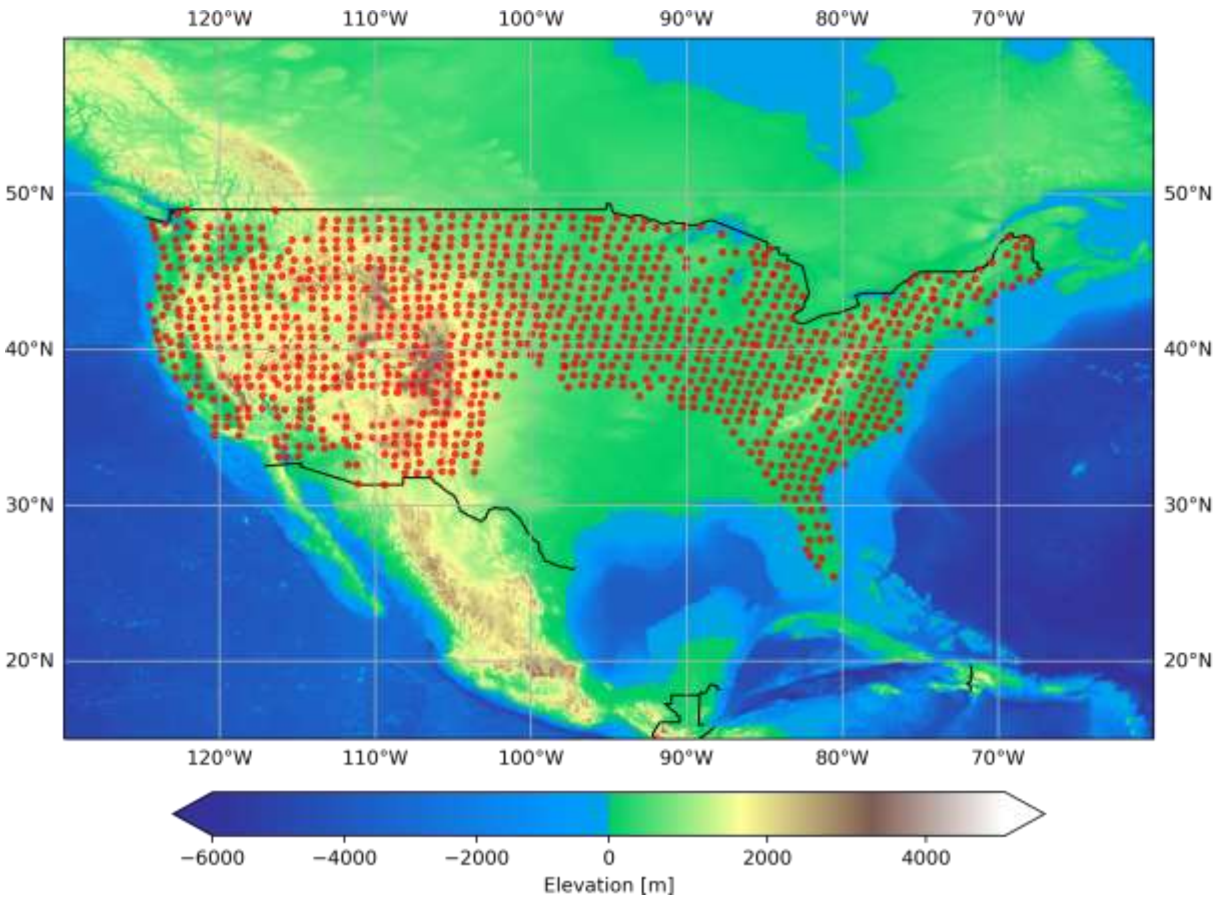
Three-dimensional MT modelling and inversion in spherical Earth: applications to continental-scale surveys

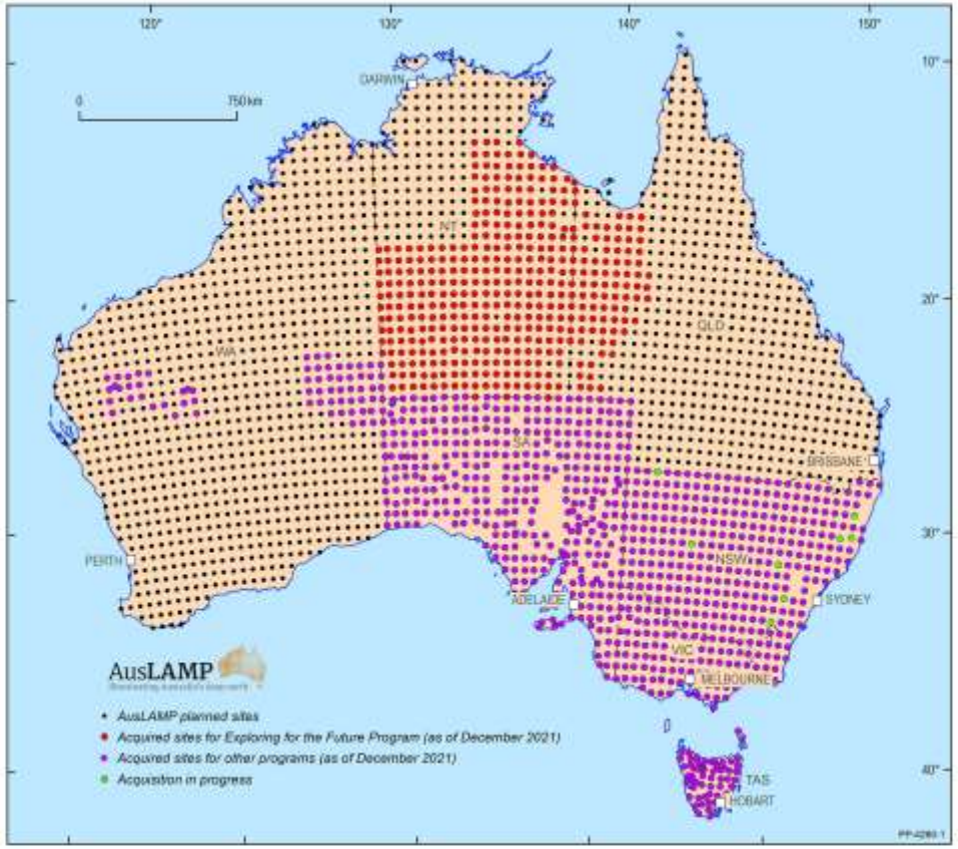
Alexander Grayver

Institute of Geophysics, ETH Zürich

With contributions from Filippo Cicchetti, Federico Munch and Alexey Kuvshinov

Motivation





- Check talks by P. Bedrosian, S. Thiel, H. Dong

Modelling perspective

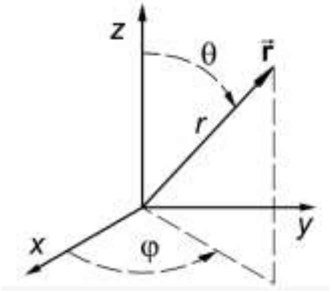
- The «flat Earth» model.
  - Drawbacks and limitations.
- Multi-scale nature of the problem
  - Ocean and sediments, small-scale distortions
- Can we resort to a spherical Earth model?
  - Advantages of a spherical frame.
- How to resolve multiple scales within one model?

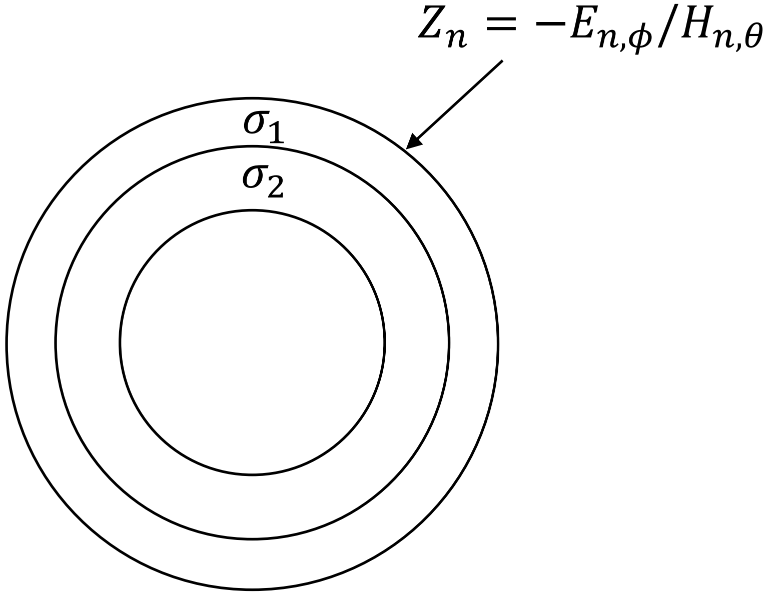
Magnetotelluric response of a 1D Earth

Faraday's law: $\nabla \times \vec{E} = -i\omega\mu\vec{H}$

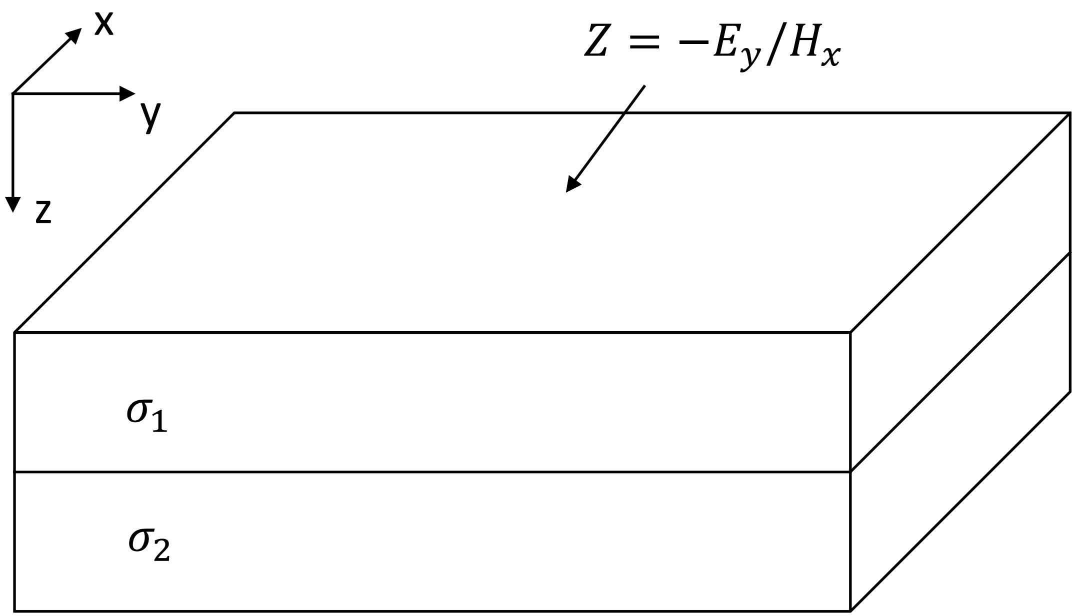
Ampere's law: $\nabla \times \vec{H} = \sigma\vec{E}$

Spherical model:





Flat (plane) model:



Magnetotelluric response of a 1D Earth

- Srivastava 1966: systematic study on and comparison of plane and spherical impedances.
- Weidelt 1972: functional relation between plane and spherical models ("Weidelt transform").
- Dmitriev and Berdichevsky 1979: proof of validity of impedance for non-homogeneous "source" fields.
- Many more works… (check references therein)

Theory of the Magnetotelluric Method for a Spherical Conductor**

S. P. Srivastava*

(Received 1964 September 24. Revised 1965 August 19)

The Inverse Problem of Geomagnetic Induction

P. WEIDELT, Göttingen[1])

Eingegangen am 24. März 1972

The Fundamental Model of Magnetotelluric Sounding

VLADIMIR I. DMITRIEV AND MARK N. BERDICHEVSKY

PROCEEDINGS OF THE IEEE, VOL. 67, NO. 7, JULY 1979

3-D modelling of MT transfer functions

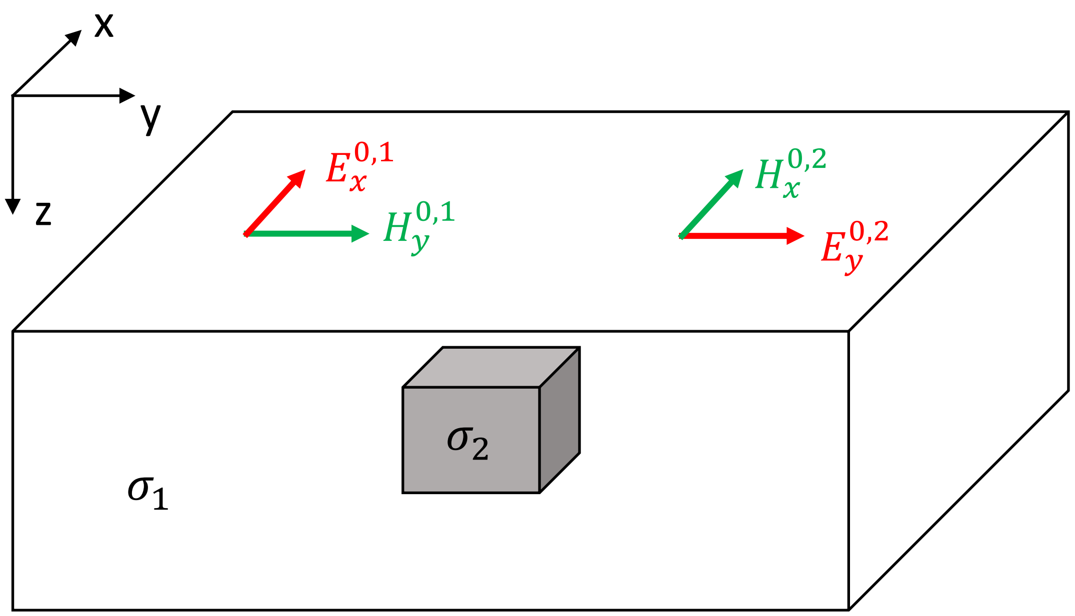
- Plane wave MT transfer functions can be simulated with two orthogonal polarizations:

Faraday's law: $\nabla \times \vec{E} = -i\omega\mu\vec{H}$

Ampere's law: $\nabla \times \vec{H} = \sigma\vec{E}$

$$\begin{bmatrix} E_x^1 & E_x^2 \\ E_y^1 & E_y^2 \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} H_x^1 & H_x^2 \\ H_y^1 & H_y^2 \end{bmatrix}$$

$$\begin{bmatrix} H_z^1 & H_z^2 \end{bmatrix} = \begin{bmatrix} T_{zx} & T_{zy} \end{bmatrix} \begin{bmatrix} H_x^1 & H_x^2 \\ H_y^1 & H_y^2 \end{bmatrix}$$



3-D modelling of MT transfer functions

- Plane wave MT transfer functions can be simulated with two orthogonal polarizations:

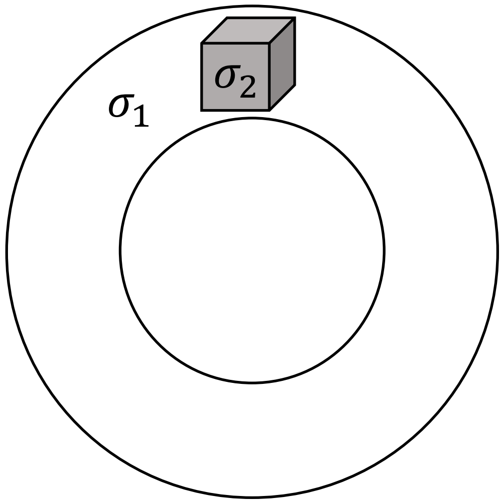
$$\begin{bmatrix} E_x^1 & E_x^2 \\ E_y^1 & E_y^2 \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} H_x^1 & H_x^2 \\ H_y^1 & H_y^2 \end{bmatrix}$$

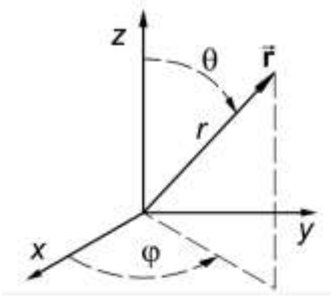
$$\begin{bmatrix} H_z^1 & H_z^2 \end{bmatrix} = \begin{bmatrix} T_{zx} & T_{zy} \end{bmatrix} \begin{bmatrix} H_x^1 & H_x^2 \\ H_y^1 & H_y^2 \end{bmatrix}$$

- Is there an equivalent source representation on a sphere?

Faraday's law: $\nabla \times \vec{E} = -i\omega\mu\vec{H}$

Ampere's law: $\nabla \times \vec{H} = \sigma\vec{E}$

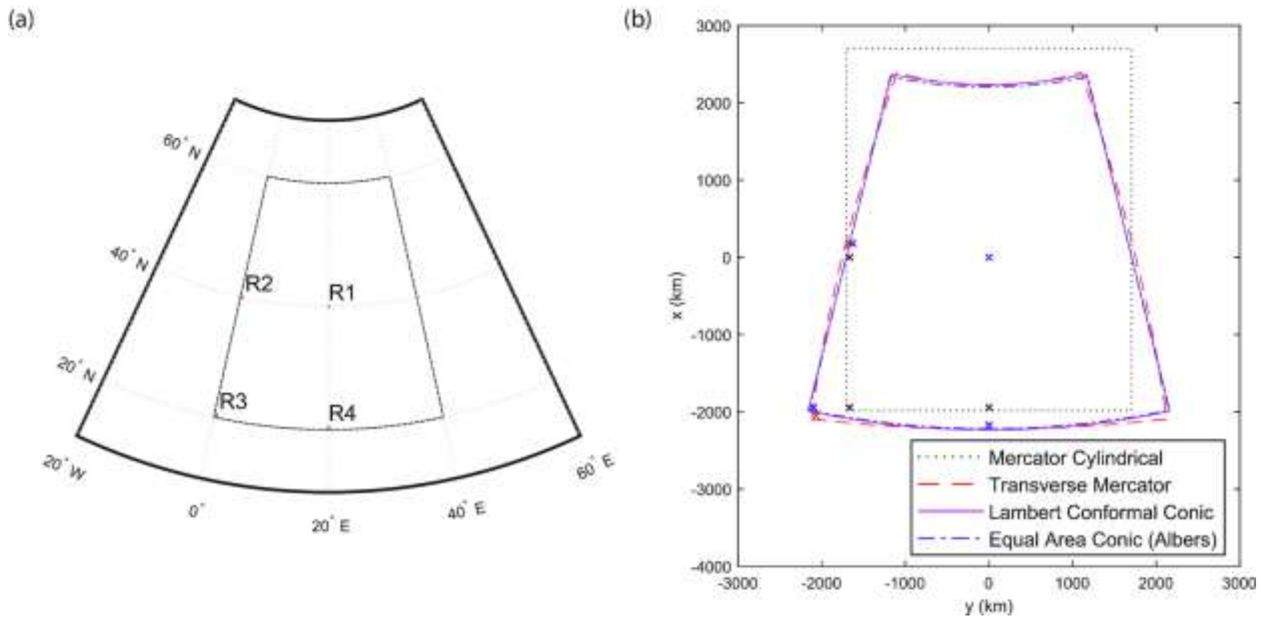




Pros and Cons of a spherical model

- No geographic projection is needed.
  - Free of potential distortions or other "flattening" effects.
  - Easier to archive and exchange.

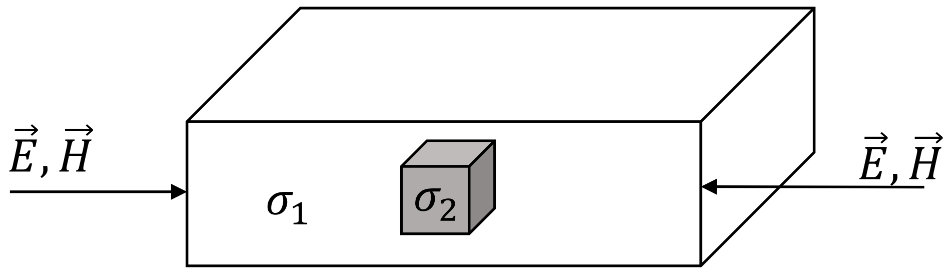
Grayver et al. 2019, GJI

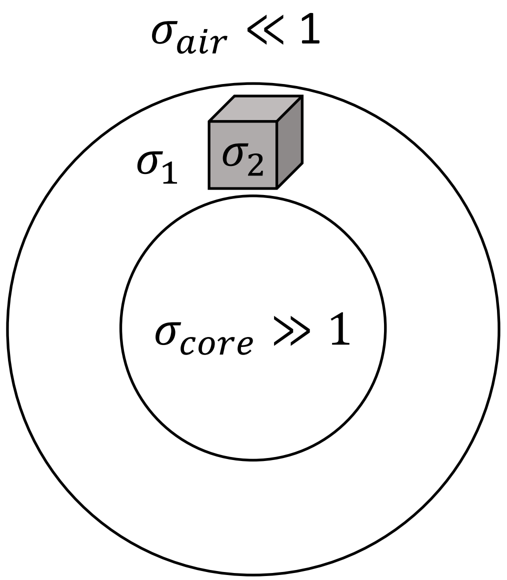


Pros and Cons of a spherical model

- No geographic projection is needed.
  - Free of potential distortions or other "flattening" effects.
  - Easier to archive and exchange.

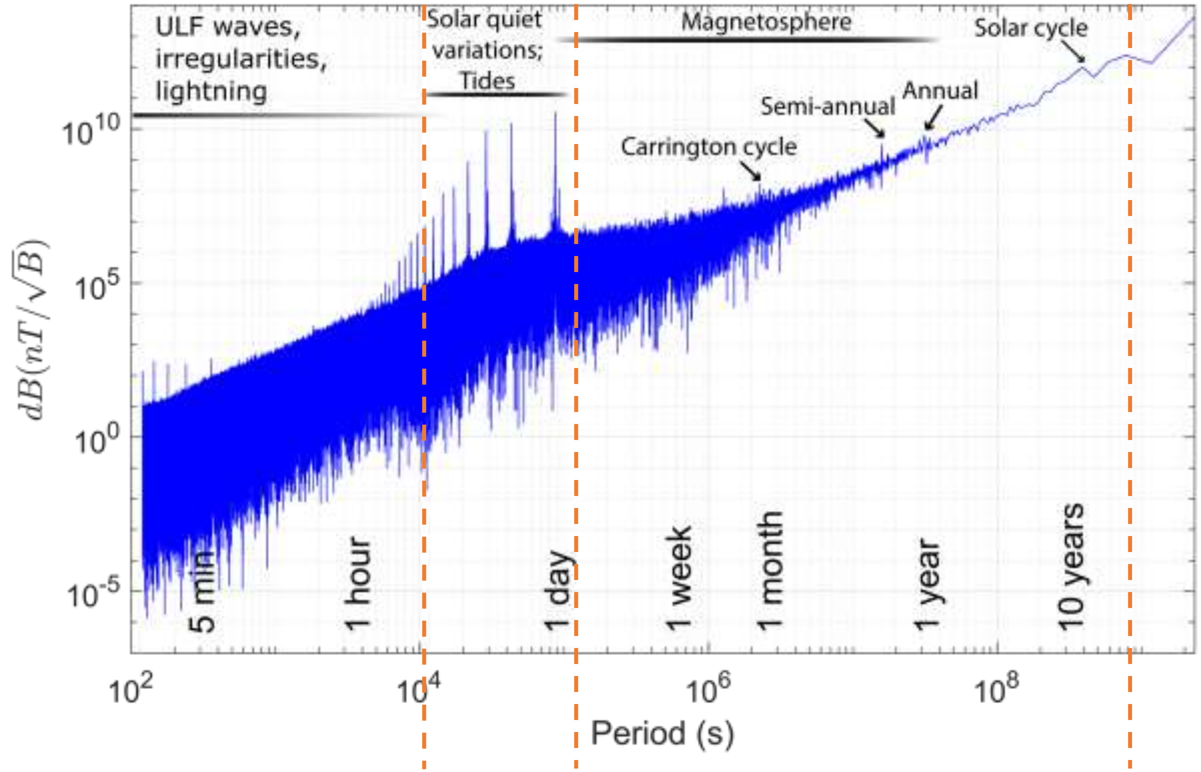
- Minimize or avoid boundary ("edge") effects.





Pros and Cons of a spherical model

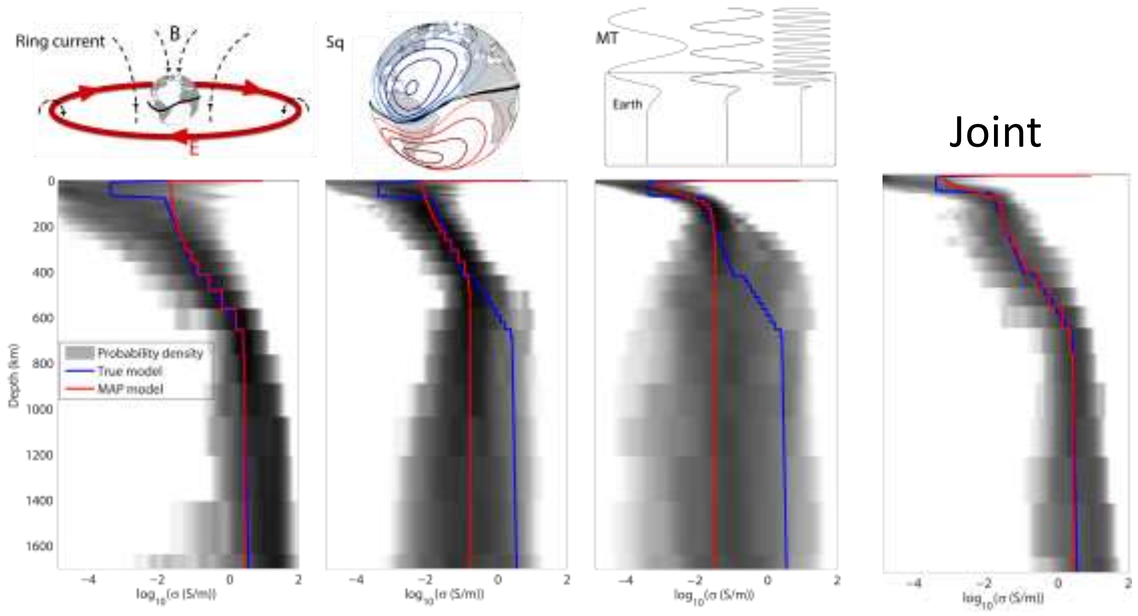
- No geographic projection is needed.
  - Free of potential distortions or other "flattening" effects.
  - Easier to archive and exchange.
- Minimize or avoid boundary ("edge") effects.
- Integration with other (intrinsically) global sources.



Band:	"Plane wave"	"Daily"	"Magnetospheric"
Sounding depth (km):	< 300	100-500	> 400

Pros and Cons of a spherical model

- No geographic projection is needed.
  - Free of potential distortions or other "flattening" effects.
  - Easier to archive and exchange.
- Minimize or avoid boundary ("edge") effects.
- Integration with other (intrinsically) global sources.



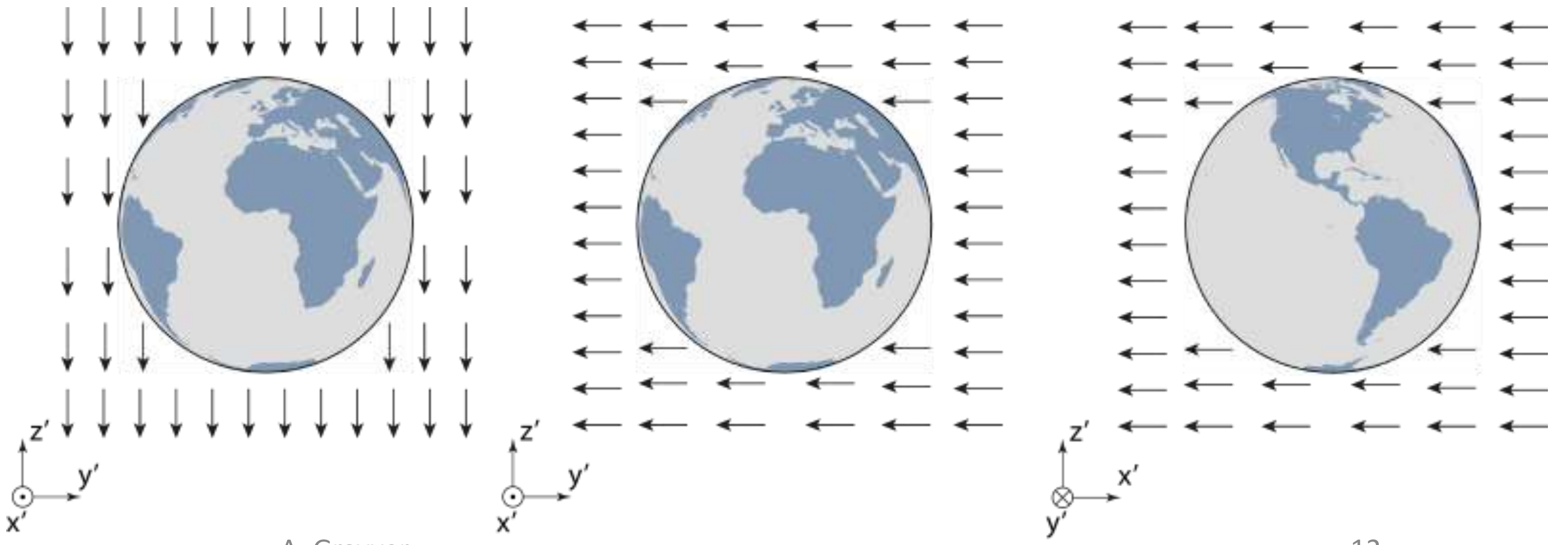
Grayver et al. 2019, GJI

3-D modelling of MT transfer functions in a sphere

- Uniform planetary fields (Fainberg et al. 1983).
- Described by degree 1 Spherical Harmonic functions.
- Reproduces plane wave impedance in a relevant period range.
- No tippers due to non-zero B_r.

Grayver et al. 2019

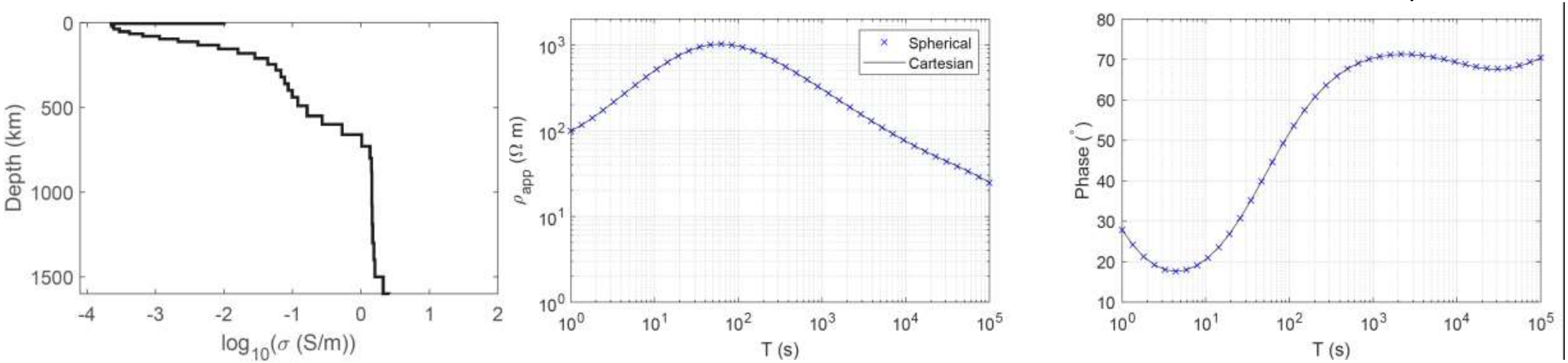
$$\begin{bmatrix} -E_\theta^1 & -E_\theta^2 & -E_\theta^3 \\ E_\phi^1 & E_\phi^2 & E_\phi^3 \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} -H_\theta^1 & -H_\theta^2 & -H_\theta^3 \\ H_\phi^1 & H_\phi^2 & H_\phi^3 \end{bmatrix}$$



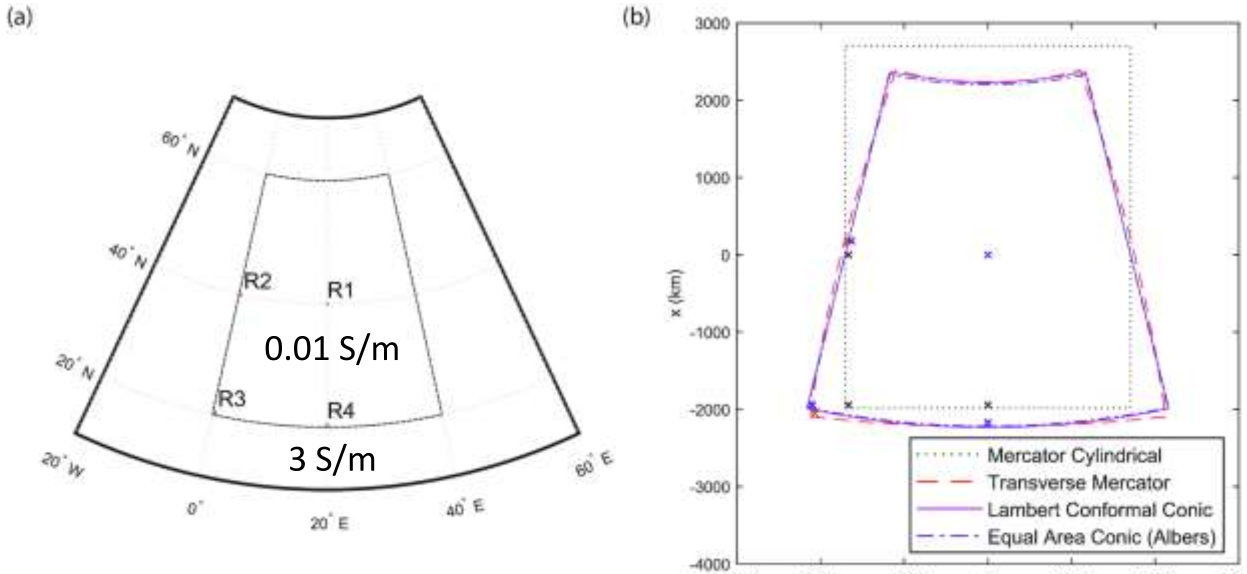
3-D modelling of MT transfer functions in a sphere

- Uniform planetary fields (Fainberg et al. 1983).
- Described by degree 1 Spherical Harmonic functions.
- Reproduces plane wave impedance in a relevant period range.
- No tippers due to non-zero B_r.

Grayver et al. 2019

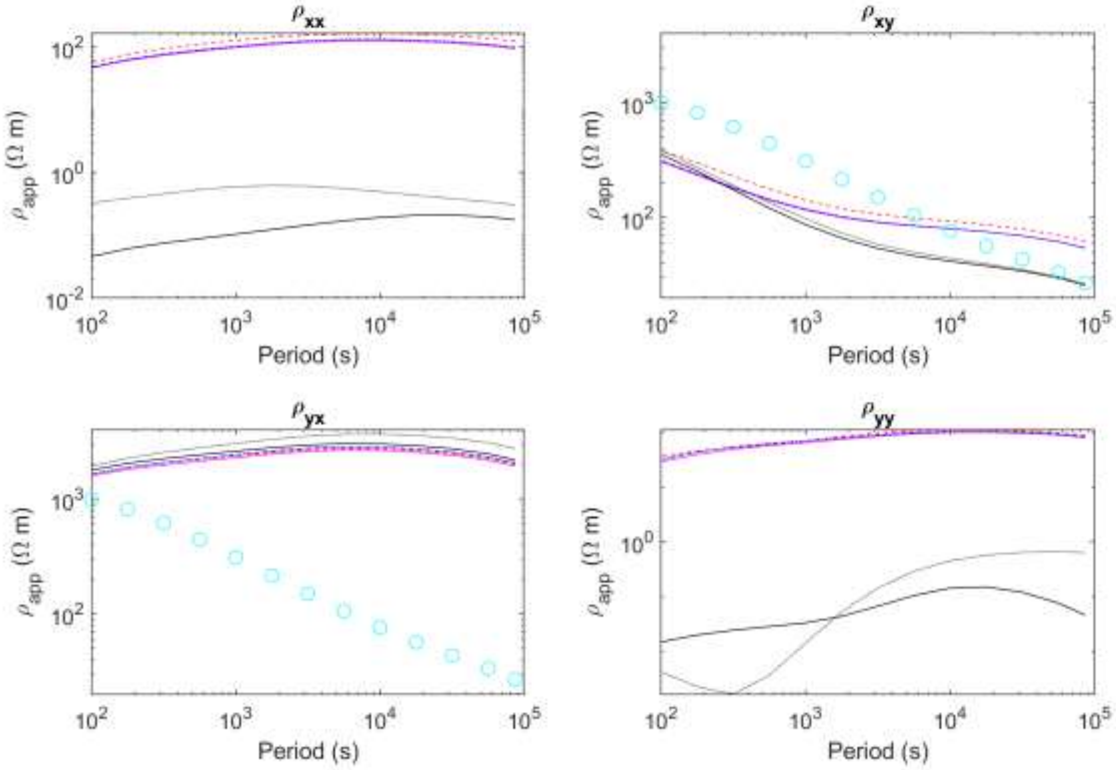


Effect of geographic projections



+ 1D profile below

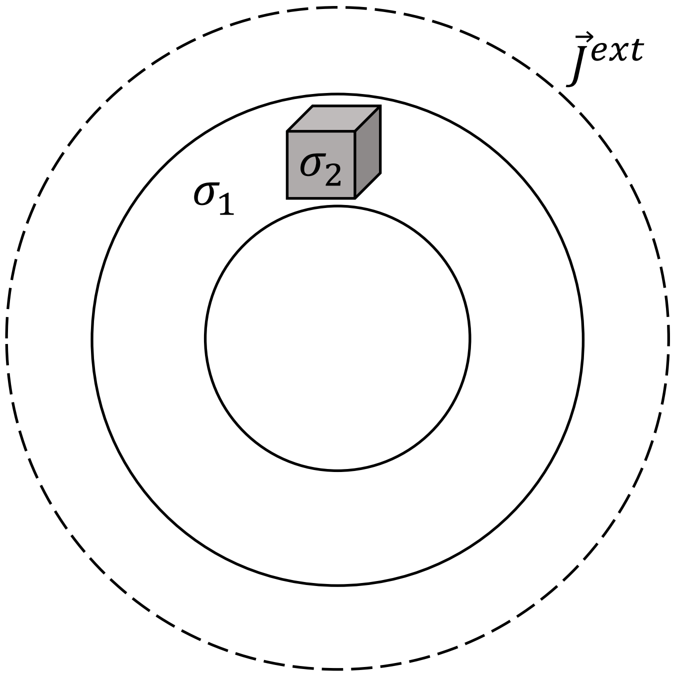
Apparent resistivity at R2:

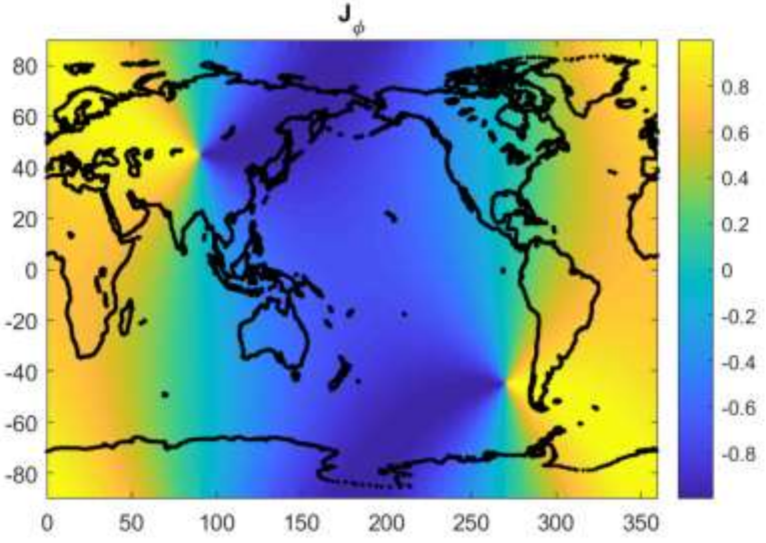


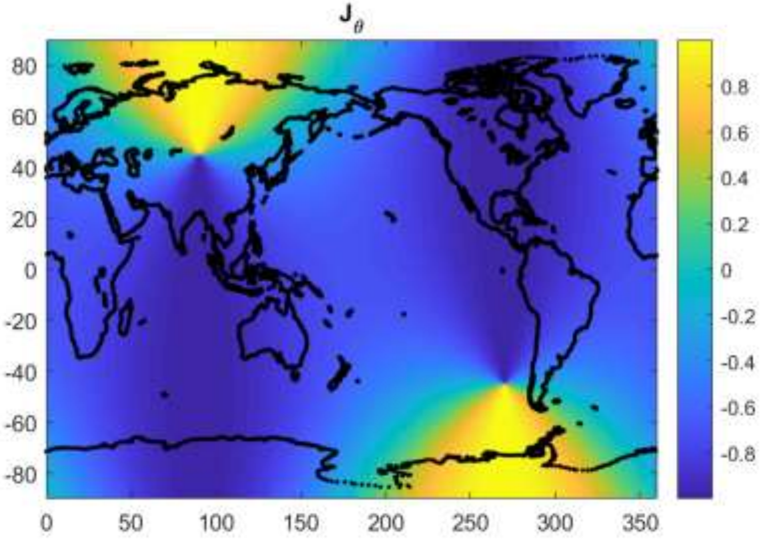
- The magnitude of the effect depends on the adopted projection and conductivity.
- Fields can be rotated (“corrected”) to account for the geographic projection effect.

3-D modelling of MT transfer functions in a sphere

- Alternative source model based on a sheet current $\vec{J}^{ext}$ flowing in ϑ-direction placed above the Earth's surface + plus two rotated orthogonal polarizations (Kruglyakov and Kuvshinov, in review).
- Radial (vertical) field is zero for any 1-D model, thus it can be used to calculate tippers.

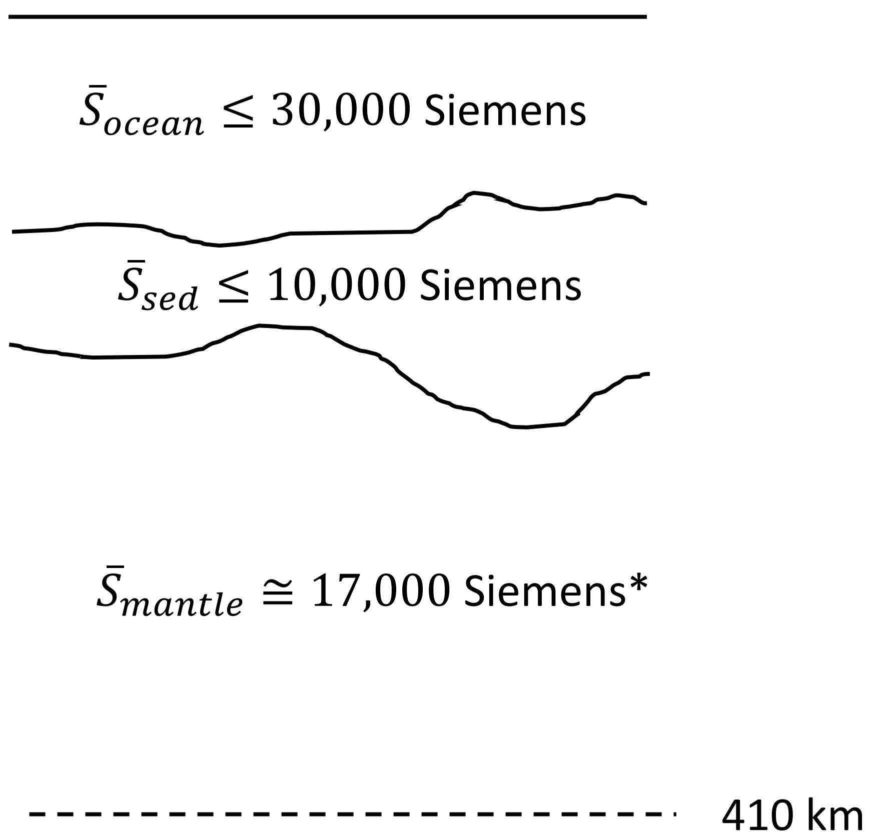






Ocean and (marine) sediments

- Average conductance of the ocean and marine sediments is equivalent to that of the entire upper mantle.
- Complex non-linear effect due to ocean and marine sediments in a wide range of periods.



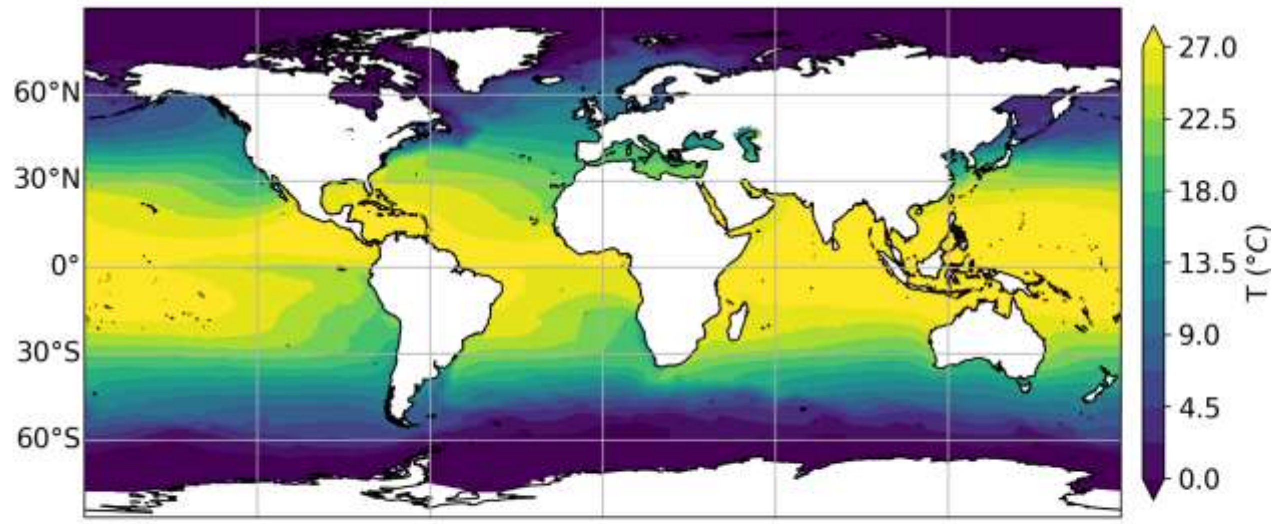
** Based on the model from Grayver et al. 2017*

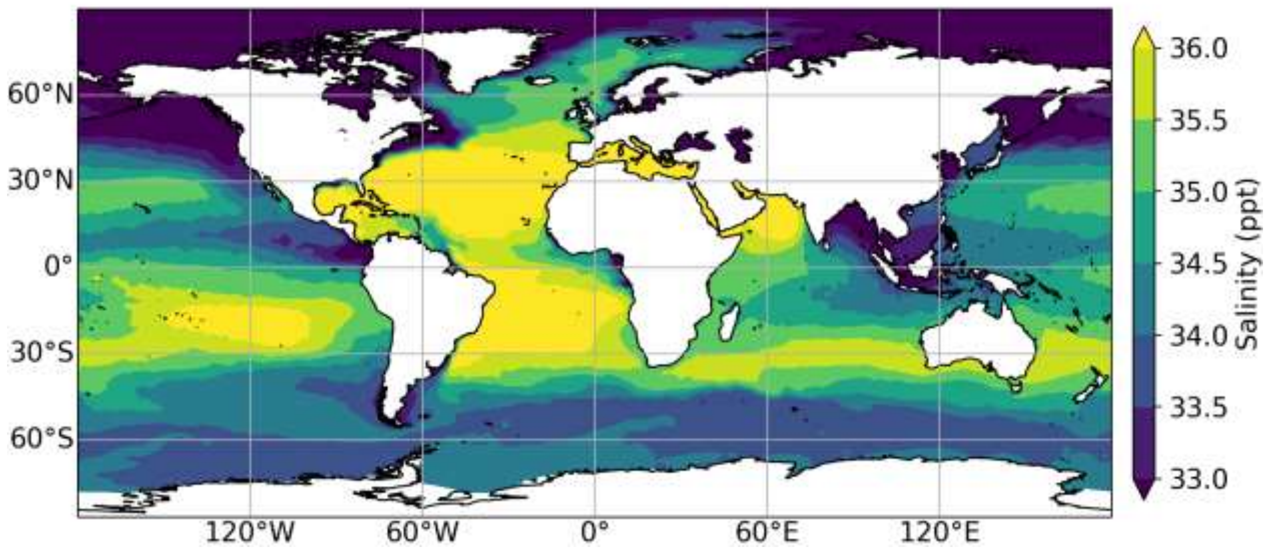
Ocean conductivity

- Equation of state of seawater (TEOS-10, Milero 2010) as a function of ocean temperature, salinity and in-situ pressure:

$$\sigma_{sw}(\vec{r}) \equiv \sigma_{TEOS}(T(\vec{r}), S(\vec{r}), P(\vec{r}))$$

- Annual mean temperature and salinity at the sea surface from World Ocean Atlas (Boyer et al. 2018):





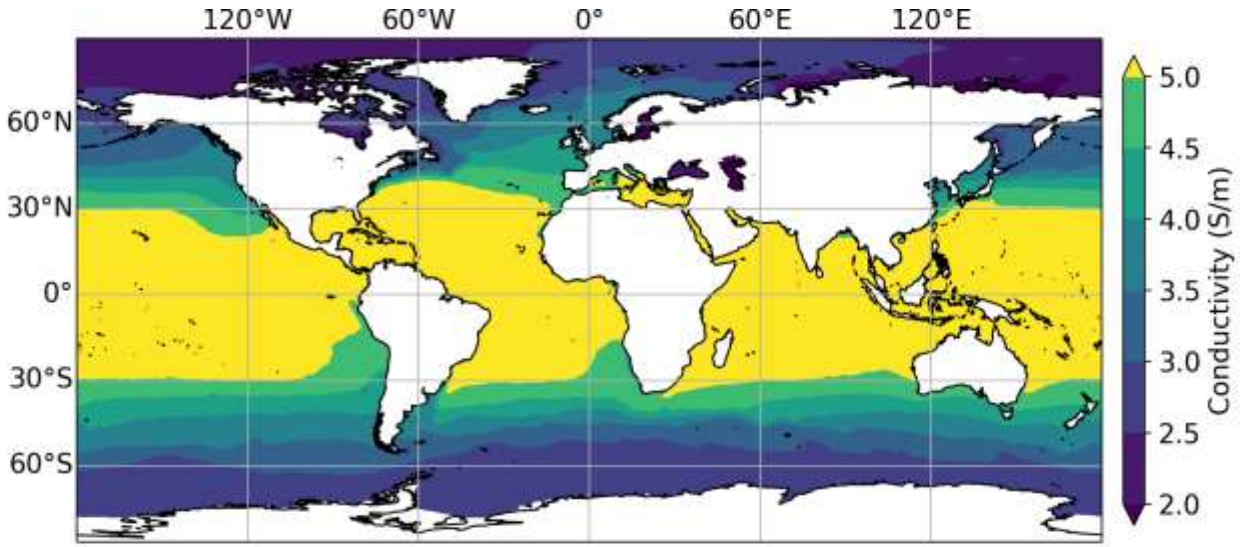
Grayver 2021, G^3

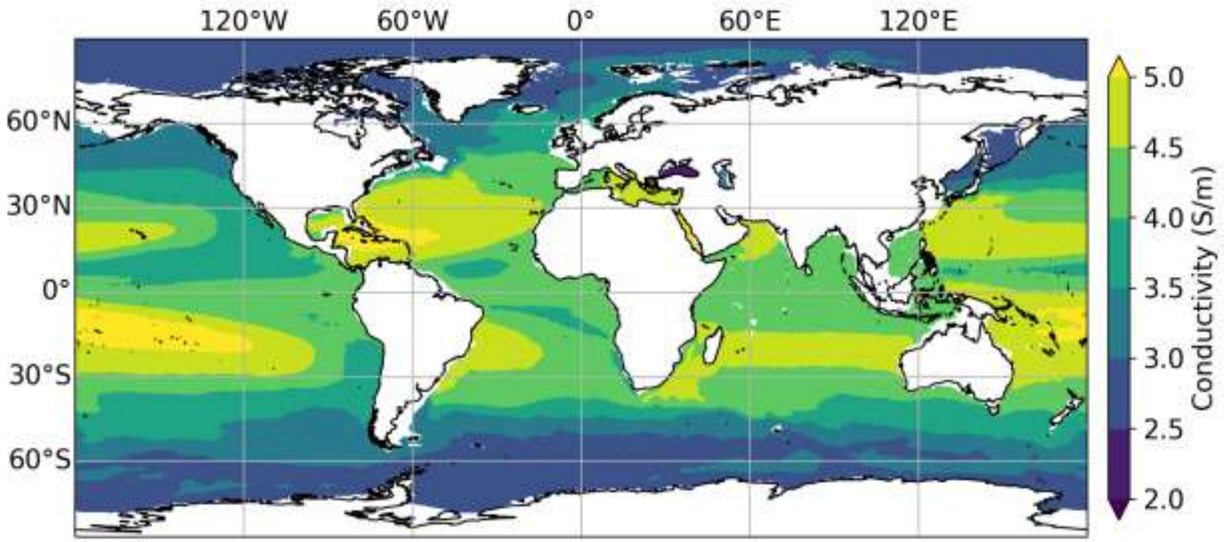
Ocean conductivity

- Equation of state of seawater (TEOS-10, Milero 2010) as a function of ocean temperature, salinity and in-situ pressure:

$$\sigma_{sw}(\vec{r}) \equiv \sigma_{TEOS}(T(\vec{r}), S(\vec{r}), P(\vec{r}))$$

- Derived conductivity at the sea surface and at 200 m depth:





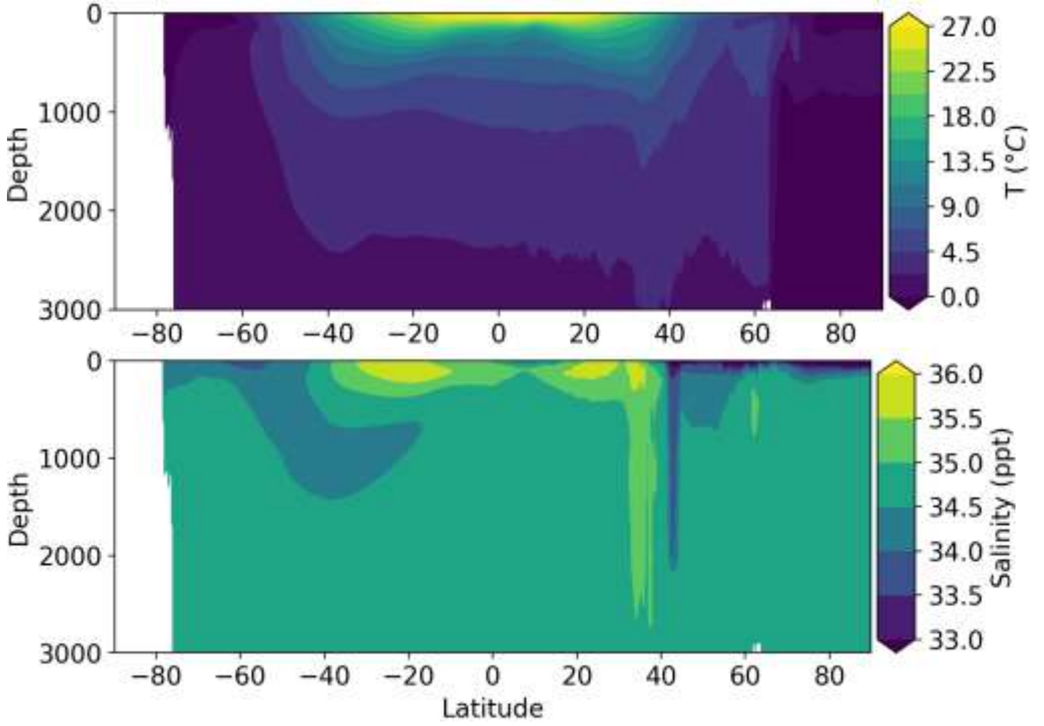
Grayver 2021, G^3

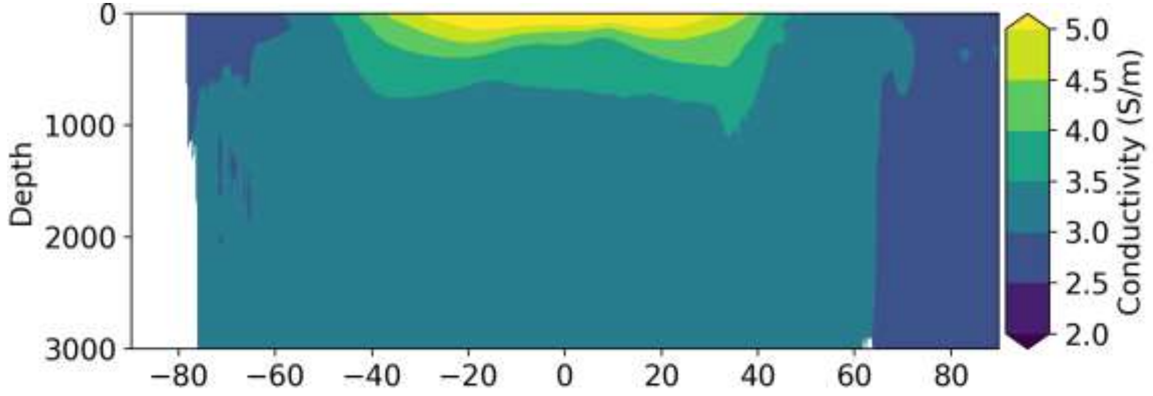
Ocean conductivity

- Equation of state of seawater (TEOS-10, Milero 2010) as a function of ocean temperature, salinity and in-situ pressure:

$$\sigma_{sw}(\vec{r}) \equiv \sigma_{TEOS}(T(\vec{r}), S(\vec{r}), P(\vec{r}))$$

- Cross-sections of σ, T and S averaged along longitude:

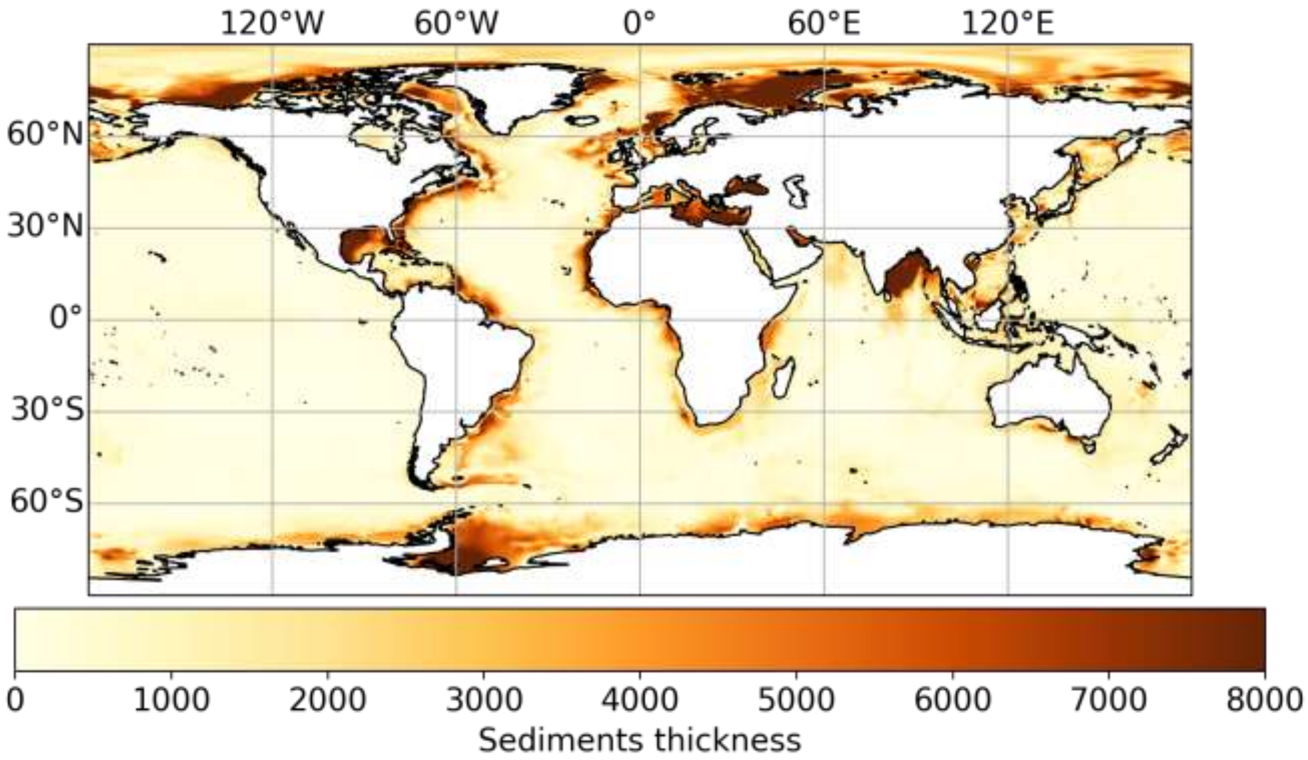




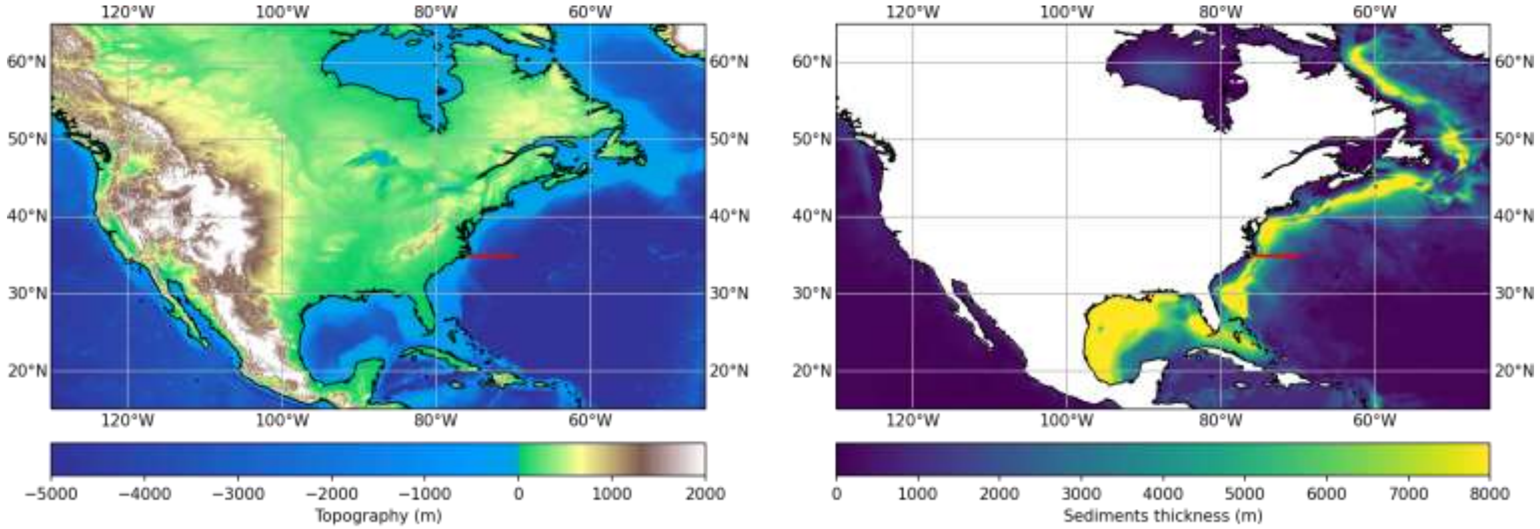
Grayver 2021, G^3

Marine sediments

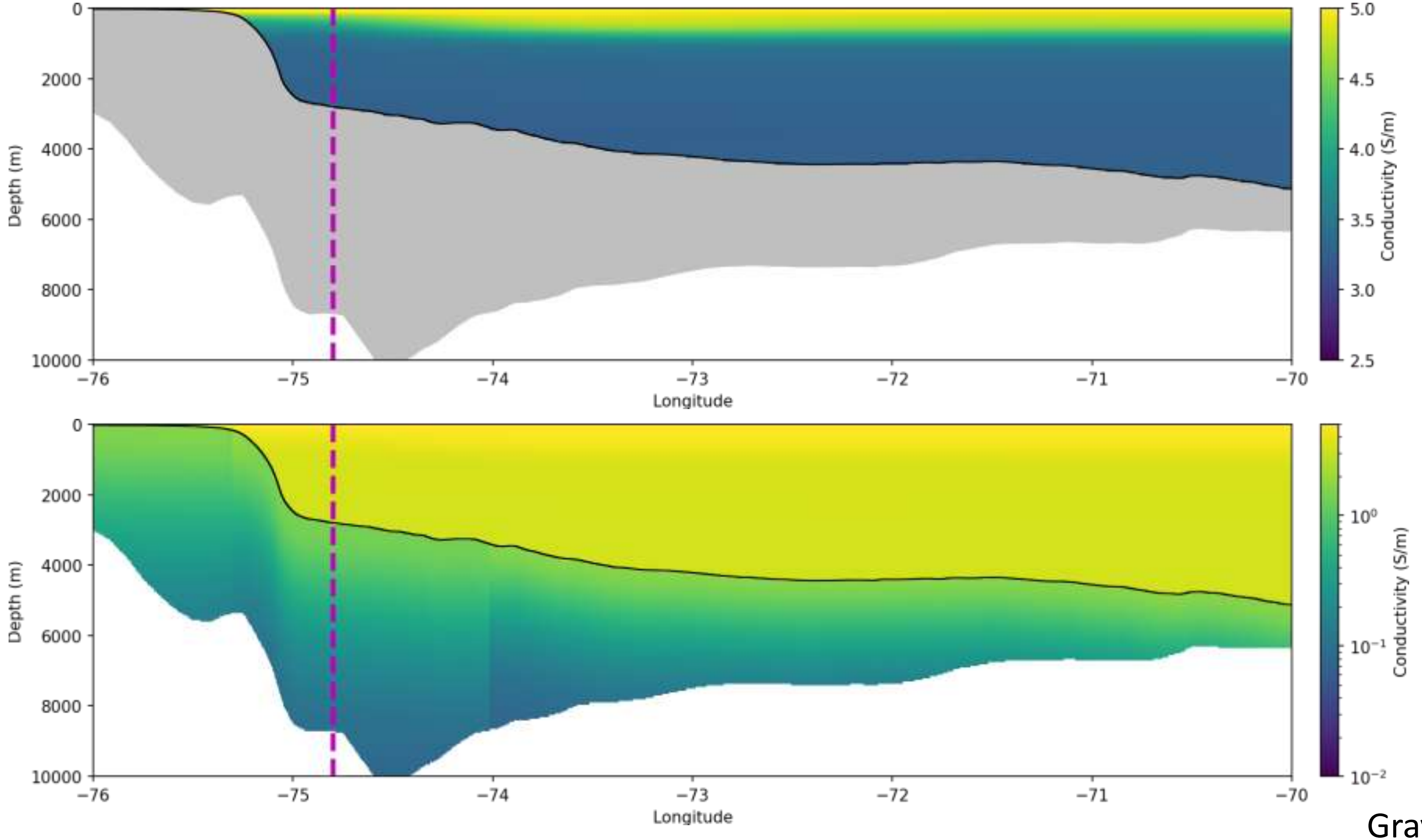
- Marine sediments are generally conductive due to penetration of seawater.
- Their thickness can exceed 10 km (Straume et al. 2019).
- May become significant part of the «ocean» effect.



Ocean and sediment conductivity

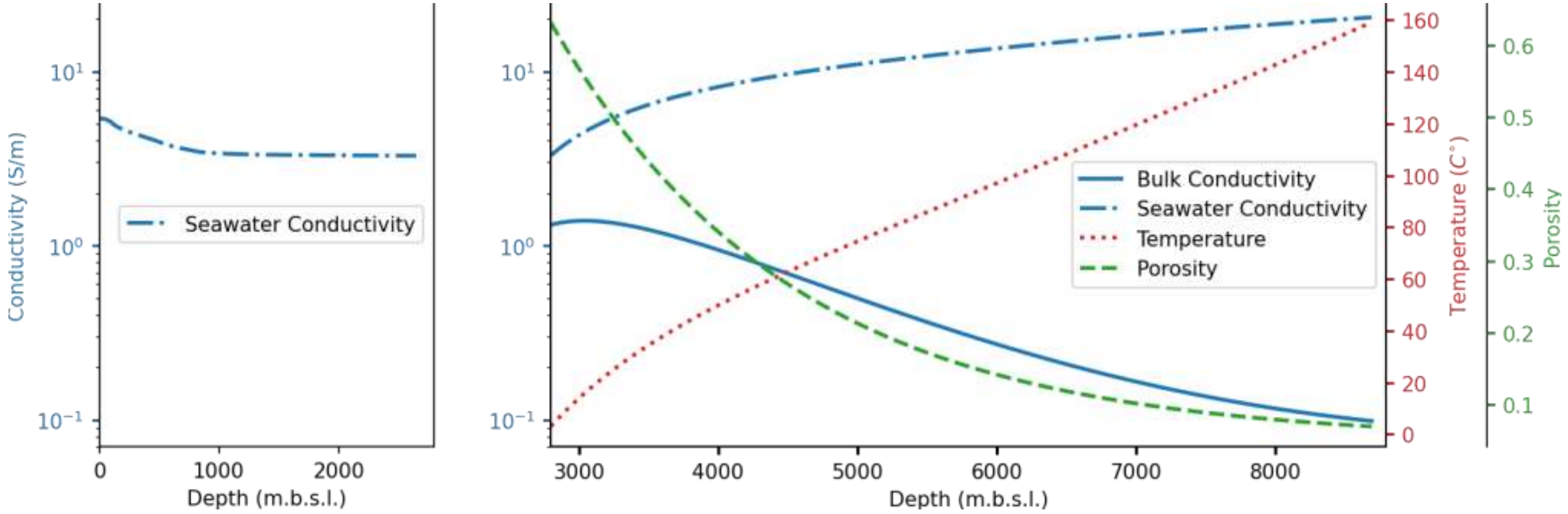


Ocean and sediment conductivity



Grayver 2021, G^3

Ocean and sediment conductance



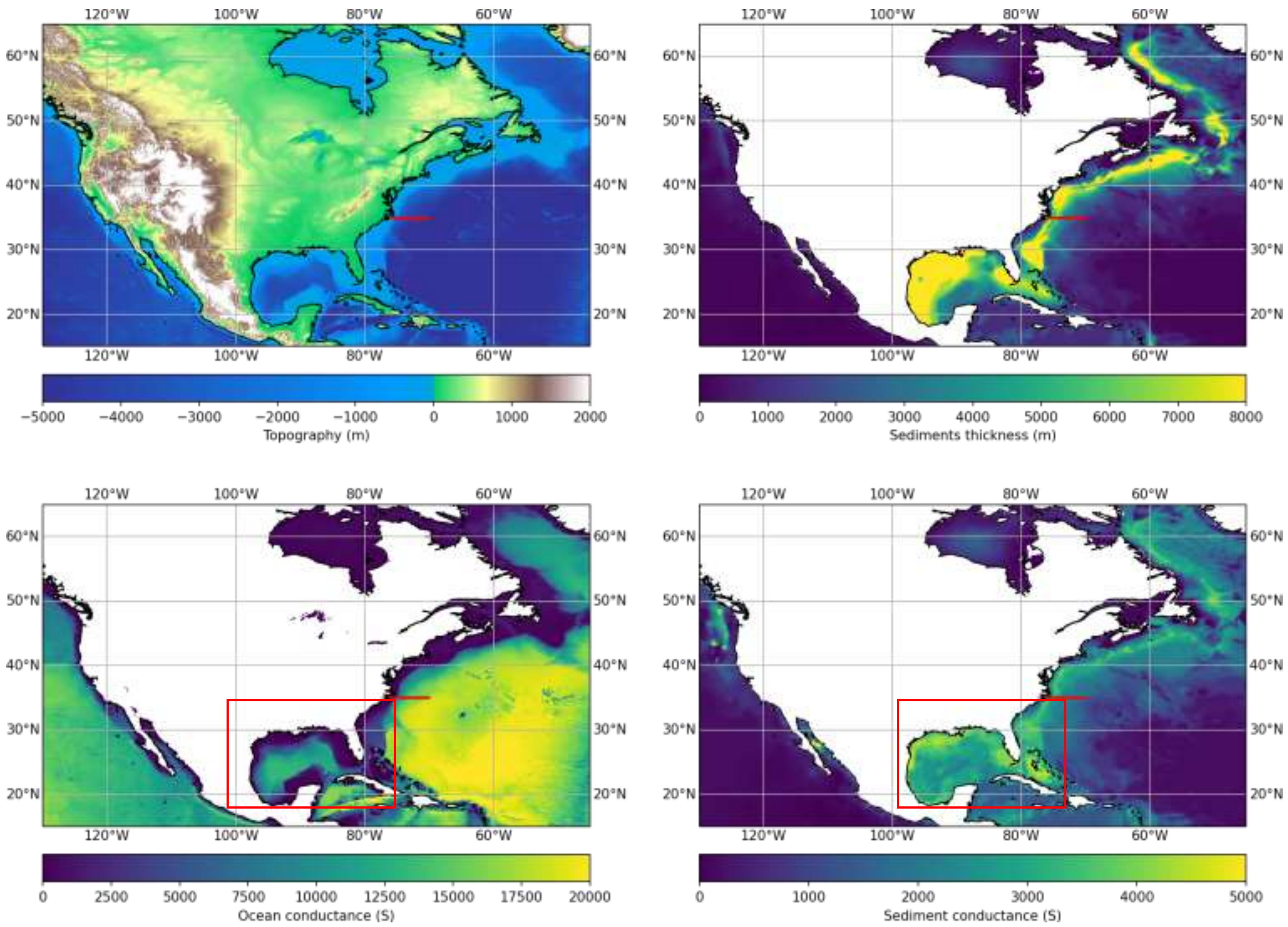
Porosity model: $\phi(z) = \phi_{min} + (\phi_0 - \phi_{min})e^{-c_0 z}$

Bulk conductivity of sediments: $\sigma_m(\vec{r}) = \sigma_f(\vec{r})\phi(\vec{r})^{\beta}$

Temperature model: $T_{sed}(z) = T_{SWI} + z\frac{q}{\lambda_{sed}^{1-\phi(z)} \cdot \lambda_f^{\phi(z)}}$

Grayver 2021, G^3

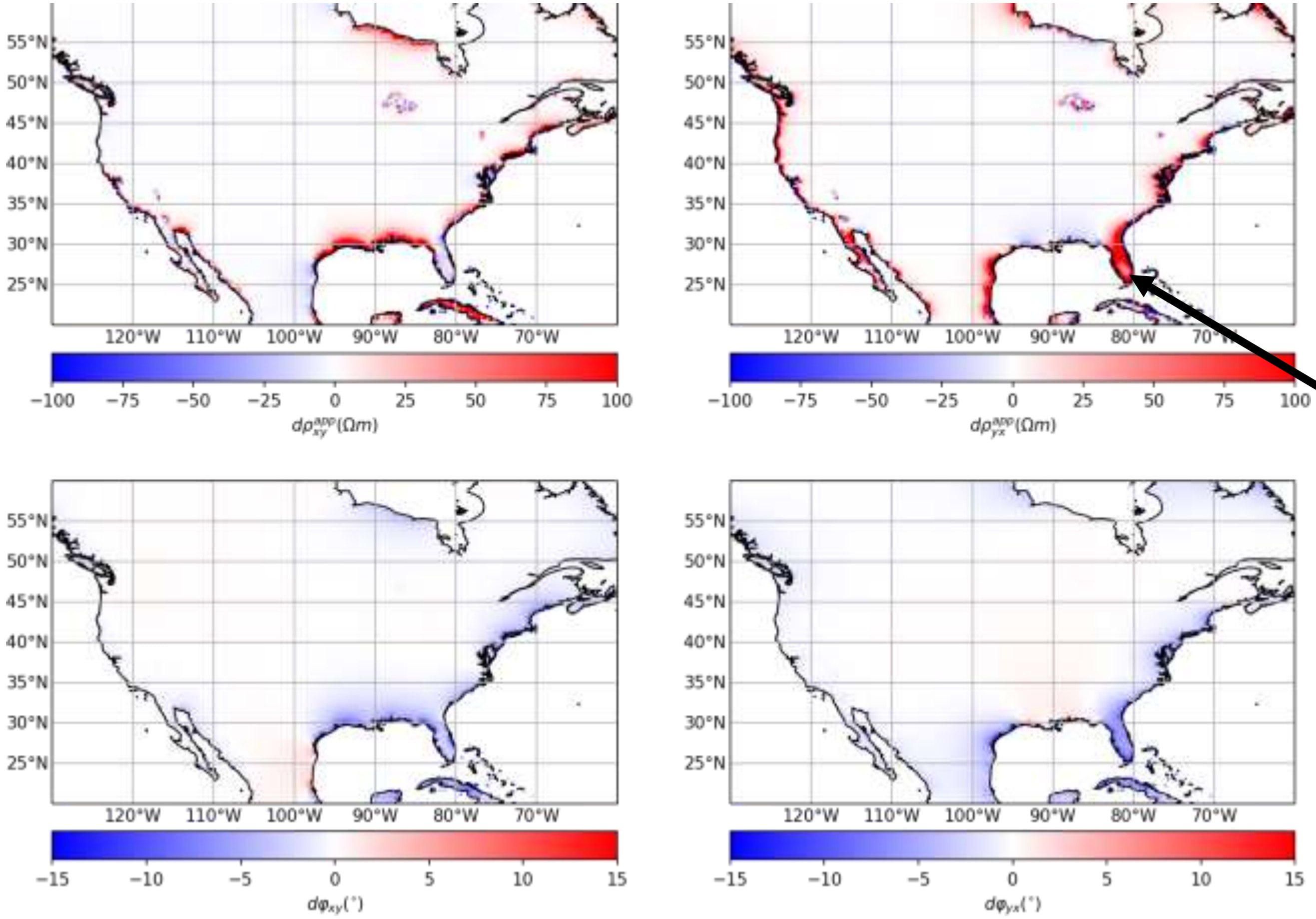
Ocean and sediment conductance



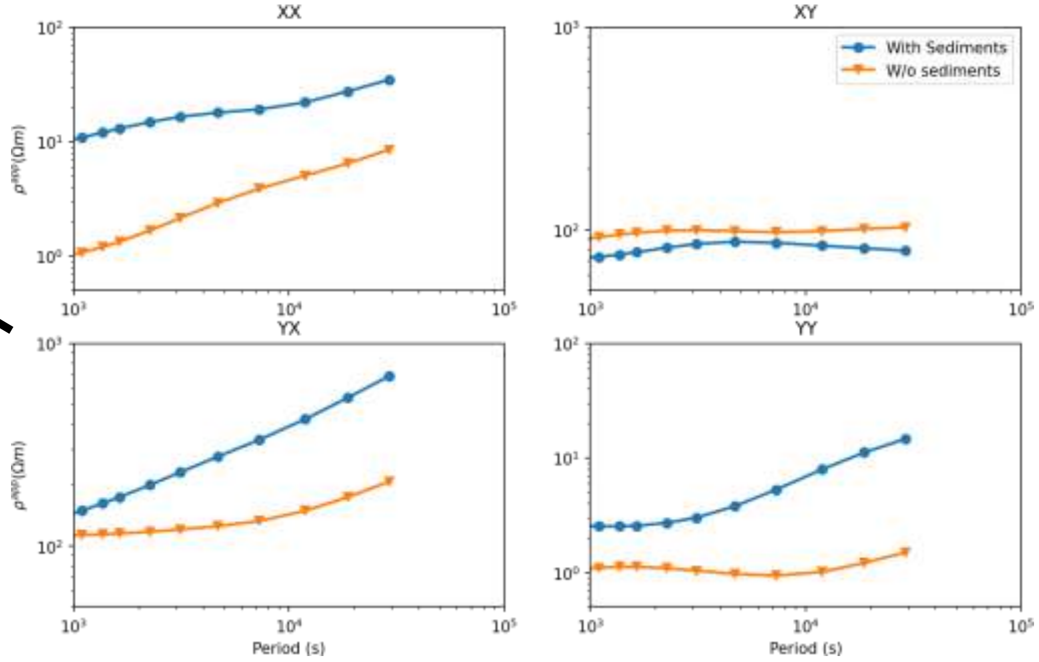
Grayver 2021, G^3

Effect of marine sediments on MT transfer functions

Period 11000 s

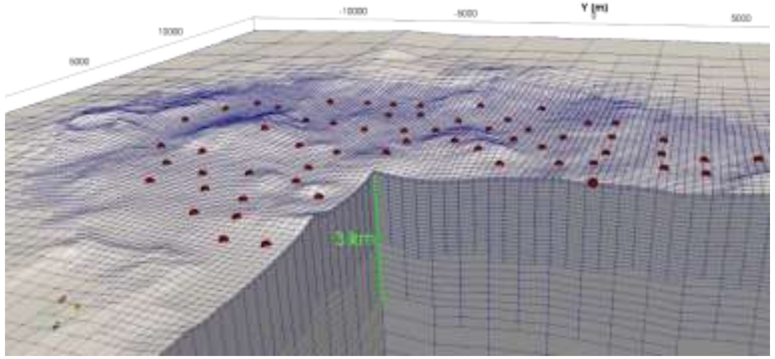


FL022



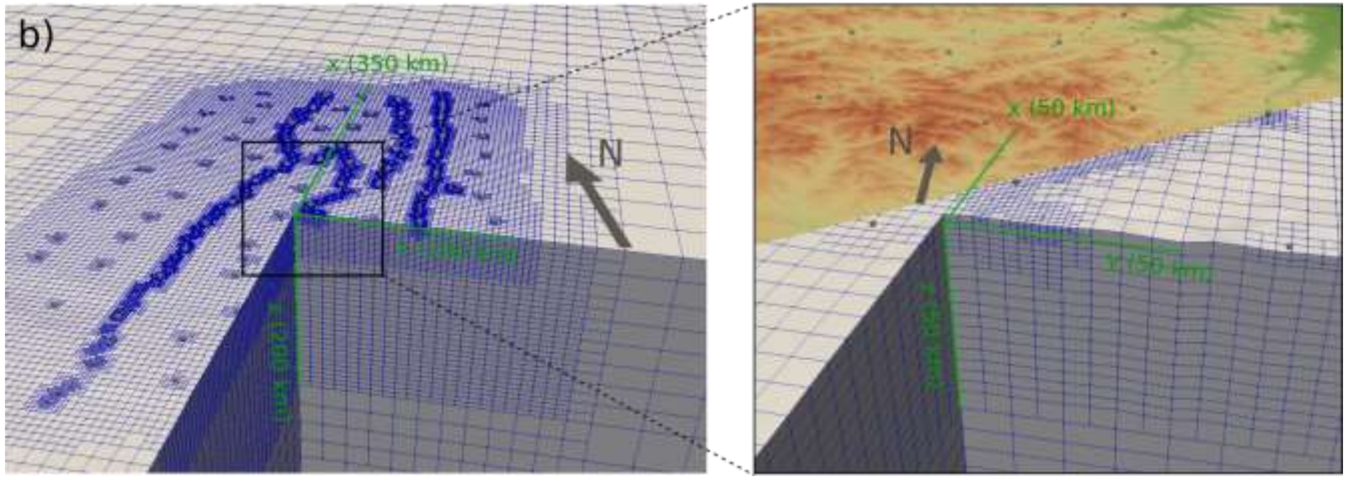
Grayver 2021, G^3

Incorporating multiple scales



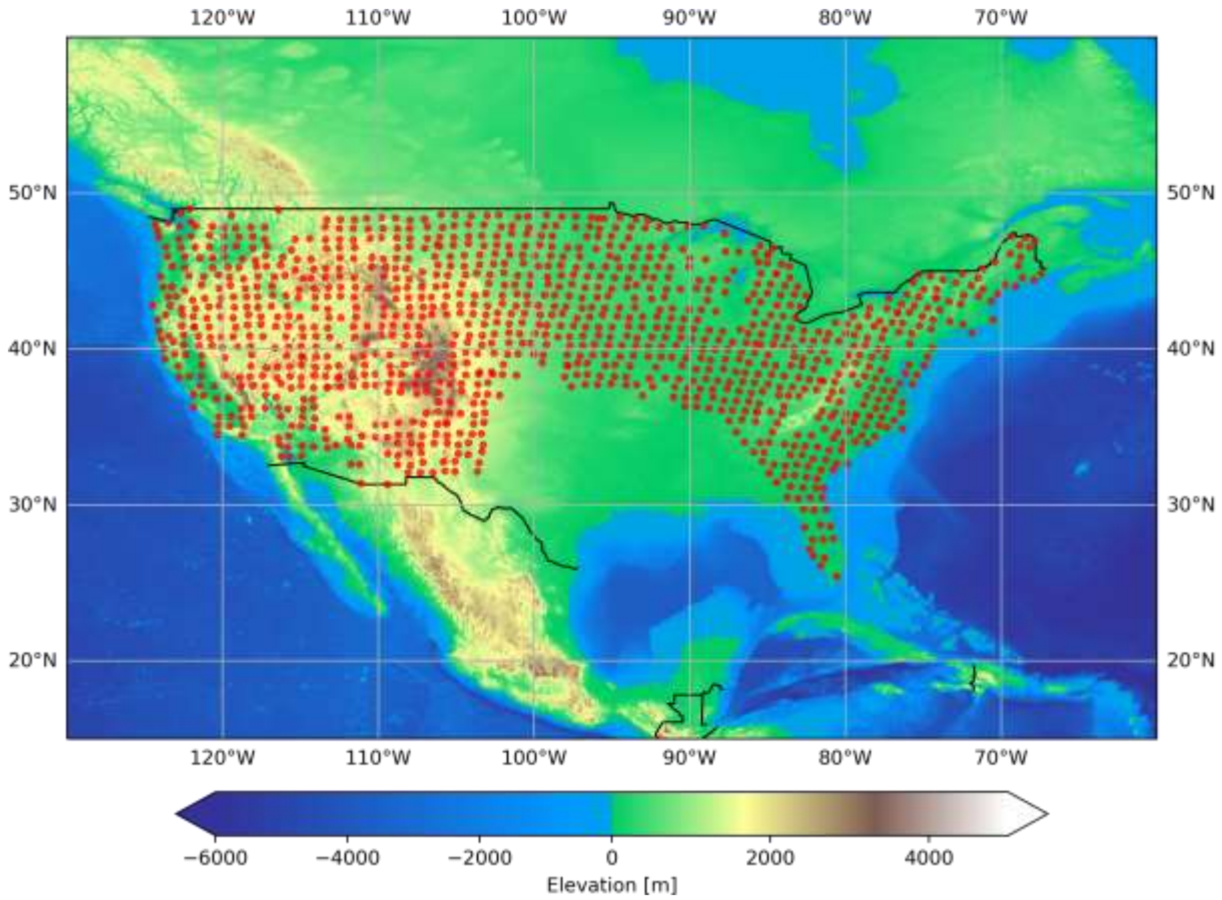
Samrock et al. 2018

Cicchetti et al., in prep



Käufl et al. 2020

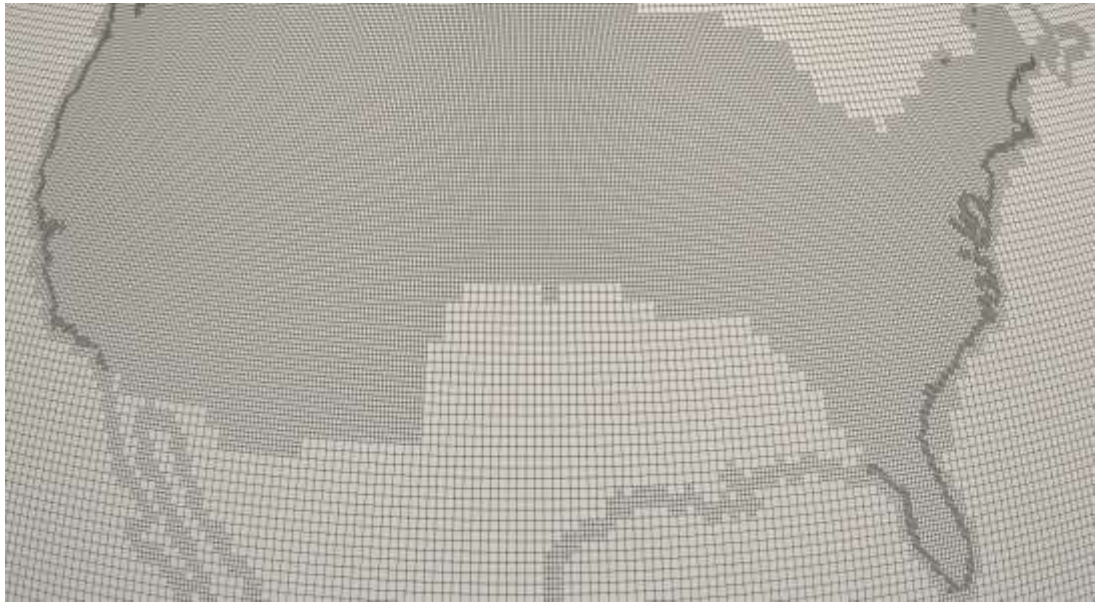
Inversion of USArray MT: data

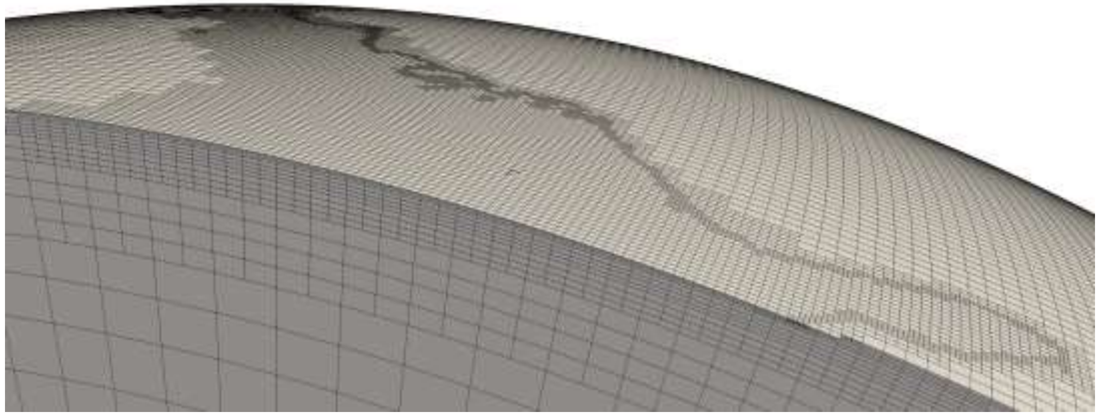


- Full Impedance tensor at ~1080 stations
- Period range: 15 - 29,000 s
- Error floor: 5% of impedance rows
- Half-space 100 Ω.m + 3-D ocean and marine sediments

(joint work with Federico Munch)

Inversion of USArray MT: multi-scale mesh

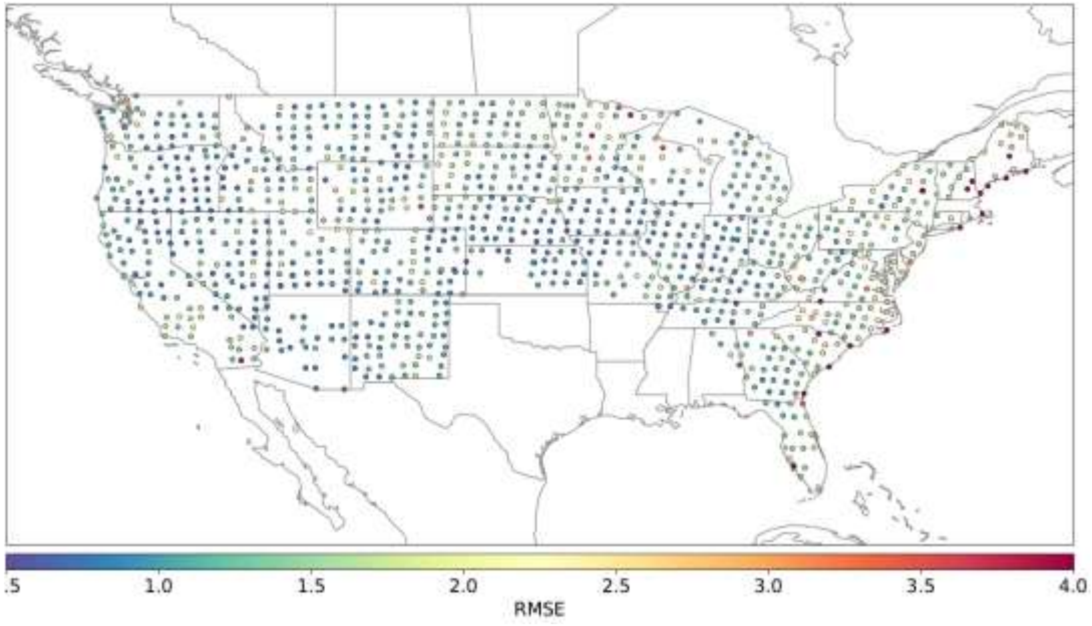


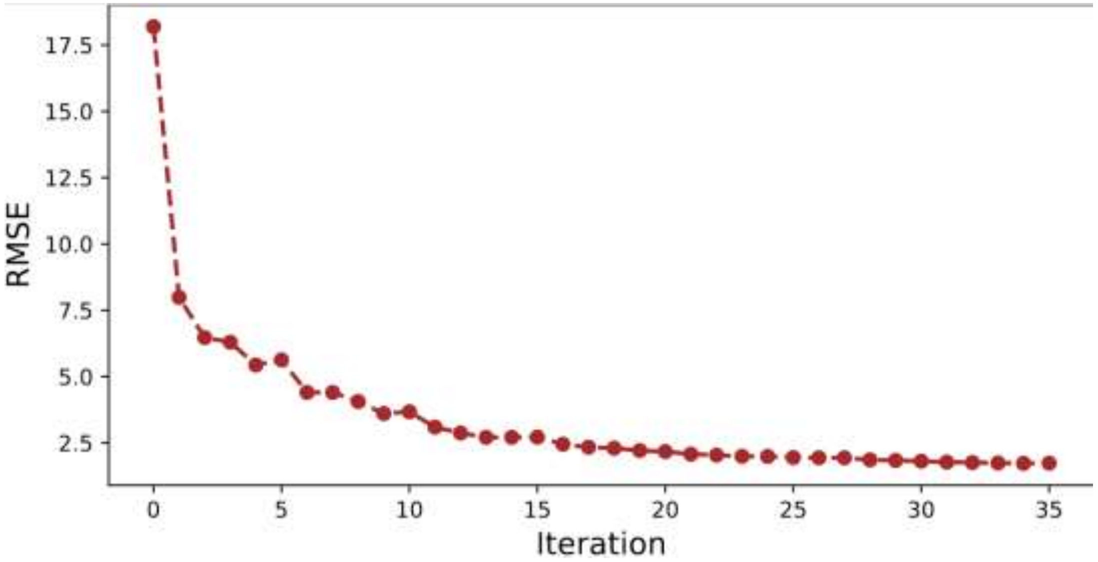


Smallest cells at the coast are ~2 km in diameter

(joint work with Federico Munch)

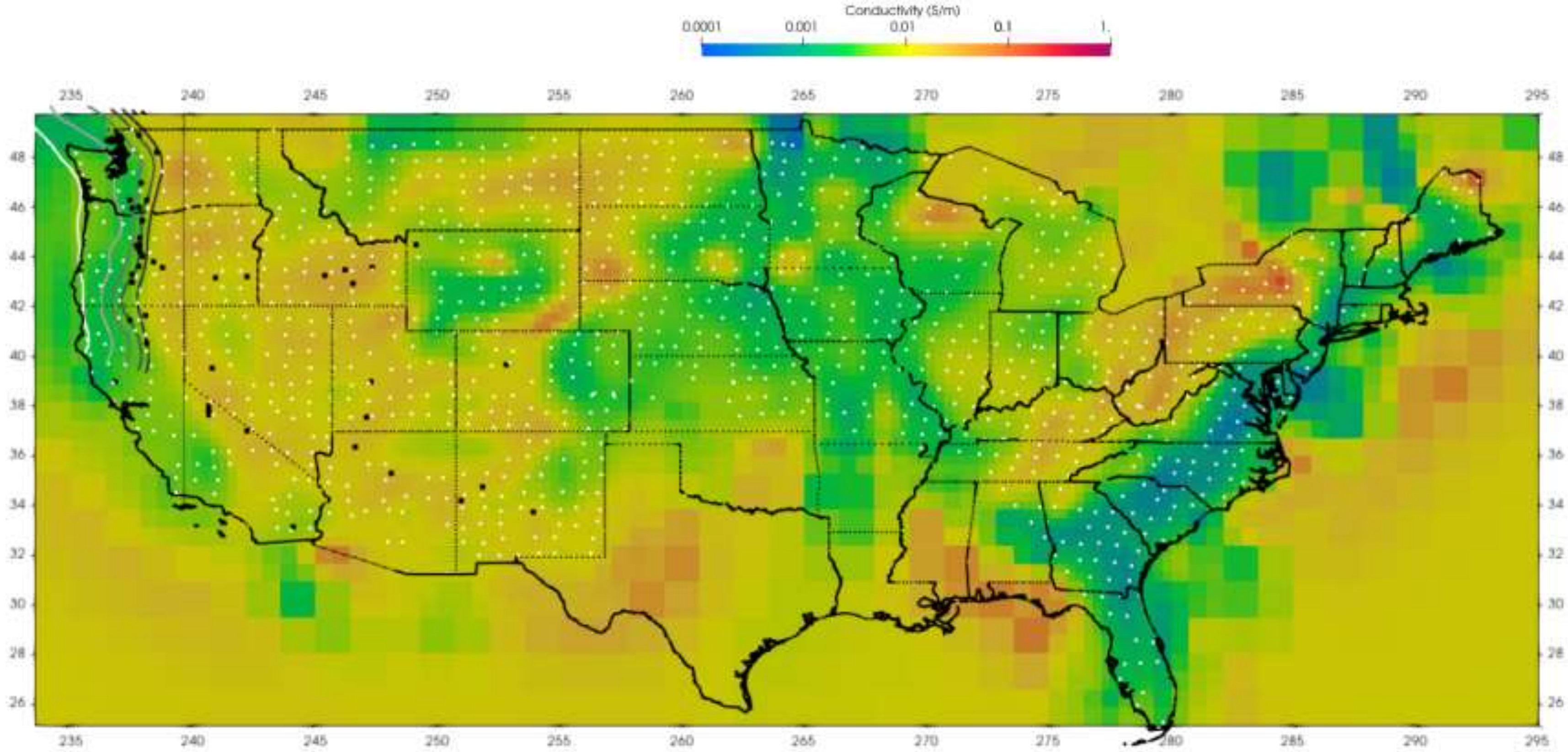
Inversion of USArray MT: data fit





(joint work with Federico Munch)

Inversion of USArray MT: preliminary model



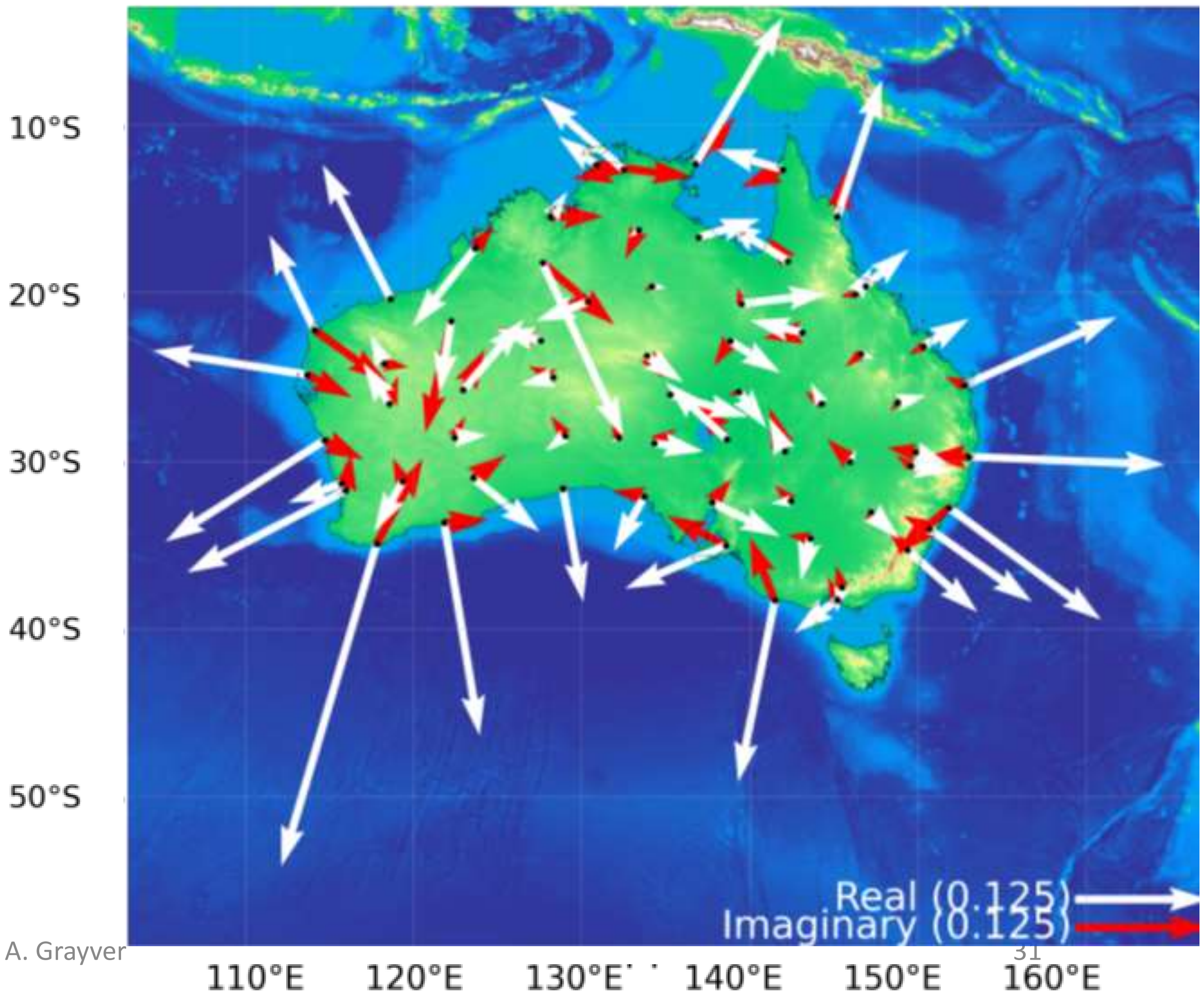
Depth 100 km

Munch and Grayver, in prep

Inversion of Australian tippers

- ✓ We used minute-data magnetic field time series from different datasets covering the Australian continent:
  - AWAGS (Australian Wide Array of Geomagnetic Stations)
  - MAGDAS (MAGnetic Data Acquisition System)
  - BGS (British Geological Survey)

Estimated tippers at 1200s

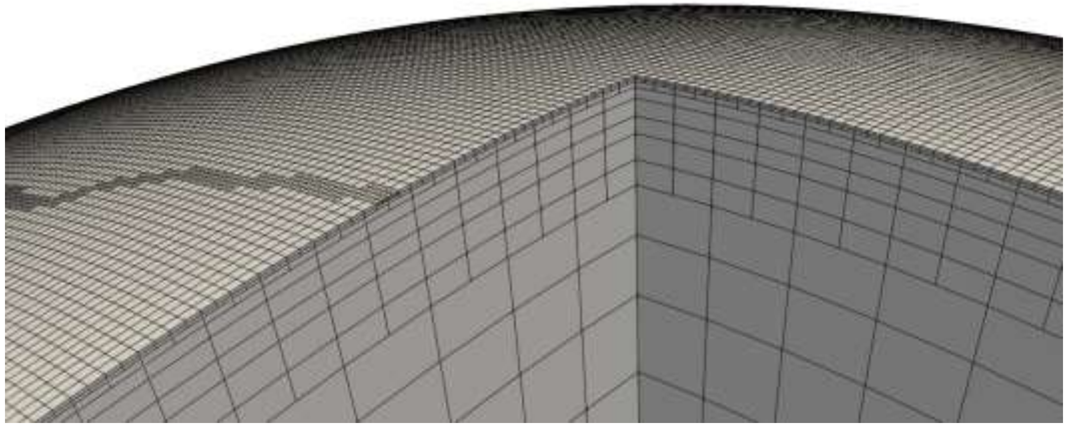


(joint work with F. Cicchetti)

Inversion of Australian tippers

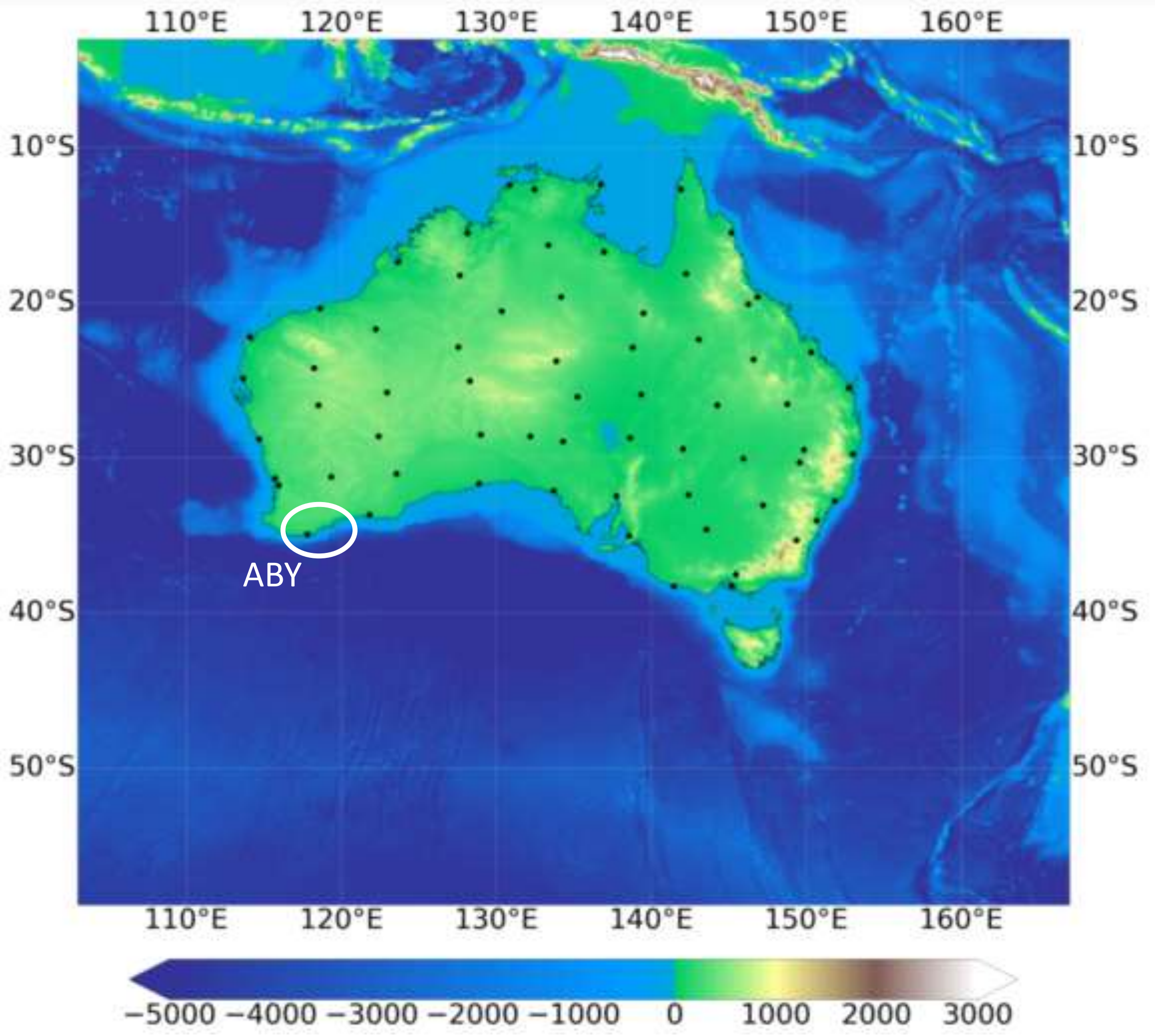
✓ We used minute-data magnetic field time series from different datasets covering the Australian continent:

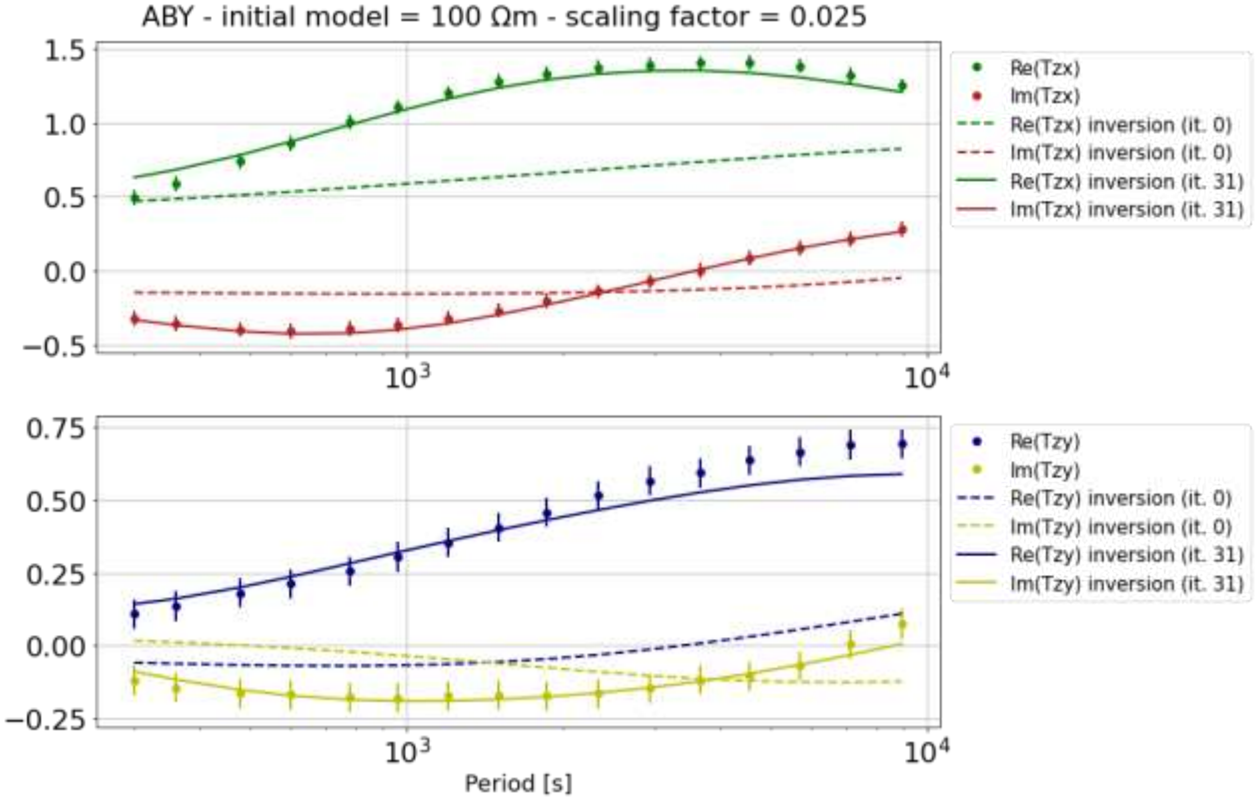
- AWAGS (Australian Wide Array of Geomagnetic Stations)
- MAGDAS (MAGnetic Data Acquisition System)
- BGS (British Geological Survey)



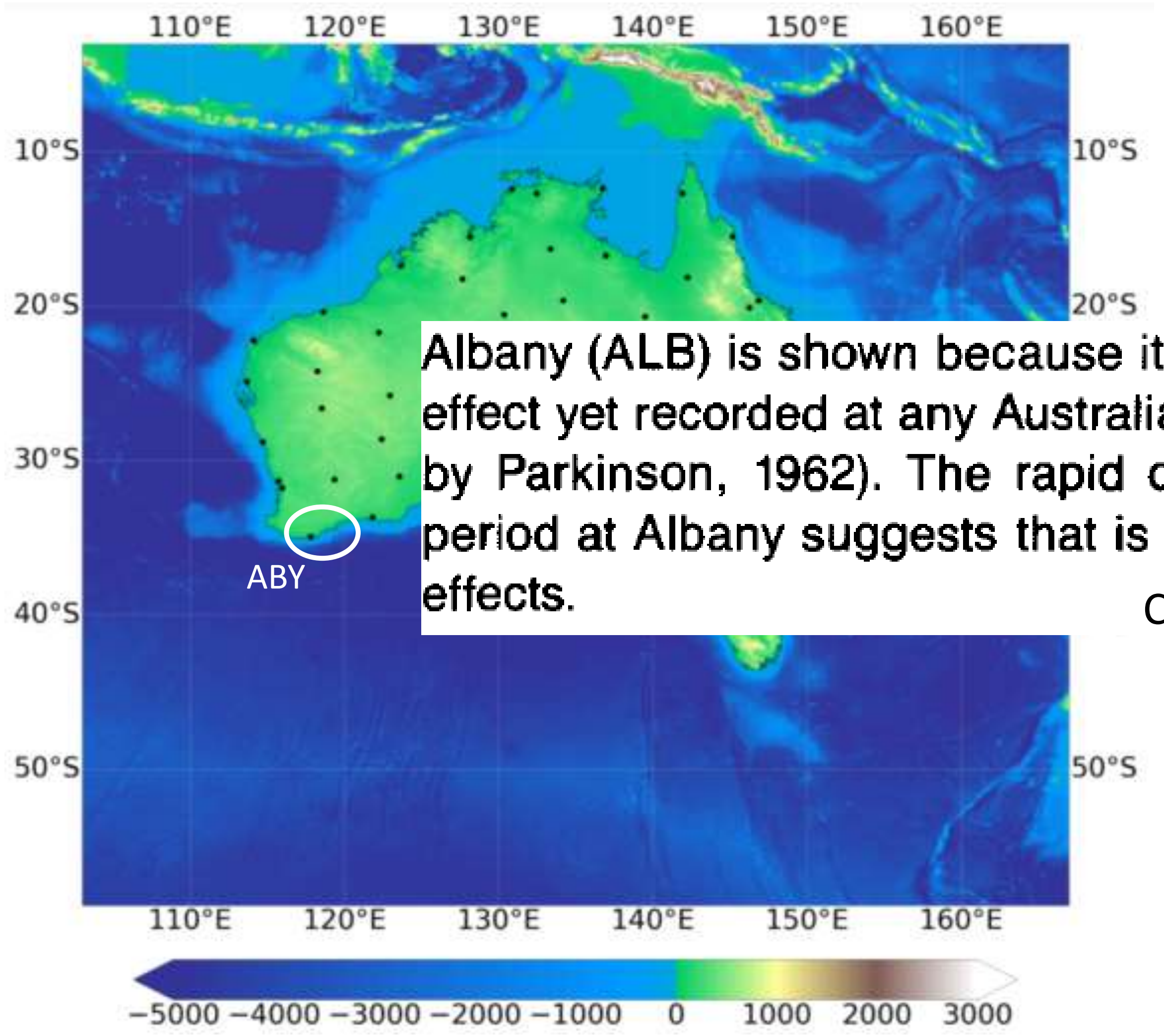
(joint work with F. Cicchetti)

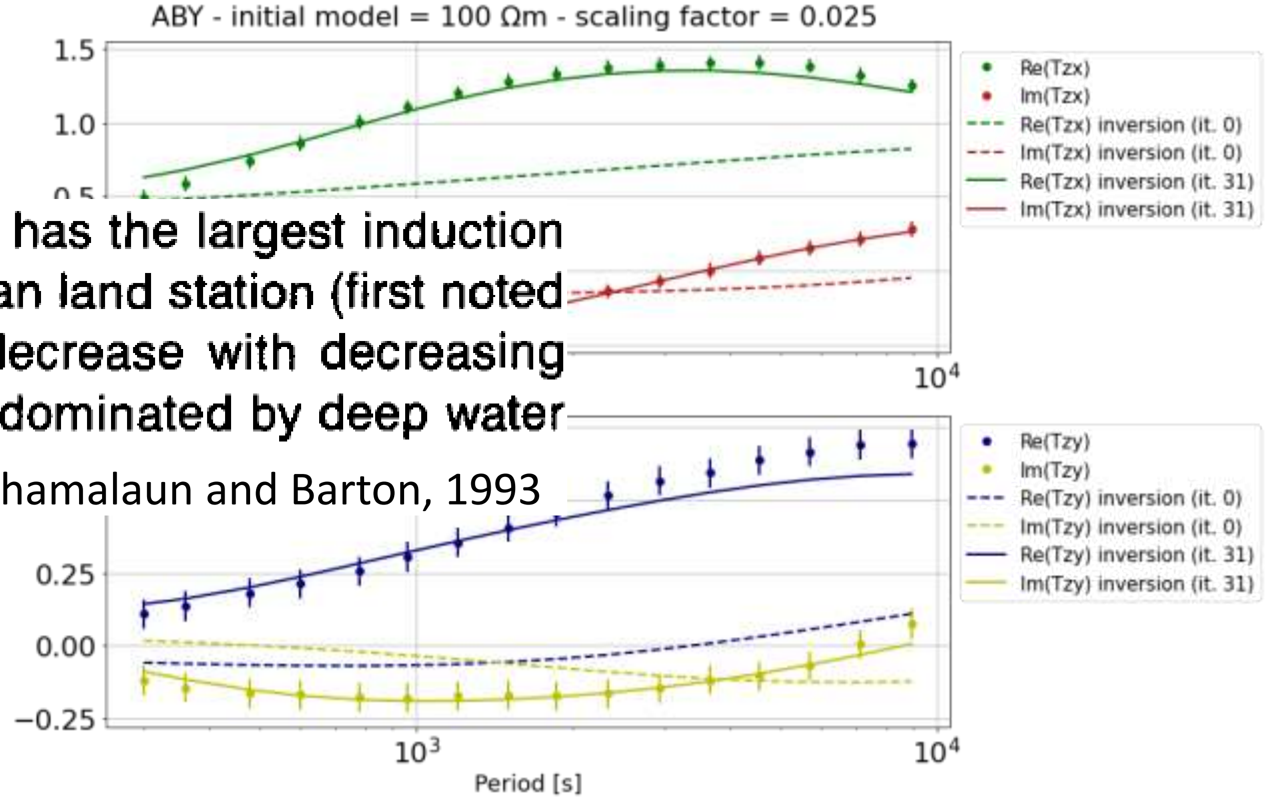
Example of tipper at Albany





Example of tipper at Albany



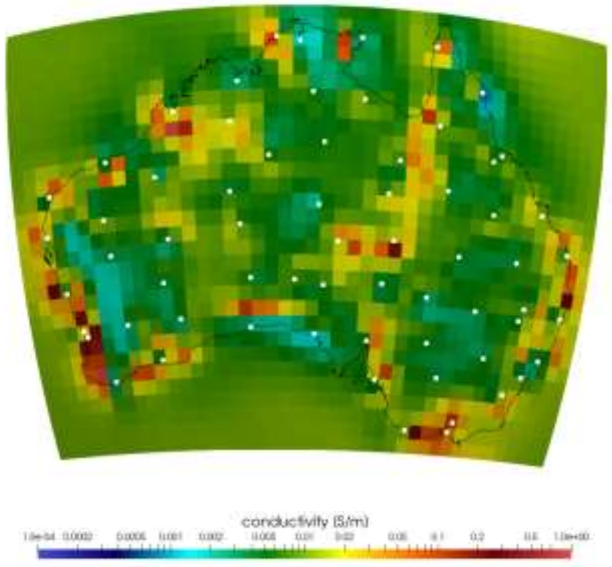


Albany (ALB) is shown because it has the largest induction effect yet recorded at any Australian land station (first noted by Parkinson, 1962). The rapid decrease with decreasing period at Albany suggests that is dominated by deep water effects.

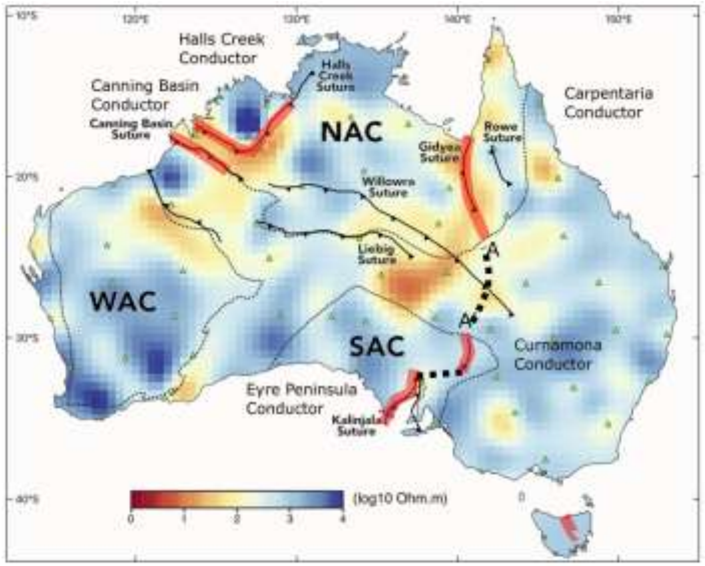
Chamalaun and Barton, 1993

Inversion of AWAGS tippers

36 km



Cicchetti et al., in prep



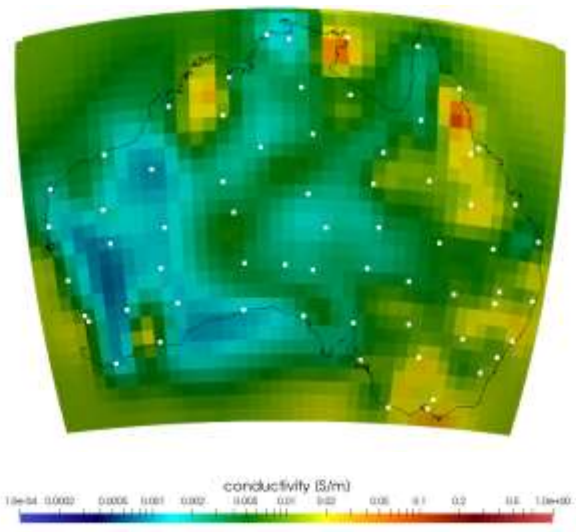
Kay, Heinson, Brand, 2021

Conductivity from the model L.Wang, et al. 2014, 3-D Conductivity model of the Australian continent using observatory and magnetometer array data.

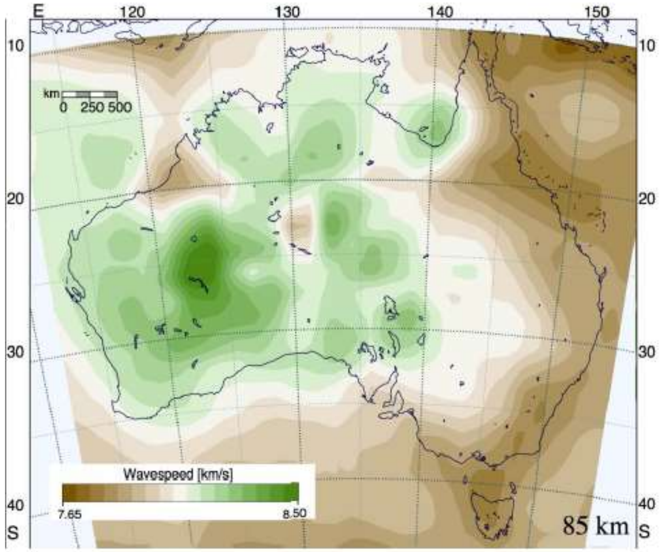
- Sutures in Eastern Australia proposed to have formed a contiguous boundary during Paleoproterozoic accretion of continental material (Betts et al., 2016).

Inversion of AWAGS tippers

90 km



Cicchetti et al., in prep



Kennett et al, 2018

Concluding remarks

- Elaborated the 3-D MT modelling in a spherical shell.
- Discussed ways to tackle multi-scale nature of the problem.
- Showed applications to continental scale arrays.