Mono-model Gramian constrained multi-physics inversion of EM data

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I

Introduction

Challenges inherent in inversion

2 Basic geophysical preoccupations

- **Basically**
  - **Modeling**
  - **Inversion**

Table 1: Modeling and Inversion with their issues

Procedure	In Control	Operator	Output
Modeling	Parameters	Forward Operator	Data
Inversion	Data	Inverse Operator	Parameters

Procedure	Associated Cost Issues		
Modeling	Computational Efficiency	Accuracy improvement	
Inversion	Existence	Stability	Uniqueness

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My contemplation today in inversion constituency: ill-posed problems

- Fundamental issues/question (Zhdanov, 2015)

- **Existence/non-existence**
  - Does a solution exist, first question? Deals mathematical formulation of the inverse problem
  - Physical point of view: there should be real geological structures
  - Mathematical view: no adequate numerical model fit observed **noisy** data
  - Noise in the data has no common ground with geophysical field equations.
- **Stability/Instability**
  - If a small perturbation of data gives arbitrary large perturbation of the solution=**Unstable**
- **Uniqueness/non-uniqueness**
  - Non-uniqueness (seems to be inherent problem):
  - A situation where two or more different models/sources sources fit the same data
  - Jackson (1972): inaccurate, insufficient, inconsistent data lead to non-uniqueness

- **Ill-posed inversion problem**
  - With any of these problems occurs, the inverse problem is ill-posed.
  - **And how do we solve an ill-posed problem**?

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Practical existence (Zhdanov, 2015)

- **Existence**
  - Noise cannot be described by the same operator for data
  - No need to completely fit the noisy data
  - Hence ***Practical Existence*** is possible where data is fitted within **measurement error bound**
  - Solution to non-existence problem is understandably by ***practical existence***.

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Tikhonov Regularization Theory

- **Stability**
- Mathematically speaking, instability occurs when the **inverse operator is not continuous**, making the inverse problem ill-posed.

- Stabilizer main application is to
  - select from the set of possible solutions
  - The solutions that continuously depend on the data
  - and which possesses a specific property depending on the choice of stabilizer

- **The solution to instability is regularization**,
  - which conditions an ill-posed problem to a well-posed problem, making the inverse operator continuous.
  - Regularization algorithm aims to consider, instead of one ill-posed (unregularized) inverse problem, a family of well-posed problems.

Regularized inversion

- To stabilize such an inversion, a regularized parametric functional can be written as lin ear combination of the misfit and stabilizing functionals (Zhdanov, 2015)

$$P^{\alpha}(m) = \mu_D + \alpha s(m), \qquad (1)$$

- where $P^{\alpha}(m)$ is the Tikhonov parametric functional for model parameter m; μ_D is the misfit functional, $s(m)$ is the stabilizing functional

- And to solve for the model parameter m

$$P^{\alpha}(m) = \min. \qquad (2)$$

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Practical uniqueness

- **Uniqueness**
  - There are uniqueness theories that work for only certain geophysical models.
  - Uniqueness theories are limited to certain geophysical models
  - ***Practical uniqueness*** is proposed by Zhdanov (2015),
    - where the geometrical dimension for data acquisition is at least the same as that for inverted models e.g. 3D for both data and model;
    - but if the data dimension is 4D for 3D model, that is even better.

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Multi-physics inversion Philosophy implementation

Multi-modal, Mono-model multi-physics inversion

Multi-physics/joint inversion tackle non-uniqueness

- The **practical uniqueness is not the general solution** to non-uniqueness problem as it is restrictive and costly

- However, recently, researchers have attempted to tackle the non-uniqueness problem by joint inversion of
  - Multiple data sets
  - Multiple geophysical approaches/techniques of methods

- For n data sets and model parameters, a third term (coupling term) will be added to the Tikhonov Parametric Functional in equation (1), with the basic form given as:

$$P^{\alpha}(m^{i}) = \sum_{i=1}^{n} \mu_{D}^{i} + \alpha \sum_{i=1}^{n} s^{i}(m^{i}) + \beta \sum_{i=1}^{n} c(m^{1}, m^{2}, \dots, m^{n}) \qquad (3)$$

Where c is a general representation of the joint inversion or multi-physics constraints or coupling term, weighted by β.

Some multi-physics constraints

- Constraints help to couple certain feature between interacting models and/or data.
- There is a growing list, but only a few will be mentioned here
- **Cross-gradient**
  - Popular and useful where physical properties are not correlated but nevertheless have similar str uctural constraints)
- **Petrophysical relations**
- **Fuzzy c-means**
- **Gramian**
  - Minimization of the determinant of the Gram matrix

Implementation of joint inversion within its philosophy

Table 2: Multi-modal and mono-model multi-physics inversion

Joint inversion	Data/approach	Model	Reference
Mono-model	**Similar** (DC, EM)	**Same** (resistivity)	Vozoff and Jupp (1975)
Mono-model	**Multiple approaches** on *same data set* (refraction traveltime migration and tomography	**Same** (velocity)	Zhang (1997)
Multi-modal	**Different** (seismic and EM)	**Different** (velocity and resistivity)	Ogunbo et al. (2018)

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Multi-modal joint inversion (a few examples)

Table 3: Multi-modal multi-physics inversion examples

Authors	Method 1	Method 2	Constraint
Gallardo and Meju (2003; 2004)	DC resistivity	Seismic traveltime	Cross-gradient
Ogunbo et al. (2018)	Seismic traveltime	Electromagnetic	Cross-gradient
Hu et al. (2009)	EM	Seismic	Cross-gradient
Carter-McAuslan et al. (2015)	Seismic tomography	Gravity	Fuzzy c-means
Gao et al. (2010; 2012)	EM	Full waveform seismic	Petrophysical
Abubakar et al. (2012)	CSEM	Seismic full waveform	Petrophysical
Zhdanov et al. (2012)	Gravity	Magnetic	Gramian
Zhu et al. (2013)	Airborne Gravity	Magnetic	Gramian

Authors	Method 1	Method 2	Method 3	Constraints
Moorkamp et al. (2011)	MT	Gravity	Seismic refraction	Mathematical relation; Cross-gradient

Mono-model joint inversion

Table 4: Mono-model multi-physics inversion examples

Authors	Method 1	Method 2	Constrain t	Remark
Vozoff and Jupp (1975)	DC resistivity	Magnetotelluric	None	
Raiche et al. (1985)	TEM	Schlumberger DC	None	
Sunwall et al. (2013)	Time-domain AEM	Frequency-domain AEM	None	Deep and shall ow Resolution complements
Ogunbo (2019)	Time-domain AEM	Frequency-domain AEM	**Gramian**	
Ogunbo et al. (2020)	Gravity	Magnetic	**Gramian**	Inverting coinci dent geometry
Ogunbo et al. (2021)	Gravity	First horizontal derivative of Gravity	**Gramian**	Inverting coinci dent geometry

Although there are several applications of the mono-model Gramian constrained joi nt inversion that I have worked on, I present results from EM-EM case from Ogunbo (2019)

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III

Gramian constrained multi-physics inversion

Synthetic and Field data Examples

Results from Ogunbo (2019)

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Key Points:
- The gramian-constrained joint inversion of RESOLVE and SkyTEM data of salinized Bookpurnong Irrigation District, South Australia is performed
- The Gramian constraint exploits the linear correlation and enforces it between similar methods
- The correlation coefficient is an additional measure of confidence on the inverted results

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Mono-Model Parameter Joint Inversion by Gramian Constraints: EM Methods Examples

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Abstract Joint inversions of coincident geophysical data are usually constrained to produce more reliable subsurface models. Structural, petrophysical, model parameter correlation, empirical, and transforms are some of the published constraints. The Gramian constraint provides a broad mathematical framework for implementing the aforementioned constraints. The Gramian constraint is formed from the determinant of the inner products of the model parameters involved. Previous works have used the Gramian constraint to invert multimodal parameters of different geophysical methods. But there has not been any extension of Gramian-constrained joint inversion to mono-model parameter from similar geophysical

Gramian constrained joint inversion (Yue et al., 2013)

- Operator relationships between multiple data sets and model parameters is given as

$$d^i = A^i\left(m^i\right)\; i = 1, 2, 3, \ldots, n, \qquad (4)$$

where A^i are the nonlinear forward operators; d^i are the different observed data sets, an d m^i are the model parameters

For convenience, we can use the dimensionless weighted model parameters

$$\tilde{m}^i = W_m^i m^i, \qquad (5)$$

W_m^i is the corresponding linear operator of model weighting (Zhdanov, 2015)

Gramian constrained joint inversion (Yue et al., 2013)

- The Gramian of a system of model parameters $(\tilde{m}^1, \tilde{m}^2, \dots, \tilde{m}^{n-1}, \tilde{m}^n)$ is the determinant, $G(\tilde{m}^1, \tilde{m}^2, \dots, \tilde{m}^{n-1}, \tilde{m}^n)$, of the Gram matrix of a set of functions, $(\tilde{m}^1, \tilde{m}^2, \dots, \tilde{m}^{n-1}, \tilde{m}^n)$,de fined as:

$$G(\tilde{m}^1, \tilde{m}^2, \dots, \tilde{m}^{n-1}, \tilde{m}^n) = \begin{vmatrix} \langle \tilde{m}^1, \tilde{m}^1 \rangle & \langle \tilde{m}^1, \tilde{m}^2 \rangle & \cdots & \langle \tilde{m}^1, \tilde{m}^n \rangle \\ \langle \tilde{m}^2, \tilde{m}^1 \rangle & \langle \tilde{m}^2, \tilde{m}^2 \rangle & \cdots & \langle \tilde{m}^2, \tilde{m}^n \rangle \\ \dots & \dots & \dots & \dots \\ \langle \tilde{m}^n, \tilde{m}^1 \rangle & \langle \tilde{m}^n, \tilde{m}^2 \rangle & \cdots & \langle \tilde{m}^n, \tilde{m}^n \rangle \end{vmatrix}, \qquad (6)$$

- where $\langle \tilde{m}^1, \tilde{m}^2 \rangle$ is the dot product of $\tilde{m}^1$ and $\tilde{m}^2$
- The Gramian provides a measure of correlation between the different model parameters or their attributes
- By imposing additional requirement of the minimum of the Gramian in the regularized inversion, we gener ally obtain multimodal inverse solutions with enhanced correlations between the different model paramete rs or attributes

Method: Gramian constrained joint inversion

- Adapting the general formulation to the case of 2 different types of EM methods

- Taking m_1 and m_2 as the logarithm of resistivity of time and frequency domain airborne EM d ata respectively, the **Gram matrix is formed from the dot** m_1, and m_2 (Zhdanov, 2015):

- The parametric functional to minimize with the Gramian stabilizer is (Zhdanov, 2015; Ogunbo, 2019):

$$P^{\alpha}(\tilde{m}^1, \tilde{m}^2) = \sum_{i=1}^{2} \left(\left\| \left(\tilde{A}^i(\tilde{m}^i) - \tilde{d}^i \right) \right\|^2 \right) + \boldsymbol{\alpha} \sum_{i=1}^{2} \boldsymbol{S}^{\boldsymbol{i}}_{\boldsymbol{MN,MS,MGS}} + \boldsymbol{\beta} \sum_{i=1}^{2} \boldsymbol{G}(\boldsymbol{m_1}, \boldsymbol{m_2}), \qquad (7)$$

where $\tilde{A}^i(\tilde{m}^i)$ are weighted predicted data; S^i_{MN}, S^i_{MN}, S^i_{MGS} are the stabilizing functionals based on the mini mum norm, minimum support and minimum gradient supports respectively

α is the regularization parameter, and β is the weight for the Gramian stabilizer.

We minimize equation (7) and solve by iterative regularized conjugate gradient method. Details are found in Zhdanov (2015).

Synthetic Example: Imaging Resistivity blocks

- Resistivity structure in Figure 1 has of 500 Ohm-m and 30 Ohm-m buried in a back ground resistivity of 100 Ohm-m.

- **Methods** used are
  - Time domain airborne EM (TDAEM)
  - Frequency domain airborne EM (FDA EM)

- **RESOLVE system is used to acquire FDA EM**
  - Flown at altitude of 20 m with horizo ntal receiver offset of 7.93 m.
  - Acquires horizontal coplanar in-phase and quadrature responses at 5 freque ncies
- **SkyTEM is used to acquire TDAEM**
  - Flown at an altitude of 40.55 m, stati on spacing of 13.26 m; offtime 4.167 ms.

The resistivity model has 7 data points.
Number of layers is 6 and the Layer thickness is 5 m

10 stand-alone iterations preceding joint inversion.
Noise levels: 0%, 1%, 3%, to check the robustness of the results

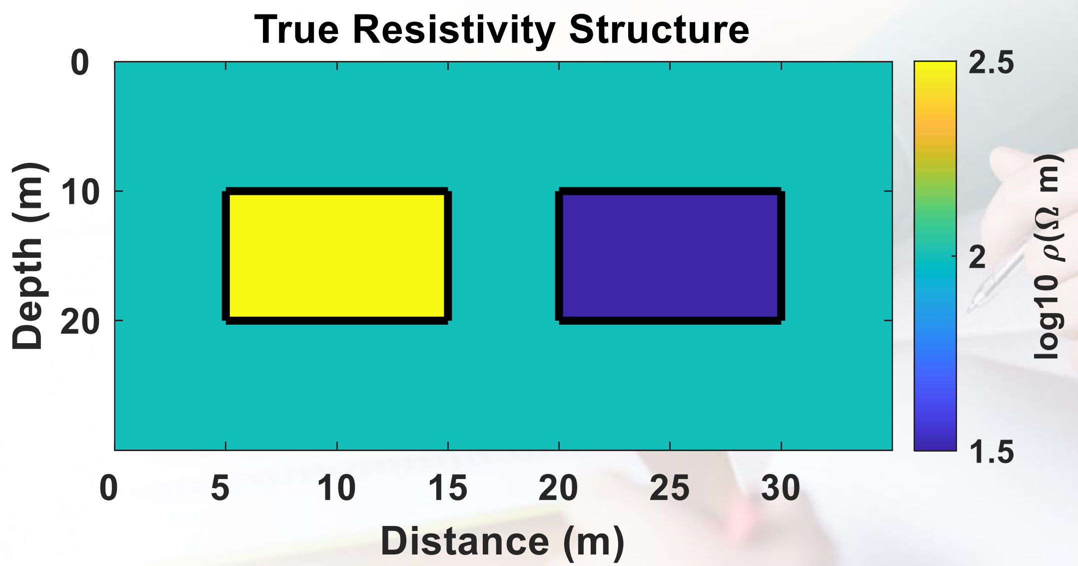

Figure 1: True synthetic resistivity model

Results: Standalone and Noiseless Joint inversion

Difficult to choose which standalone result is representative of true model.

There is a high correlation between the jointly inverted models, helping

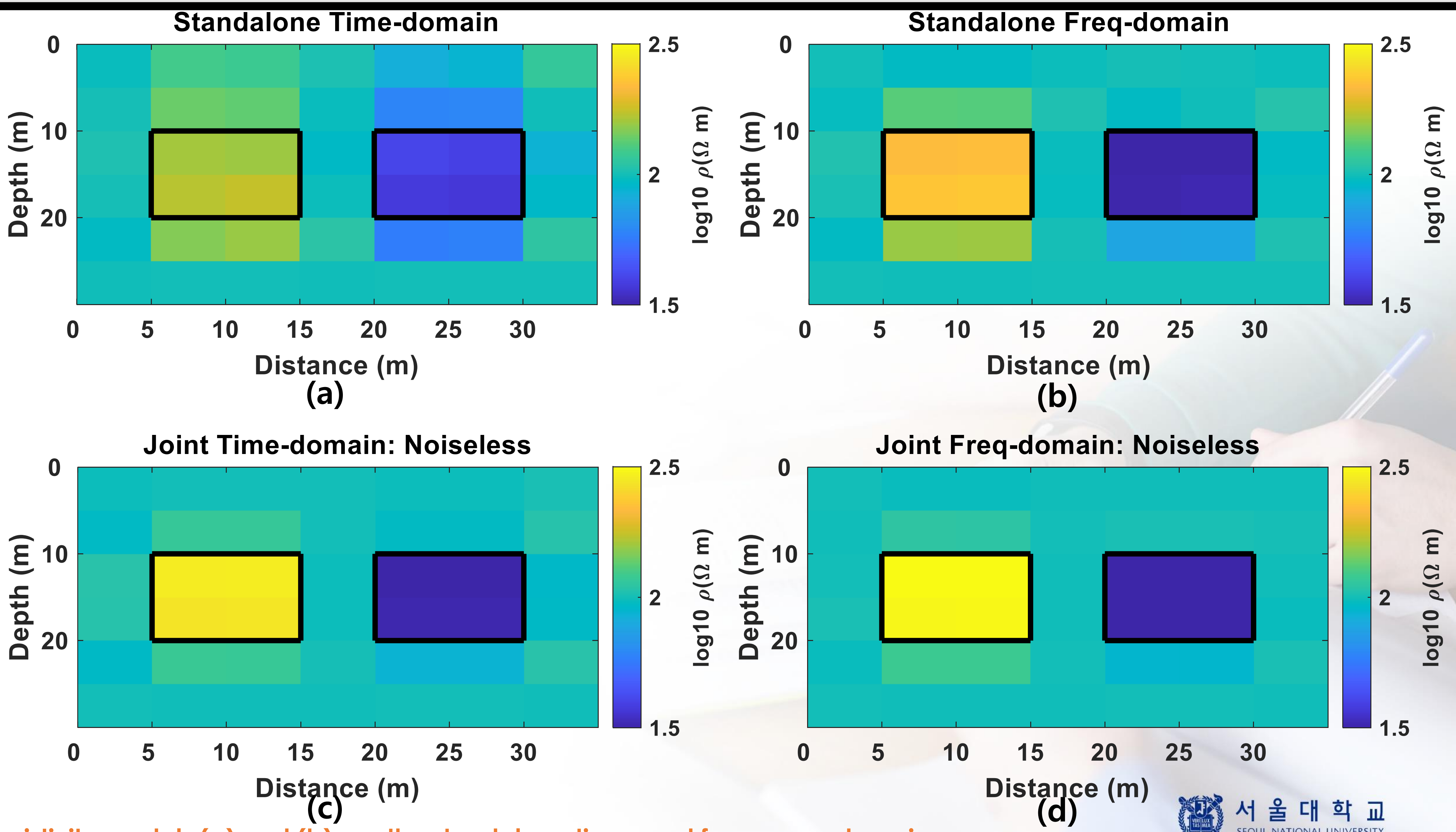

Figure 2: Inverted resistivity models (a) and (b) are the standalone time- and frequency-domain respectively; (c) and (d) are the corresponding jointly inverted resistivity models for noiseless data

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Joint inversion: high correlation even up to 3 % noise

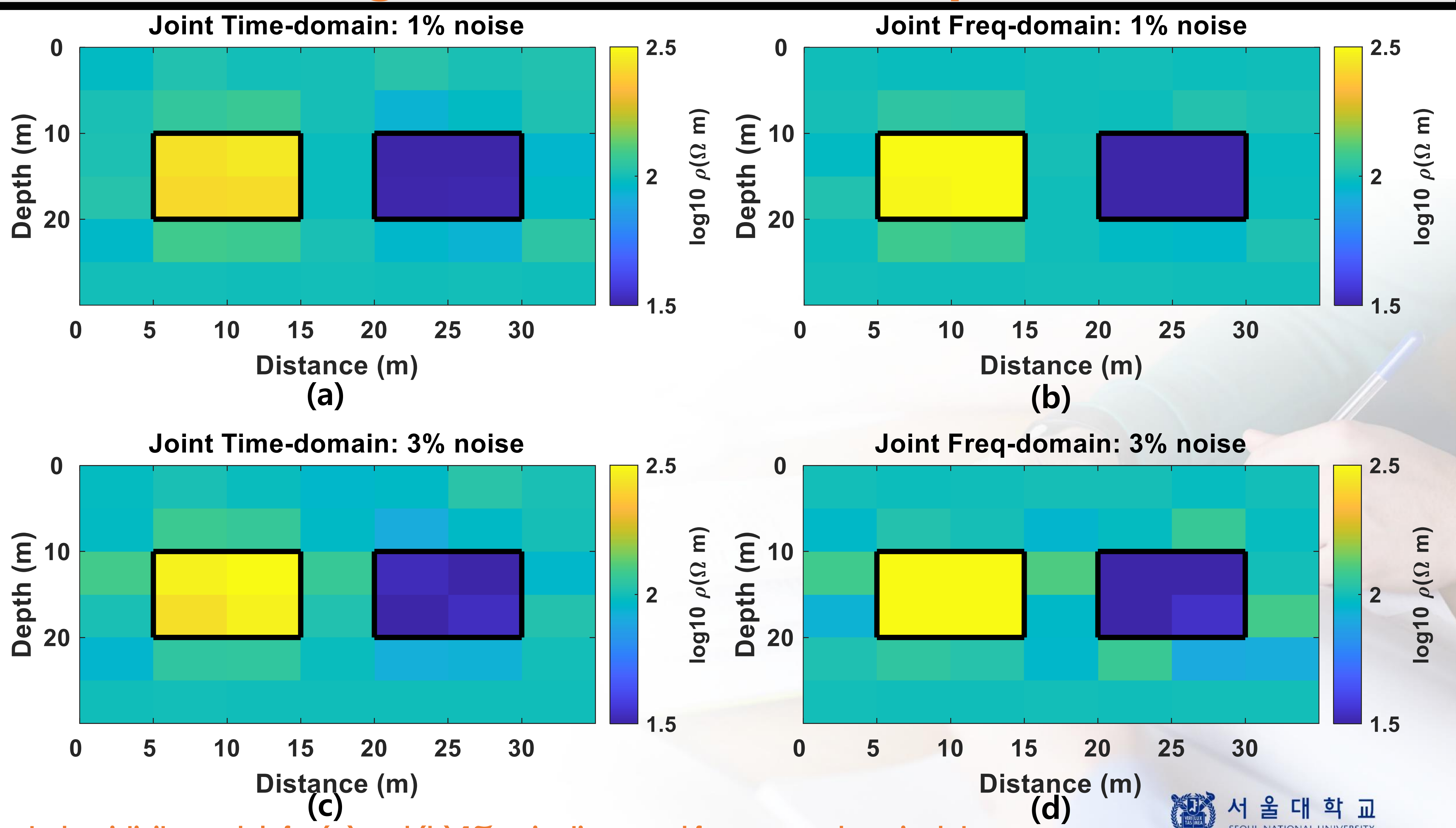

Figure 3: Jointly inverted resistivity models for (a) and (b) 1% noisy time- and frequency-domain data resp ectively; (c) and (d) are the corresponding inverted resistivity models for 3% noisy data

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Correlation coefficient, convergence, cross plots, plots with iterations

We observe that the Gramian constraint indeed increases the correlation coefficient between the jointly inverted resistivity models; although higher noise percentage has lower correlati on coefficient value; while the data misfit reduces and the res ult approach the true solution;

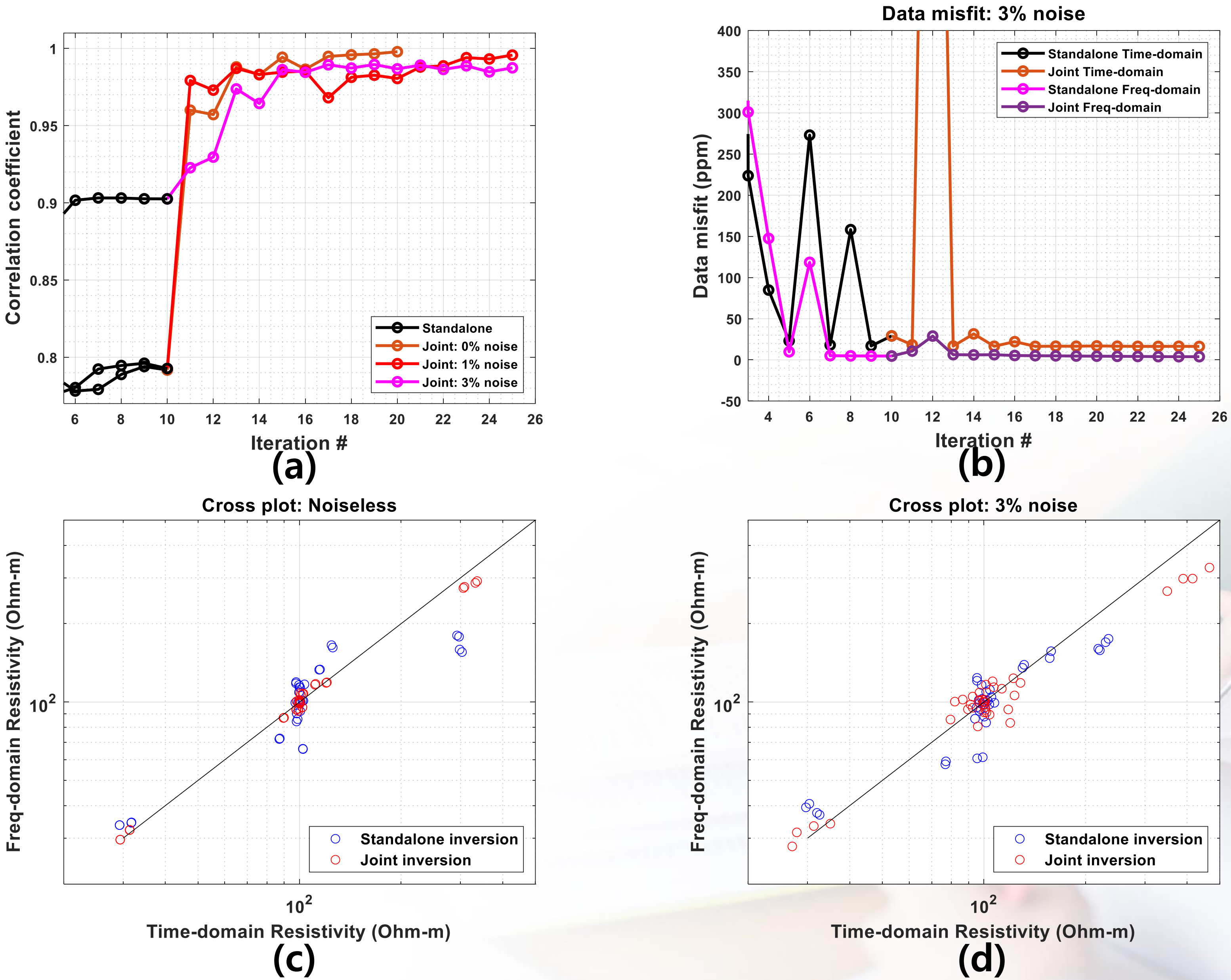

Figure 4: (a) Correlation coefficient; (b) data misfit for 3% noisy data; cross plots of jo intly inverted resistivity models (c) from noiseless data and (d) from 3% noisy data

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Data fit for the Synthetic Airborne EM data overlay

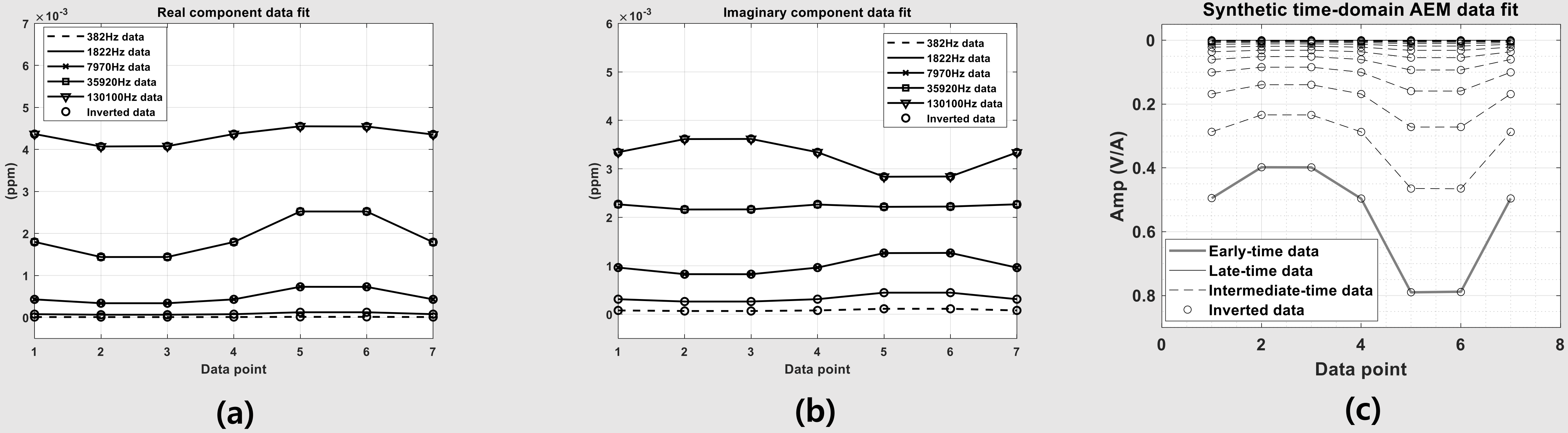

Figure 5: Synthetic data fit for (a) real component (b) imaginary component of the fre quency-domain AEM data; (c) time-domain data

Field EM Data: Bookpurnong Irrigation District

- **RESOLVE system (FDAEM)**
  - Flown at altitude of 33.46 m with horizontal receiver offset of 7.86 m.
  - Acquires horizontal coplanar in-phase and quadrature responses at 5 frequencies

- **High Moment SkyTEM (TDAEM)**
  - Flown at an altitude of 39.8 m, horizontal receiver offset of 12.4 m; offtime 4.167 ms.

- **Methods** used are
  - Time domain airborne EM (TDAEM)
  - Frequency domain airborne EM (FDAEM)
- Data acquired over the highly salinized Bookpurnong Irrigation District, South Australia. TDAEM in 2006; FDAEM in 2008 : 2 years time lapse

- The resistivity model has 10 data points.
- Number of layers is 14.
- 6 stand-alone iterations preceding 10 joint inversion iterations.

Results: Standalone and joint inversion results

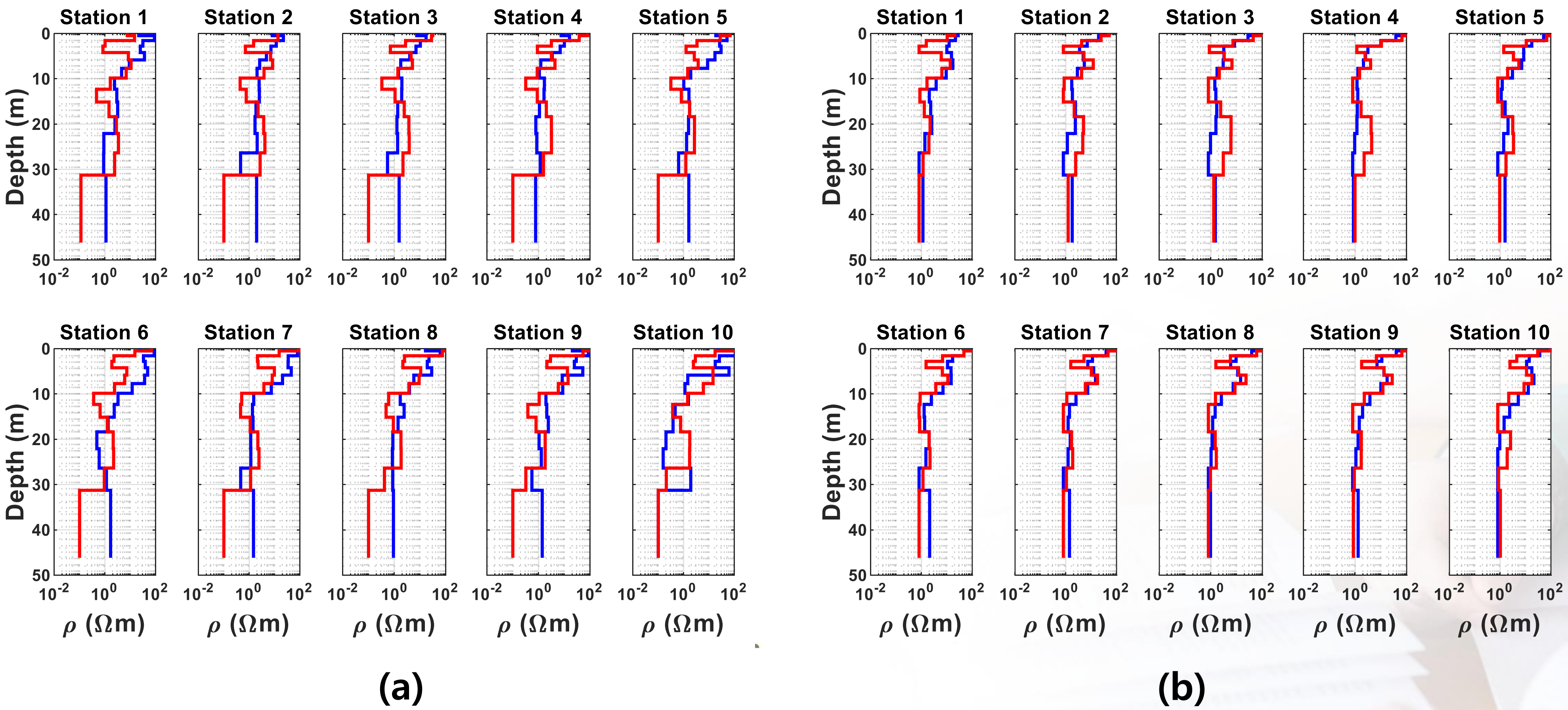

Figure 6: resistivity models from the (a) standalone inversion with visible weak correlation between the resistivity models from the frequency-domain (red) and time-domain (blue) data. (b) joint inversion results constrained by the Gramian constraint. It is apparent that the Gramian constraint synergizes the inversion towards a common unique result (Results from Ogunbo, 2019).

In the standalone results, neither the time- nor the frequency domain resistivity models can be confidently used as the final resistivity models; however, Gramian constraint enforces the correlation between the resistivity models, to focus the solution towards high confident interpretation.

The joint inversion results suggest that the Bookpurnong Irrigation District has a background resistivity of 1 Ωm which is still largely unaffected by the salinization from 20 m depth. However, in the near-surface the salinization over the two-year time lapse has increased the conductivity (decreased resistivity) as captured by the jointly inverted resistivity model from the frequency-domain data.

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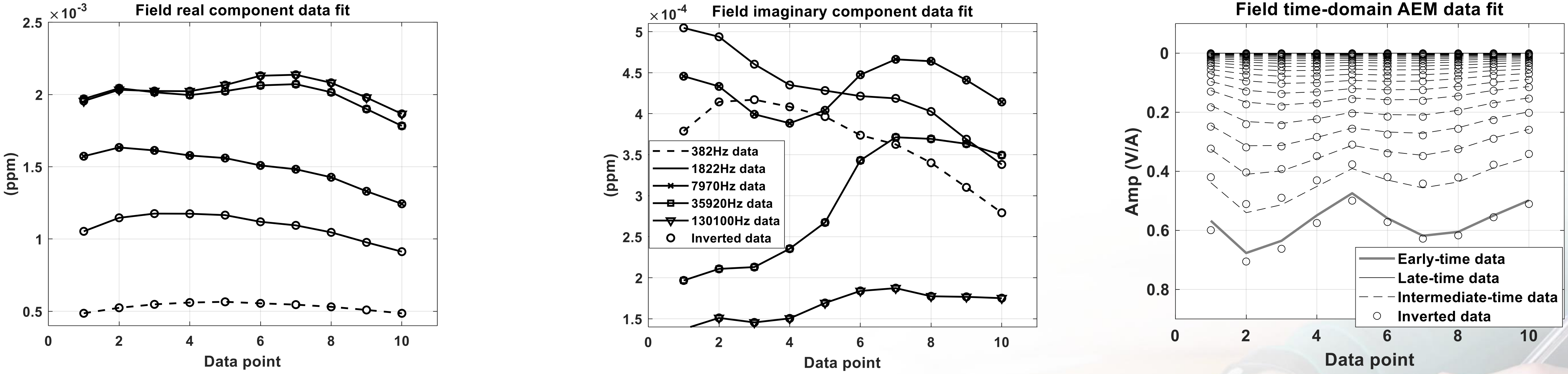
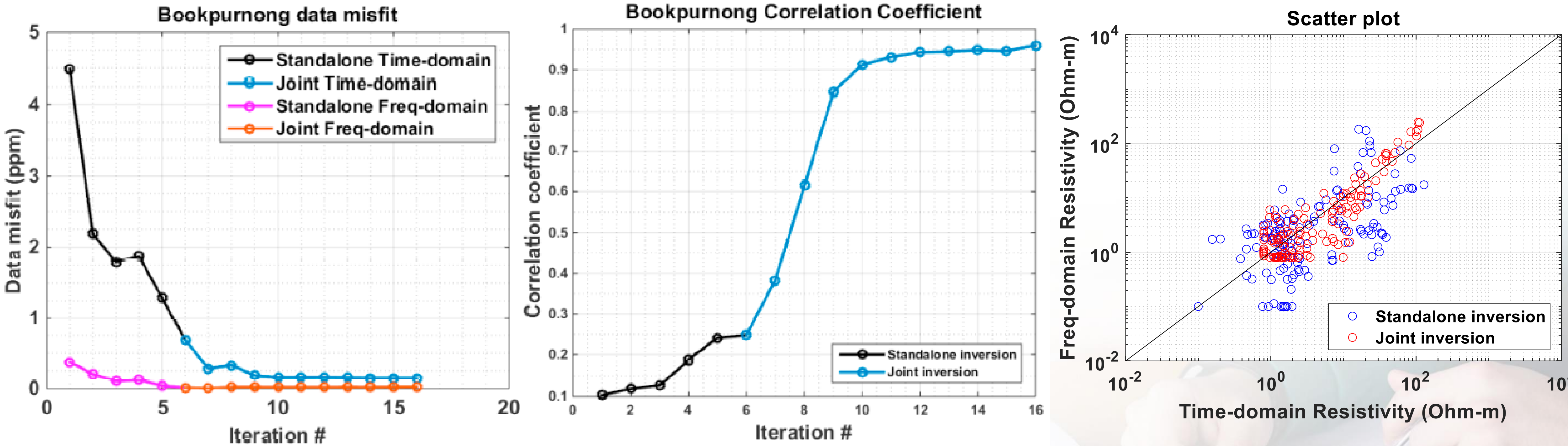
Field data fit



(a) (b) (c)

Figure 7: Field data fit for (a) real component (b) imaginary component of the frequency-domain AEM data;

Field data: data misfit, correlation coefficient, cross plot



(a) (b) (c)

Figure 8: (a) RMSE with iteration (b) correlation coefficient with iteration (c) and cross plot of jointly inverted resistivity models

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Conclusions

Conclusions

- The multi-physics from the frequency and time-domain airborne EM (AEM) data has been jointly inverted with the Gramian constraint, which is the dot product of two resistivity model vectors.

- The Gramian constraints synergize the linear correlation between the resistivity models to produce more reliable images than those from the standalone inversions.

- The application of the concept on synthetic data proves the compelling role of the Gramian influence in the joint inversion even in the presence of noise. Increasing linear correlation coefficient and decreasing data misfit with iteration is further ensured by the Gramian constraint.

- Moreover, the frequency and time-domain AEM data from Bookpurnong Irrigation District, South Australia have been jointly inverted with Gramian constraint which reveals a resistivity background of 1 Ωm with a salinized (lower resistivity values) near surface (shallow depths).

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