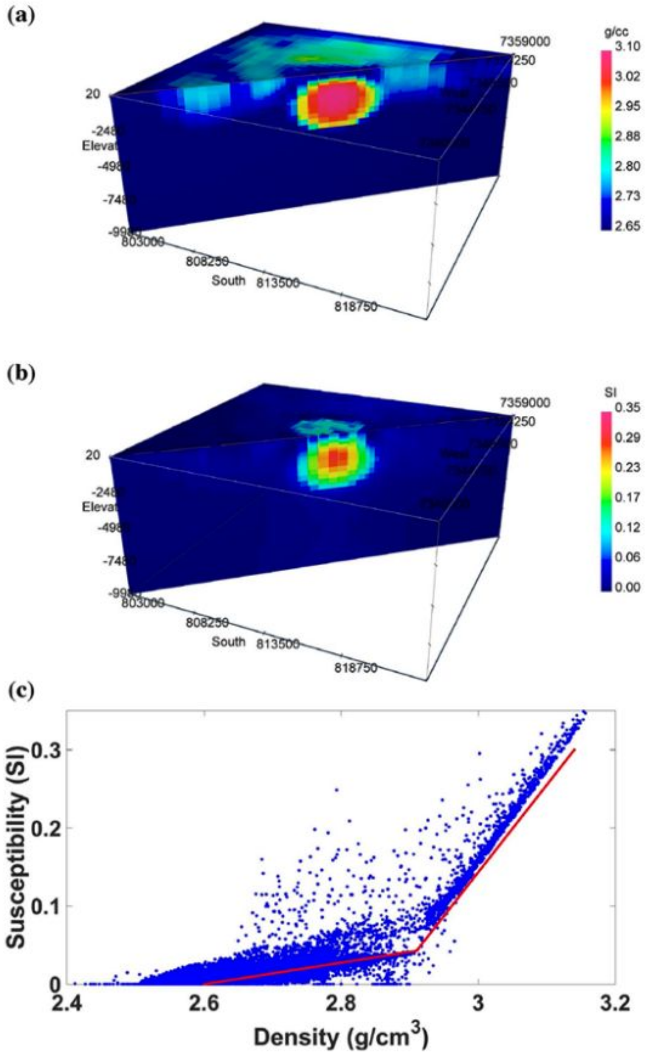
Fuzzy clustering inversions: gabbroic intrusion (Sweden)



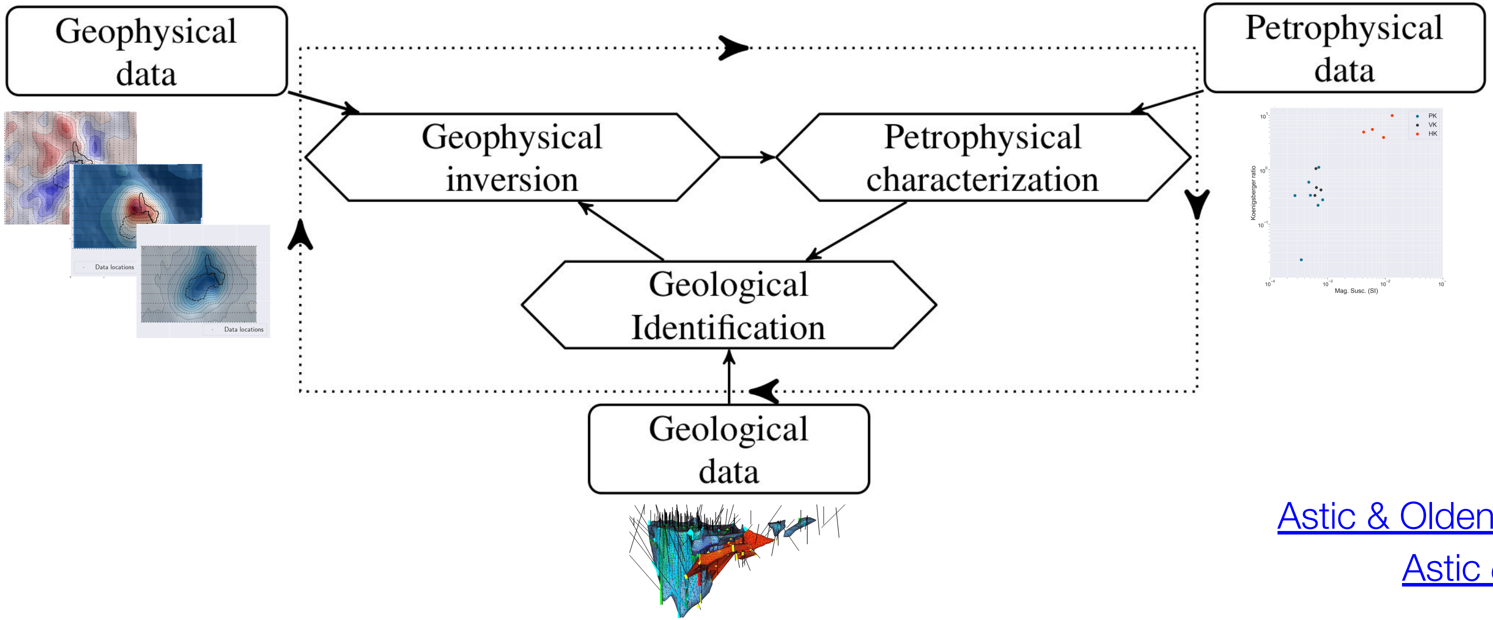




[Sun & Li 2017]



Linking geophysics, petrophysics and geology



[Astic & Oldenburg 2019]()
[Astic *et al*. 2021]()

Petrophysically and Geologically guided Inversion (PGI)
Tie and reproduce geophysical, petrophysical and geological information with a single geophysical inversion framework

A step back looking at the inverse problem

Recovering the subsurface physical property distributions **m**:

Minimize $$\Phi(\mathbf{m}) = \Phi_d(\mathbf{m}) + \beta\Phi_m(\mathbf{m})$$

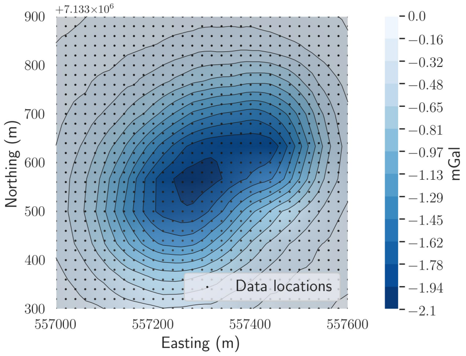
subject to $\mathbf{m}_{lower} < \mathbf{m} < \mathbf{m}_{upper}$

- Fit the geophysical data:

Data misfit: $\Phi_d(\mathbf{m}) = \frac{1}{2}||W_d(F[\mathbf{m}] - \mathbf{d}_{\text{obs}})||_2^2$

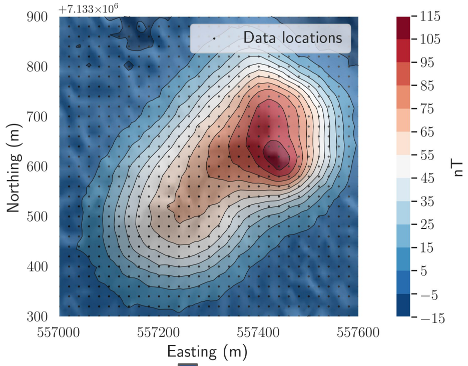
- Under-determined problem: addition of prior information through the regularizer $\Phi_m(\mathbf{m})$

Gravity data

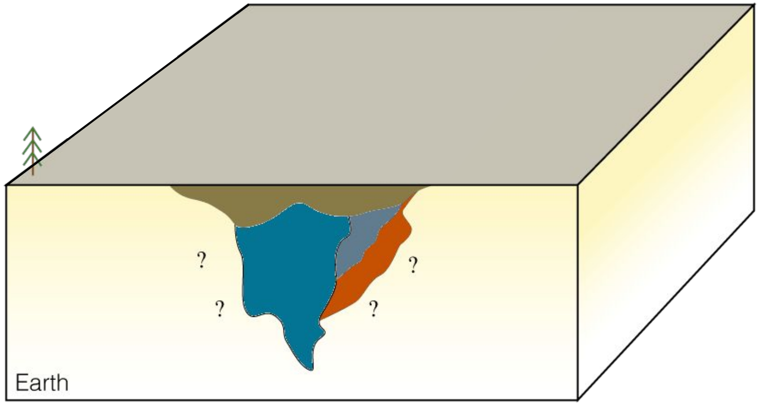


$\mathcal{F}^{-1}$

Magnetic data



$\mathcal{F}^{-1}$



L_2 inversion

Minimize $$\Phi(\mathbf{m}) = \Phi_d(\mathbf{m}) + \beta\Phi_m(\mathbf{m})$$

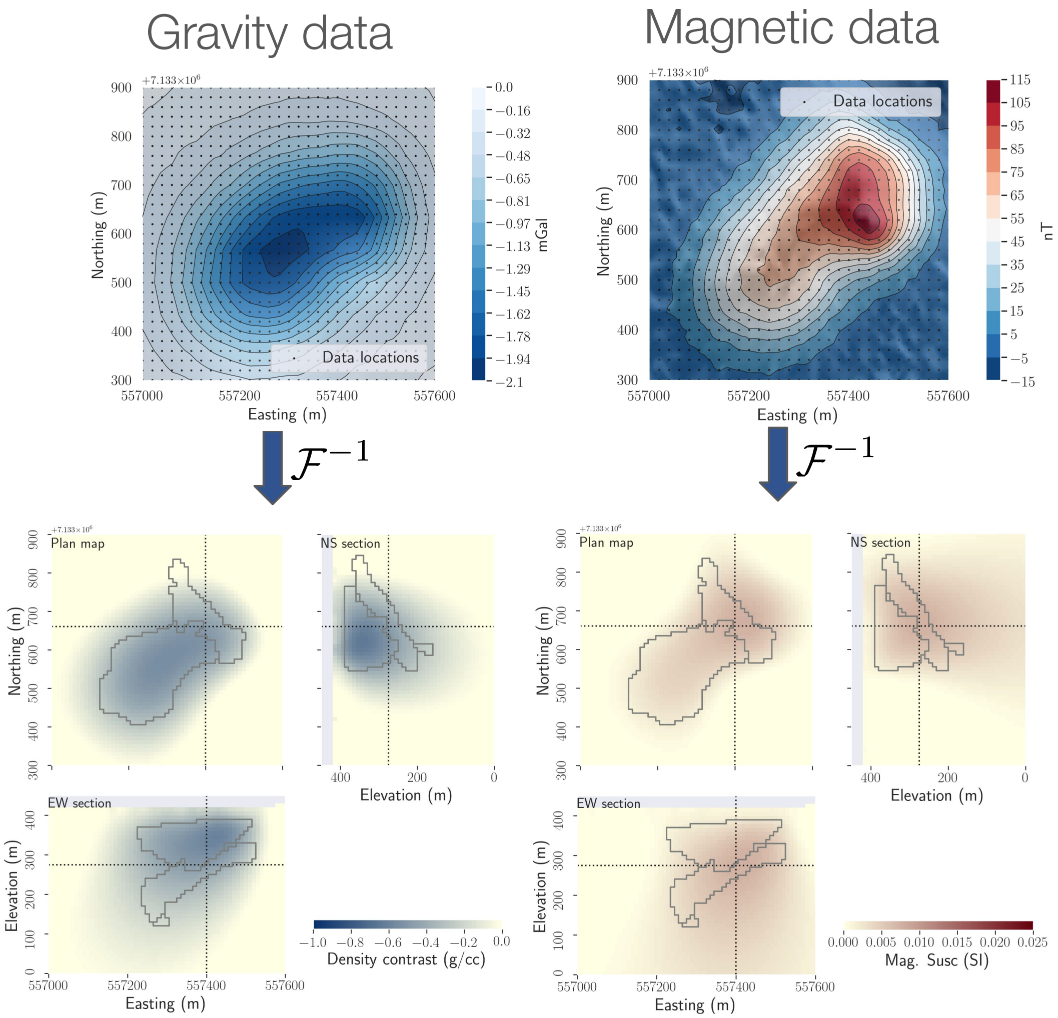
subject to $\mathbf{m}_{lower} < \mathbf{m} < \mathbf{m}_{upper}$

Data misfit $$\Phi_d(\mathbf{m}) = \frac{1}{2}||W_d(F[\mathbf{m}] - \mathbf{d}_{\text{obs}})||_2^2$$

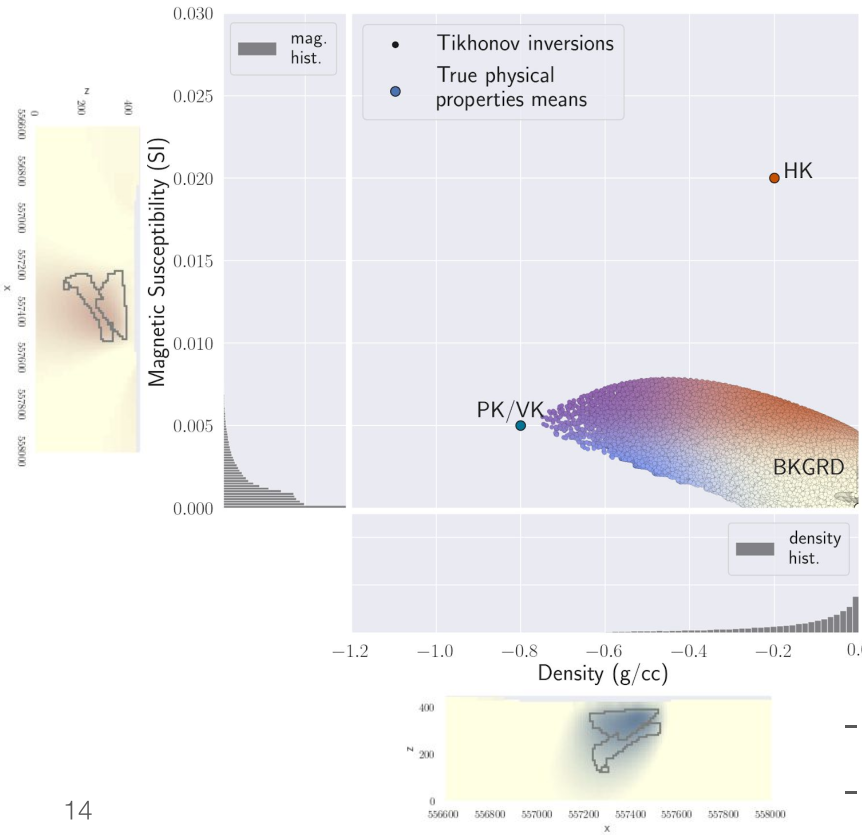
Regularization $$\Phi_m(\mathbf{m}) = \frac{1}{2}||W_m(\mathbf{m} - \mathbf{m}_{\text{ref}})||_2^2$$

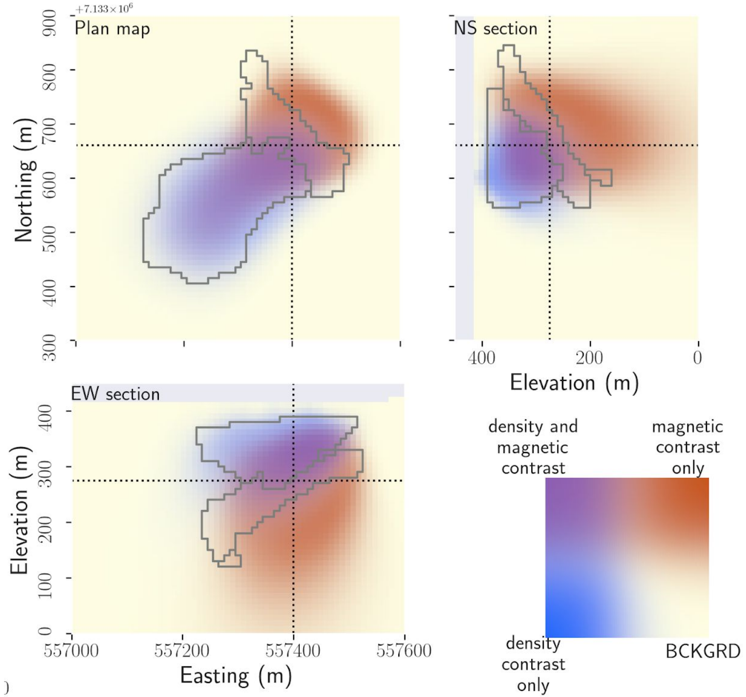
Smallness: minimum distance to a reference model

Smoothness: minimum spatial variations



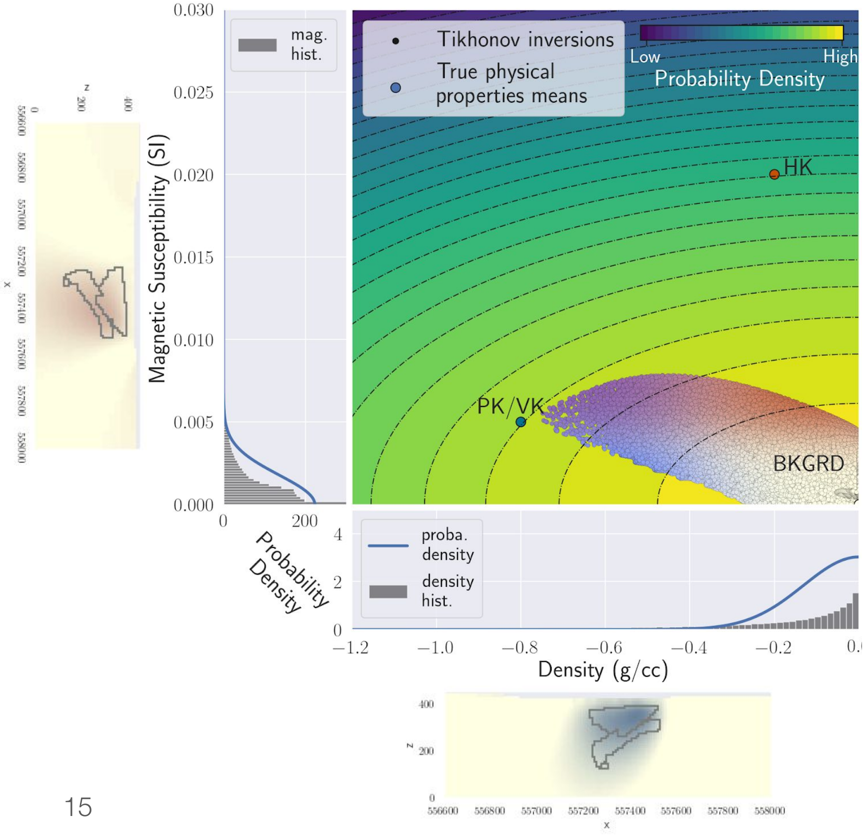
Post-inversion classification





- Petrophysical characteristics are not reproduced.
- Rock identification is hard.

Gaussian prior



L_2 inversion assumes a gaussian distribution around the reference model

$$\Phi_{\text{small}}(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{n} ||\mathbf{W}_s(\mathbf{m}_i - \mathbf{m}_{\text{ref}})||_2^2$$

$$\Longleftrightarrow$$

$$\mathcal{P}_{\text{small}}(\mathbf{m}) = \mathcal{N}(\mathbf{m}|\mathbf{m}_{\text{ref}}, (\mathbf{W}_s^\top \mathbf{W}_s)^{-1})$$

$$\Phi_{\text{small}}(\mathbf{m}) = -\log(\mathcal{P}_{\text{small}}(\mathbf{m})) + c$$

Can we include the physical properties information in our inversions?

Physics & Machine Learning

$$\Phi(\mathbf{m}) = \Phi_d(\mathbf{m}) + \beta\Phi_m(\mathbf{m})$$

$$\Phi_d(\mathbf{m}) = \frac{1}{2}||W_d(F[\mathbf{m}] - \mathbf{d}_{\text{obs}})||_2^2$$

Physics: measure how well we reproduce the geophysical observations

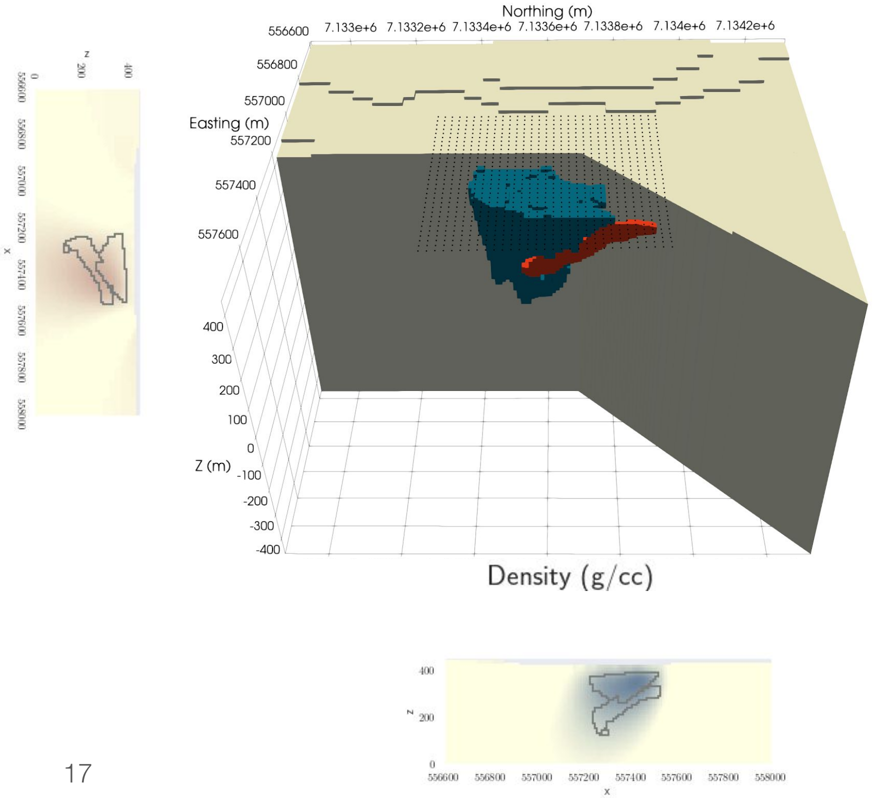
- $\boldsymbol{F}$: Physics operator
  Partial Differential Equations
- Sensitivity-based optimization

$$\Phi_m(\mathbf{m}) = ?$$

Prior expectations: measure the "goodness" of our model

- Machine learning is especially suited to capture **Empirical knowledge**
- What characteristics do we desire from the recovered model **m** ?

How to find a result that is more geologic?



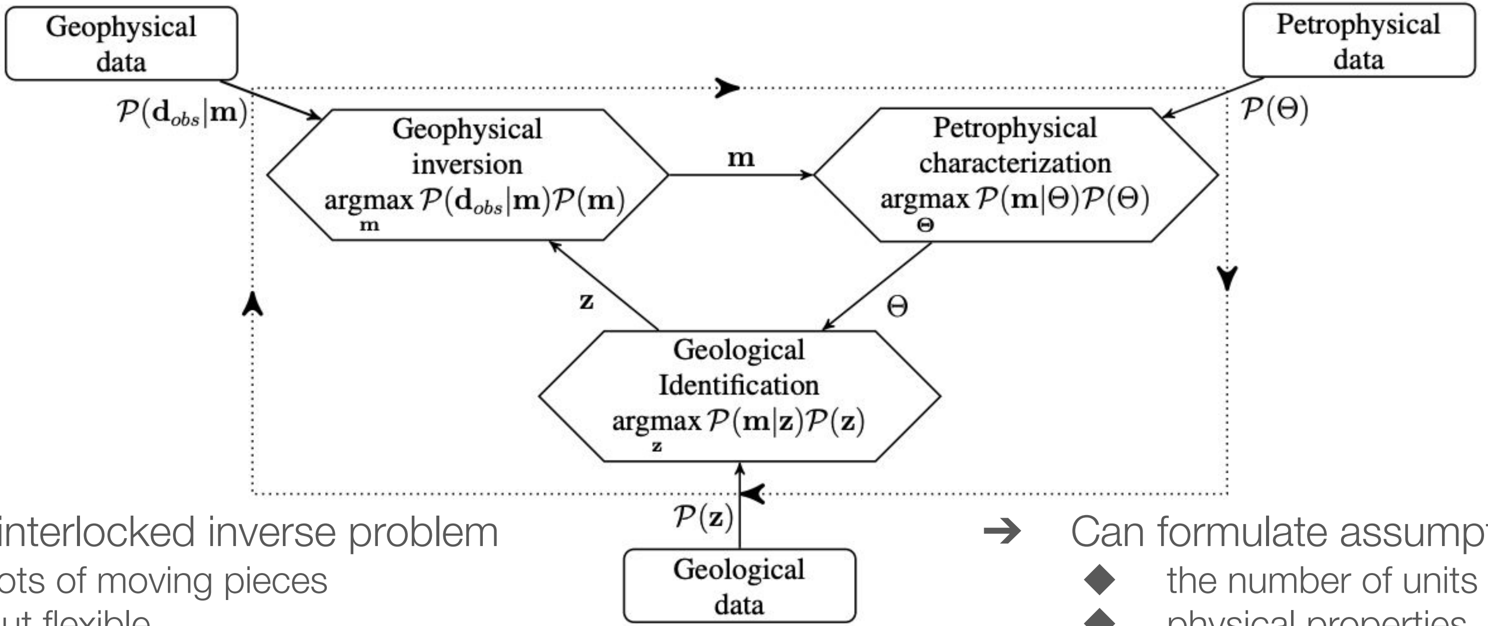
Each pixel

- needs a physical property value that is associated with a viable rock unit
- needs a geologic identifier

Globally

- Geophysical, petrophysical and geological observations must be fit

The PGI framework



➔ Three interlocked inverse problem

- Lots of moving pieces
- But flexible
- designed to reach multiple target misfits at once

➔ Can formulate assumptions on:

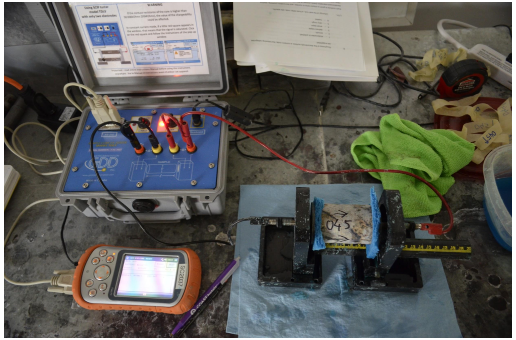
- the number of units
- physical properties
- the locations of the units

A general and formal framework defined from a probability point of view

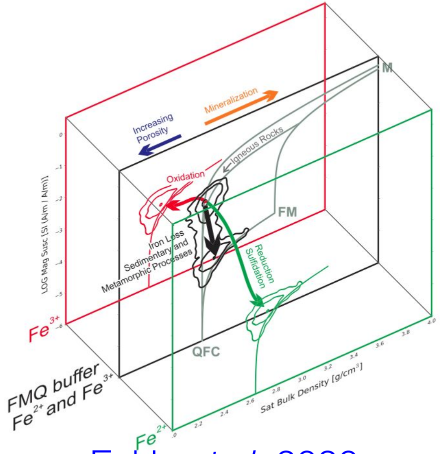
Petrophysics

On-site Measurements

Samples measurements

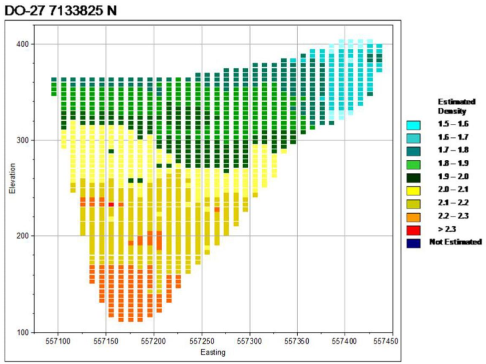


Mineralogy model

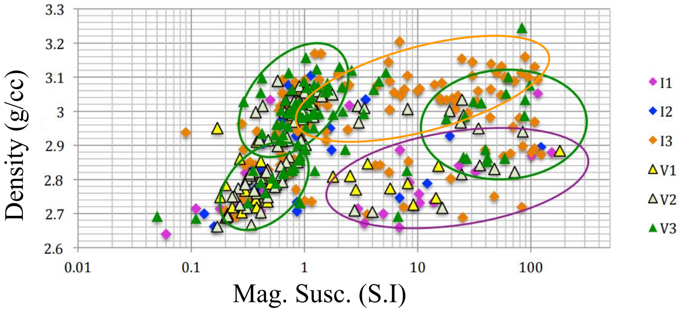


[Enkin *et al.* 2020]()

Trends

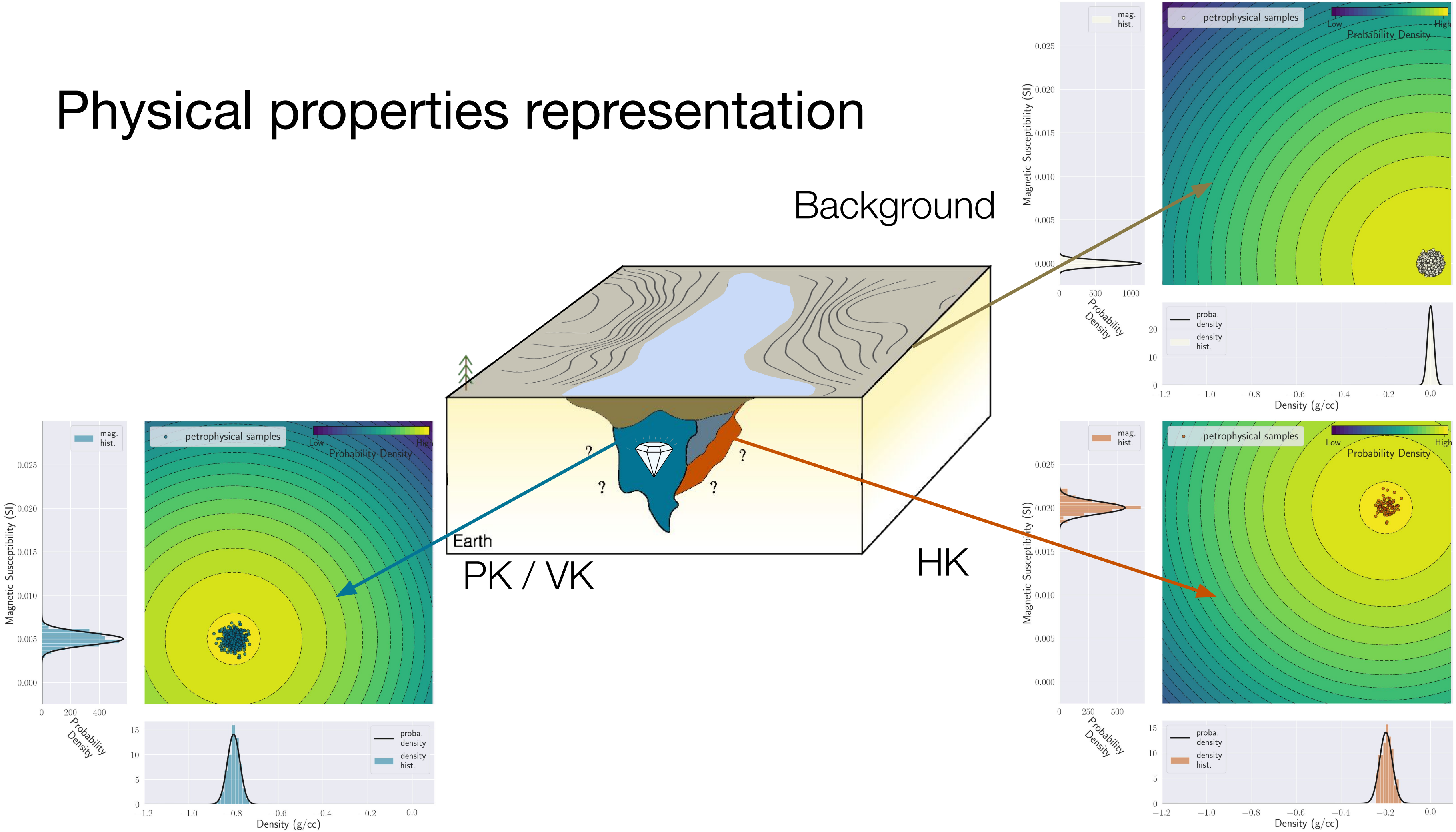


Interdependencies / correlation



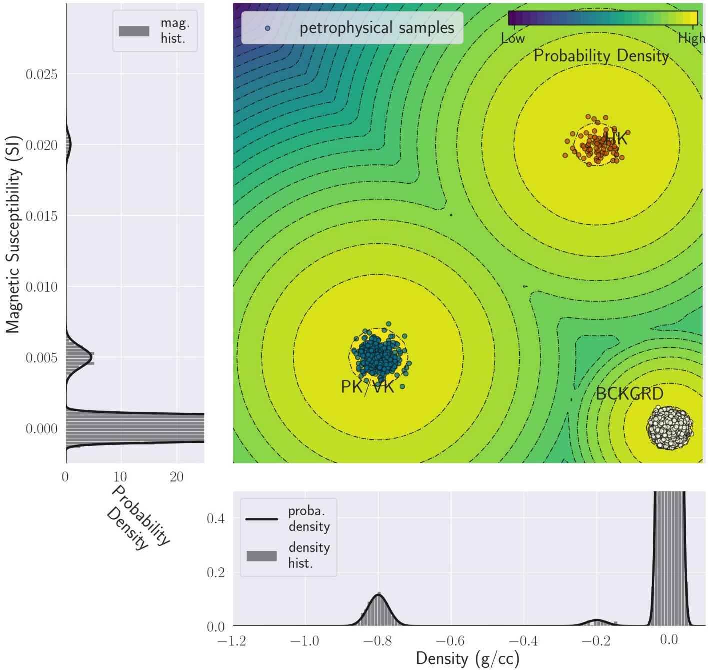
How do we include this in our inversions?

Physical properties representation



Gaussian mixture model (GMM) for physical properties

Synthetic rock samples and physical properties distribution



- Physical properties for each rock unit as a probability distribution $\mathcal{N}$

$$\mathcal{M}(\mathbf{m}_i) = \sum_{j=1}^{c} \pi_j \mathcal{N}(\mathbf{m}_i | \boldsymbol{\mu}_j, \boldsymbol{\Sigma}_j)$$

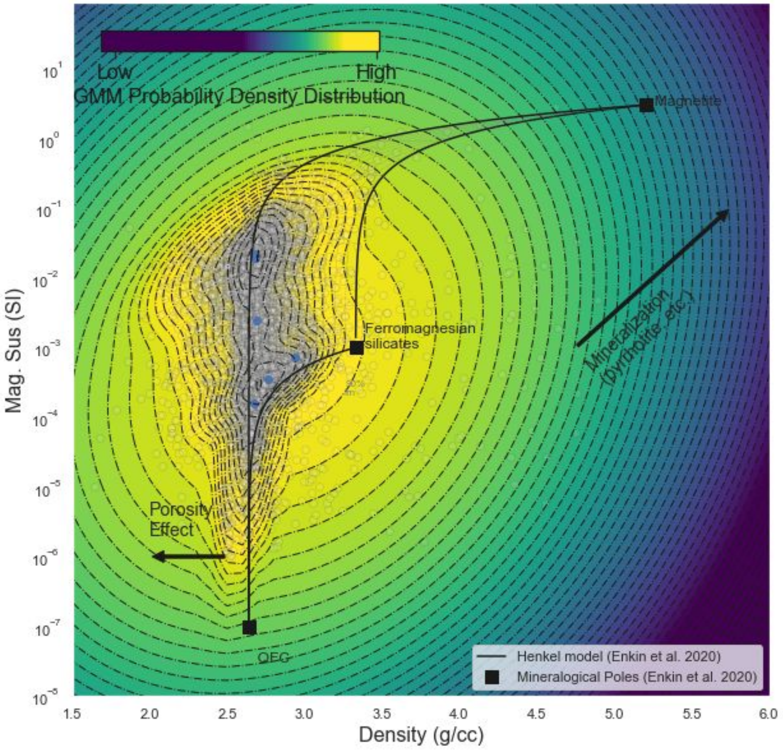
Number of expected units (c)

Means ($\boldsymbol{\mu}_j$)

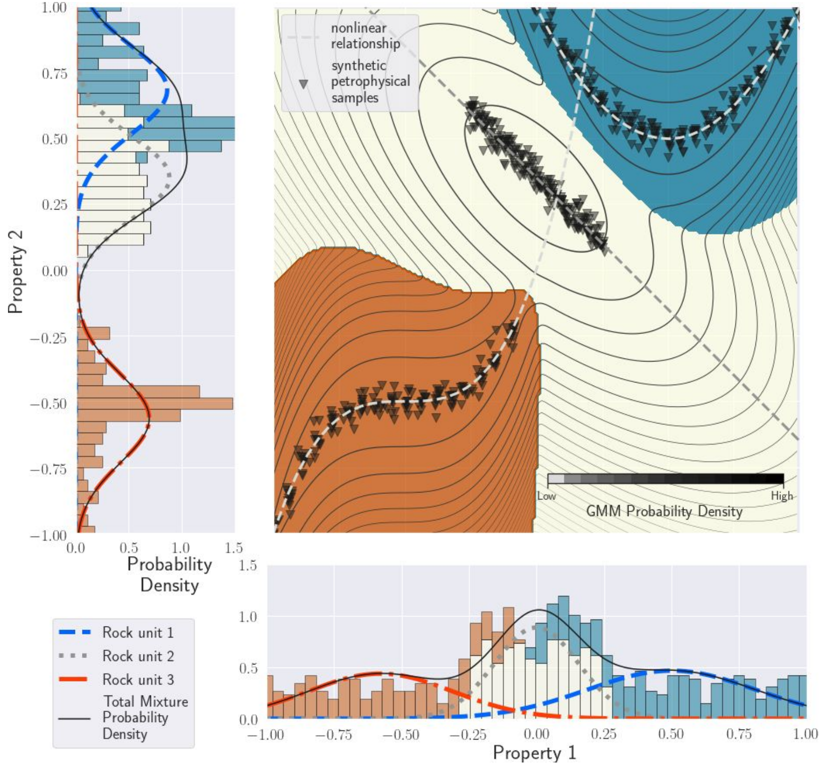
Proportions (π_j)

Covariance Matrix ($\boldsymbol{\Sigma}_j$)

Flexible formulation



Approximating complex distributions
([Enkin *et al.* 2020]())

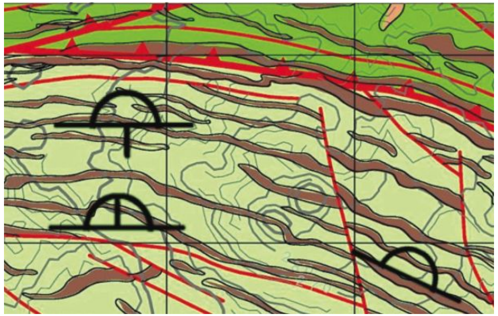


Nonlinear relationships

Geology

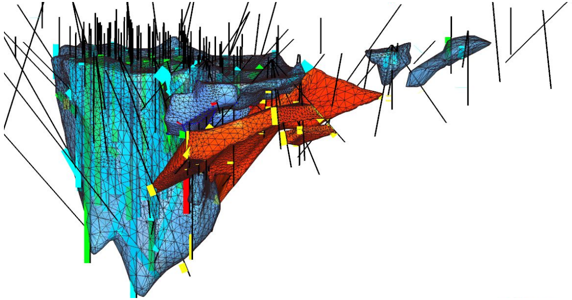
Outcrops

Structural measurements



Stratigraphy

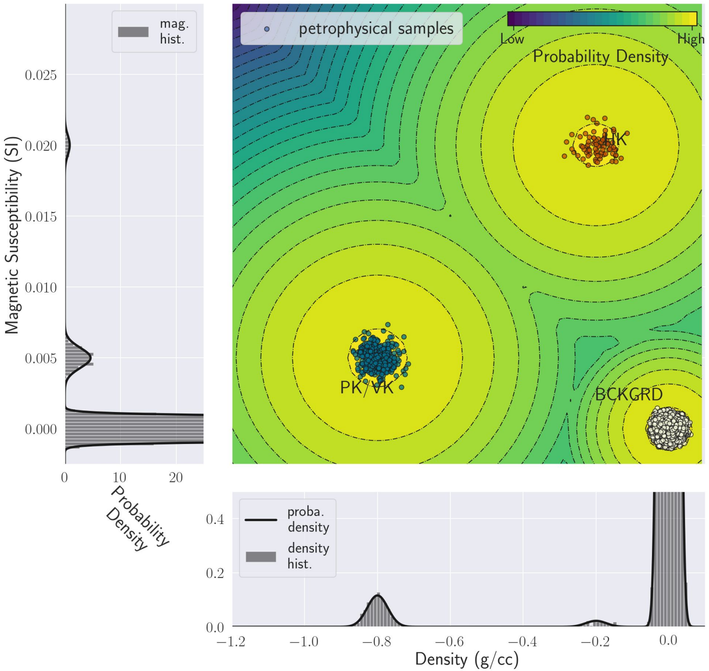
Borehole model



How can we represent this information in a geophysical inversion framework?

GMM with petrophysics and geology information

Synthetic rock samples and physical properties distribution



- Geology as prior expectations (between 0 and 1) of finding rock unit *j* at location *i*
  - z: Quasi-geology model

Number of expected rock units

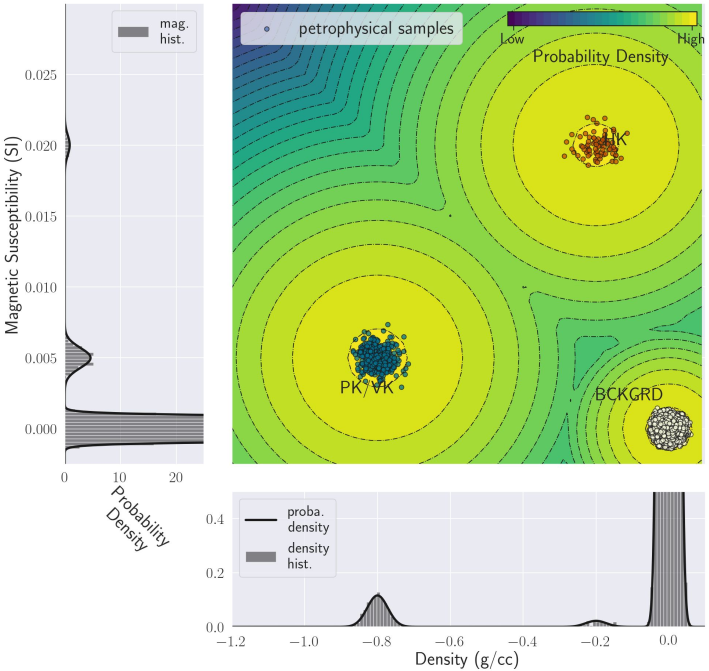
$$\mathcal{M}(\mathbf{m}_i) = \sum_{j=1}^{c} \mathcal{P}(z_i = j)\mathcal{N}(\mathbf{m}_i | z_i = j)$$

Proportions: geology information

Petrophysical Information (+ geophy. weights)

GMM prior

Synthetic rock samples and physical properties distribution



$$\mathcal{P}_{\text{small}}(\mathbf{m}|\Theta) = \prod_{i=1}^{n} \sum_{j=1}^{c} \mathcal{P}(z_i = j)\mathcal{N}(\mathbf{m}_i | \boldsymbol{\mu}_j, \mathbf{W}_i^{-\top} \boldsymbol{\Sigma}_j \mathbf{W}_i^{-1})$$

Number of expected units (c)

Loop over the mesh ($\prod_{i=1}^{n}$)

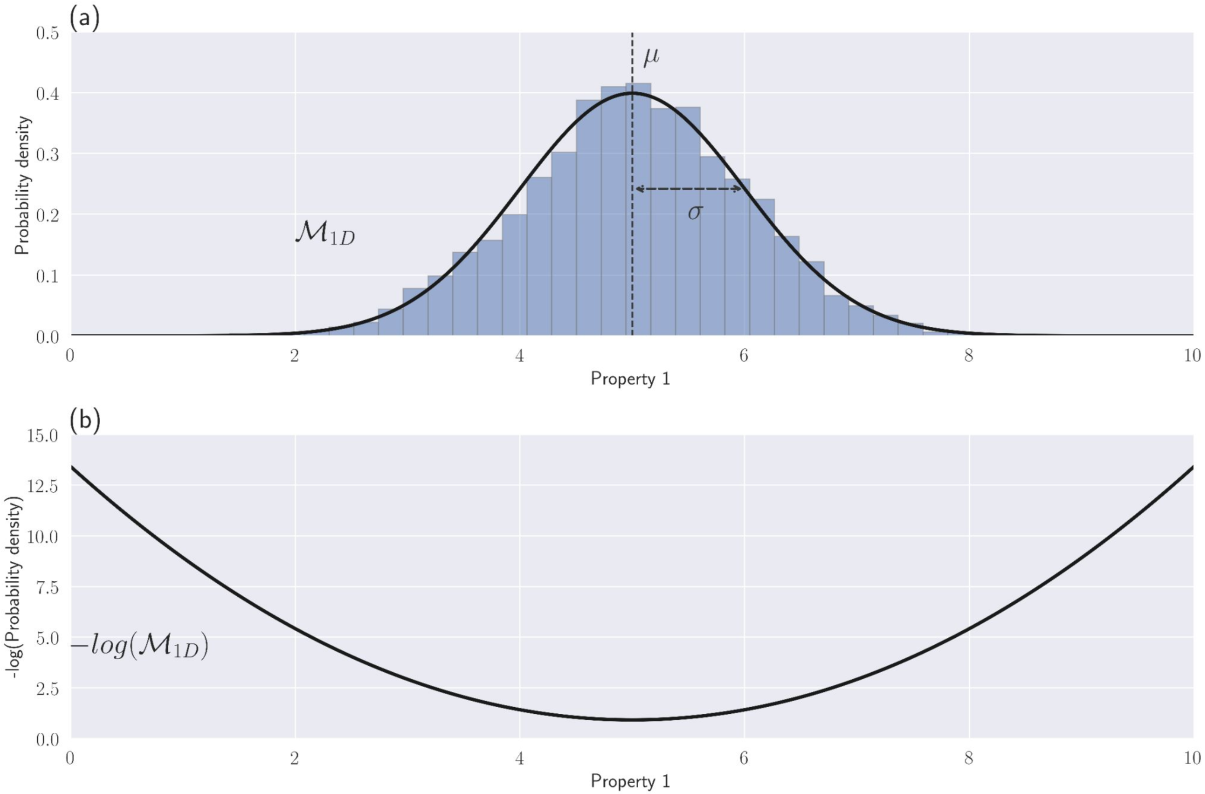
petrophysical information:
- Updatable $\mathbf{m}_{\text{ref}}$ ($\boldsymbol{\mu}_j$)
- Covariance of each unit ($\boldsymbol{\Sigma}_j$)

Geological information:
- Lithology occurrence ($\mathcal{P}(z_i = j)$)
- Depth/Sensitivity weighting ($\mathbf{W}_i$)

With: $\mathbf{W}_i = \mathbf{diag}(\mathbf{w}_i)$

Define: $\Phi_{\text{small}}(\mathbf{m}) = -\log\left(\mathcal{P}_{\text{small}}(\mathbf{m})\right)$

Link between probabilistic and objective function formulation



L_2 approximation

Valid when units are “distinct enough”.

Motivations:

- Practical: compatible with standard compiled codes
- Pedagogical: easier formulation to understand.

The implementation can handle both the exact and approximated regularizations.

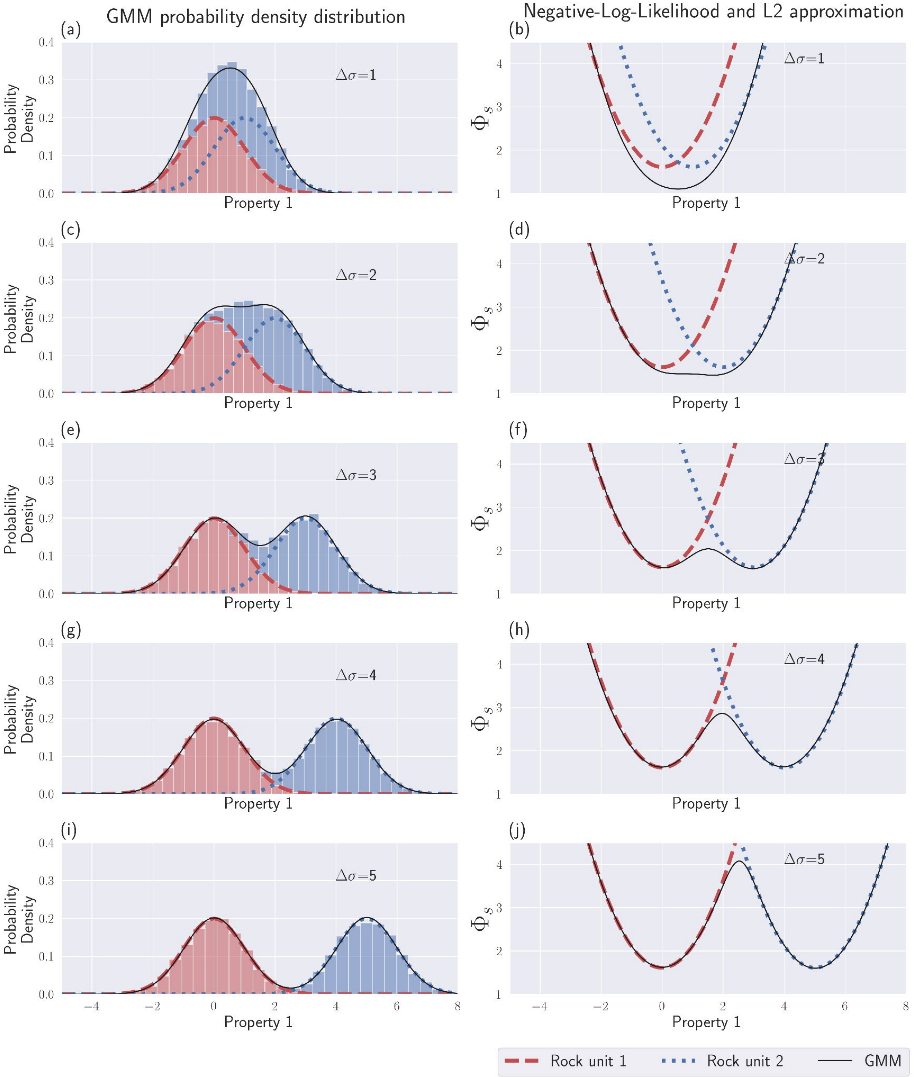
$$\Phi_{\text{small}}(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{n} ||\mathbf{W}_s(\Theta, z_i)(\mathbf{m}_i - \mathbf{m}_{\text{ref}}(\Theta, z_i))||_2^2,$$

with:

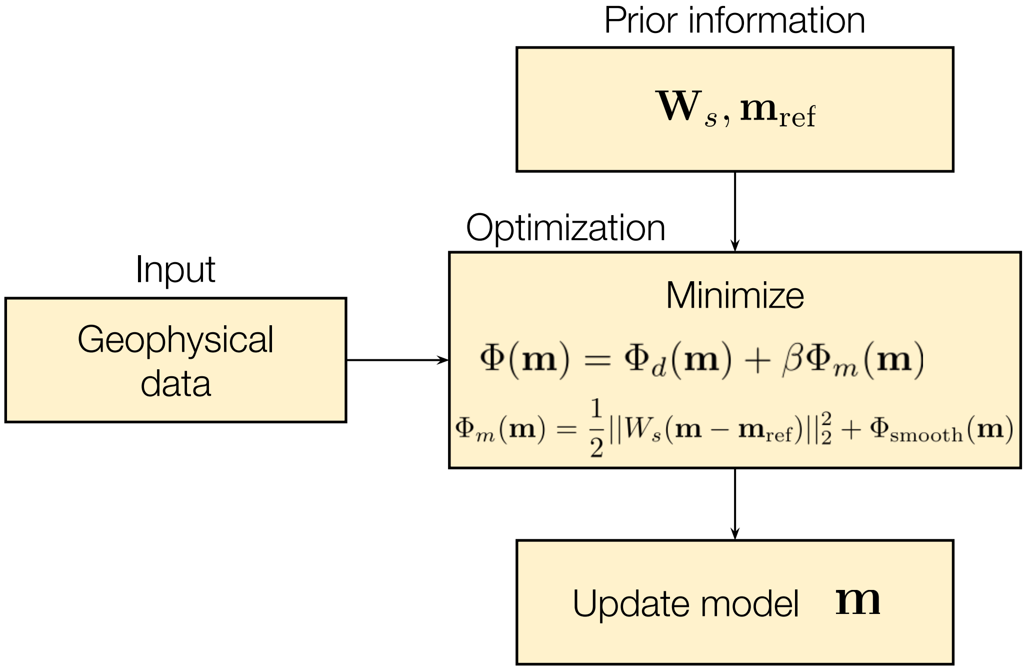
$$z_i = \underset{\tilde{z}_i \in \{1..j\}}{\operatorname{argmax}} \mathcal{N}(\mathbf{m}_i|\tilde{z}_i)\mathcal{P}(\tilde{z}_i),$$

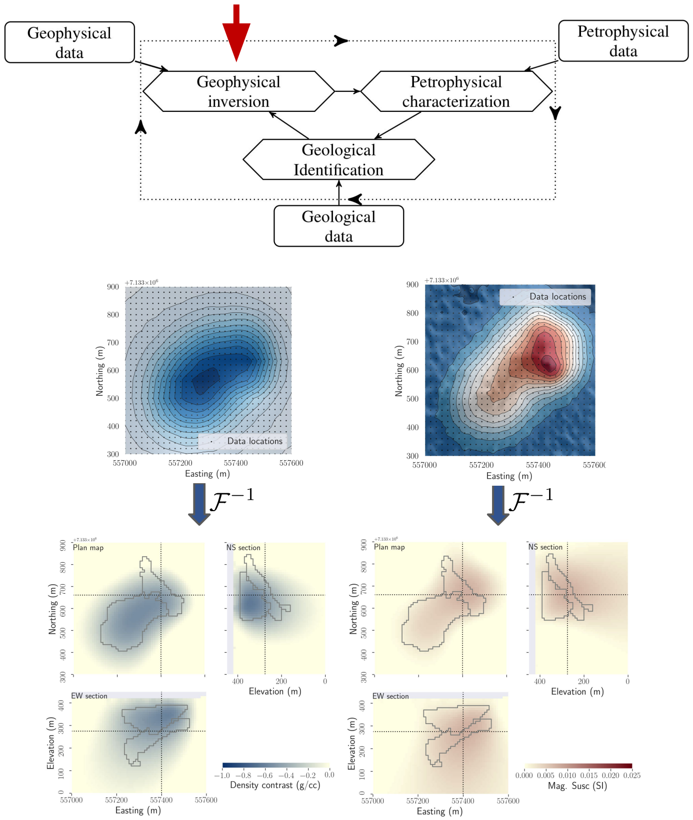
$$\mathbf{m}_{\text{ref}}(\Theta, z_i) = \boldsymbol{\mu}_{z_i},$$

$$\mathbf{W}_s(\Theta, z_i) = \boldsymbol{\Sigma}_{z_i}^{-1/2}\mathbf{W}_i$$



Geophysical inversion

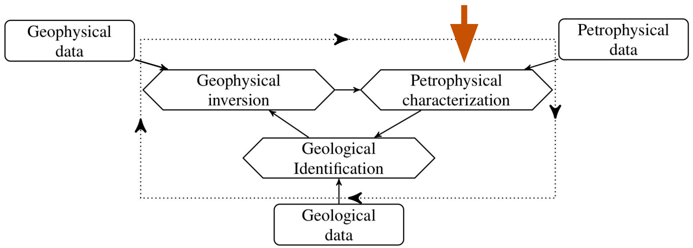




L_2 approximation for “distinct enough” units:

$$\Phi_{\text{small}}(\mathbf{m}) = -\log(\text{GMM}) \approx \frac{1}{2}\sum_{i=1}^{n} ||\mathbf{W}_s(\Theta)\left(\mathbf{m}_i - \mathbf{m}_{\text{ref}}(\Theta)\right)||_2^2$$

Petrophysical characterization



Gaussian Mixture Model (GMM)

$$\mathcal{M}(m|\theta) = \sum_{j=1}^{c} \pi_j \mathcal{N}(m|\mu_j, \Sigma_j)$$

Prior information

Petrophysical data
$\mathcal{P}(\Theta)$

Optimization

Input

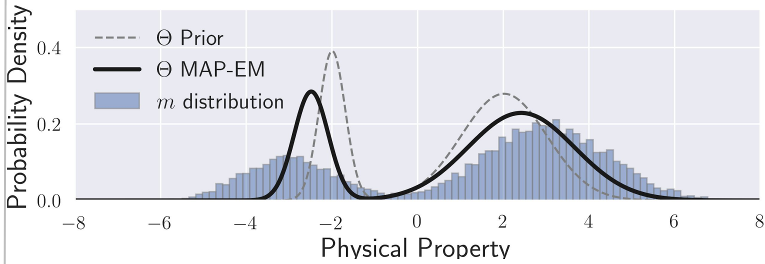
Geophysical model
$\mathbf{m}$

MAP-EM
Solution

$$\underset{\Theta}{\operatorname{argmax}} \mathcal{M}(\mathbf{m}|\Theta)\mathcal{P}(\Theta)$$

$$\Theta \begin{cases} \pi_j : & \text{proportion} \\ \mu_j : & \text{mean} \\ \Sigma_j : & \text{covariance} \end{cases}$$

New GMM
update π_j, μ_j, Σ_j



Learn from the geophysics missing petrophysical information

Geological identification

Prior information

Geological data
$\mathcal{P}(\mathbf{z})$

Input

GMM and model
π_j, μ_j, Σ_j $\quad \mathbf{m}$

Optimization

Each pixel assigned a rock unit

$$\mathbf{z}^* = \underset{\mathbf{z}}{\text{argmax}}\, \mathcal{P}(\mathbf{m}|\mathbf{z})\mathcal{P}(\mathbf{z})$$

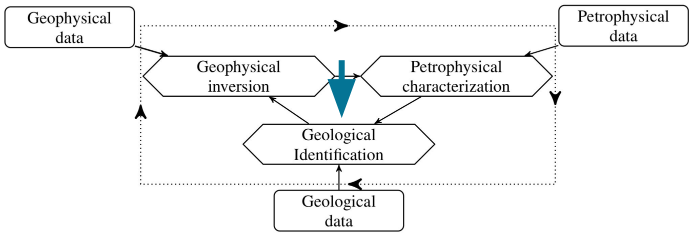
$$\mathbf{m}_{\text{ref}}(\Theta, \mathbf{z}^*) = \mu_{\mathbf{z}^*}$$

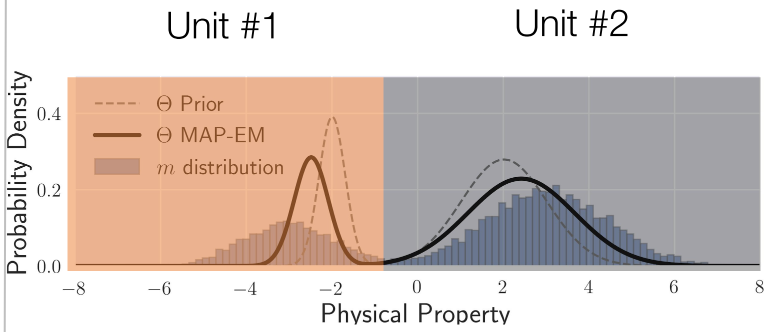
$$W_s(\Theta, \mathbf{z}^*) = \text{diag}(\mathbf{w} \circ \sigma_{\mathbf{z}^*}^{-1})$$

Each pixel

(1) Expected value $\quad \mathbf{m}_{\text{ref}}$

(2) covariance $\quad \mathbf{W}_s$





Petrophysically and Geologically guided Inversion (PGI)

After each iteration on **m**:

1. Learn a new physical properties distribution $\boldsymbol{\Theta}$, averaging the prior information and current inversion model
2. Update the quasi-geology model **z** according to **m**, $\boldsymbol{\Theta}$, and prior geology information
3. Update $\mathbf{m}_{\text{ref}}$ and $\mathbf{W}_s$ according to $\boldsymbol{\Theta}$ and **z**

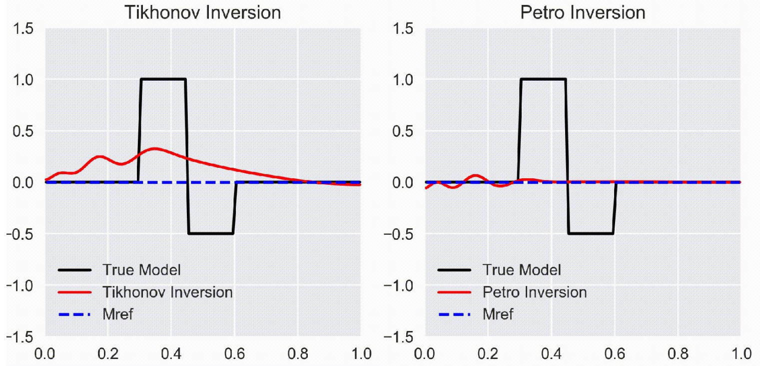
$$\Phi_{\text{small}}(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{n} ||\mathbf{W}_s(\Theta, z_i)(\mathbf{m}_i - \mathbf{m}_{\text{ref}}(\Theta, z_i))||_2^2,$$

with:

$$z_i = \underset{\tilde{z}_i \in \{1..j\}}{\text{argmax}}\, \mathcal{N}(\mathbf{m}_i|\tilde{z}_i)\mathcal{P}(\tilde{z}_i),$$

$$\mathbf{m}_{\text{ref}}(\Theta, z_i) = \boldsymbol{\mu}_{z_i},$$

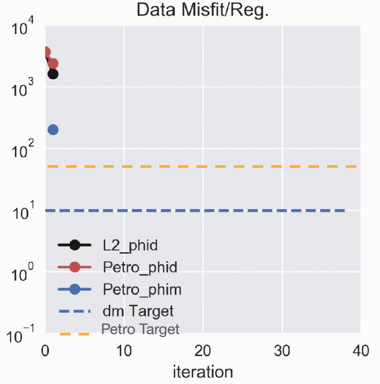
$$\mathbf{W}_s(\Theta, z_i) = \boldsymbol{\Sigma}_{z_i}^{-1/2}\mathbf{W}_i$$



Target misfits:

$$\Phi_d^* = \frac{\#d}{2}$$

$$\Phi_{\text{small}}^* = \frac{\#n}{2}$$



Petrophysically and Geologically guided Inversion (PGI)

After each iteration on **m**:

1. Learn a new physical properties distribution **Θ**, averaging the prior information and current inversion model
2. Update the quasi-geology model **z** according to **m**, **Θ**, and prior geology information
3. Update $\mathbf{m}_{\text{ref}}$ and $\mathbf{W}_{\text{s}}$ according to **Θ** and **z**

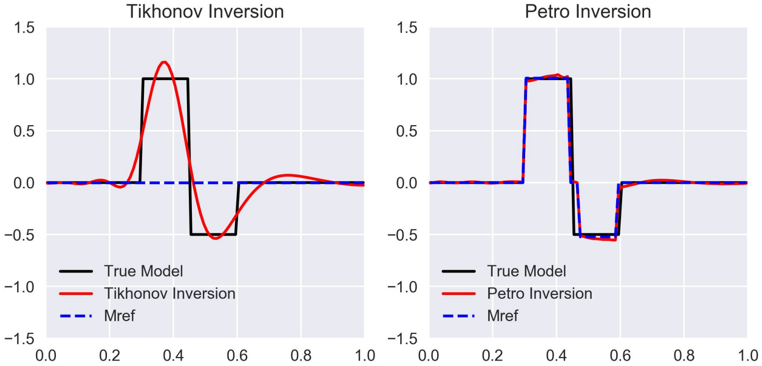
$$\Phi_{\text{small}}(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{n} ||\mathbf{W}_s(\Theta, z_i)(\mathbf{m}_i - \mathbf{m}_{\text{ref}}(\Theta, z_i))||_2^2,$$

with:

$$z_i = \underset{\tilde{z}_i \in \{1..j\}}{\text{argmax}}\, \mathcal{N}(\mathbf{m}_i|\tilde{z}_i)\mathcal{P}(\tilde{z}_i),$$

$$\mathbf{m}_{\text{ref}}(\Theta, z_i) = \boldsymbol{\mu}_{z_i},$$

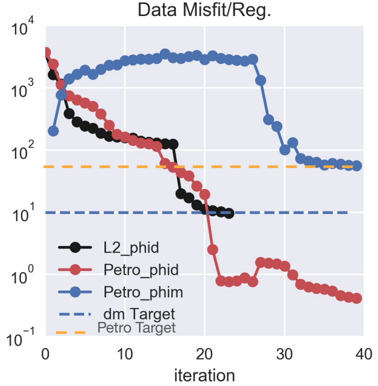
$$\mathbf{W}_s(\Theta, z_i) = \boldsymbol{\Sigma}_{z_i}^{-1/2}\mathbf{W}_i$$



Target misfits:

$$\Phi_d^* = \frac{\#d}{2}$$

$$\Phi_{\text{small}}^* = \frac{\#n}{2}$$



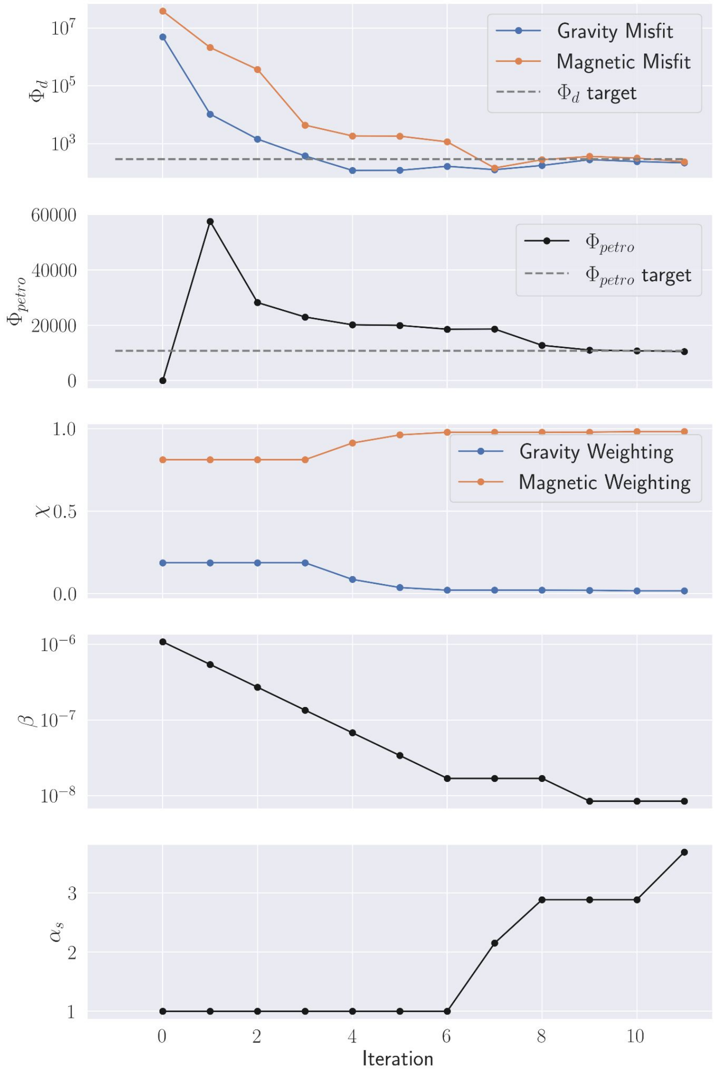
Convergence considerations

Dynamic, heuristic, approach to reweight an intricate Objective Function:

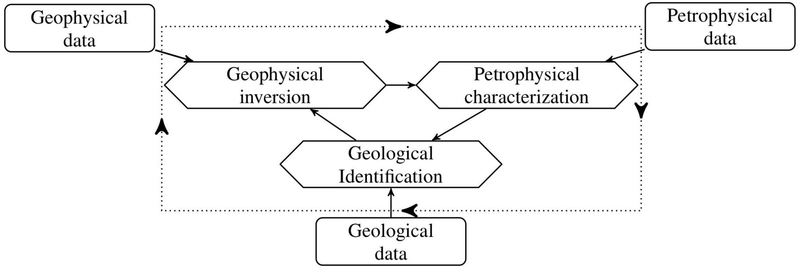
$$\Phi(\mathbf{m}) = \Phi_d(\mathbf{m}) + \beta \left(\alpha_s \Phi_s(\mathbf{m}) + \sum_{p=1}^{q} \alpha_p \Phi_{\text{smooth}}(\mathbf{m}^{\{p\}}) \right),$$

with:

$$\Phi_d(\mathbf{m}) = \sum_{k=1}^{r} \chi_k \Phi_d^k(\mathbf{m}) = \frac{1}{2} \sum_{k=1}^{r} \chi_k ||\mathbf{W}_d^k(\mathbb{F}^k[\mathbf{m}^{\{k\}}] - \mathbf{d}_{\text{obs}}^k)||_2^2$$



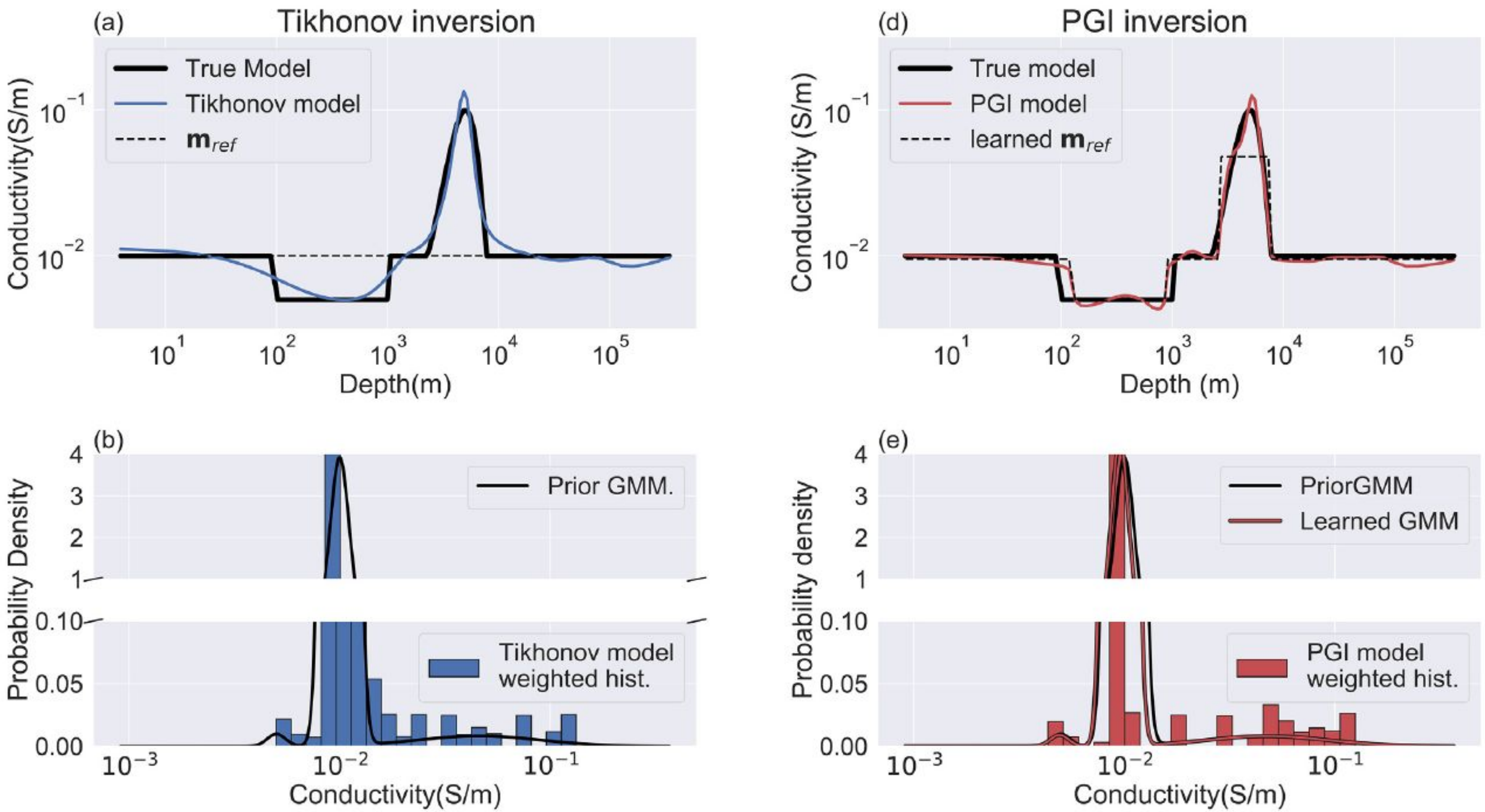
PGI, an extended toolkit

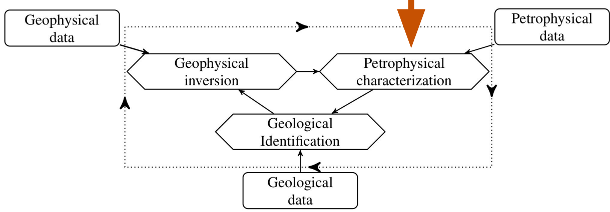


PGI provides advanced tools to **adapt the inverse problem to the geologic questions**

- Recover sharp or smooth features
- Learn the petrophysical model to work with missing information
- Incorporate local expectations about geology
- Reduce dependence on initial reference models from standard approaches
- Make geologic assumptions through the petrophysical characterization
- Implement geologic rules within the construction of the quasi-geology model
- etc.

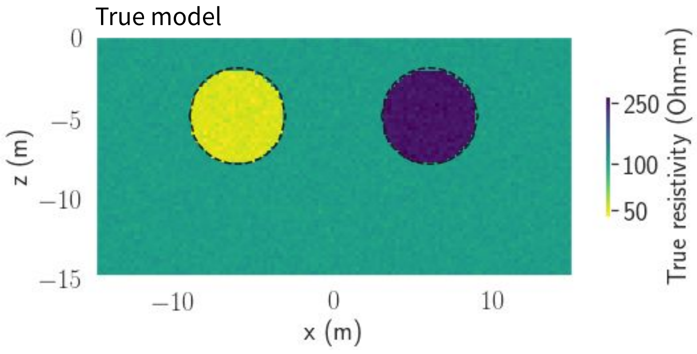
Recovering sharp and smooth features (MT 1D example)

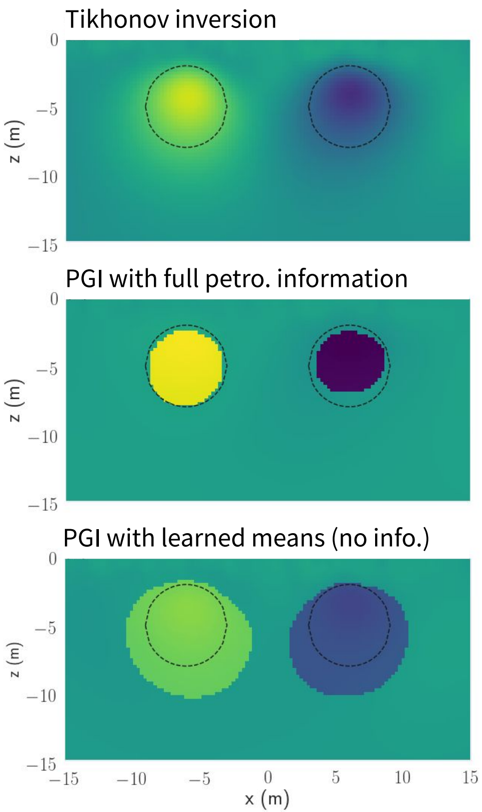


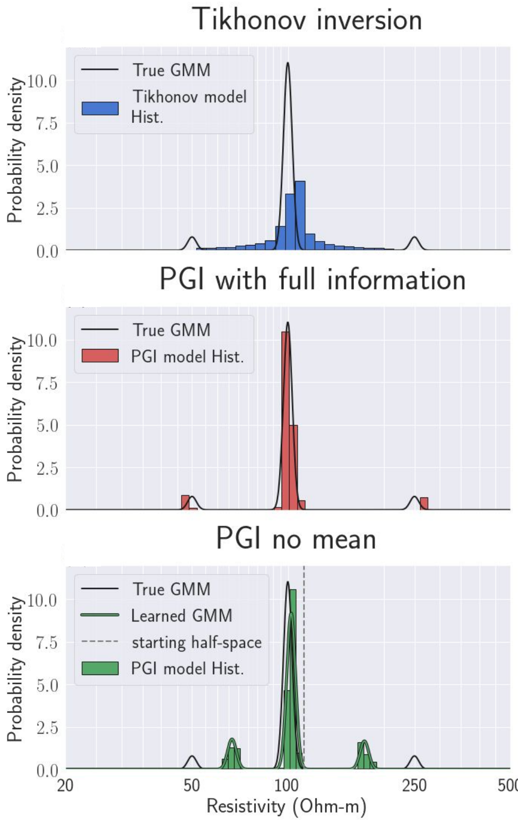


Learning a new petrophysical model

- We can work with partial, incomplete or biased information
    - no mean value information



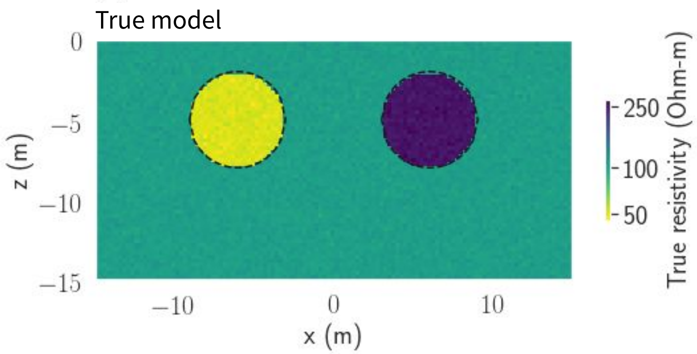


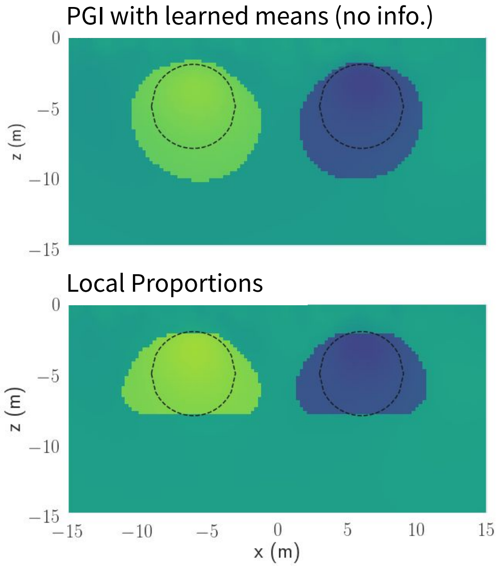


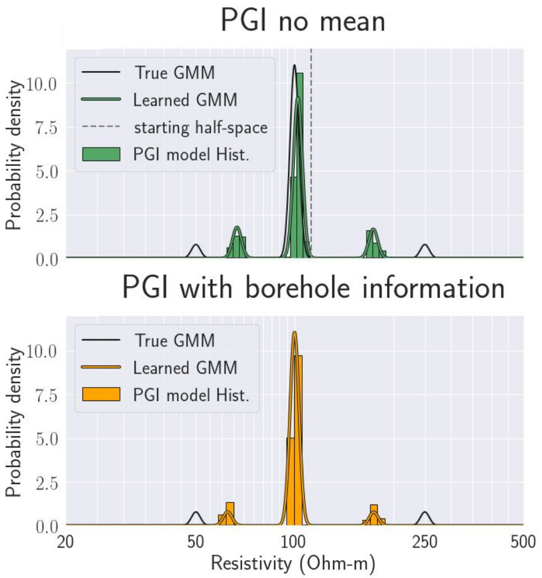
Defining local prior expectations to constrain the geology

$$\mathcal{P}_{\text{small}}(\mathbf{m}|\Theta) = \prod_{i=1}^{n} \sum_{j=1}^{c} \underbrace{\mathcal{P}(z_i = j)}_{} \mathcal{N}(\mathbf{m}_i | \boldsymbol{\mu}_j, \mathbf{W}_i^{-\top} \boldsymbol{\Sigma}_j \mathbf{W}_i^{-1})$$

Prior expectation of finding rock unit *j* at location *i*







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Bookpurnong case study (hydrology, RESOLVE)

Reducing ambiguity for Saline Contamination Characterization by EM Surveys

Problematic:

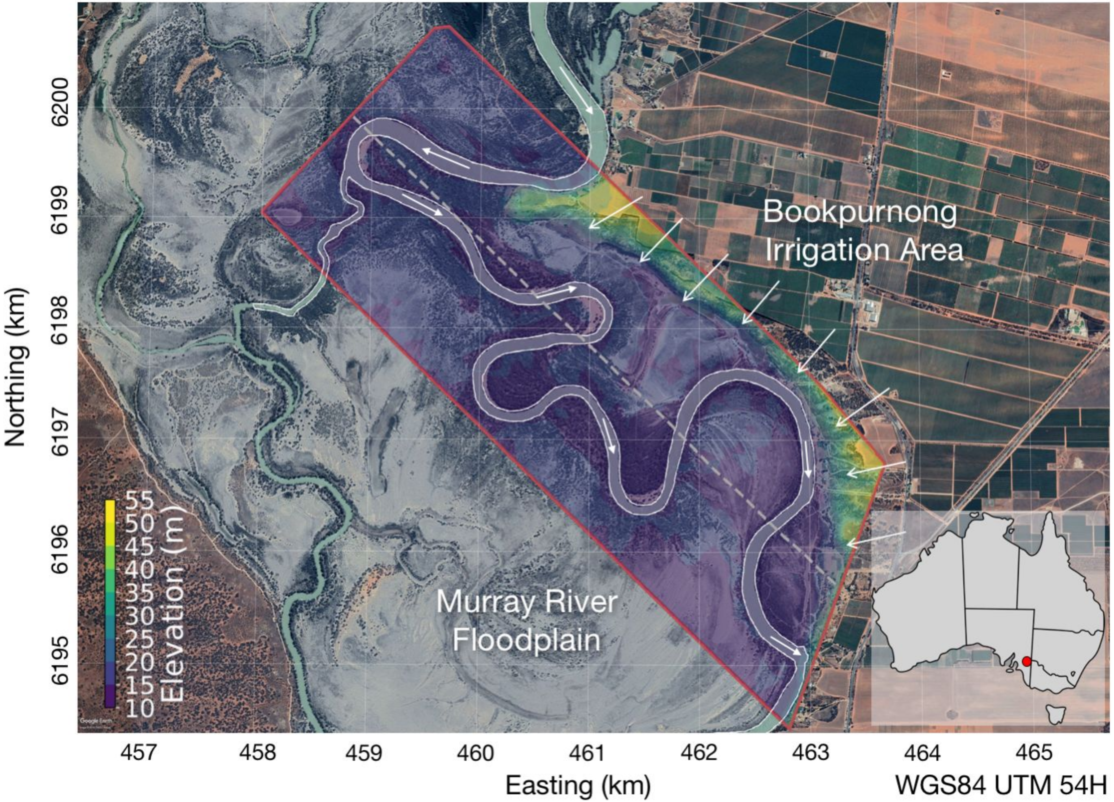
- Irrigation has lead to the salinization of the floodplain soil.

Goal:

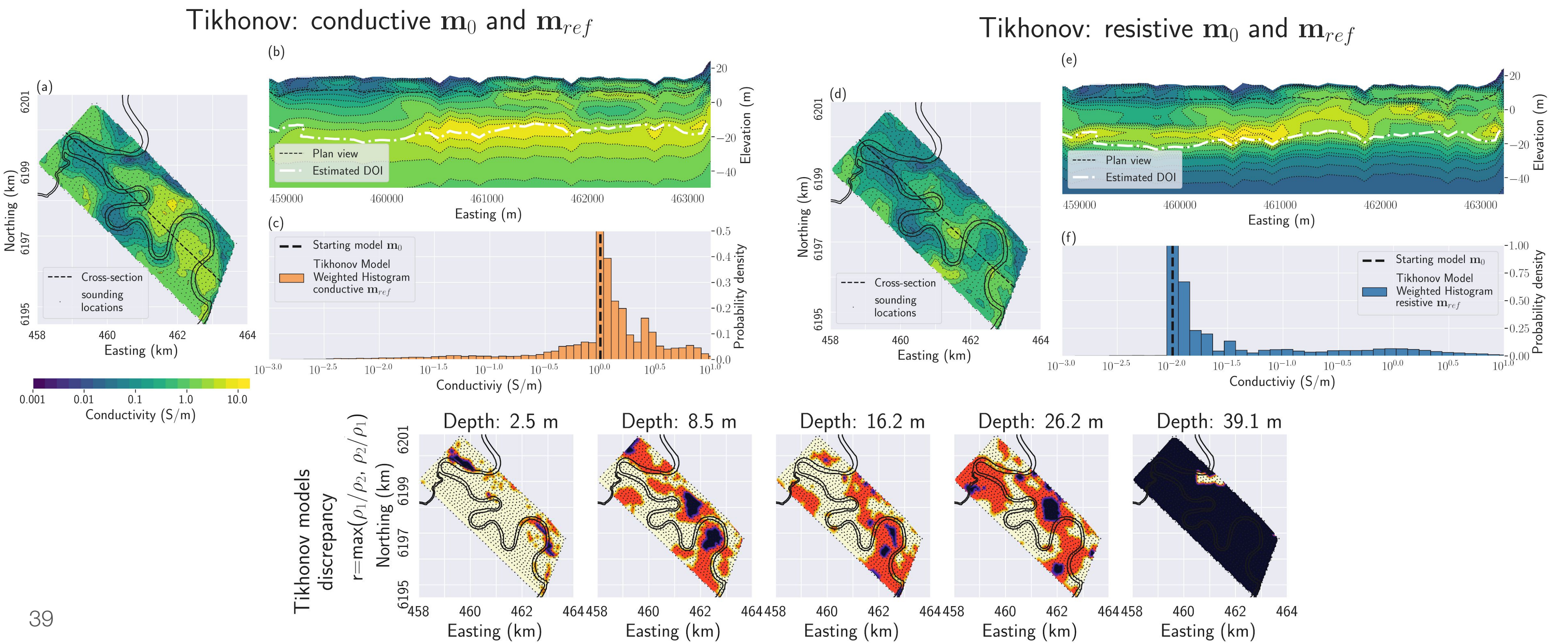
- determine if the freshwater river is charging the aquifer (healthy scenario) or if the saline aquifer is charging the river.

Method:

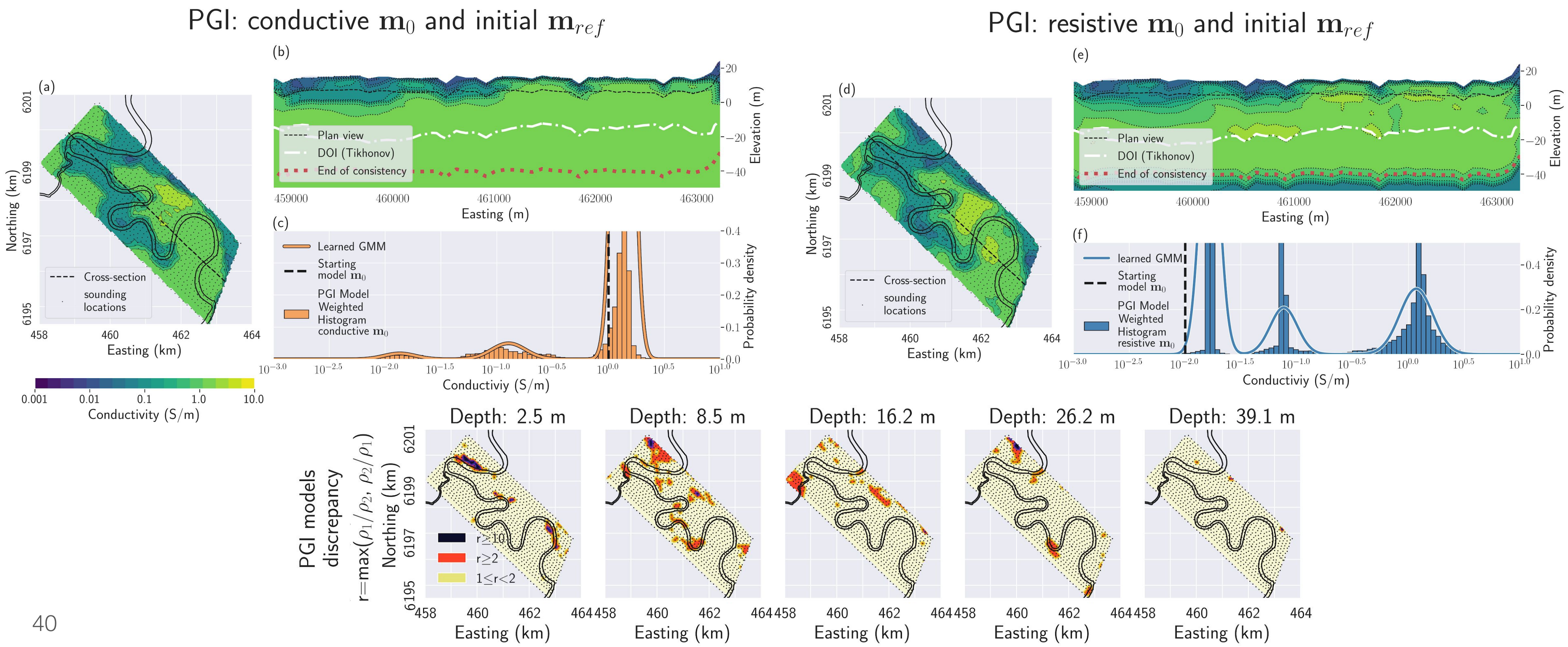
- 1D laterally constrained inversions



Bookpurnong: Tikhonov inversions with various starting and reference models



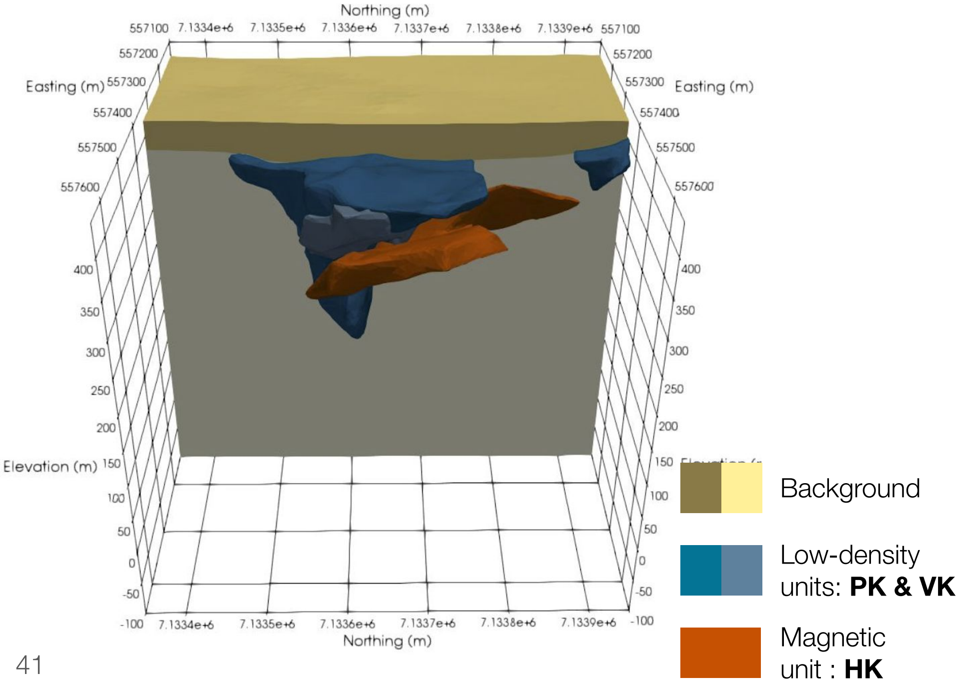
Bookpurnong: PGI with various starting models



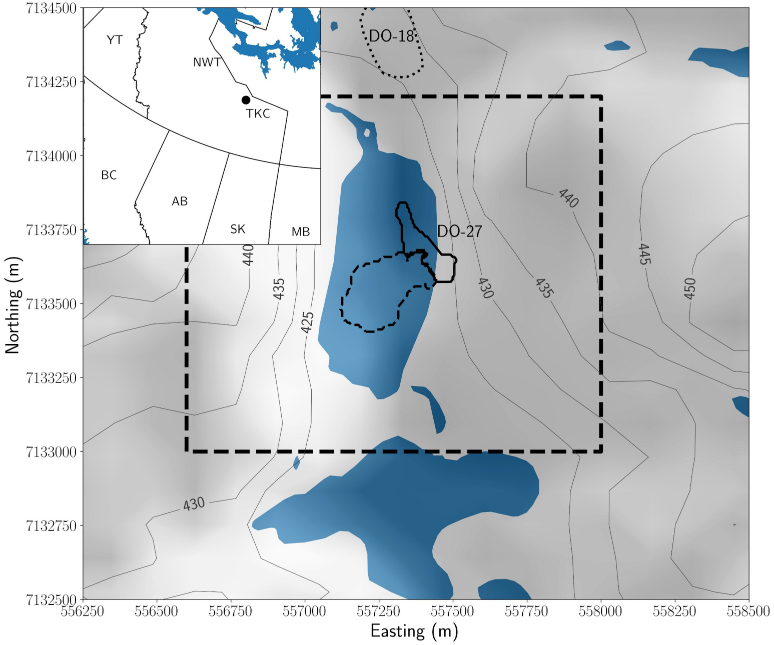
Multi-physics case study: the DO-27 kimberlite pipe

Diamondiferous kimberlite pipe in the Northwest Territories, Canada

Geology model built from boreholes



Location



Astic *et al*. 2020

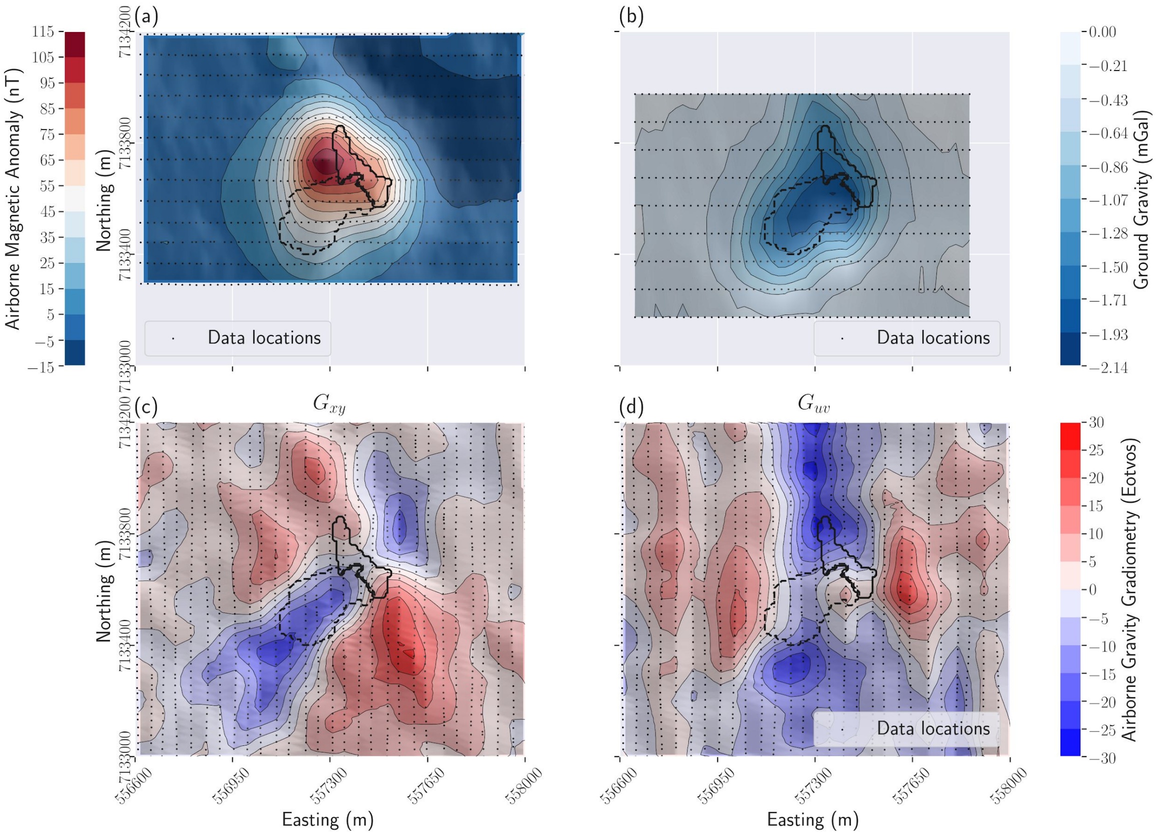
Potential field data sets

3 surveys of interest:

(a) airborne magnetic (VTEM)
(b) ground gravity
(c) and (d) airborne gravity gradiometry (Falcon)

Processing:

- downsample to 25 m
- remove linear trends from:
  - magnetic data
  - ground gravity data



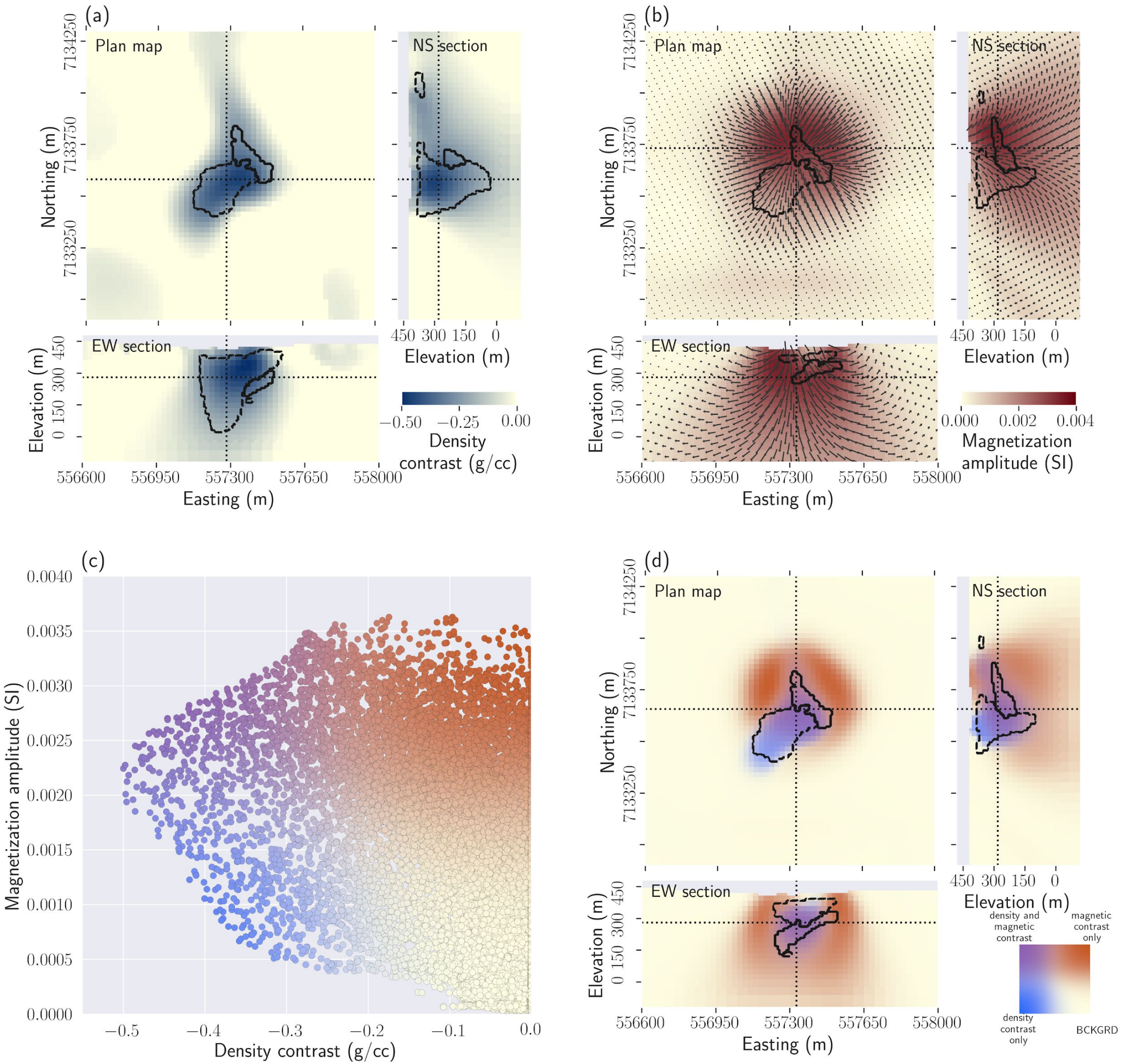
L2 inversions

(a) Joint ground gravity and airborne gravity gradiometry inversion

(b) Magnetic Vector Inversion (MVI)

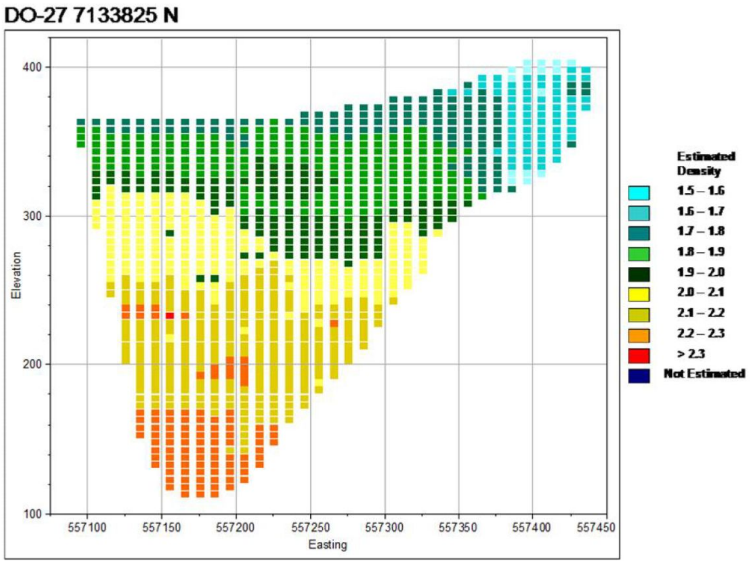
(c) Cross-plot density / magnetic amplitude

(d) Density vs magnetic amplitude

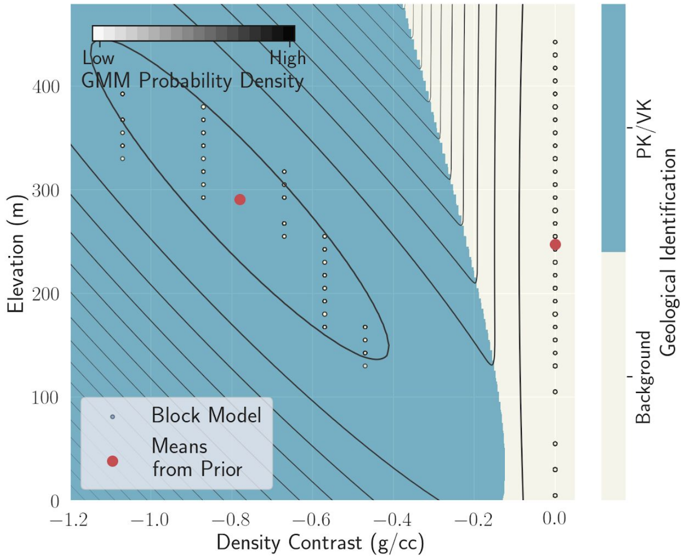


Physical properties: density representation

Petrophysical information:
PK density block model from drilling

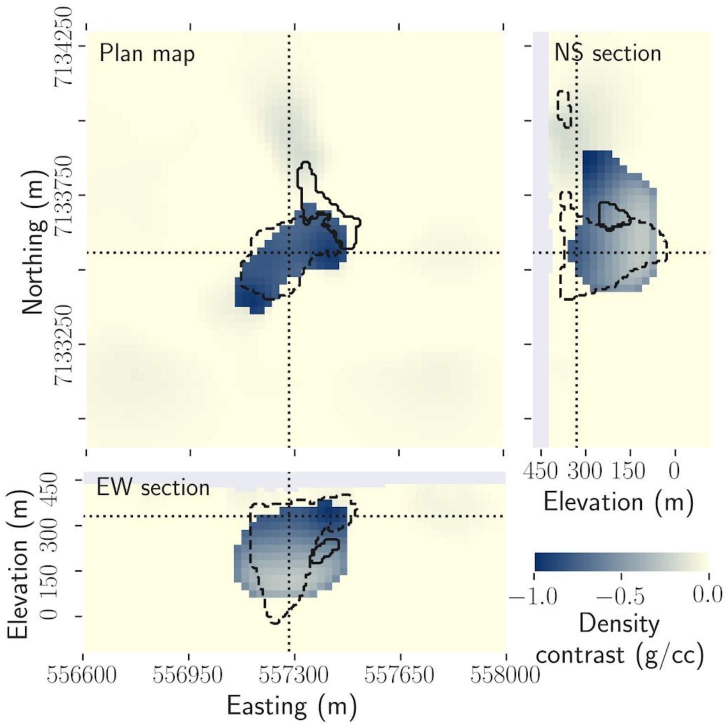


GMM representation:
density means, spreads, and trends
for all rock units

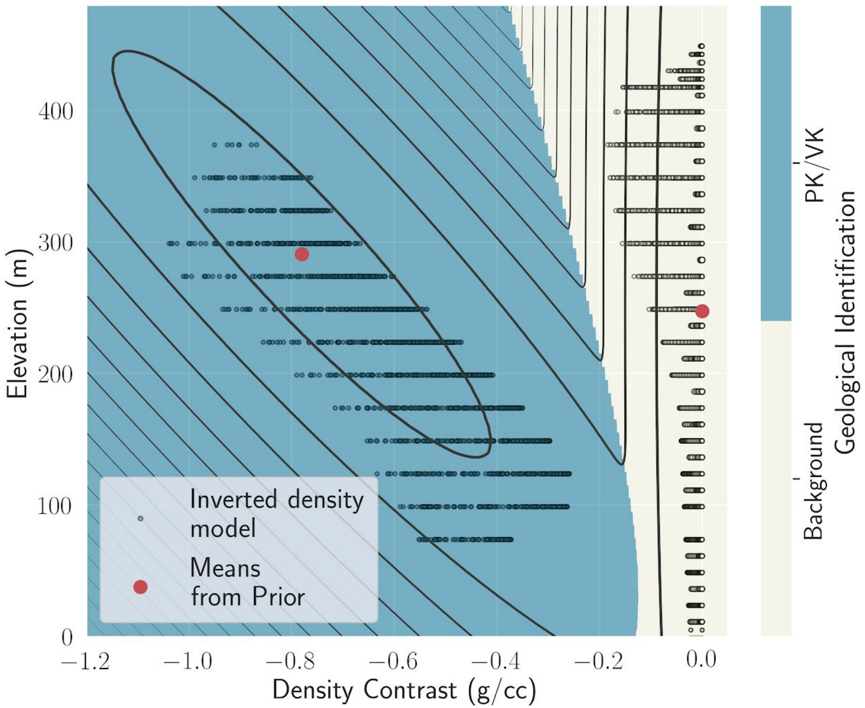


Single-physics PGI: gravity surveys

PGI gravity & gravity gradiometry

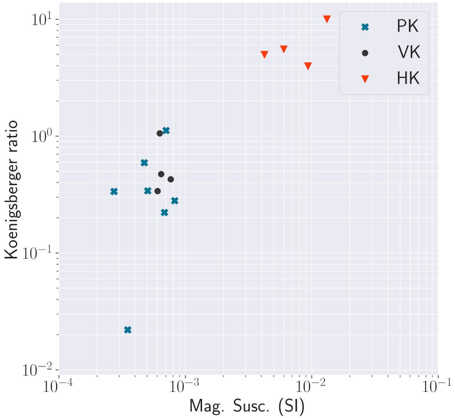


GMM petrophysical fit

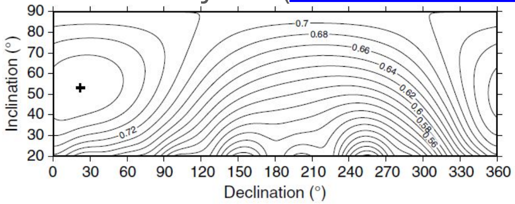


Physical properties: magnetization representation

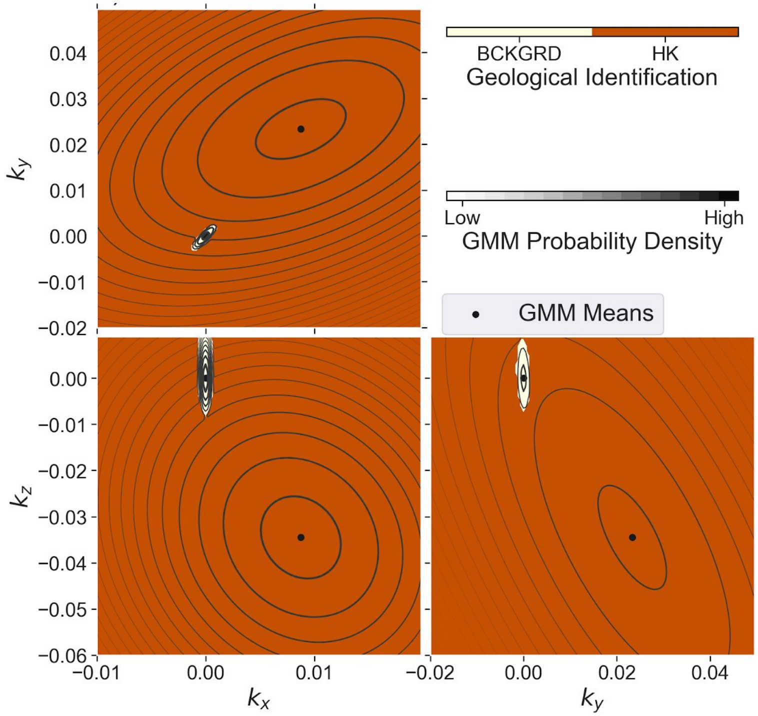
Petrophysical information:
susceptibilities and remanence amplitude from samples (GSC)



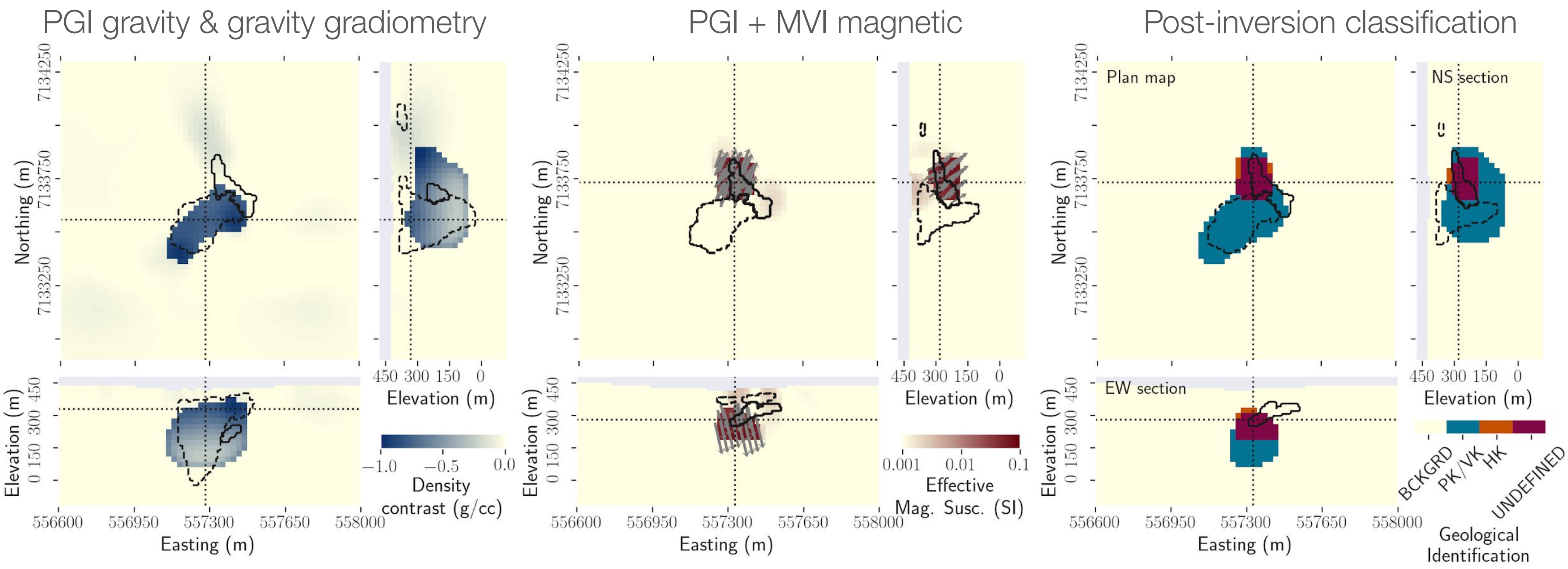
remanence orientation estimation from magnetic data analysis (Devriese *et al.* 2017)



GMM representation:
magnetization means, spreads, and trends in all three directions for all rock units



Single-physics PGIs: post-inversion classification

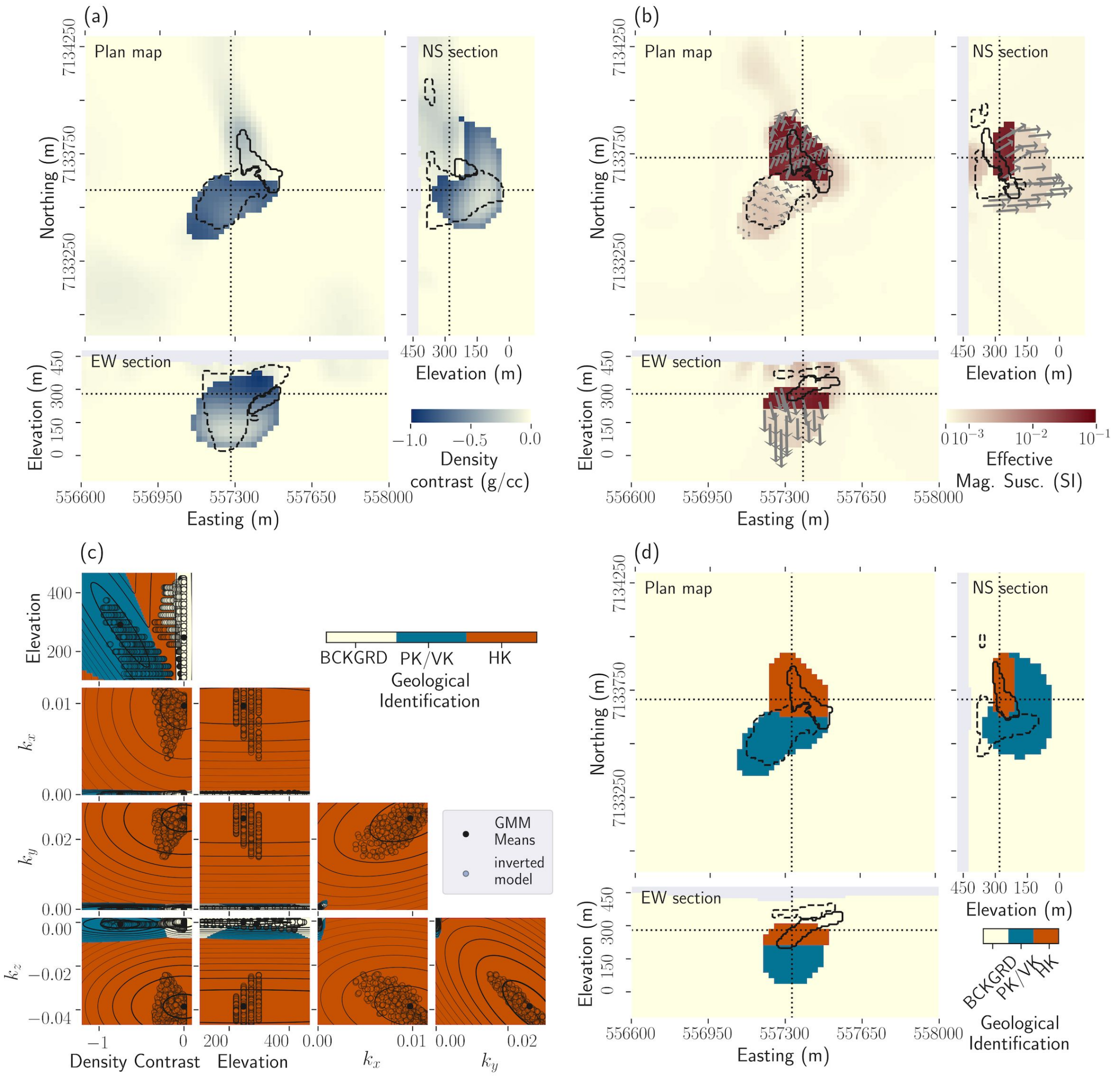


Multi-physics PGI

- 5 parameters (density, magnetic vector 3 components, elevation).
- All 3 geophysical surveys are fitted.
- Petrophysical signatures are reproduced.

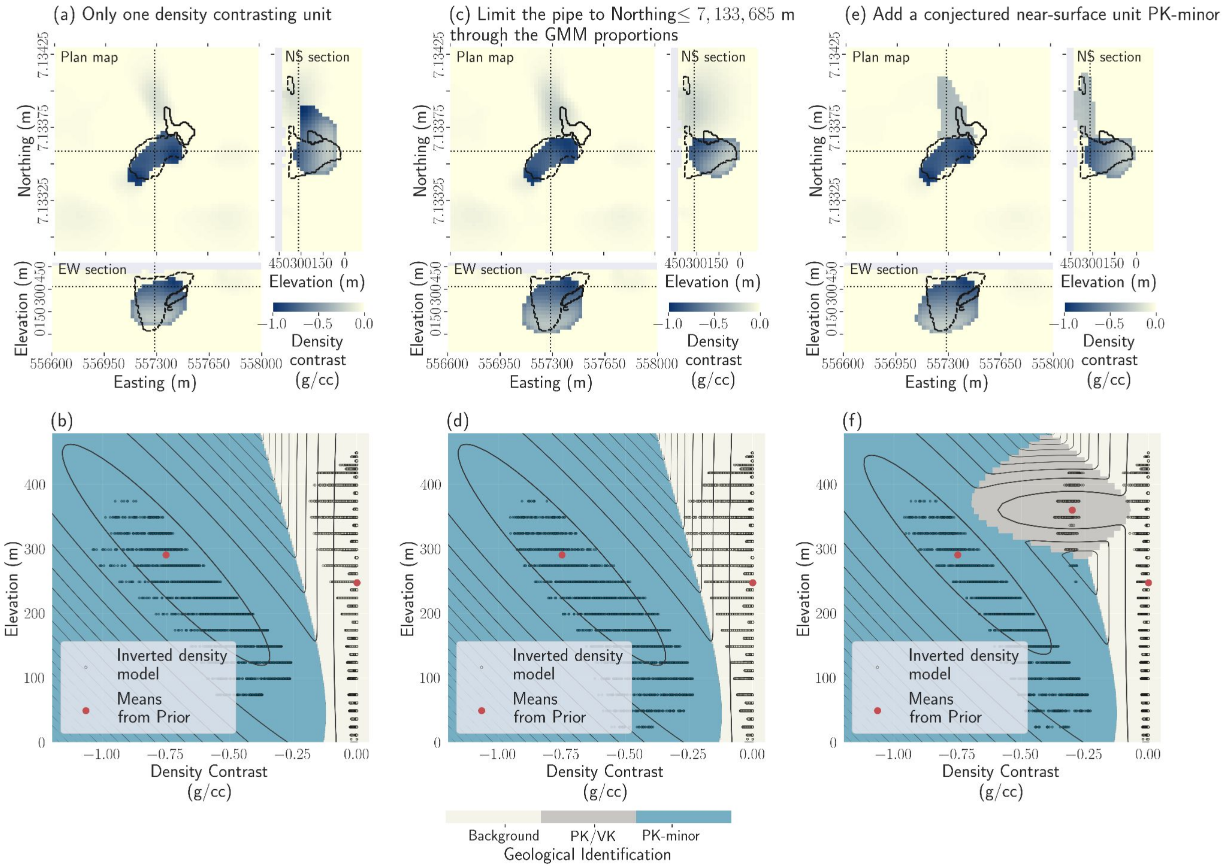
Observations:

- Drillholes in the area have not encountered any PK/VK unit below HK.
- Smooth near-surface anomalies are visible.
- **PK occurrences outside the pipe may have a different density signature.**

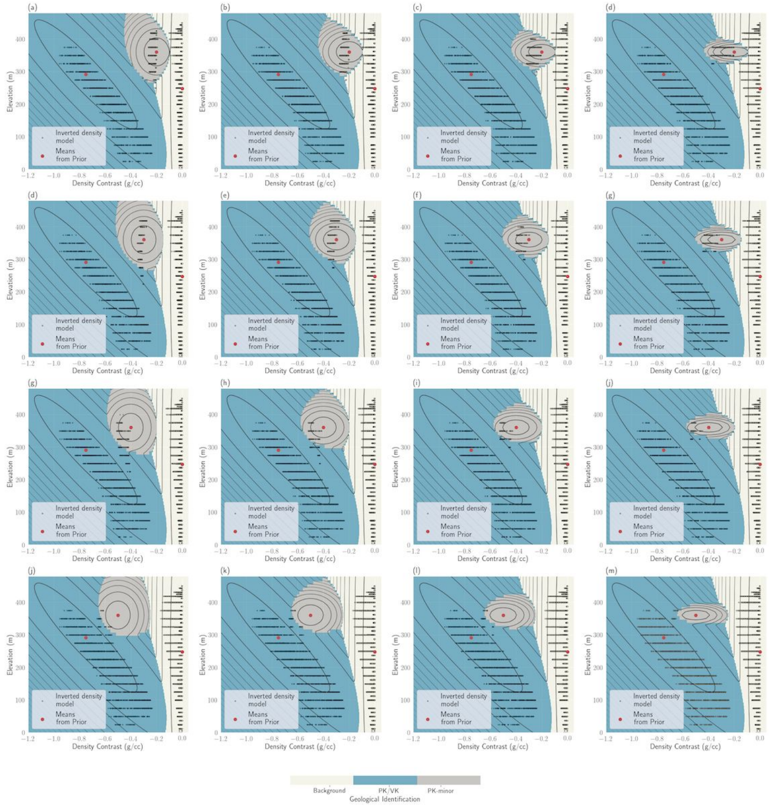


Gravity PGIs to define third kimberlite signature

- Run multiple PGIs with various characteristics for a fourth unit:
  - Density mean
  - Density variance
  - Elevation variance
  - Magnetic: same as PK
  - Limited to the near-surface
  - Limit its occurrence north of the pipe (through local proportions in the GMM)



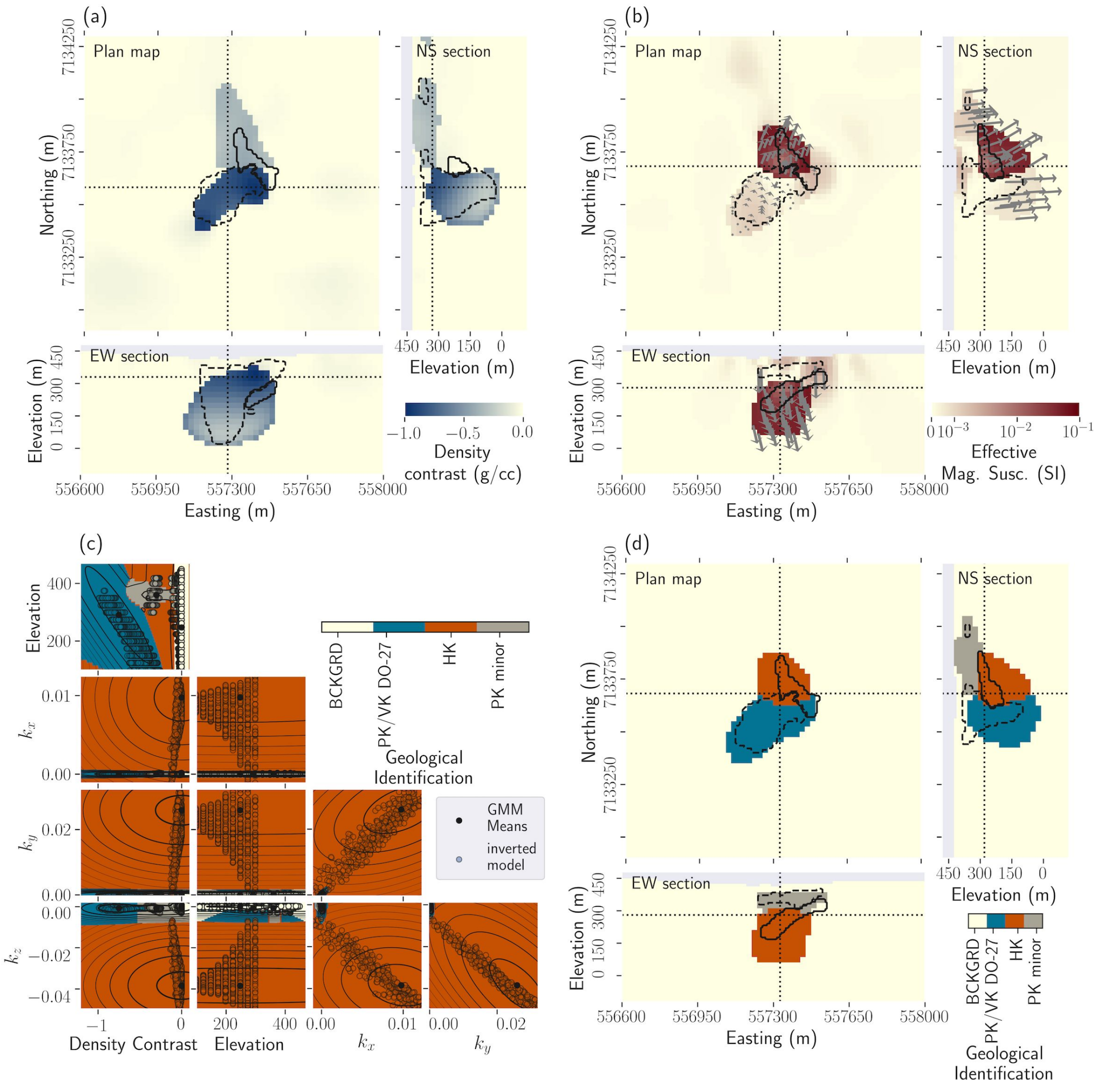
Gravity PGIs to define third kimberlite



Multi-physics PGI assuming an additional kimberlite unit

Geological assumptions
- Additional rock unit (PK-minor)
- Favoured in the near-surface
- Limited to the north (Northing > 7,133,680 m)

Petrophysical assumptions
- Density: -0.3 g/cm^3
- Magnetic: same as PK



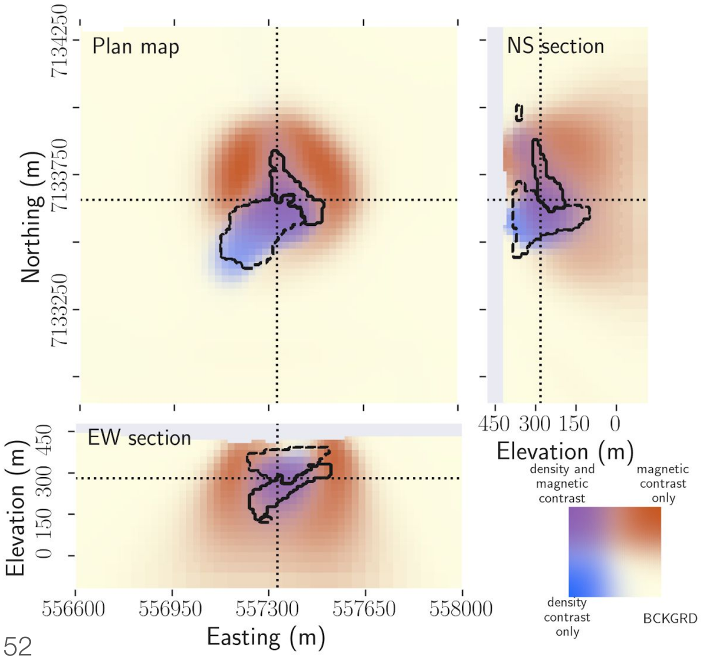
Case study summary

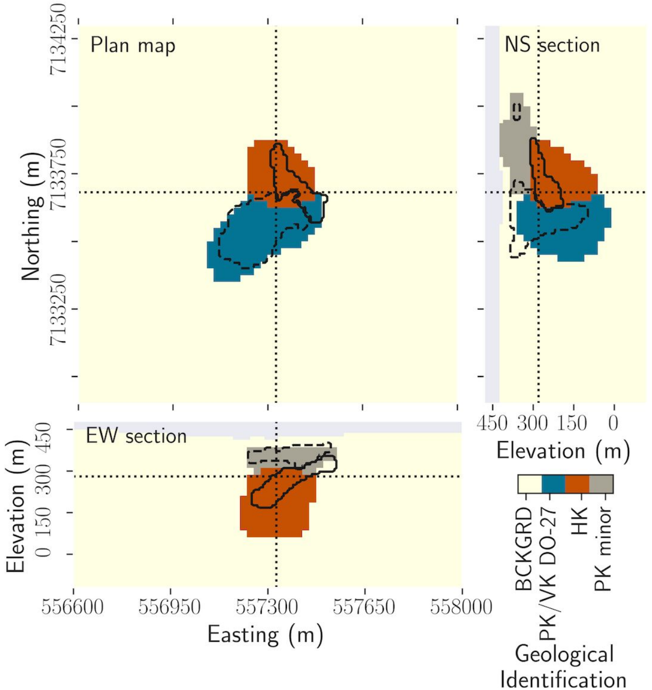
Where we started:

- Inferences from inversions of single data sets can be deficient.

PGI:

- Fit all of the potential fields data and physical property information.
- Can impose geologic assumptions.

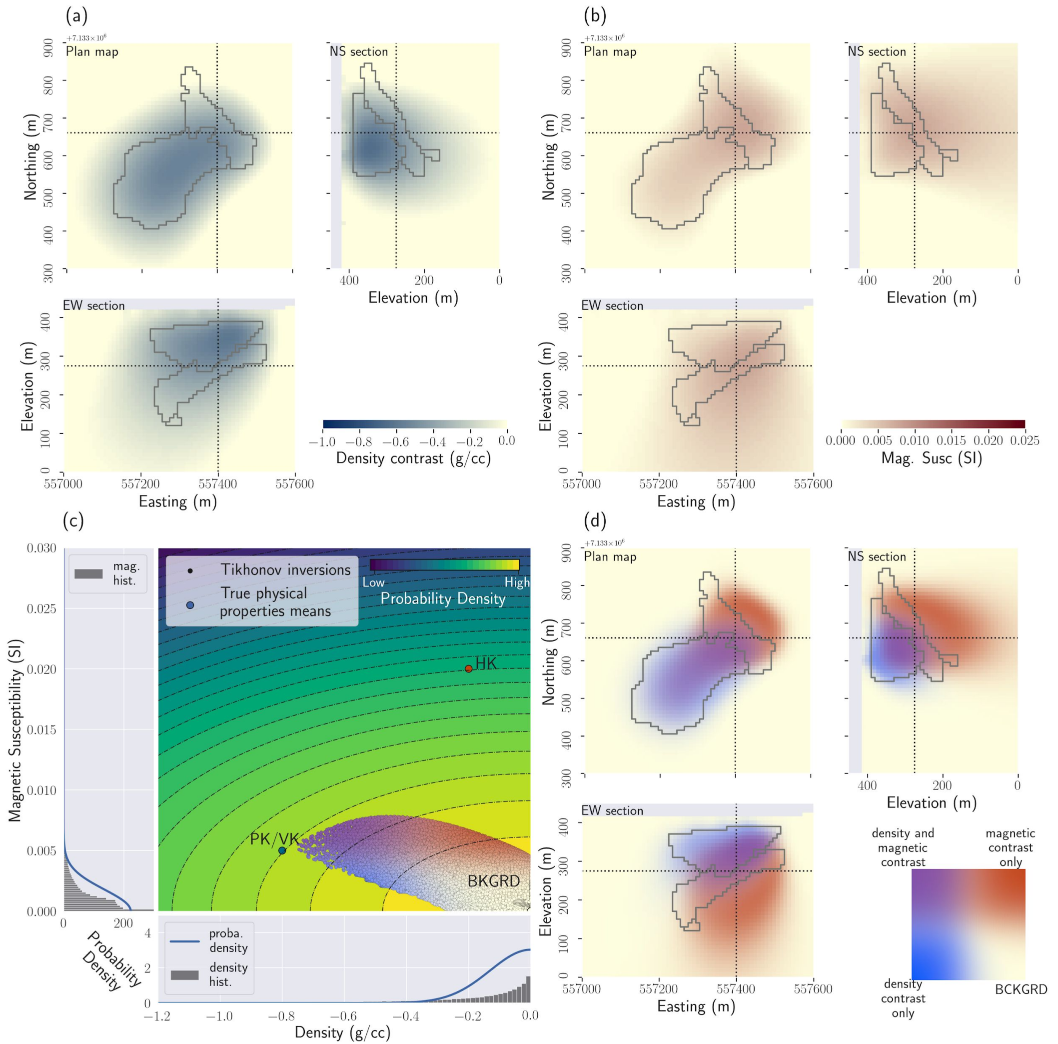


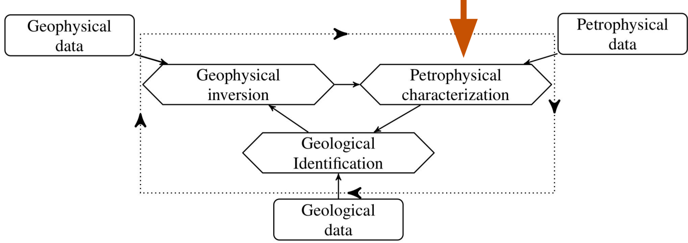


About making geologic assumptions (synthetic)

What if we did not know the petrophysical signatures?

- From the L2 inversion, we can assume 3 units with different characteristics:
  - Background
  - One with a low density and no mag. susc.
  - One with a high mag. susc. but no density contrast.
- **PGI can learn a suitable petrophysical distribution (MAP-EM algorithm).**

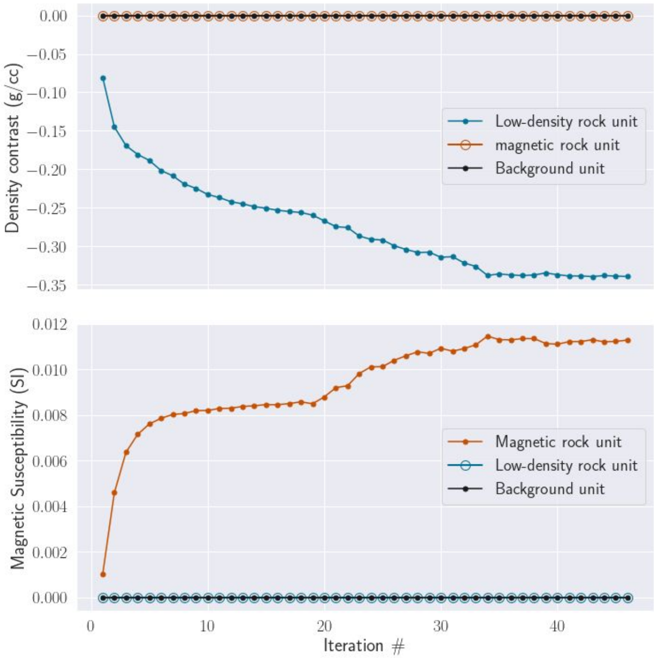


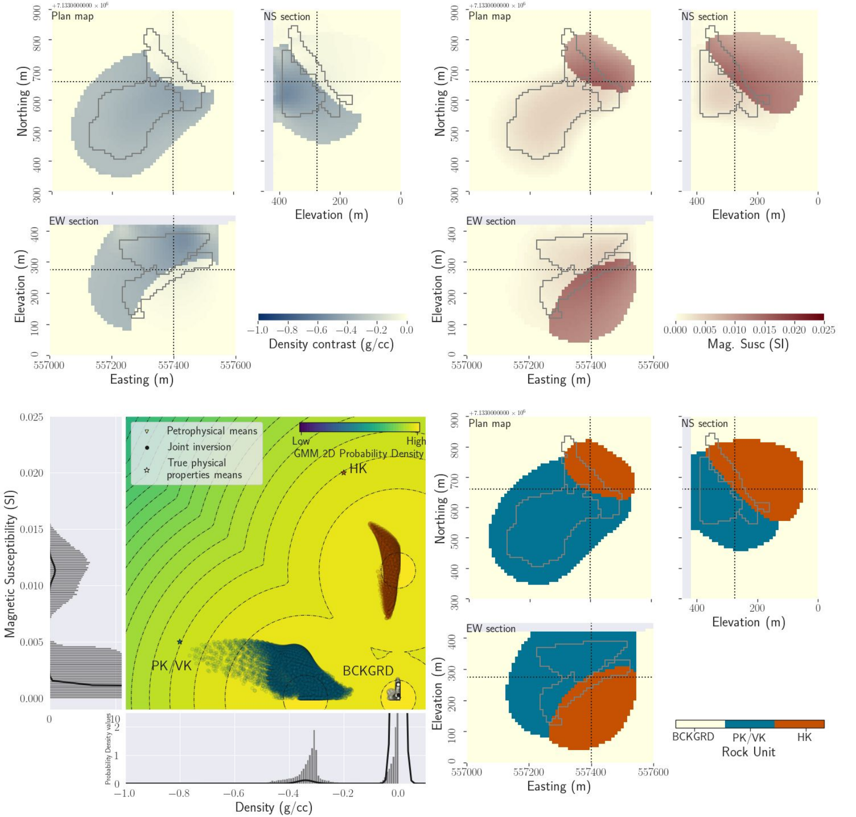


Making geologic assumptions

PGI learns the petrophysical signature of 3 units with the following constraints:

1. Background (*fixed at 0*)
2. One with a low density (*movable*) and no mag. susc. (*fixed at 0*)
3. One with a high mag. susc. (movable) but no density contrast. (*fixed at 0*)

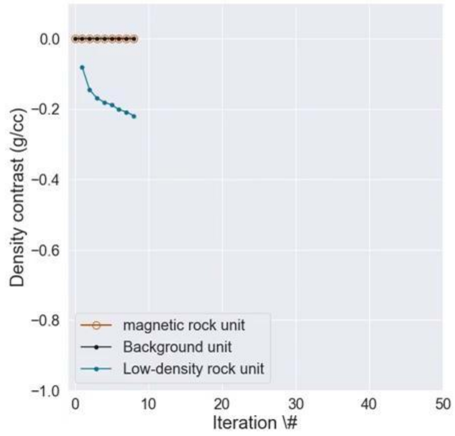


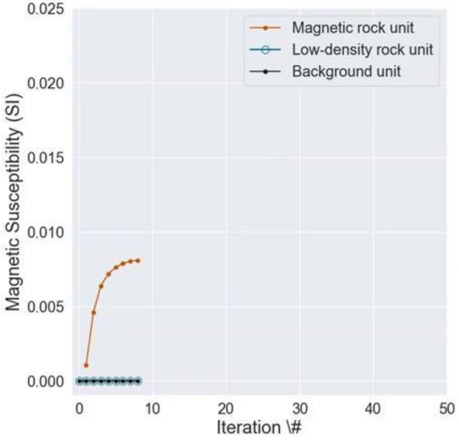


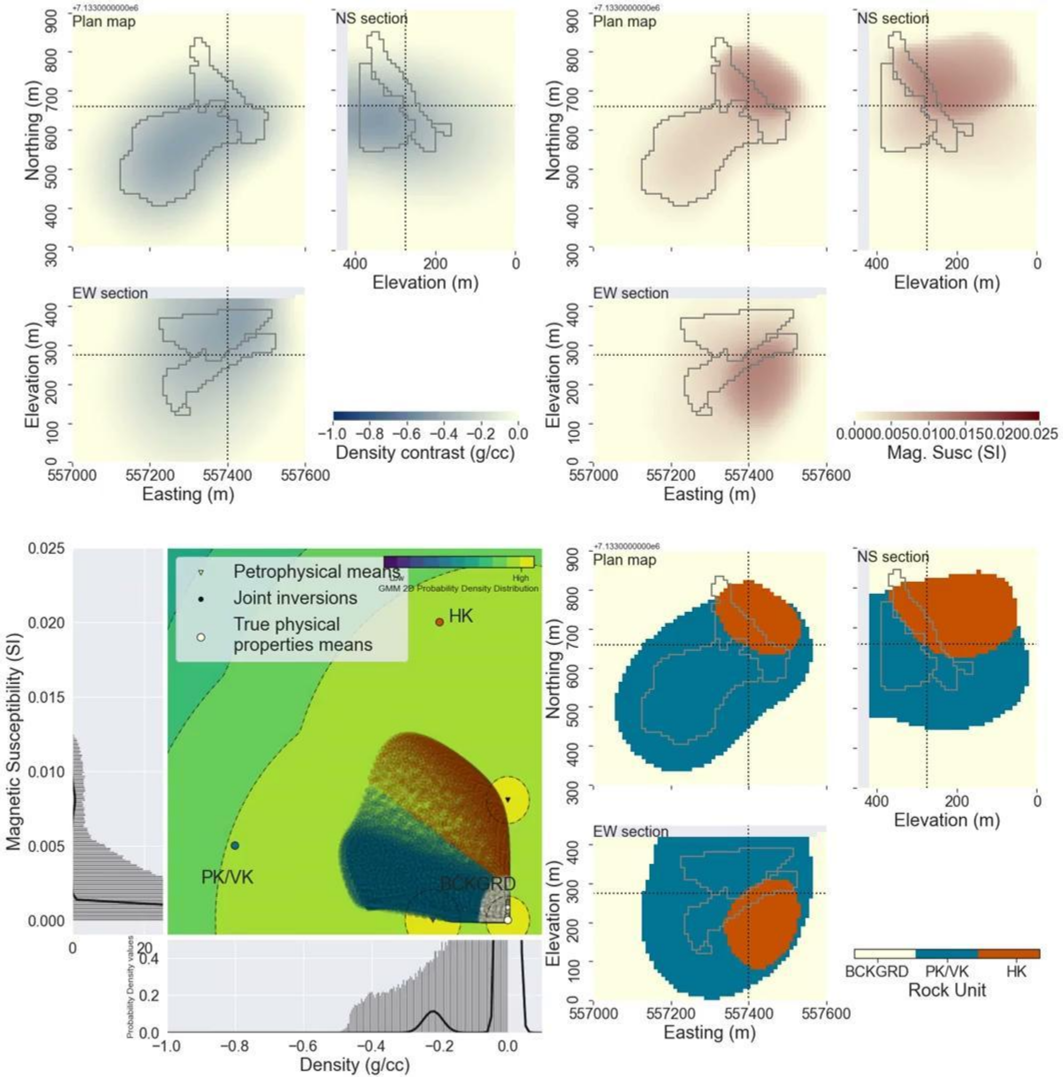
Making geologic assumptions (video)

PGI learns the petrophysical signature of 3 units with the following constraints:

1. Background (*fixed at 0*)
2. One with a low density (*movable*) and no mag. susc. (*fixed at 0*)
3. One with a high mag. susc. (movable) but no density contrast. (*fixed at 0*)







Geological identification

Limits of the current implementation:

- Cell-by-cell classification.
- No information about geology is shared across neighboring cells.
- Continuity, orientation, etc. are ensured by the smoothness on the physical properties.

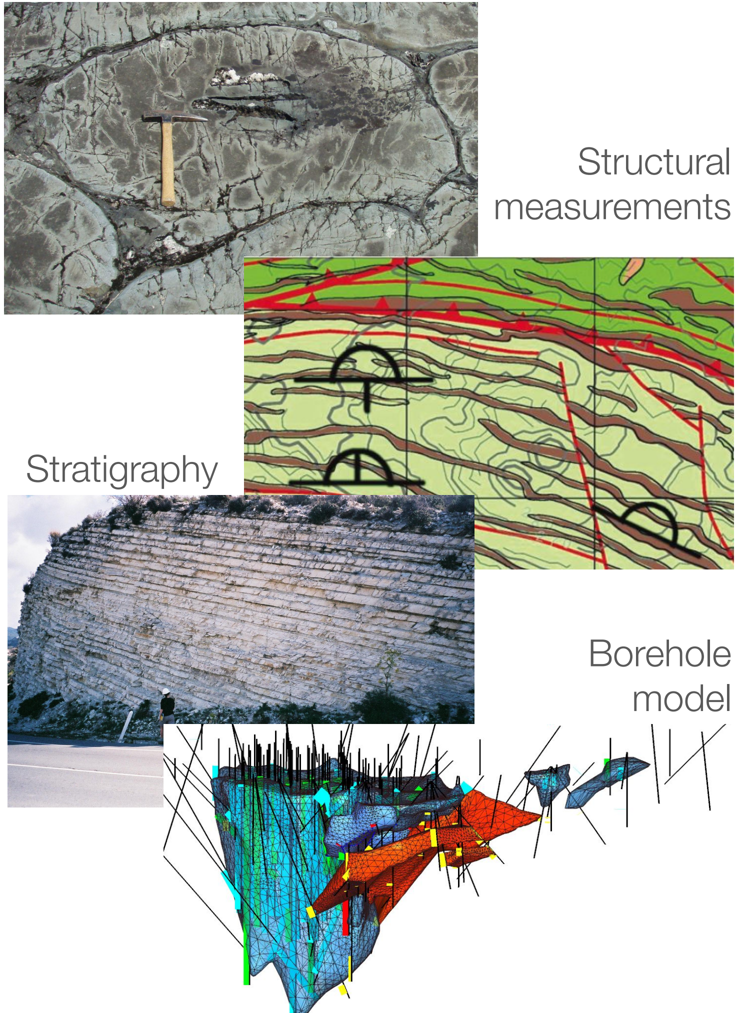
> Change the prior information formulation to propagate geology information across cells

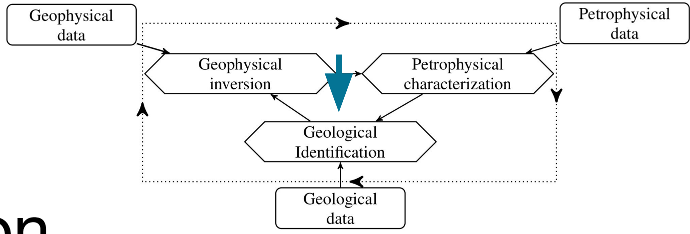
Outcrops or borehole log

Structural measurements

Stratigraphy

Borehole model



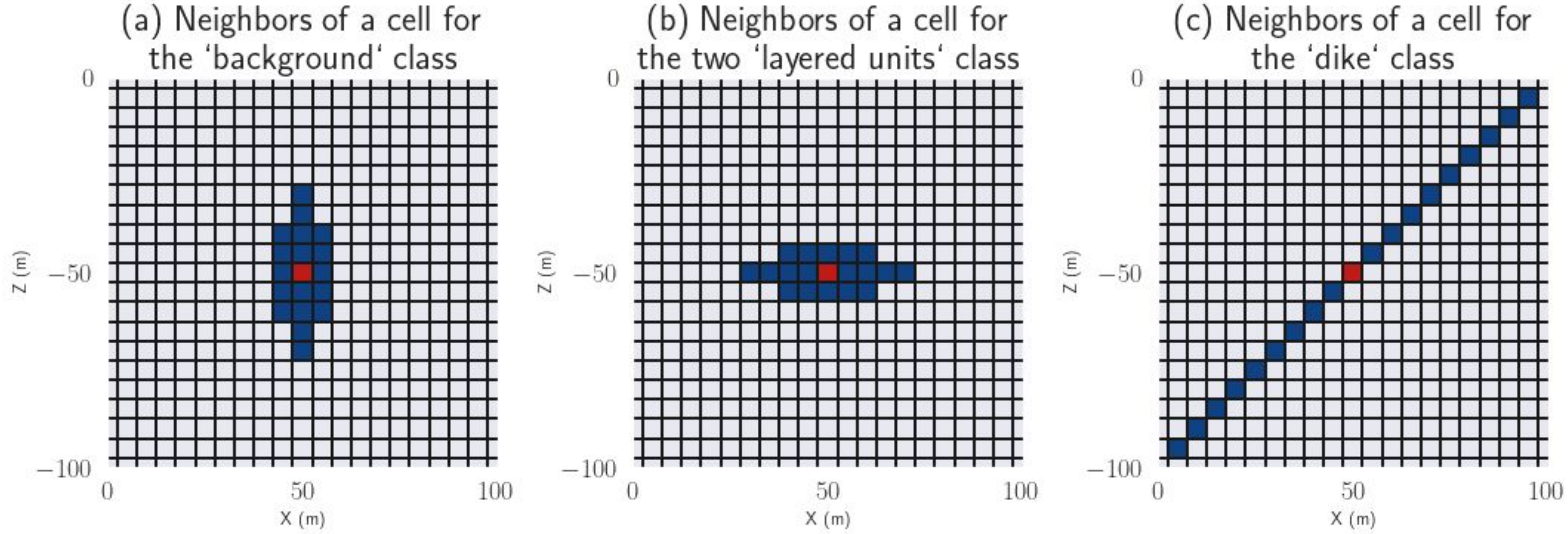


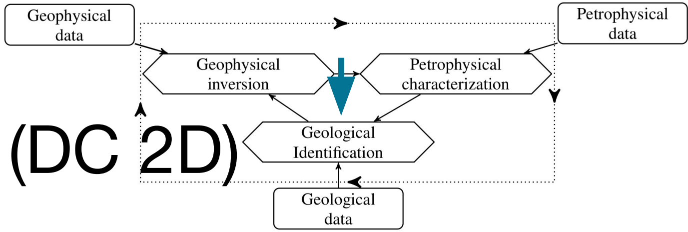
Implementing geology rules within inversion thanks to Image Segmentation

Geology/classification information is shared between neighboring cells.

Nguyen & Wu, 2012

Add structural information through defining neighbors



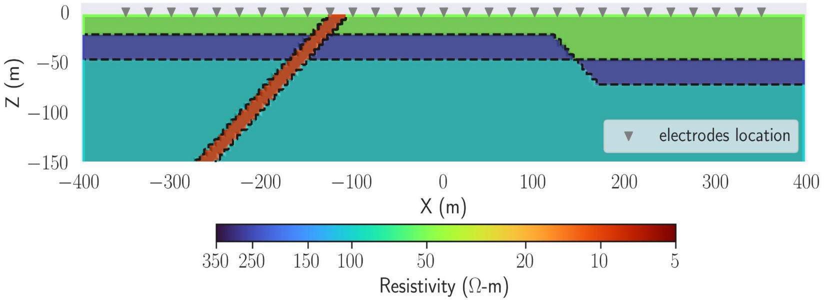


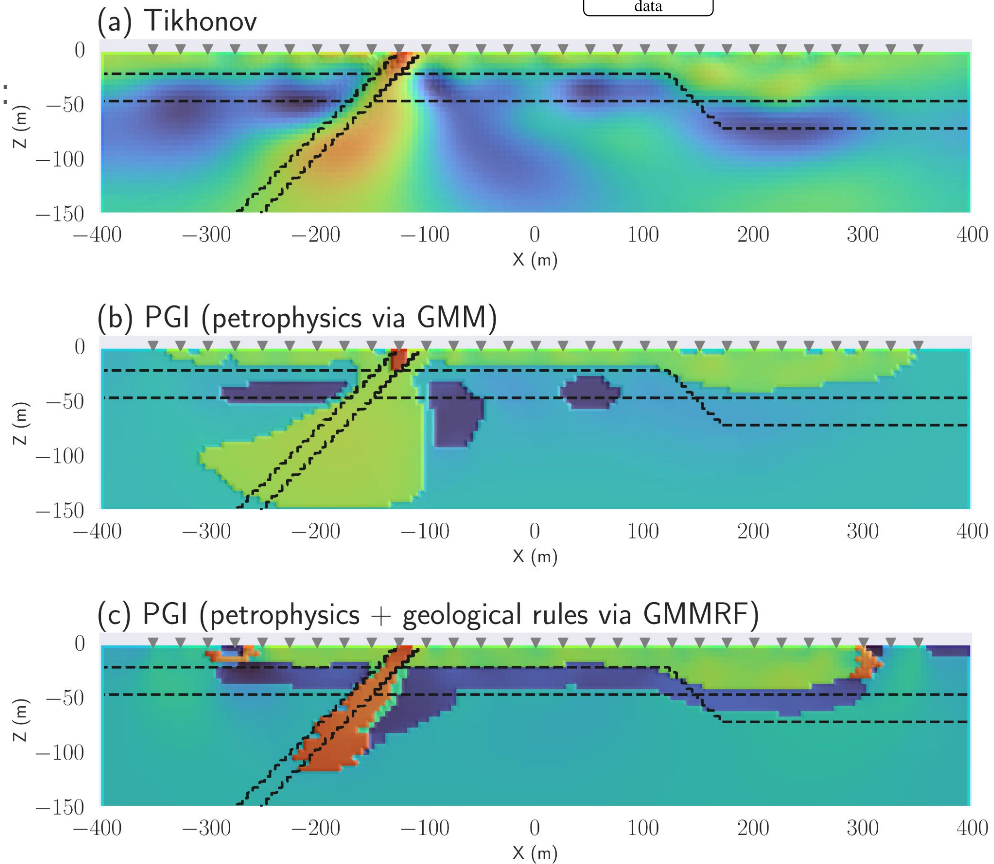
Building geology rules within inversion (DC 2D)

- Adding geology rules through iteratively updated local proportions ([Astic *et al*. 2021](), [online seminar]()):
  - Units continuity
  - Stratigraphic order
  - Structural orientations

$$\mathcal{P}_{\text{small}}(\mathbf{m}|\Theta) = \prod_{i=1}^{n} \sum_{j=1}^{c} \boxed{\mathcal{P}(z_i = j)} \mathcal{N}(\mathbf{m}_i|\boldsymbol{\mu}_j, \mathbf{W}_i^{-\top}\boldsymbol{\Sigma}_j\mathbf{W}_i^{-1})$$

True model

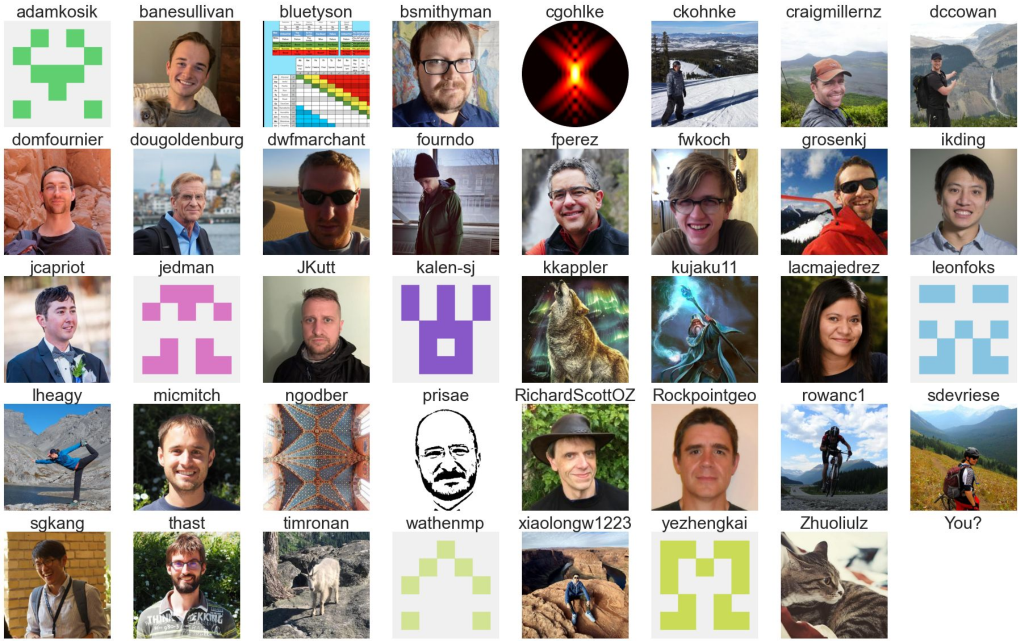




Geology/classification information is shared between neighboring cells.

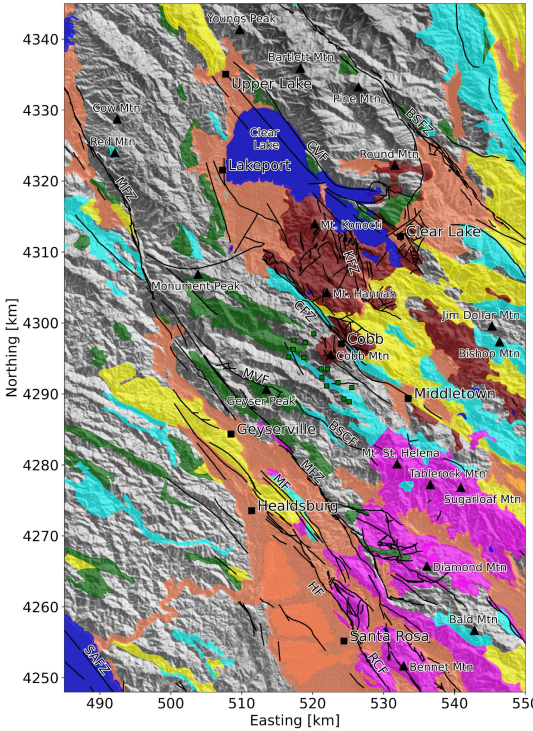
SimPEG:
open source framework for **Sim**ulation and **P**arameter **E**stimation in **G**eophysics in Python.

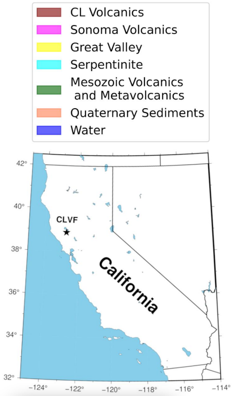
PGI has been part of the main distribution of SimPEG since May, 15, 2021 (version >= 0.15.0)

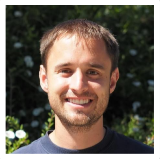


https://simpeg.xyz

Imaging the Magmatic Plumbing of the Clear Lake Volcanic Field (USGS)

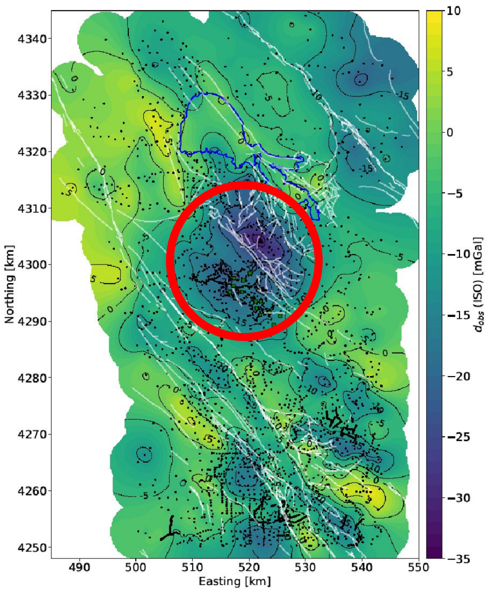


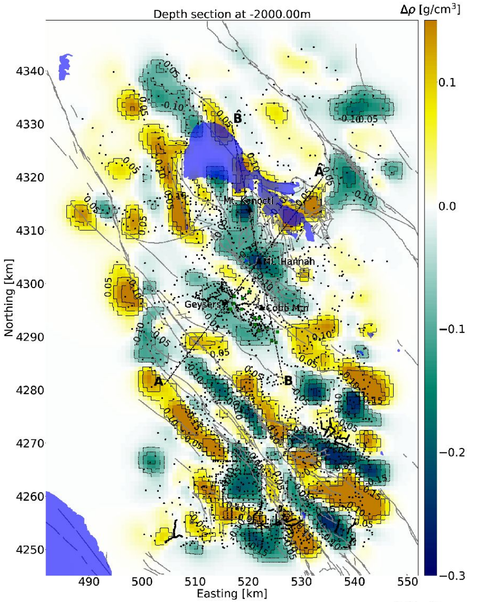


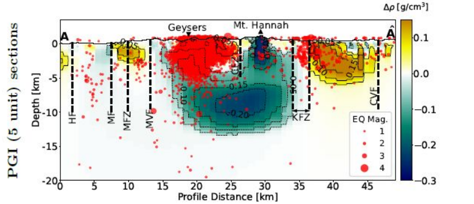


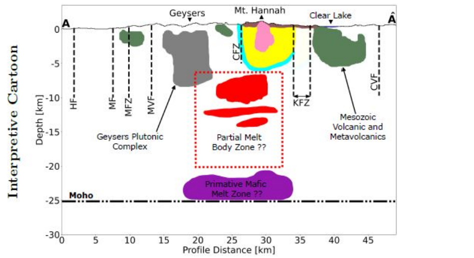
Michael Mitchell (USGS)

Jared Peacock (USGS)







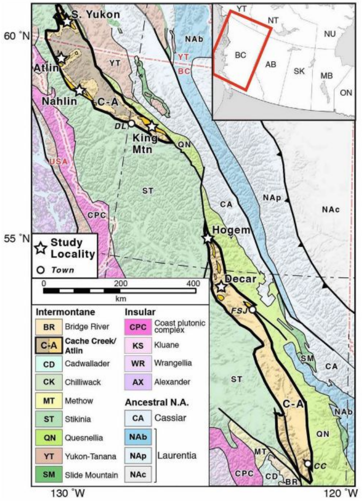


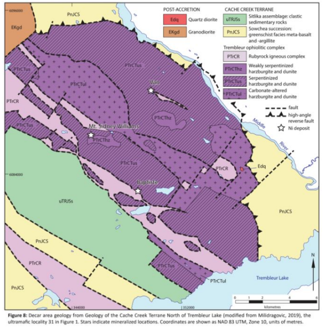
- Deep melt required to fit gravity anomaly?
- Heat source for the Geysers geothermal field?
- Hazard implications

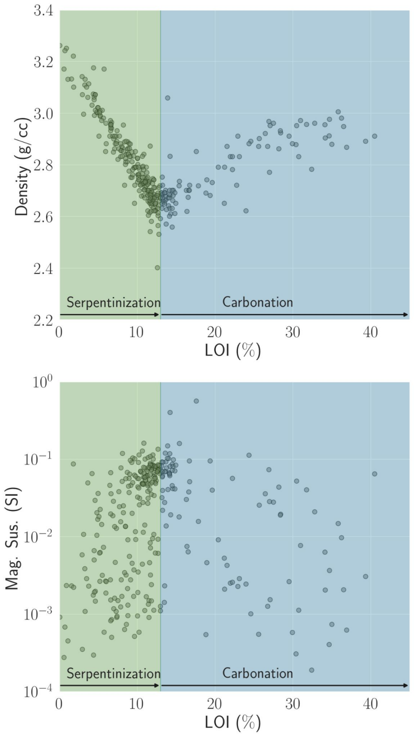
Next Steps:

- Joint inversion of potential field and MT data using PGI

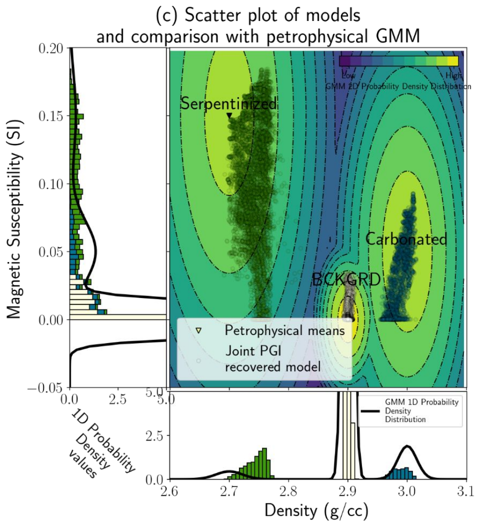
Mapping Carbon Sink resources (UBC - Mira Geoscience Mitacs)

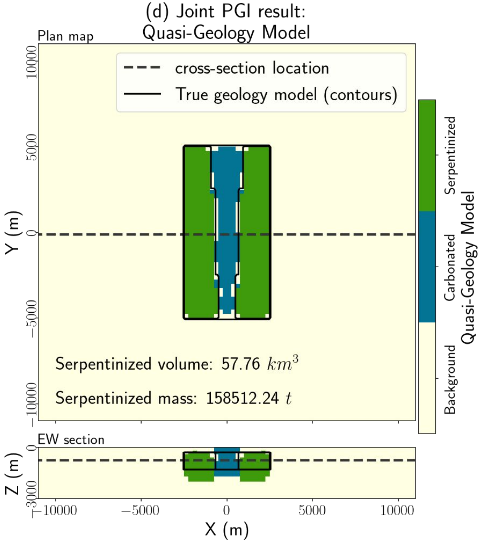




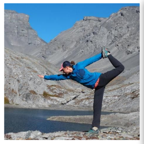


[Cutts *et al.* 2021](#) & [Mitchinson et al. 2020](#)





First proof of concept for the use of PGI: Heagy *et al.* (submitted)

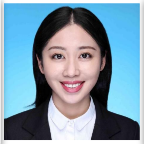

Lindsey Heagy

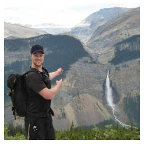

Doug Oldenburg


Thibaut Astic


Joe Capriotti


John Weis


Jingrong (Mimi) Lin


Devin Cowan

How to use / Where to start ?

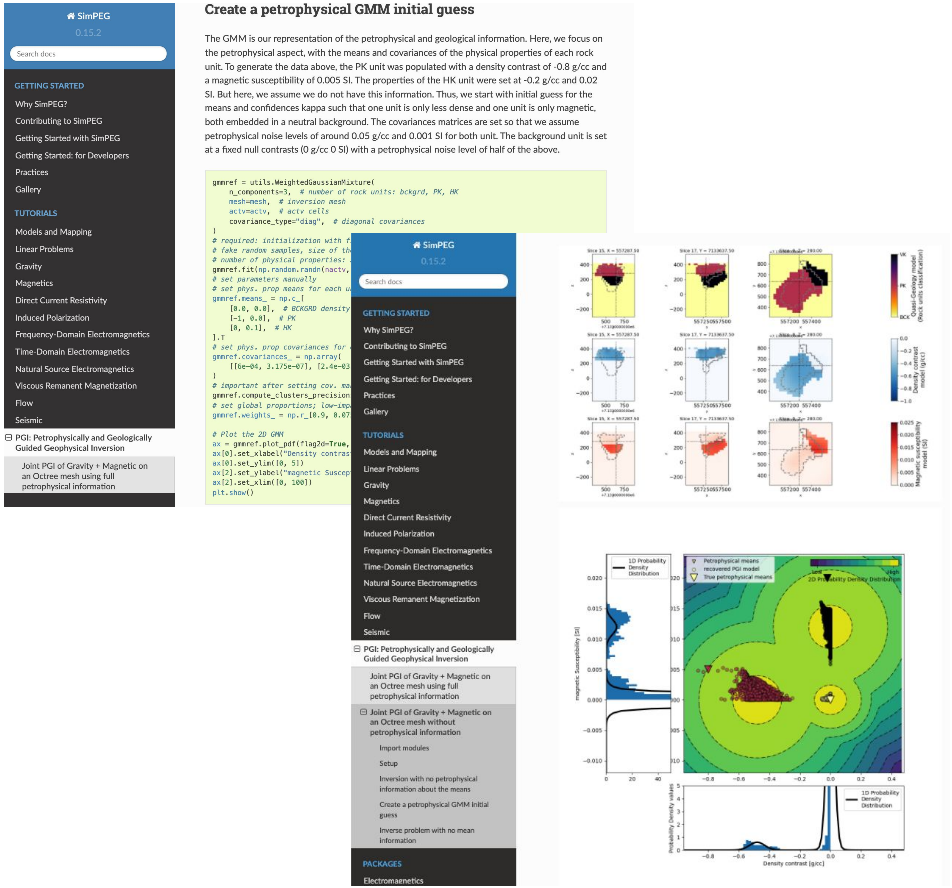
For SimPEG:

- Installation and documentation

```
conda install -c conda-forge simpeg
```

For PGI:

- Online code tutorials
- Online Gallery
- Reproducible examples on the cloud (MyBinder)
- Review / Tutorial manuscript in preparation



Summary

- Analyzing various datasets can reveal information that was not available in any individual analysis

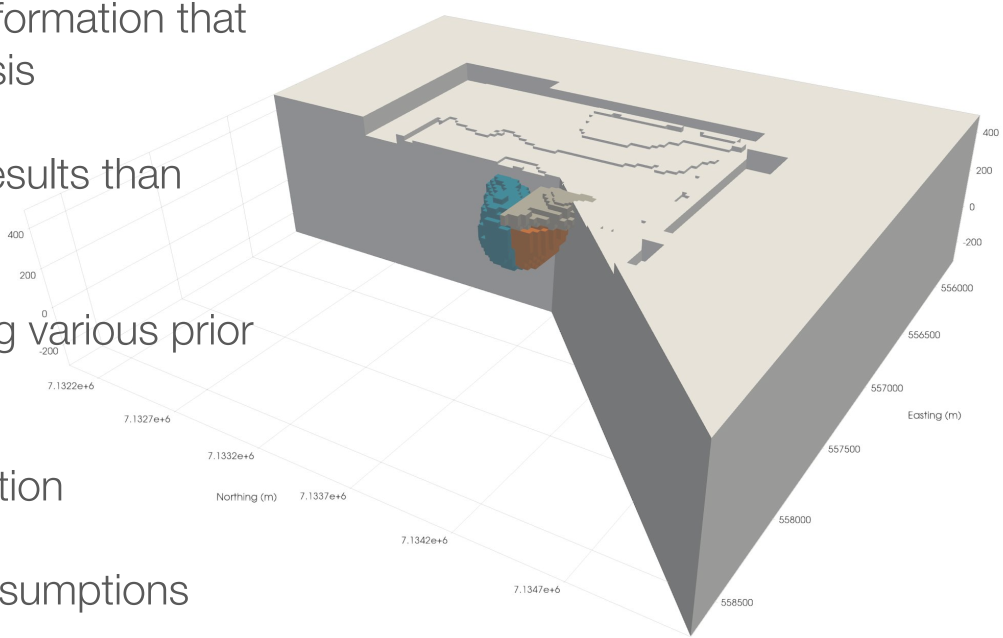
- Joint analysis generally leads to better results than merging several individual analysis.

- PGI: General framework for incorporating various prior knowledge in the inversion

- Works with partial petrophysical information

- Allows for the formulation of geologic assumptions

- Tailor the inversion to the geologic question being asked.



Acknowledgments

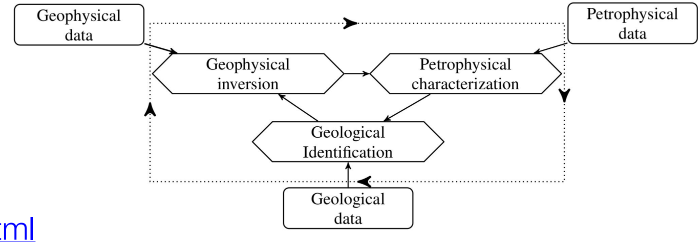
Many thanks to:

Condor Consulting Inc.

Peregrine Diamonds

Kennecott

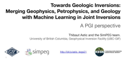
for providing the DO-27 and DO-18 data sets.



Thank you, questions?

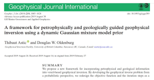
SimPEG tutorials: https://docs.simpeg.xyz/content/tutorials/13-pgi/index.html

Reproducible PGI examples:
- https://github.com/simpeg-research/Astic-2019-PGI
- https://github.com/simpeg-research/Astic-2020-JointInversion



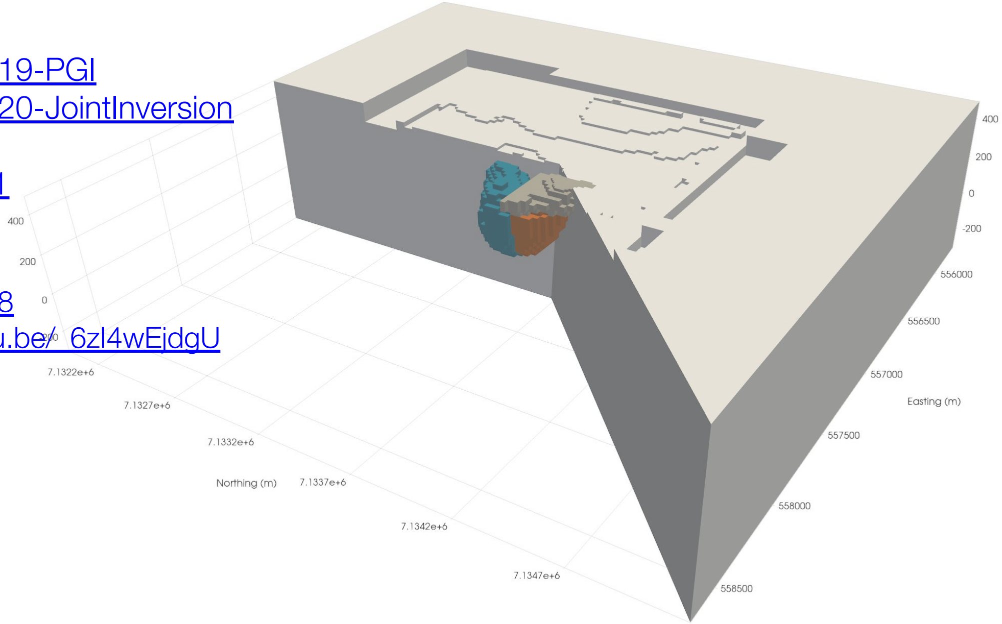
Presentation slides: http://bit.ly/astic_EMinar_2021

Youtube:
- mock PhD defence: https://youtu.be/GlUon-xyoA8
- Implementing geology rules seminar: https://youtu.be/_6zl4wEjdgU



PGI publications:
- 10.1190/segam2018-2995155.1
- 10.1093/gji/ggz389
- 10.1093/gji/ggaa378
- 10.1190/INT-2019-0283.1
- 10.1190/segam2021-3583615.1

Thesis: 10.14288/1.0394725

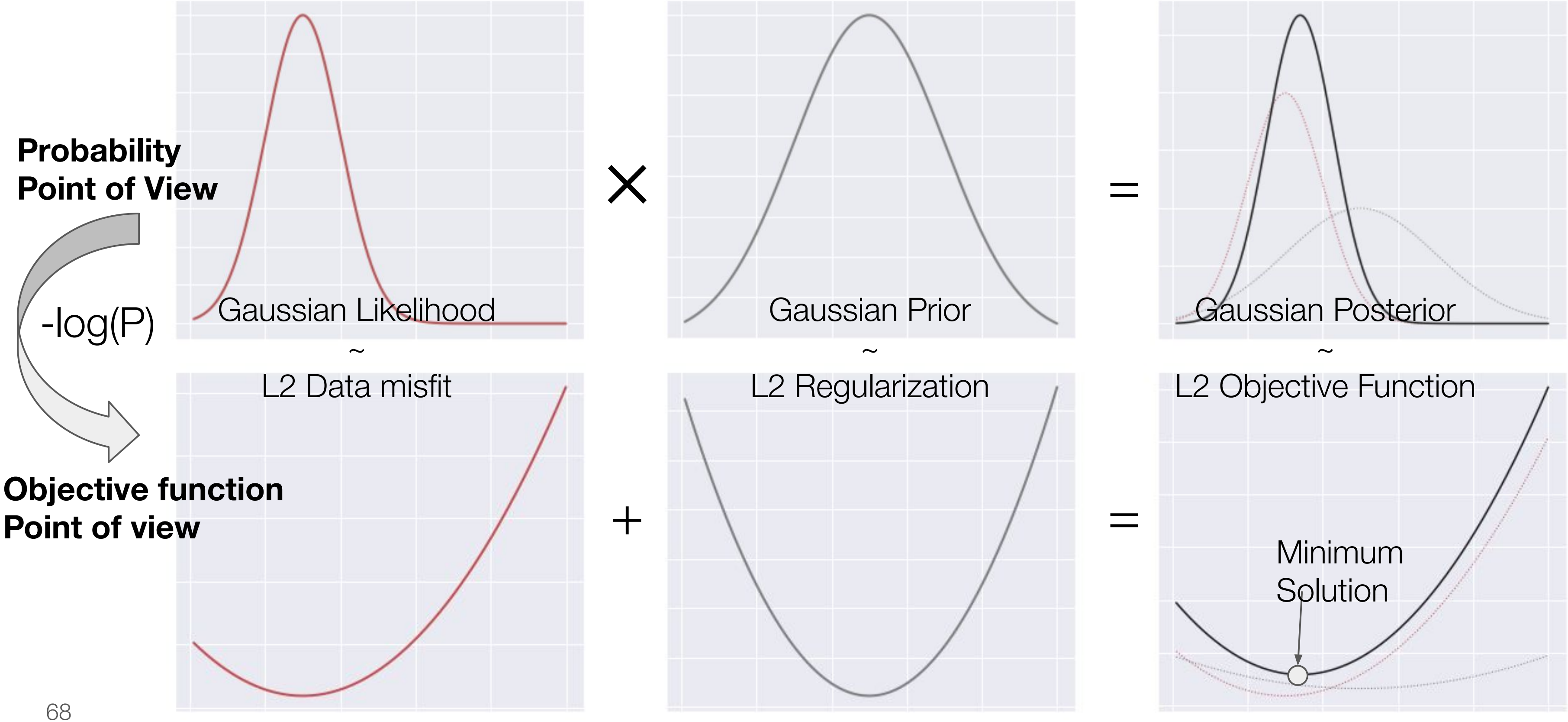


Towards Geologic Inversion

Objectives: using geophysical data, physical property information, geologic information: generate a “quasi-geology model” (Li *et al.* 2019) that facilitates answering of geologic questions.

1. **Differentiation:** “ascertain if multiple anomalous regions in inverted physical property models belong to the same type or different geologic units”
2. **Characterization:** “what geologic unit or type a given model region [it] corresponds to”
3. Other geologic questions. ...**relate the inversion to the geological questions.**

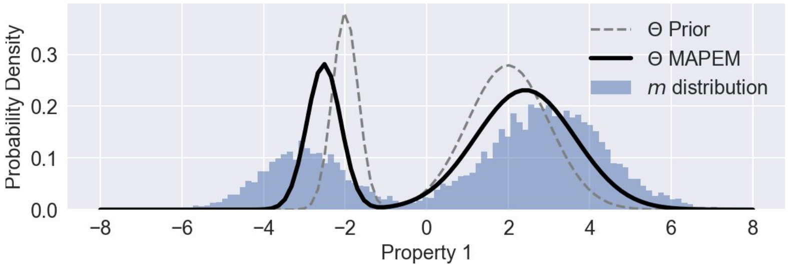
Usual inverse problem



MAP-EM

- E-step: Responsibilities

$$n_{ij}^{(k)} = \frac{p(z_i = j)^{(k-1)} \mathcal{N}((\mathbf{m}_i | \mu_j{}^{(k-1)}, \Sigma_j^{(k-1)})}{\sum_{t=1}^{c} p(z_i = t)^{(k-1)} \mathcal{N}(\mathbf{m}_i | \mu_t{}^{(k-1)}, \Sigma_t^{(k-1)})}$$



- M-step:
  - proportions
  - means
  - covariances

proportions

$$\mathcal{P}(\pi) = \mathrm{Dir}(\zeta n \pi_{prior} - 1)$$

$$\pi_j^{(k)} = \frac{n_j^{(k)} + \zeta_j n \pi_{j\,prior}}{n(1 + \sum_{t=1}^{c} \zeta_t \pi_{t\,prior})}$$

with:

$$n_j^{(k)} = \sum_{i=1}^{n} n_{ij}^{(k)}$$

means

$$\mathcal{P}(\mu|\Sigma) = \mathcal{N}(\mu | \mu_{prior}, (\kappa n \pi_{prior})^{-1})$$

$$\mu_j^{(k)} = \frac{n_j^{(k)} \bar{\mathbf{m}}_j^{(k)} + \kappa_j \pi_{j\,prior} n \mu_{j\,prior}}{n_j^{(k)} + \kappa_j \pi_{j\,prior} n}$$

with:

$$\bar{\mathbf{m}}_j^{(k)} = \frac{\sum_{i=1}^{n} n_{ij}^{(k)} \mathbf{m}_i}{n_j^{(k)}}$$

covariances

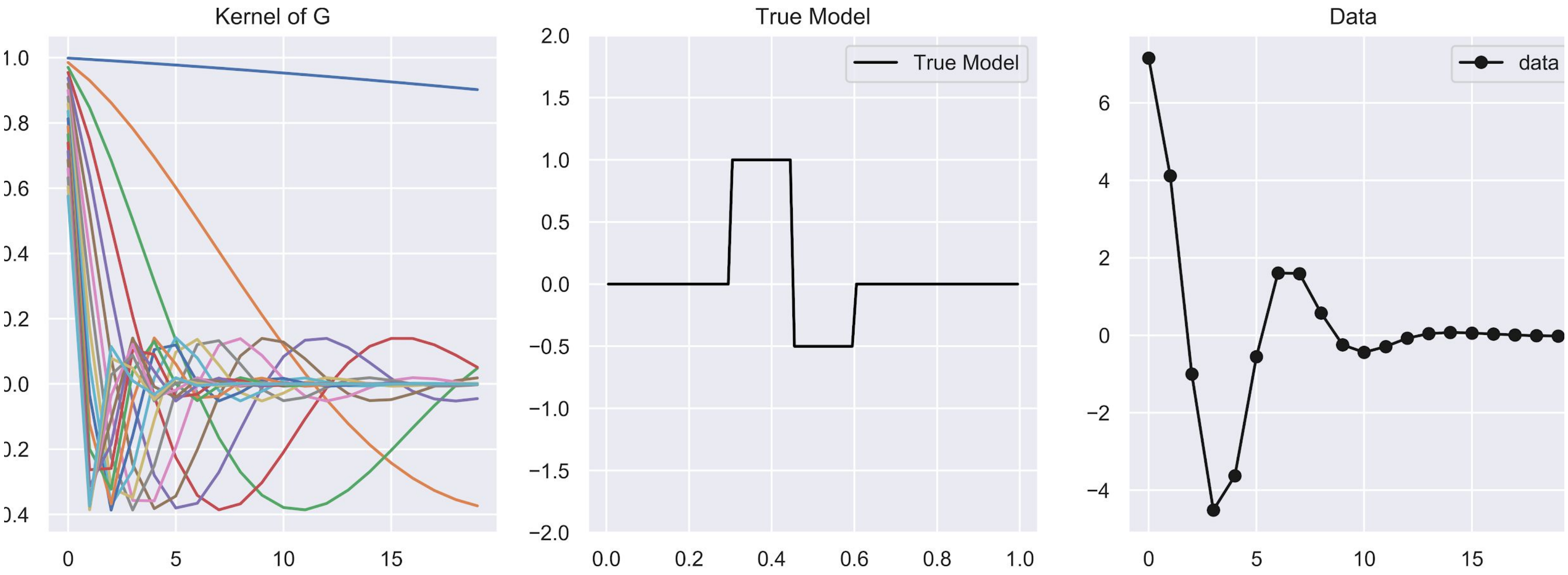
$$\mathcal{P}(\Sigma|\mu) = \mathrm{IW}(\Sigma | \mathbf{V}, \mathbf{V}\Sigma_{prior})$$

$$\Sigma_j^{(k)} = \frac{n_j^{(k)} \Sigma_{\bar{\mathbf{m}}_j}{}^{(k)} + \nu_j \pi_{j\,prior} n \Sigma_{j\,prior}}{n_j^{(k)} + \nu_j \pi_{j\,prior} n}$$

with:

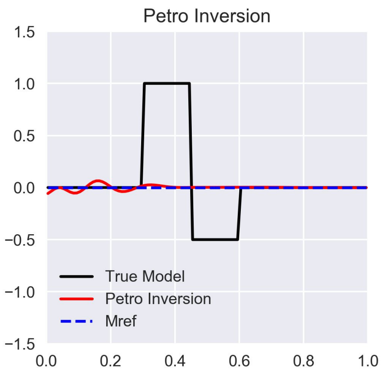
$$\Sigma_{\bar{\mathbf{m}}_j}{}^{(k)} = \frac{1}{n_j^{(k)}} \sum_{i=1}^{n} n_{ij}^{(k)} (\mathbf{m}_i - \bar{\mathbf{m}}_j{}^{(k)})(\mathbf{m}_i - \bar{\mathbf{m}}_j{}^{(k)})^T$$

UBC-GIF linear example

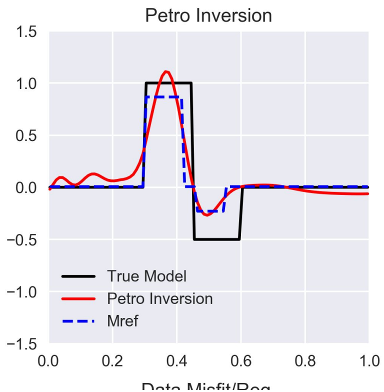


UBC-GIF linear example

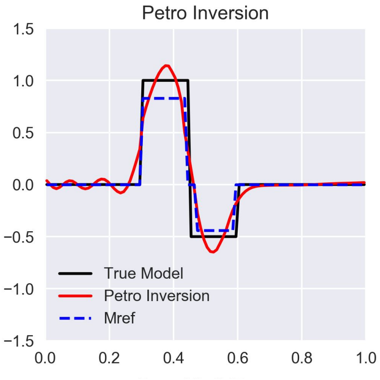
Iteration 1



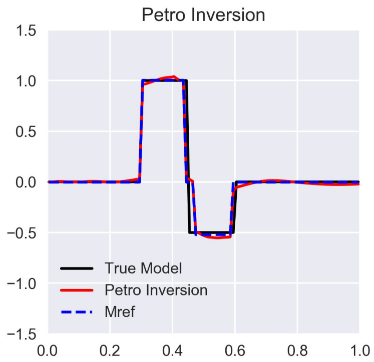
Iteration 10



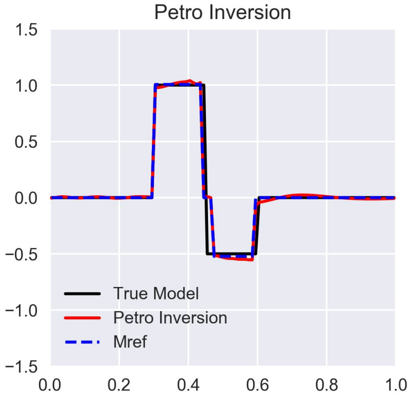
Iteration 23



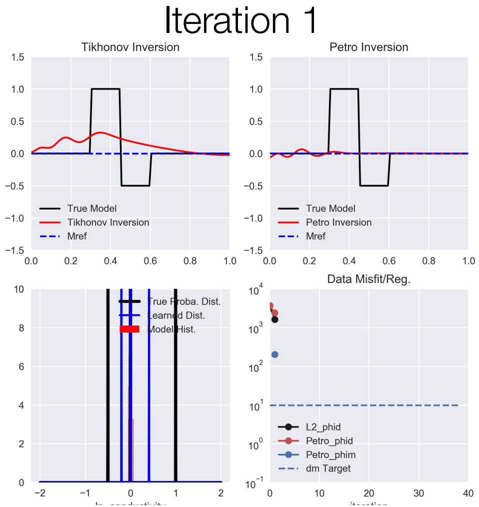
Iteration 30

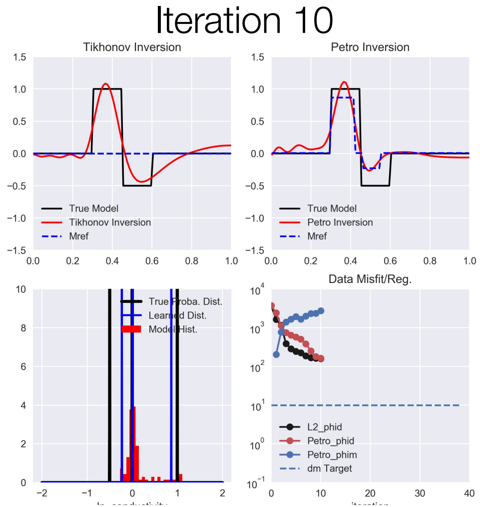


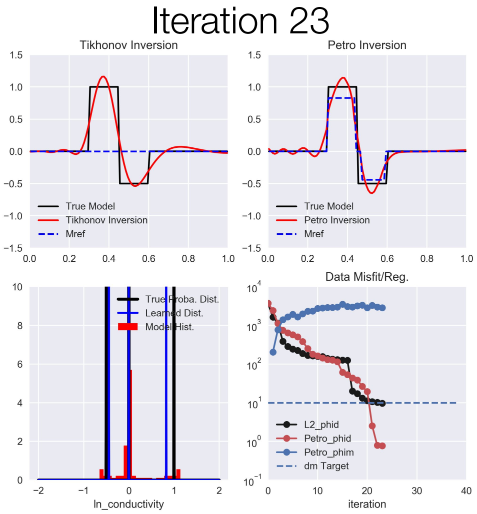
Iteration 40

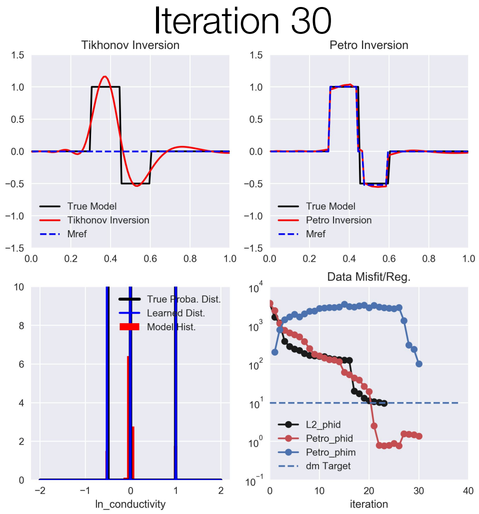


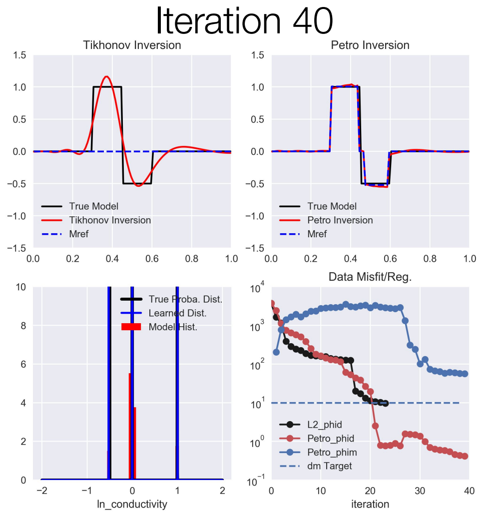
UBC-GIF linear example



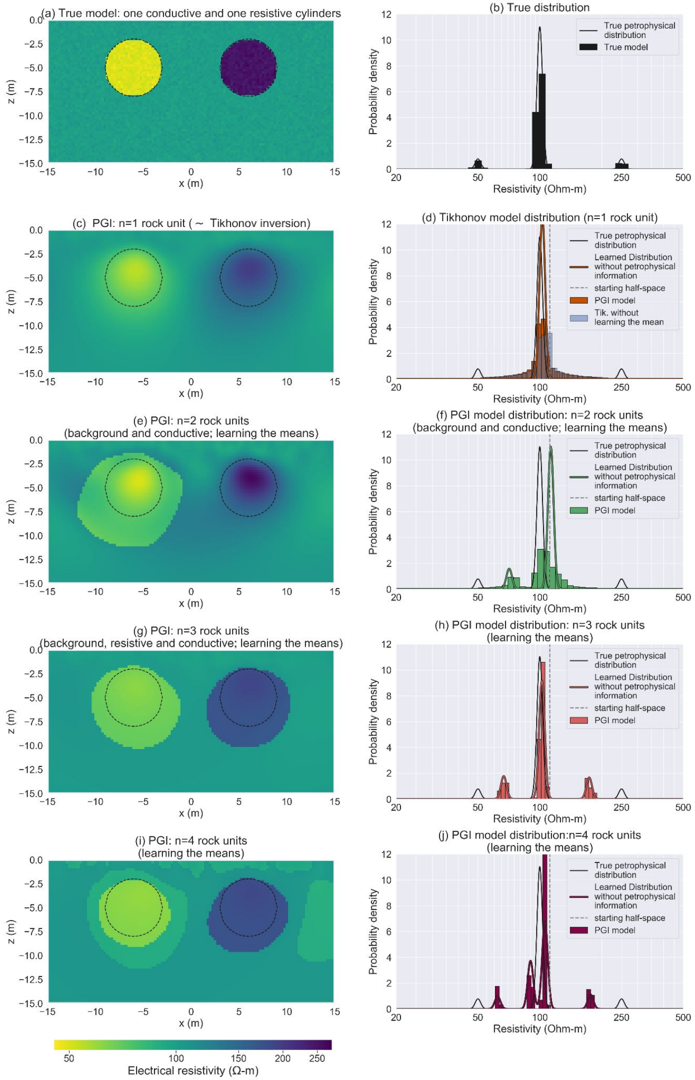




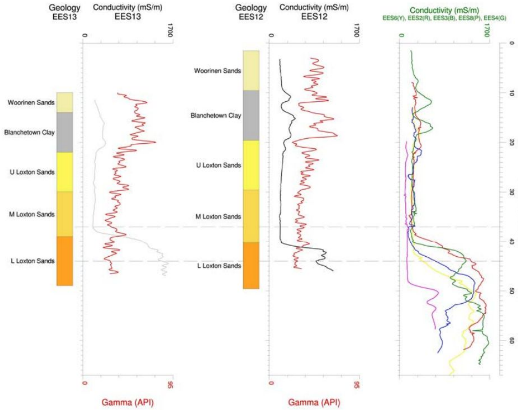




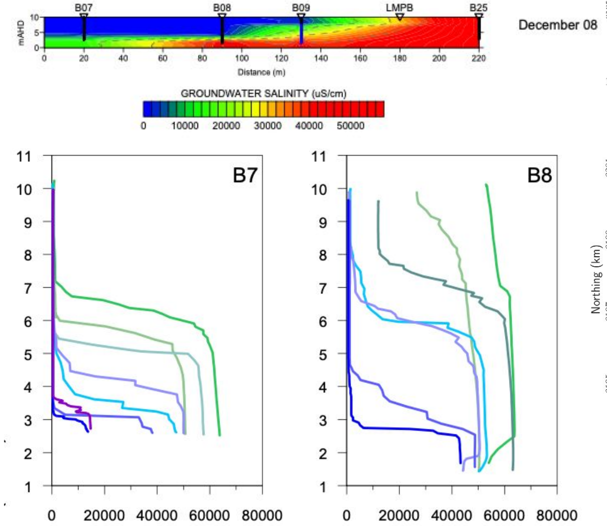
DC2D: what happened with the wrong number of units?



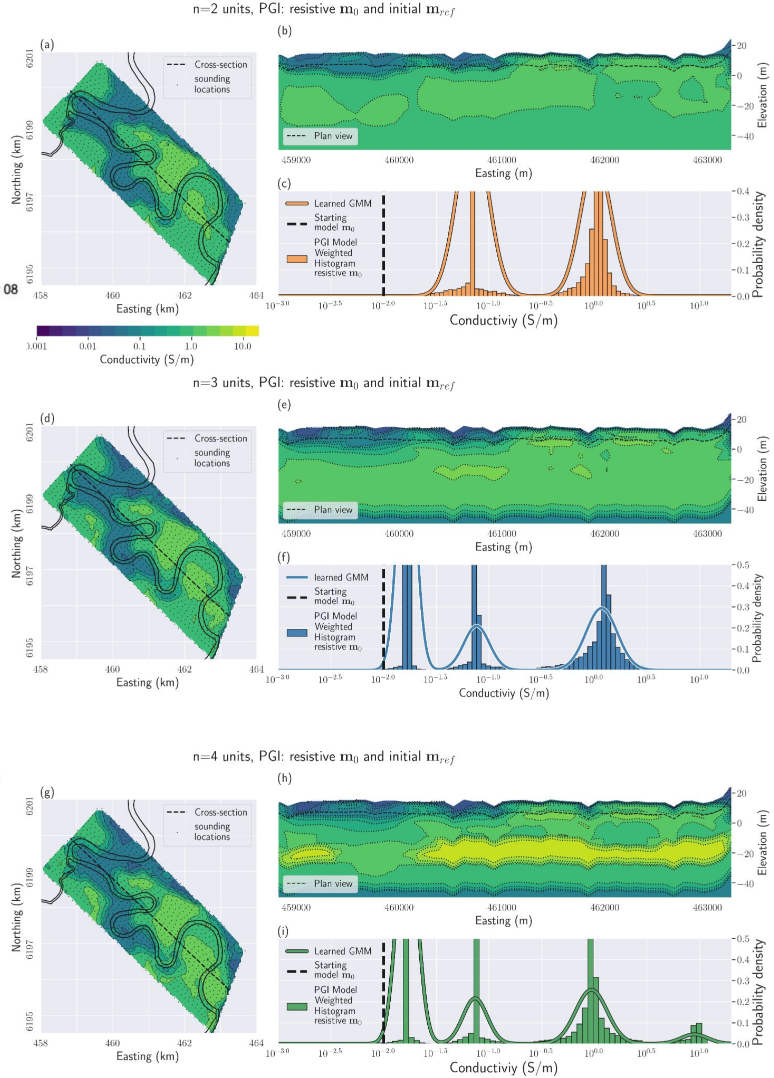
Bookpurnong: Why 3 units?



[Munday *et al.* 2019](#)



[Berens *et al.* 2009](#)



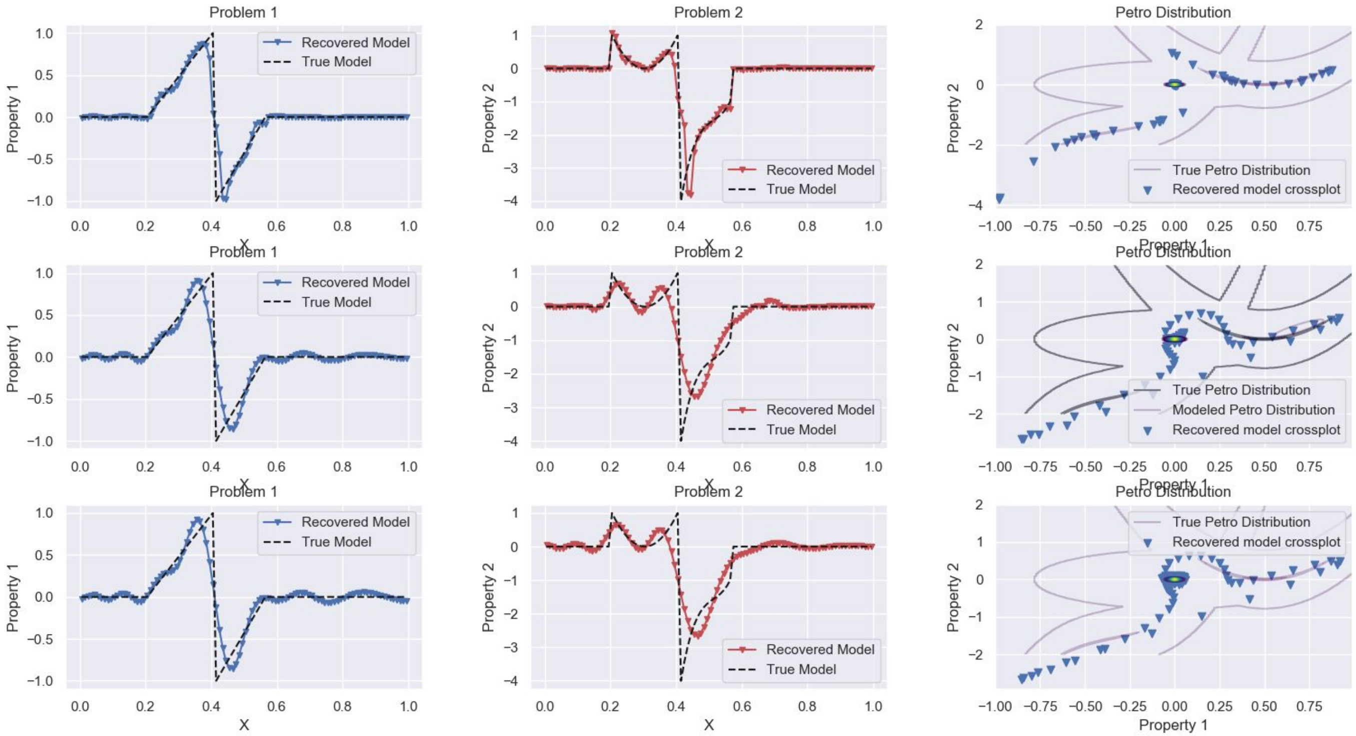
Inverting with nonlinear relationships

Doodling with Mapping: one mapping per identified rock unit
Joint inversion of 1D Linear Problems with nonlinear petrophysical relationships

Using true nonlinear petrophysics clusters

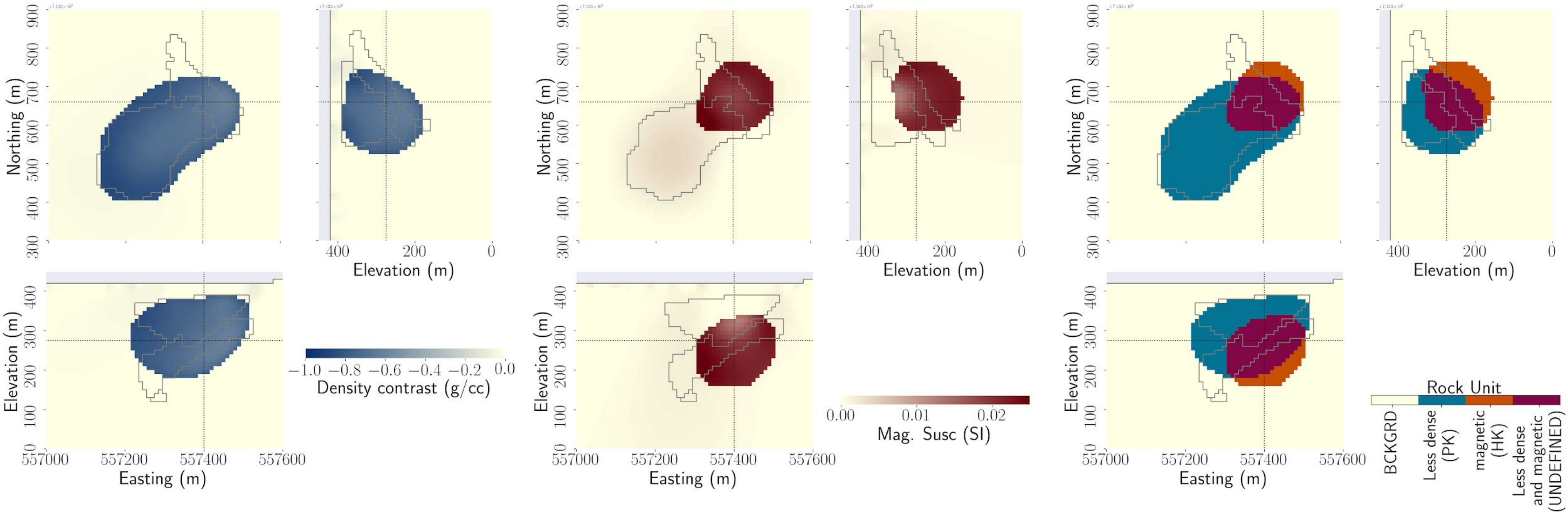
Using linear petrophysics clusters

Tikhonov ~
Using a single cluster

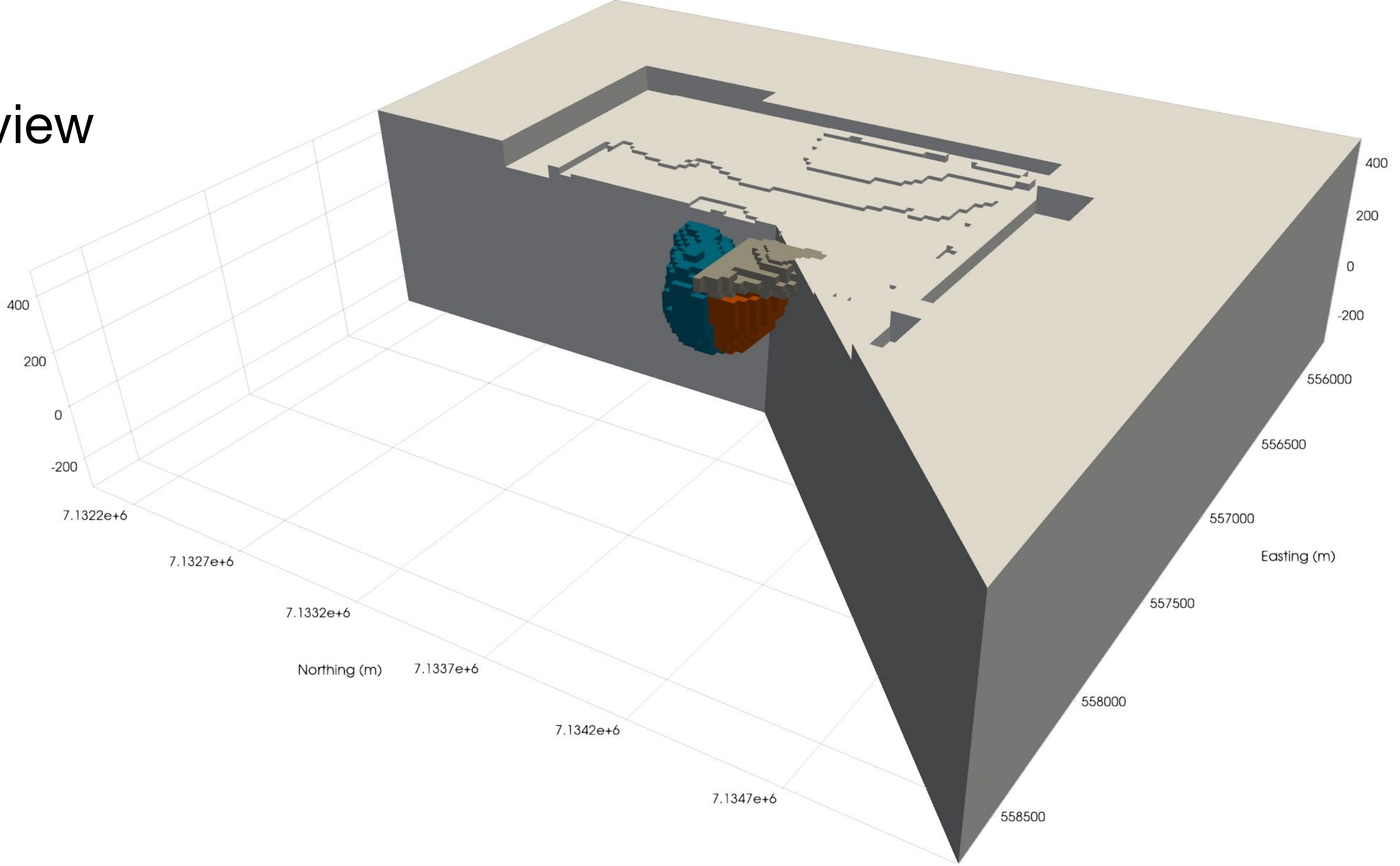


DO-27 (synthetic): Single-physics PGIs

Only one unit is necessary to explain each geophysical dataset individually.

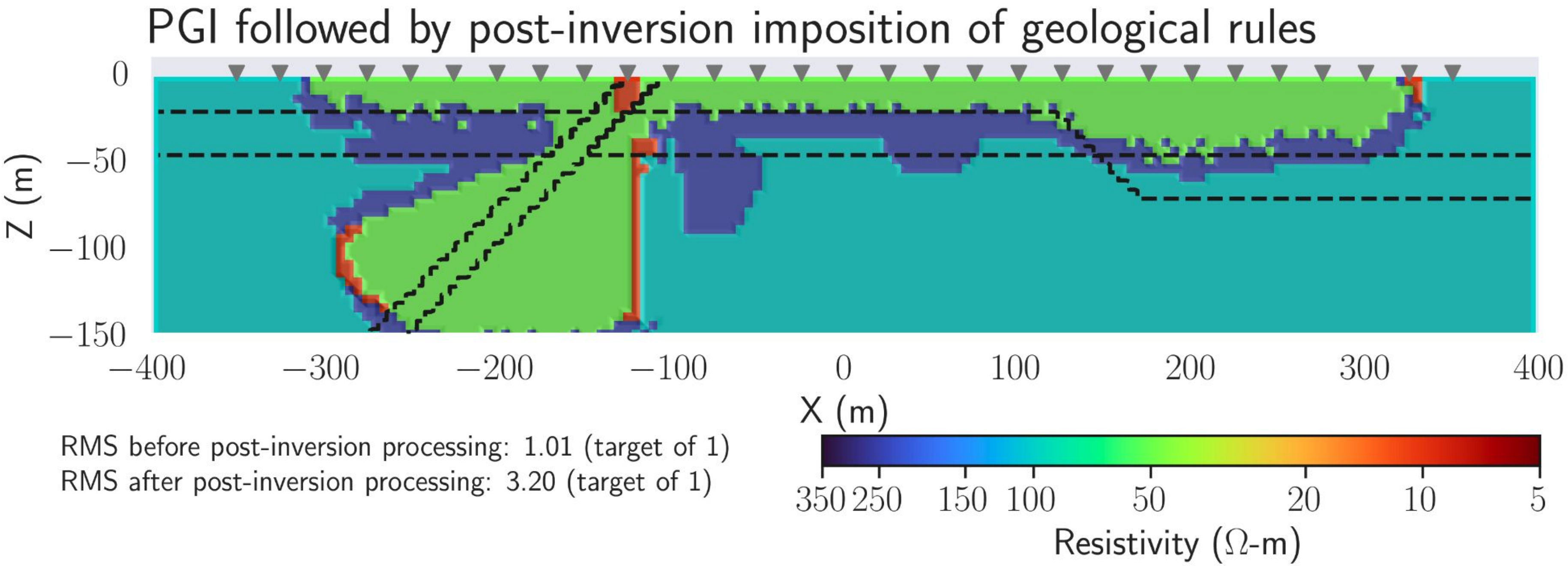


3D view



400
200
0
-200
7.1322e+6
7.1327e+6
7.1332e+6
Northing (m)
7.1337e+6
7.1342e+6
7.1347e+6
400
200
0
-200
556000
556500
557000
Easting (m)
557500
558000
558500

Post-inversion imposition of geology rules



RMS before post-inversion processing: 1.01 (target of 1)
RMS after post-inversion processing: 3.20 (target of 1)