Modeling spatial structure of external source fields for induction studies (and other things)

Gary Egbert
Oregon State University (Professor Emeritus)
ModEM Geophysics Inc.

Outline:

- Motivation and overview
- Approach, illustrated via summary of "Modeling diurnal variation magnetic fields due to ionosphere currents". (*GJI, Egbert, Alken, Maute, Zhang, 2021*)
- Application of source model to induction (*Zhang, Egbert, Huang, in review*)
- Refinements, extensions, ongoing work (incorporating transfer function ideas, satellite data, shorter and longer periods)

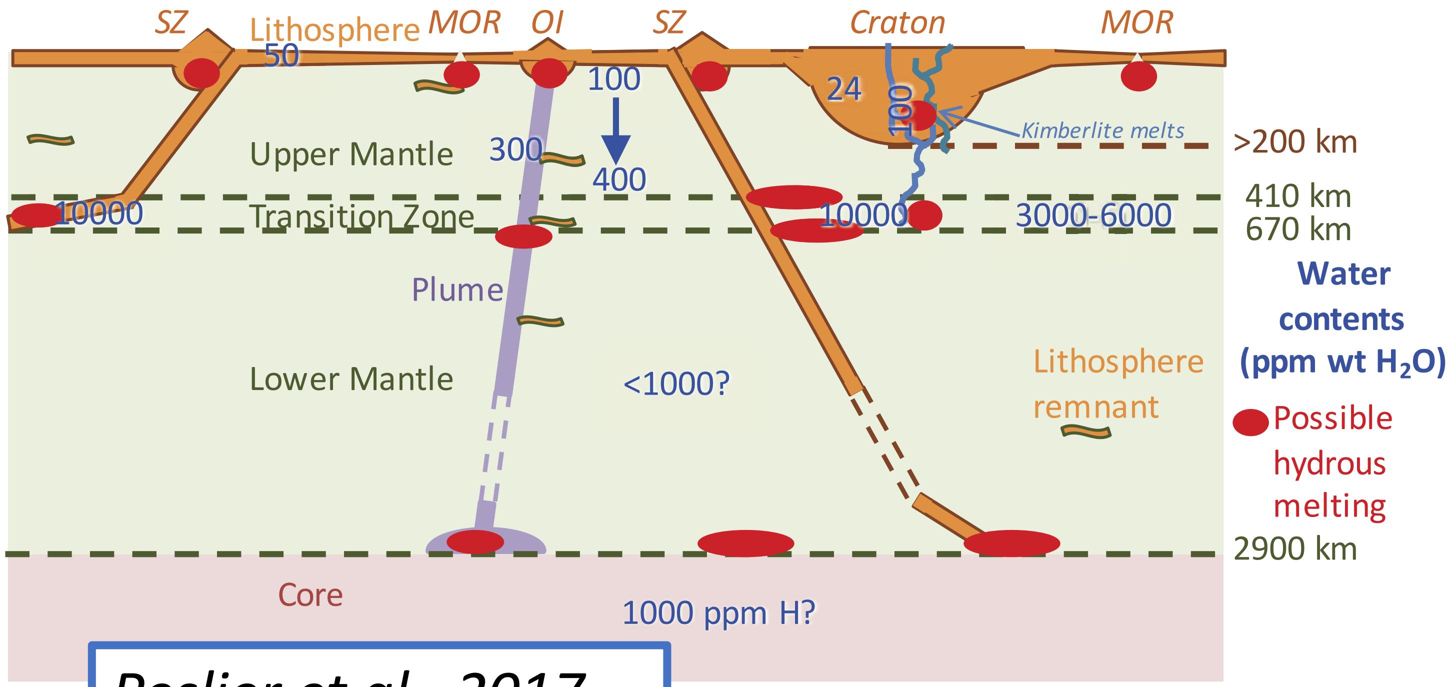
Motivation: Image Deep Mantle Electrical Conductivity:

- Provide additional constraint on spatial variations in mantle composition and physical state

- Conductivity is very sensitive to water (in contrast to seismic data)

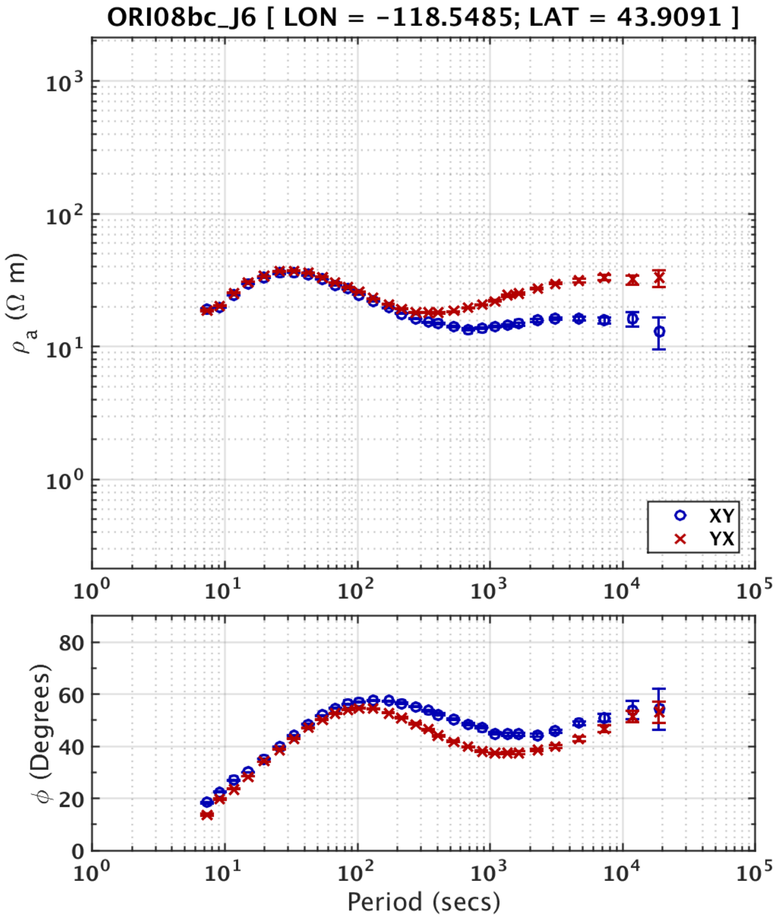
- Most of Earth's water (H^+) is in the solid Earth; distribution has significant implications for rheology, melting, geodynamics, Earth history

- Currently not well constrained!



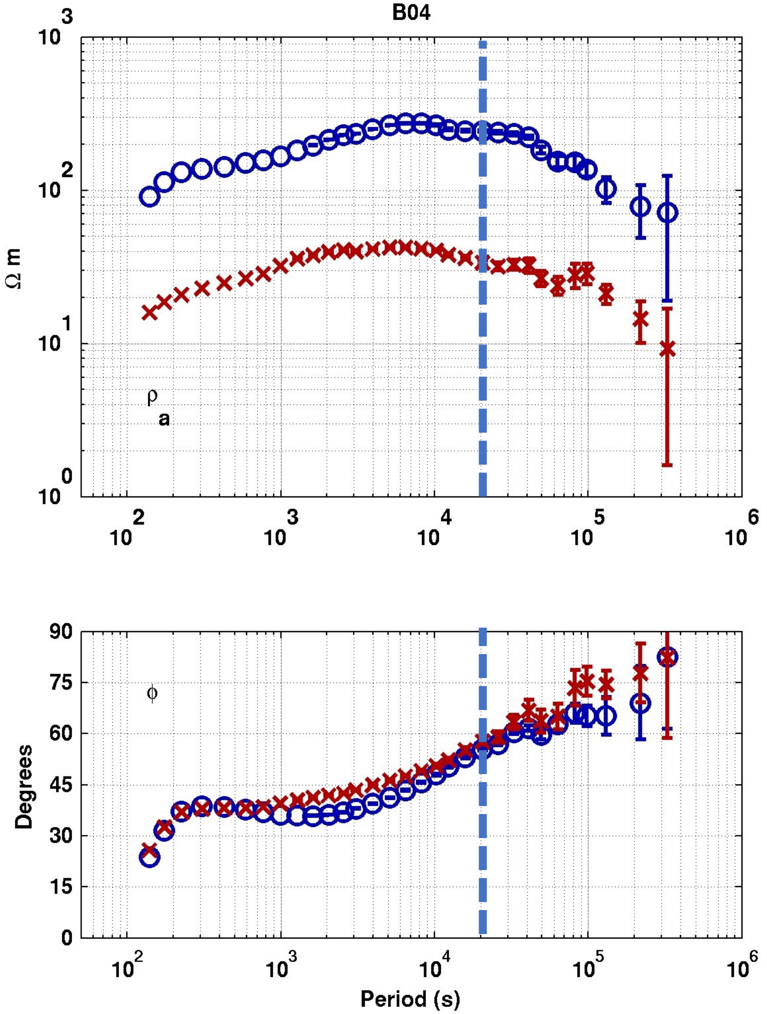
Peslier et al., 2017

MT: great for imaging the lithosphere, but getting to the very long periods (>> 10^4 s) required to image deeper into the mantle is very challenging



One of the best EarthScope TA LPMT sites (~ 1 month deployment)

A “good” EarthScope “backbone” site nominally 3-year deployment



Long periods:

- **E-fields become very small (physics of induction + increasing deep conductivity)**
- **Noise spectrum very red (temperature effects, self potential, electrode noise)**
- **Source highly polarized**

Deep conductivity imaging without electric fields: Geomagnetic Depth Sounding (GDS) or Magneto-variational (MV) approach

For 1D (layered) Earth easy to show:

$$C(\omega) = \frac{B_z}{\partial_x B_x + \partial_y B_y} = i\omega\mu_0 Z(\omega)$$

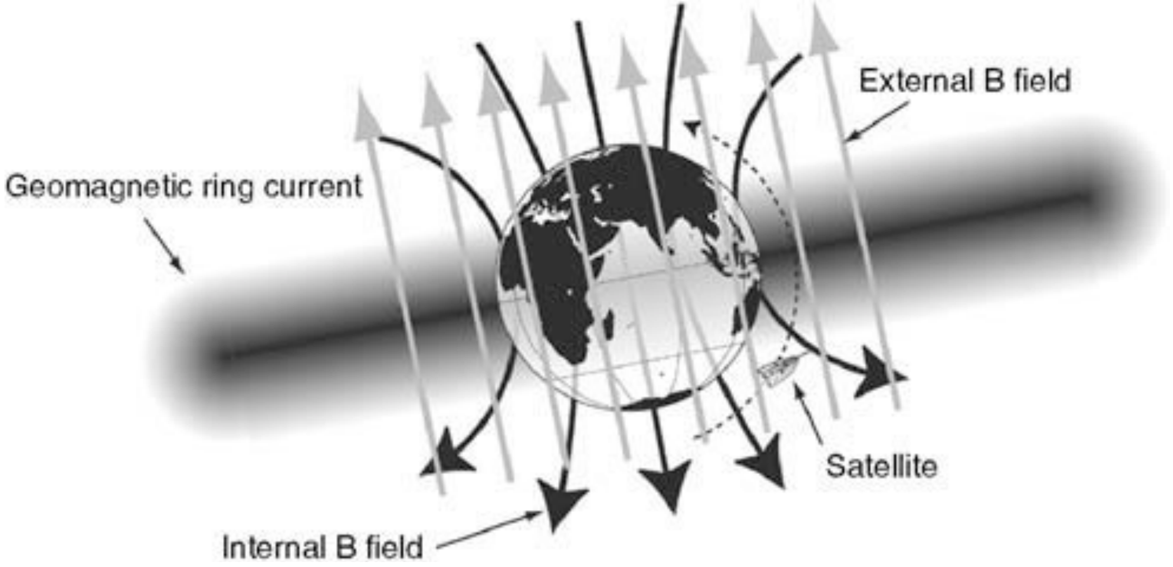
where $Z(\omega) = E_x/B_y = -E_y/B_x =$ **1-D impedance**

Thus, can get (part of) the MT impedance without electric fields (not galvanic, or TM part of response)

(will come back to this later in the presentation)

In fact, the MV approach predates MT significantly—almost a century ago, Chapman and students showed that the Earth was very conductive below ~600km depth (we now understand this is due to a phase transition to brigmanite in the lower mantle)

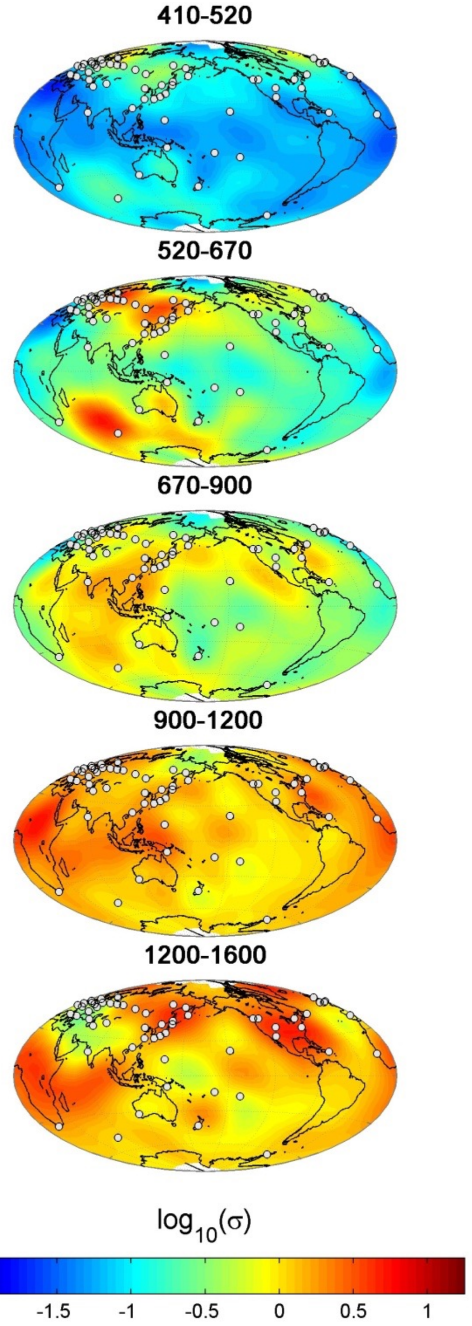
By assuming a ring current source (resulting in a zonal dipole on Earth's surface) $C(\omega)$ can be obtained from a local ratio of field components B_z/B_x



3D global electromagnetic inversion *(e.g., Kelbert et al., 2009 + others since):*

- 59 mid-latitude observatories
- 28 periods (5.12 – 107 days)
- P_1^0 (Dst) source assumption, with correction for auroral currents
- C-responses from Fujii & Schultz (2002)
- 10° x 10° numerical grid
- correction for shallow conductivity variations (oceans)

Suggested large conductivity variations (interpreted as variable hydration) in the transition zone



LIMITATIONS OF THIS EARLY STUDY

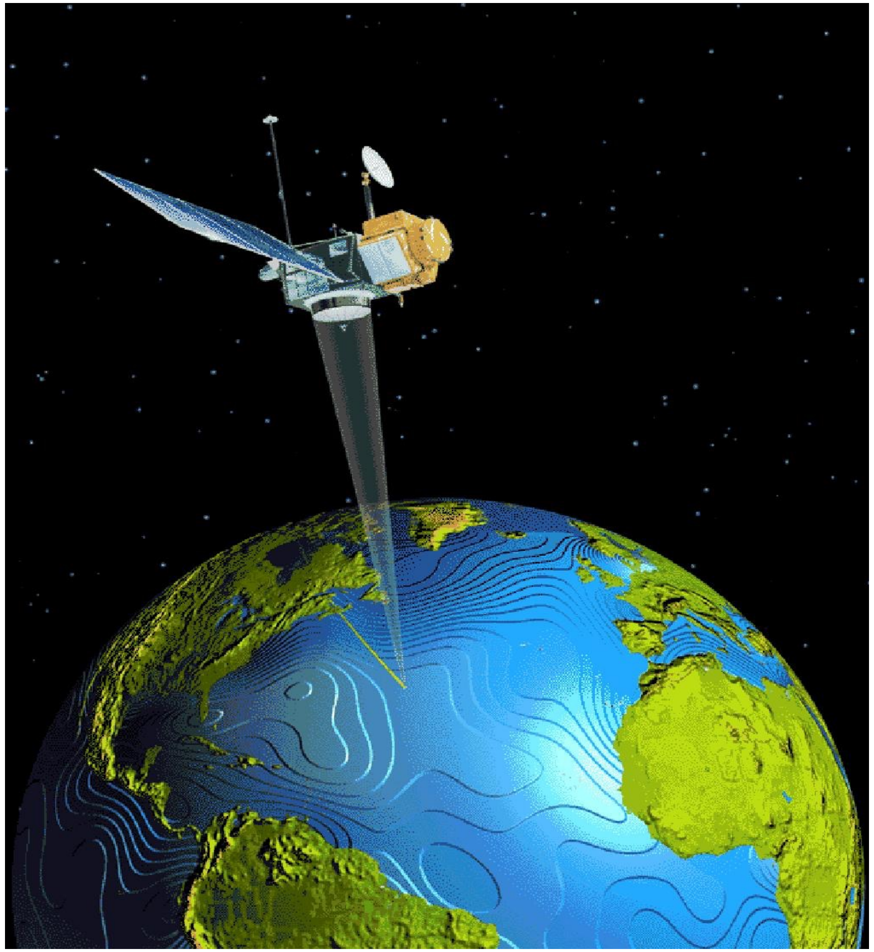
- Small number of observatories (especially in Southern hemisphere!)
- source complications at high (> 50°) latitudes (simple correction of Fujii and Schultz (2002) used)
- Limited period range (T > 5 day) severely limits resolution in upper mantle . . . . need to use daily variations to image transition zone

Although there has been significant progress since, reliable modeling of source fields is still challenging, especially for the near-Earth ionospheric fields that dominate in the daily variation band (and near the electrojets at all periods)

This is the focus of this talk

Aside: A big part of my scientific career was spent on oceanographic data assimilation, especially for ocean tides

- Very accurate tidal corrections were required to avoid aliasing with the oceanographic signal of interest
- Stimulated much effort on tidal modeling, and ultimately tidal science
- I became involved with an oceanographer (A. Bennett) who was approaching this as an inverse problem



TOPEX/POSEIDON

Altimeter

Measure low-frequency ocean surface elevations to monitor ocean currents (El Nino, climate, etc.)

Approach: Variational Data Assimilation

Estimate state (e.g., tidal height + currents) combining :

Dynamical equations $\mathbf{Su} = \mathbf{f} \; + \mathbf{df}$

Data $\mathbf{d} = \mathbf{Lu} \; + \mathbf{e}$

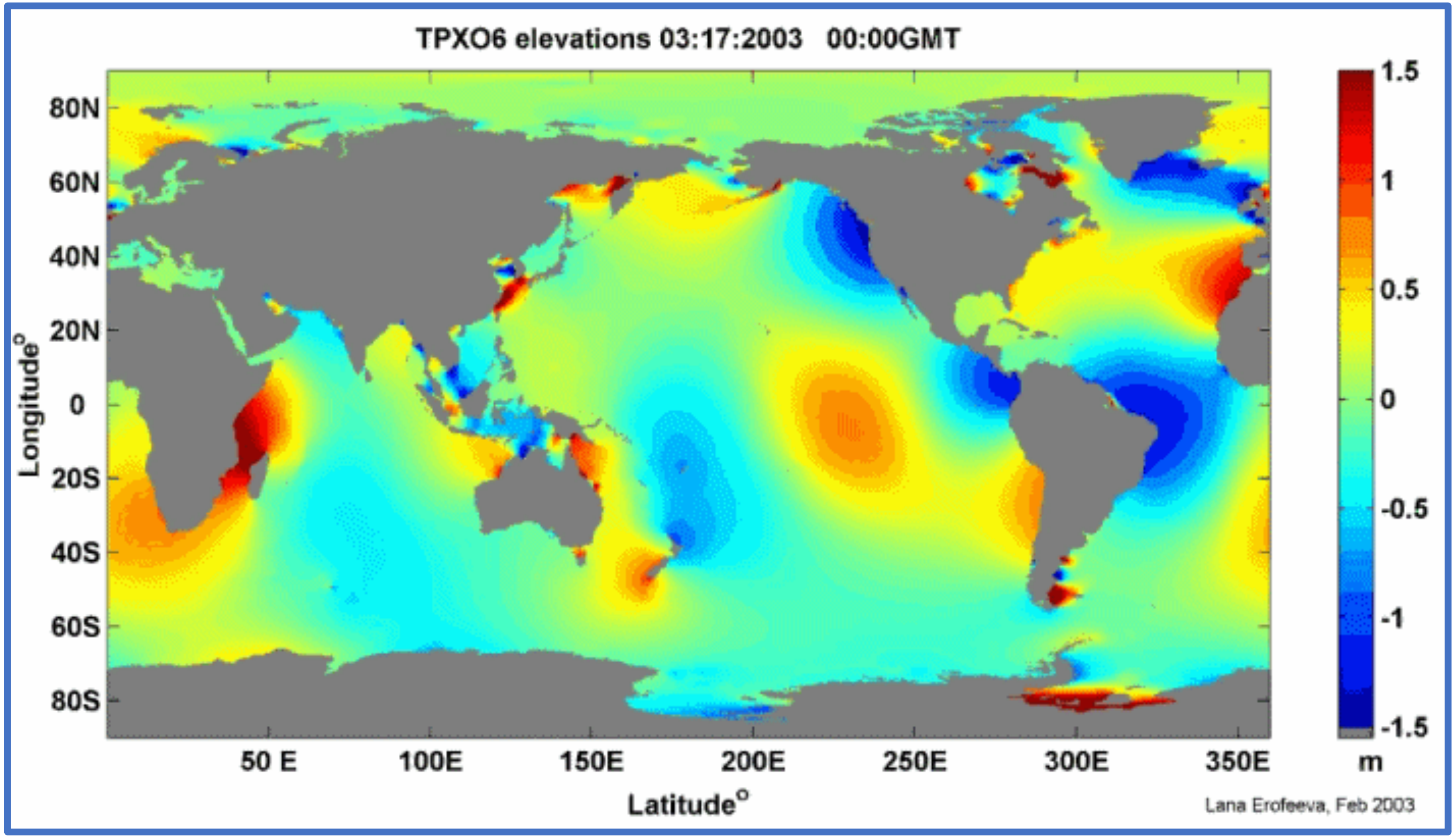
Allow for errors in dynamical equations (forcing, boundary conditions, missing physics) and data

Minimize penalty functional

$$\mathcal{J}[\mathbf{u}] = (\mathbf{d} - \mathbf{Lu})^{\dagger} \Sigma_{\mathbf{d}}^{-1} (\mathbf{d} - \mathbf{Lu}) + (\mathbf{Su} - \mathbf{f})^{\dagger} \Sigma_{\mathbf{f}}^{-1} (\mathbf{Su} - \mathbf{f})$$

Error covariances encode a priori beliefs about magnitude, spatial/temporal correlation structure of errors in forcing, boundary and initial conditions, data

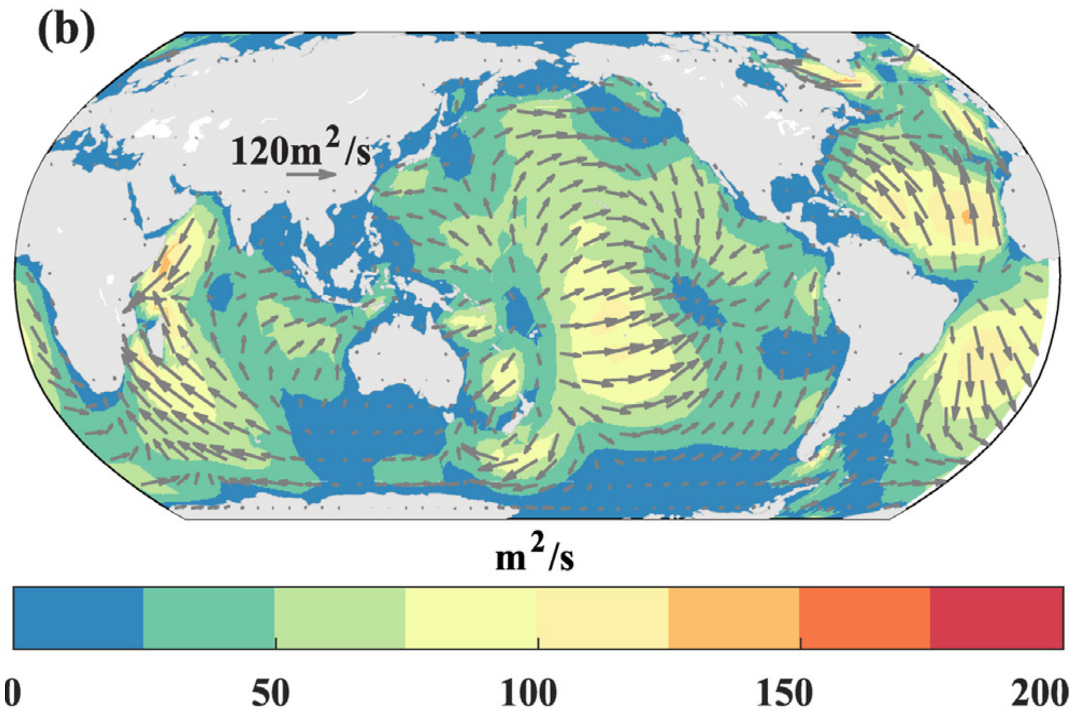
Modeling of global ocean tides: use a dynamical model (shallow water equations on the sphere) to interpolate data (altimetry, tide gauges) in a physically consistent manner *(Egbert et al., 1994)*



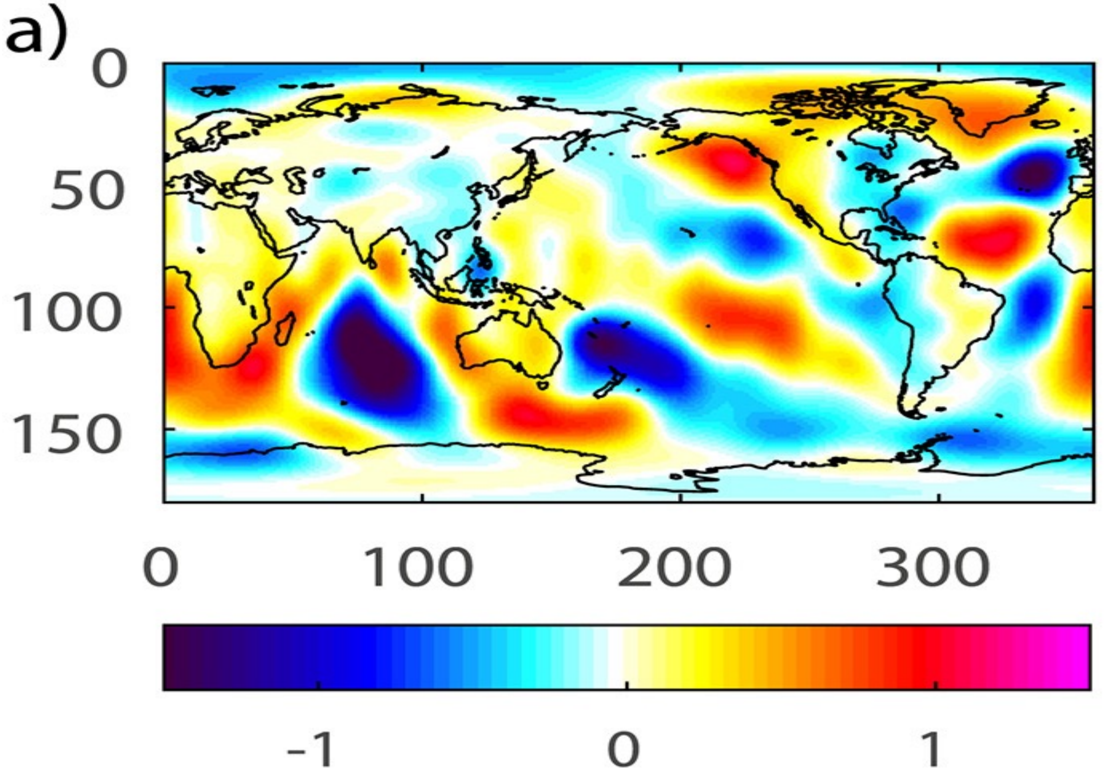
One key point: a physics-based numerical model can provide realistic basis functions for interpolation of sparse data sets

Very widely used in the atmospheric and oceanographic sciences

A second key point: he tidal models, initially developed as a "correction" for a source of "noise", have had many unexpected applications!



M2 tidal currents from assimilation of altimetry data: can be used to compute EM sources for tidal induction



Re Br (nT) at satellite altitude used in mantle conductivity study (Grayver et al., 2017)

Modelling diurnal variation magnetic fields due to ionospheric currents

G.D. Egbert , P. Alken, A. Maute and H. Zhang GJI, 2021

- global model for ionospheric source currents in the DV band (~10^4-10^5 s)—represented as an equivalent sheet current at 110 km altitude
- for all geomagnetic conditions, hourly cadence, 1997-2018
- combines ground data from observatories, physics-based ionospheric model (Thermosphere-Ionosphere-Electrodynamics General Circulation Mode (TIEGCM)—essentially a simplified data assimilation scheme

Approach: Three key steps …

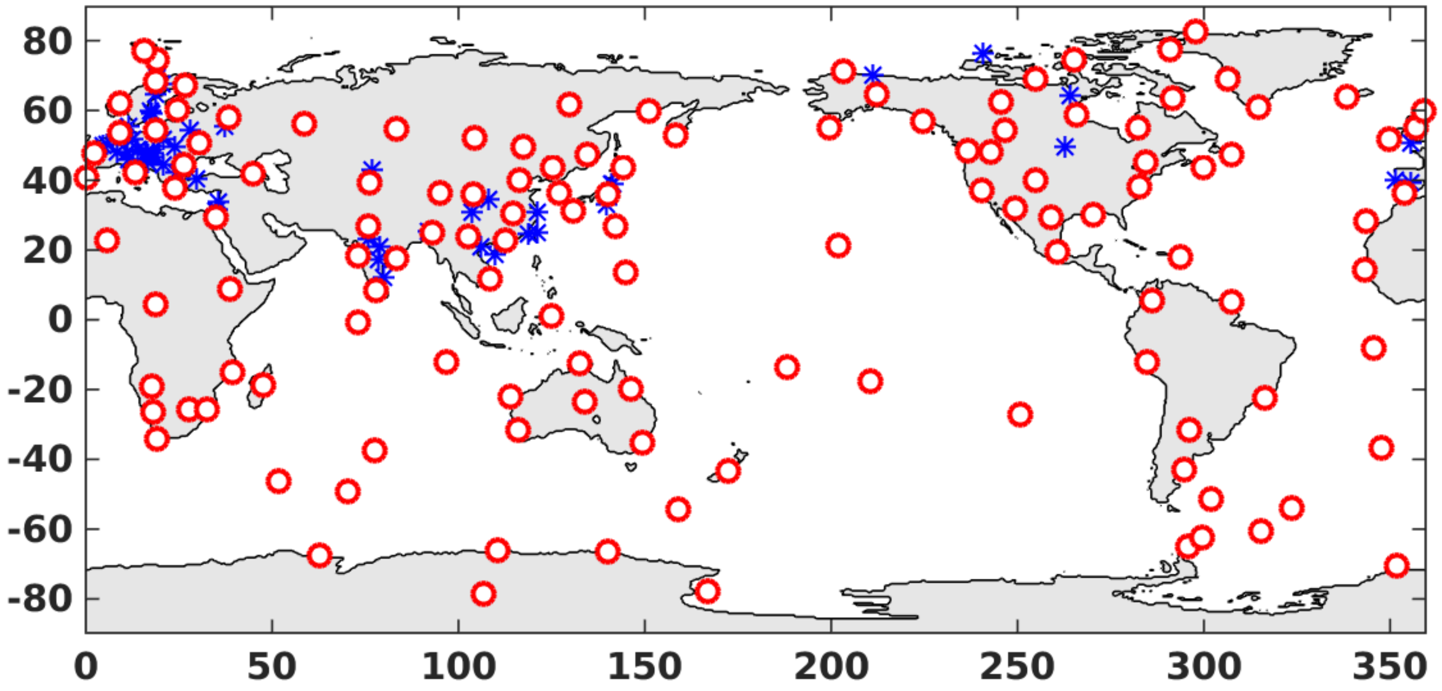
1. Frequency domain principal components analysis (PCA) of ground magnetic data → "data modes" sampled
   - sparsely in space (at ground observatories)
   - as Fourier coefficients (FC) in a continuous sequence of time windows (temporal data modes)

2. Interpolate data modes in space, using basis functions derived from a physics-based ionospheric model (TIEGCM)

3. Invert temporal data modes back to time domain

Step 1 : DATA MODES

Derived from Frequency Domain PCA of Observatory Data



Analysis based on 127 (out of 182) geomagnetic observatories, 1997-2018

Allow for missing data with "criss-cross" regression approach (Smirnov and Egbert, 2012)

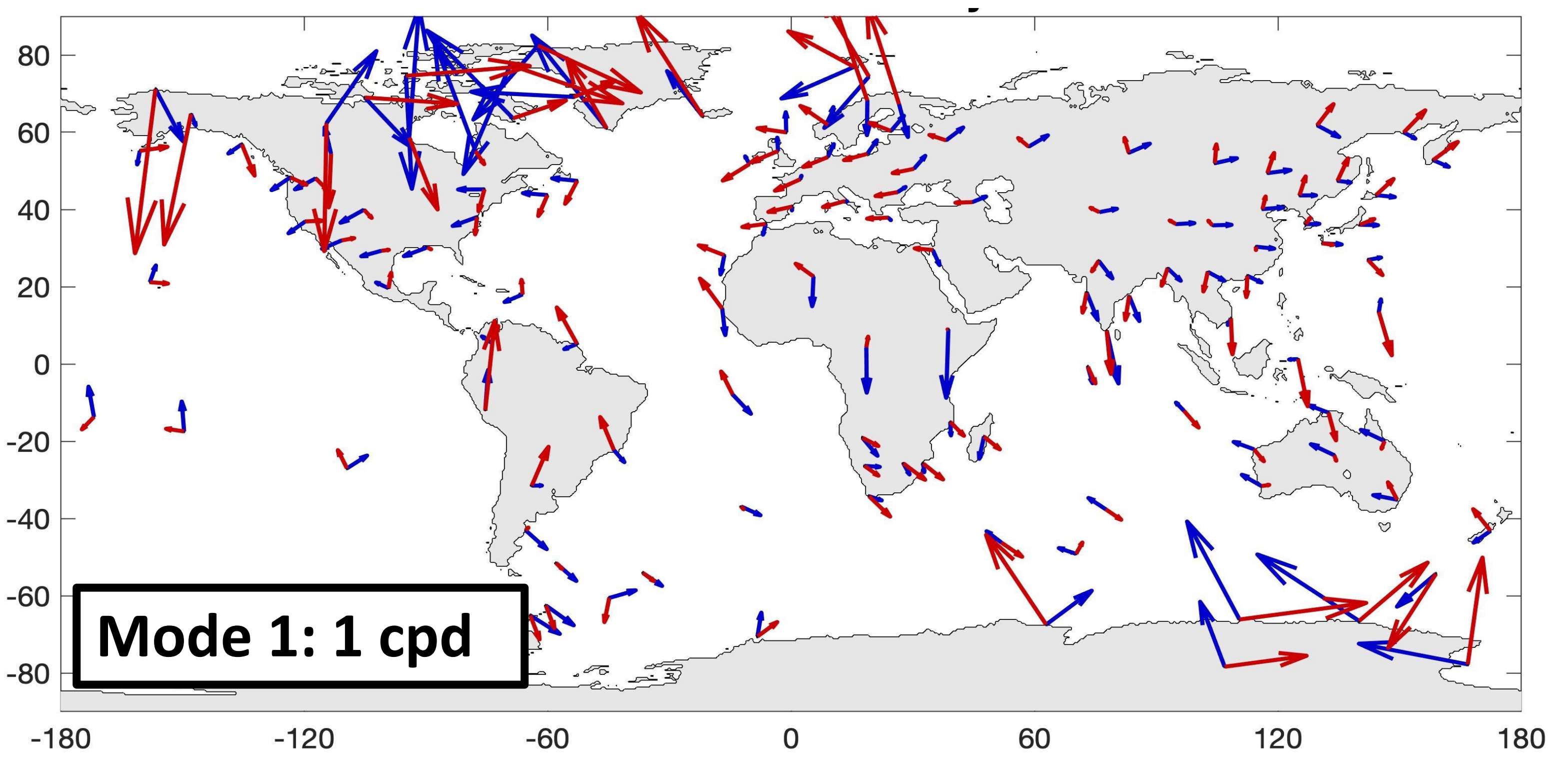
$$X_{nj} = \sum_k U_{nk} \alpha_{kj} + \varepsilon_{nj}$$

spatial modes **temporal modes**

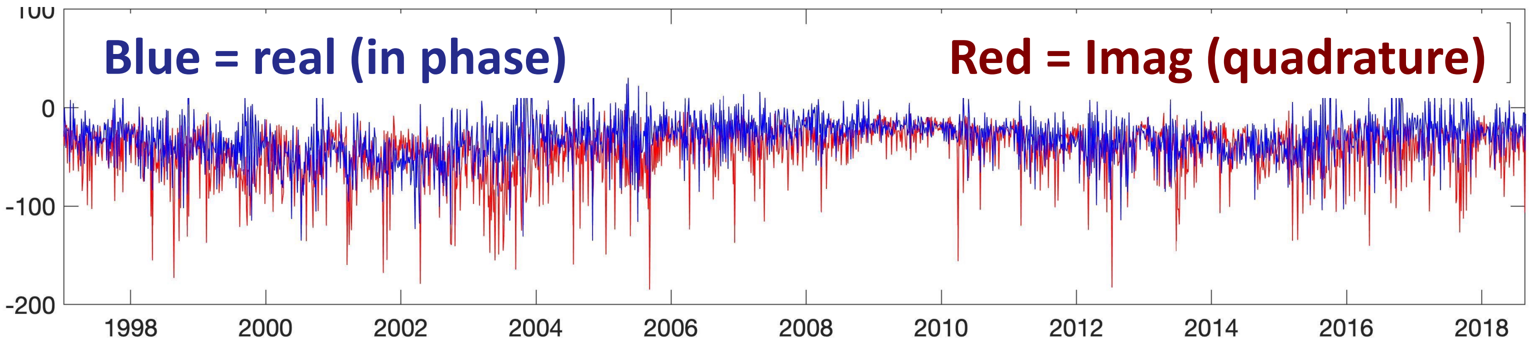
alternately fit spatial and temporal mode parameters, using robust regression

Step 1: data PCA one mode

Spatial Mode (U):
horizontal components of (complex) magnetic field plotted observatory locations



Temporal Mode (α):
Real, Imag parts of FC for each time window



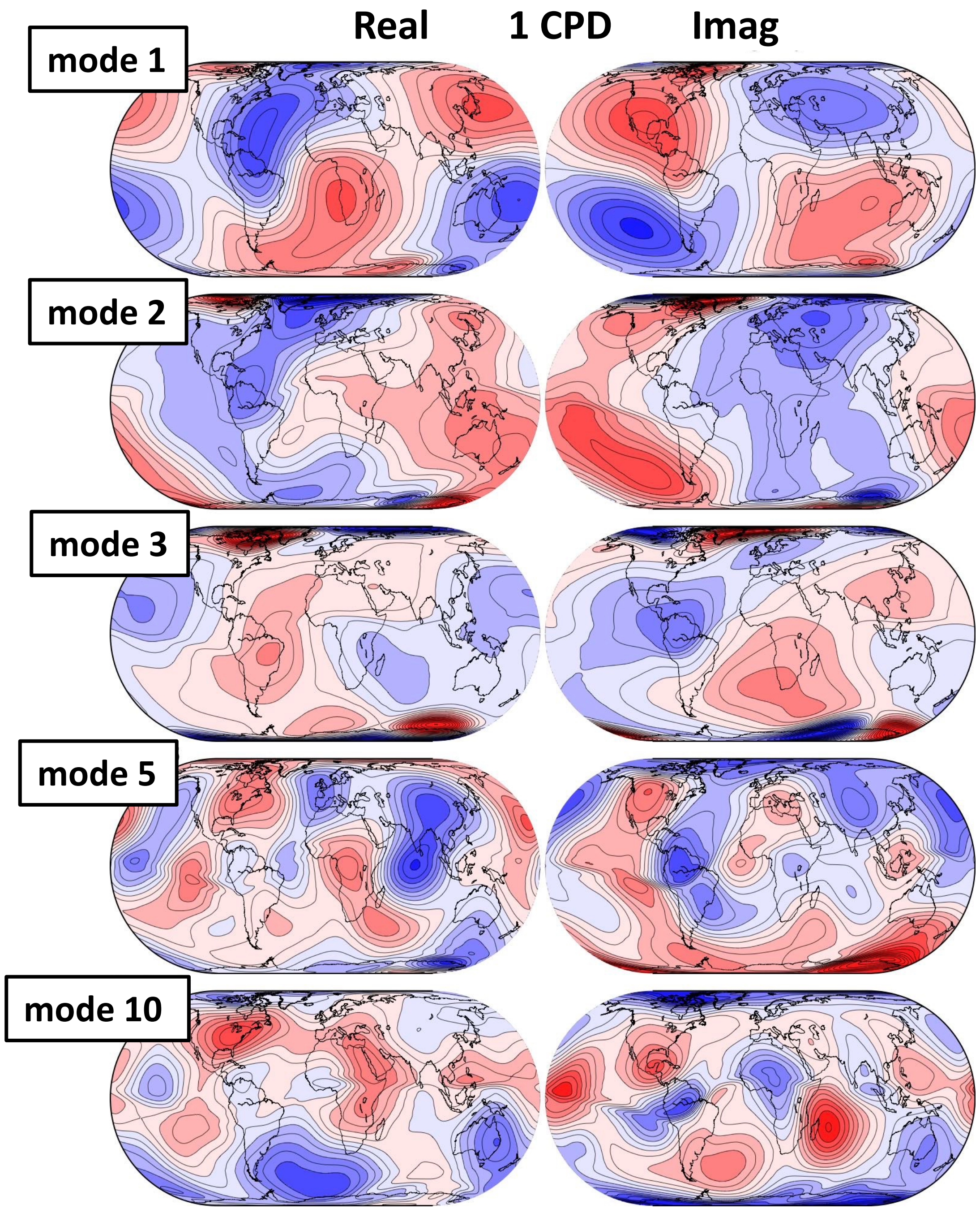
In total: 11 frequency Bands (~10^{-5} – 10^{-4} Hz), 20 modes/band

Step 2: Interpolate data modes to global grid:

→ basis functions derived from Thermosphere-Ionosphere-Electrodynamics General Circulation Model (TIEGCM)

→ 1 year runs for 2002/2009 (solar max/min)

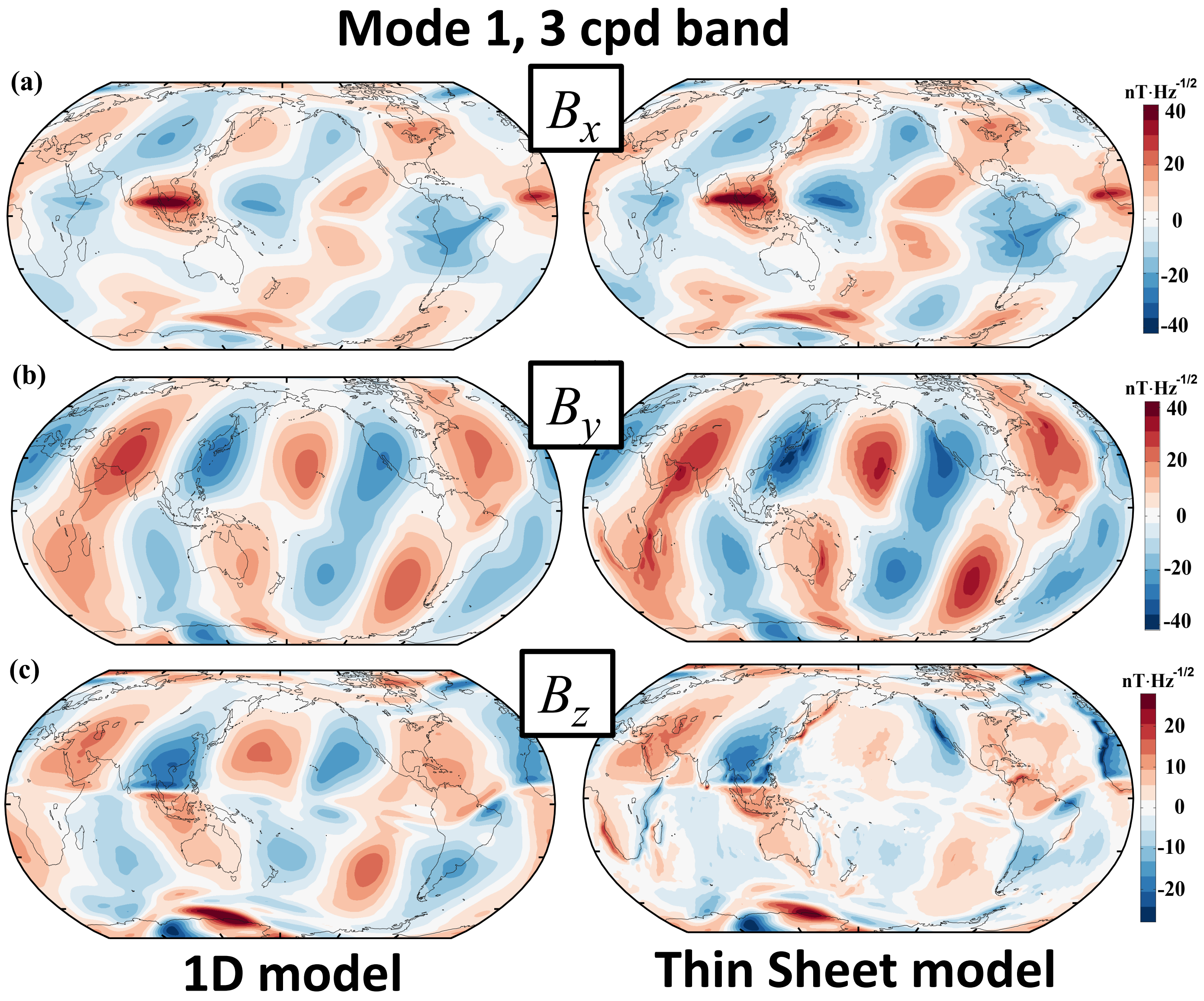
→ processed with frequency domain PCA (identical to observatory data)



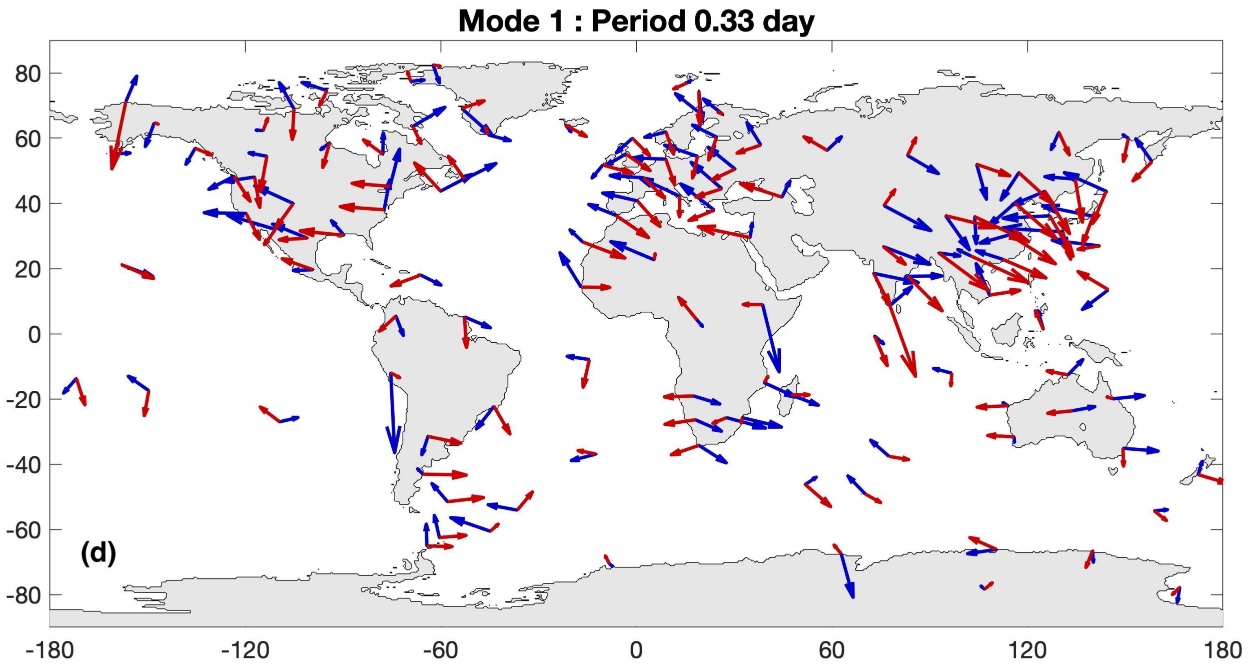
TIE modes shown as stream function for equivalent current sheet at 110 km altitude

Step 2: Interpolate data modes to global grid:

- fit data modes to TIE basis functions with damped least squares
- Induced internal fields modeled with 1D + thin-sheet surface layer
- only fit horizontal components—Bz is more sensitive to conductivity model



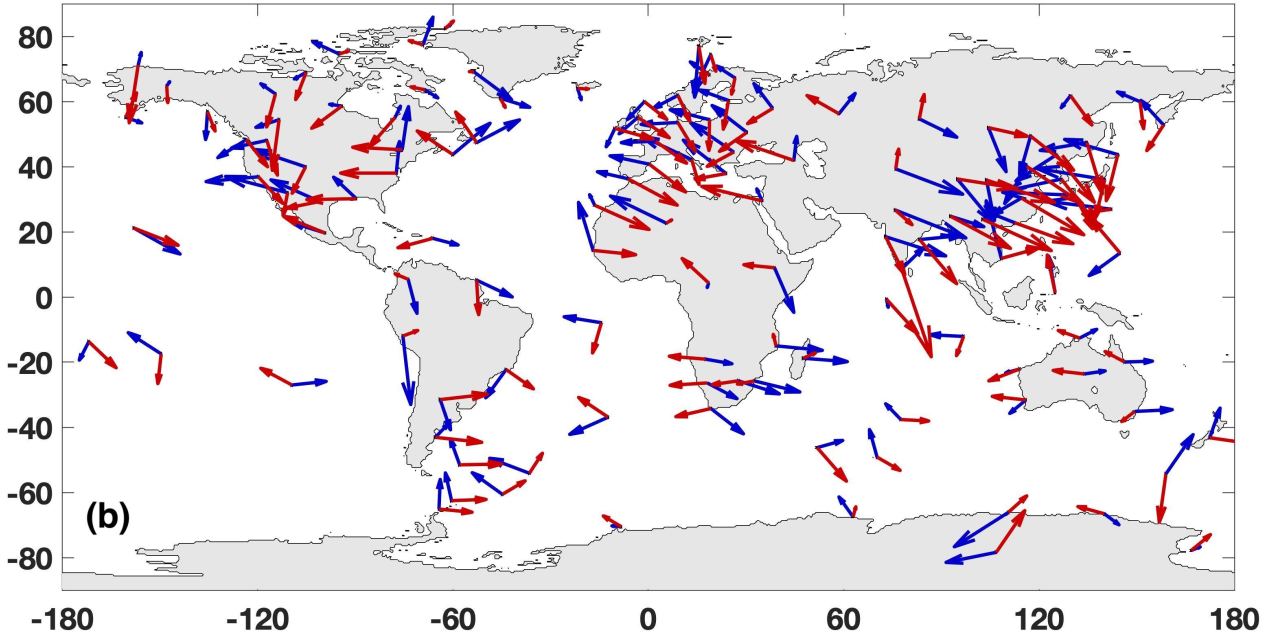
Total surface magnetic fields computed (from source estimate) with different Earth models



Spatial mode 1 : 3 cpd
(fit this using basis functions derived from TIE)

Real parts: blue

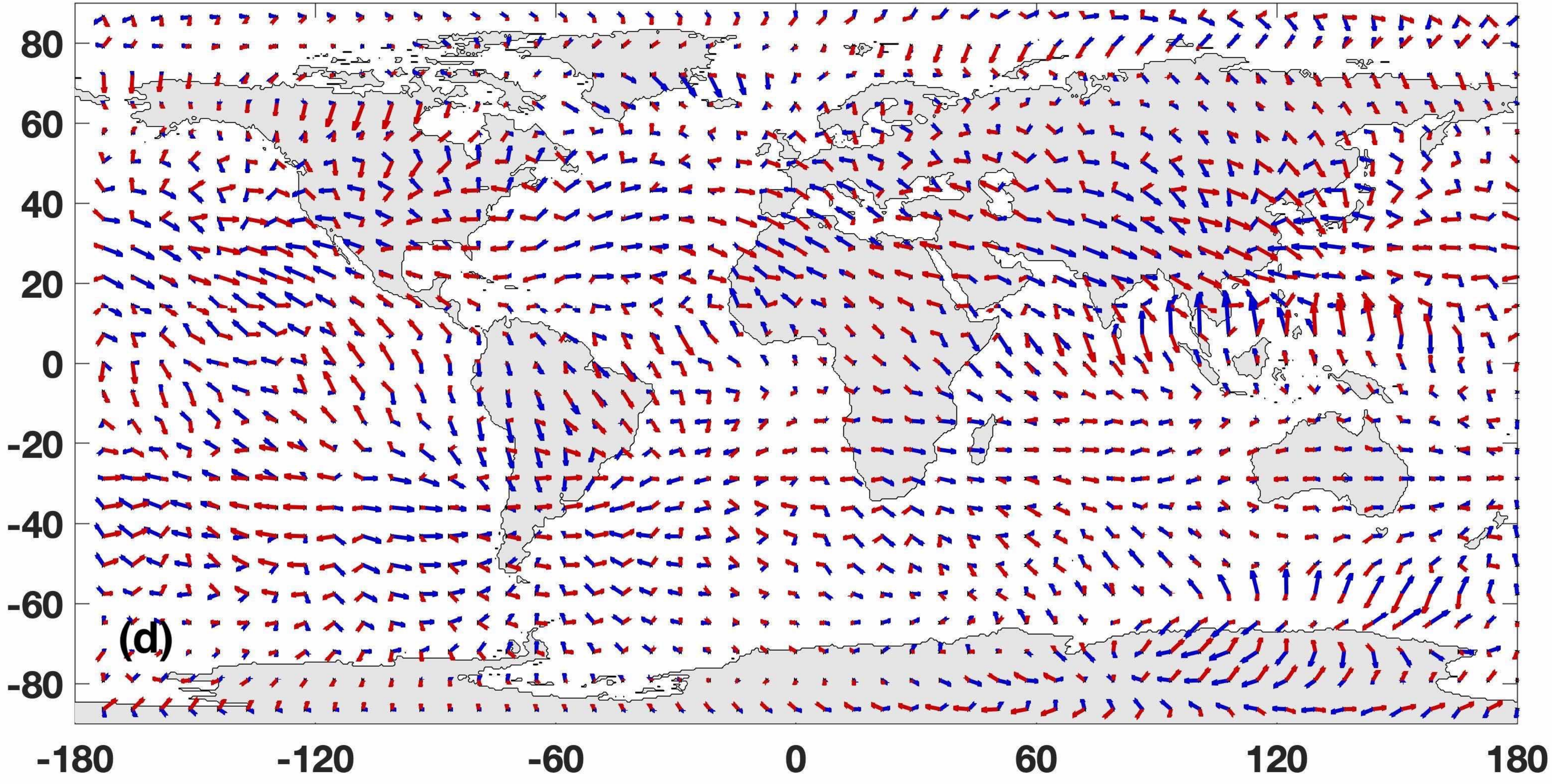
Imaginary parts: red



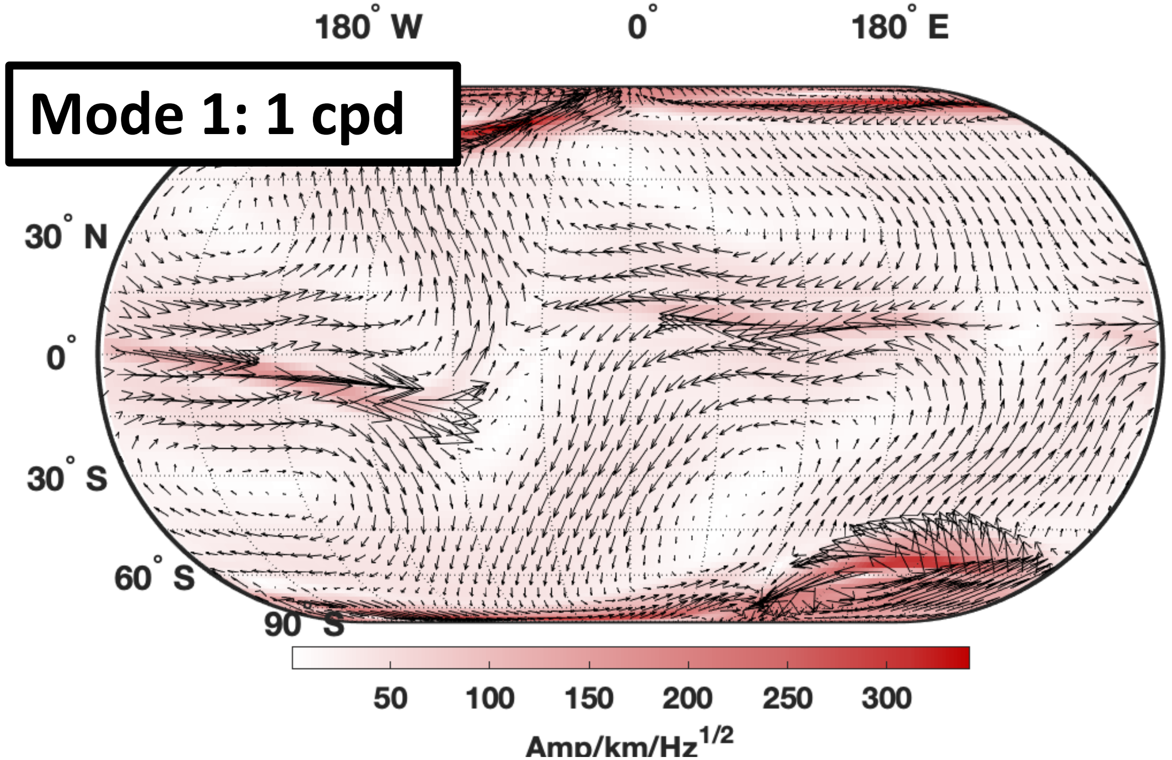
Spatial mode 1 : 3 cpd
(fitted model evaluated at observatory locations … visually very similar, but fit is not perfect!)

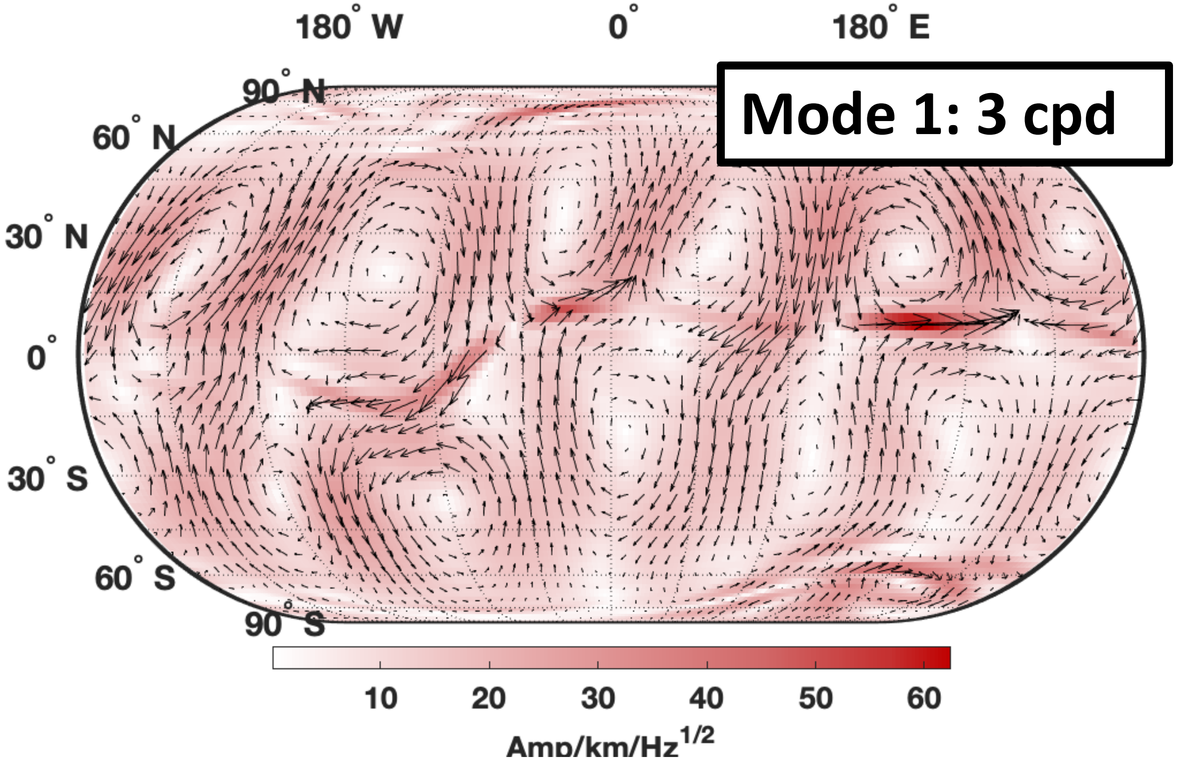
Fitted model is global – can evaluate at any lat-lon
Interpolated spatial mode 1 for 3 cpd

Real parts: blue
Imaginary parts: red



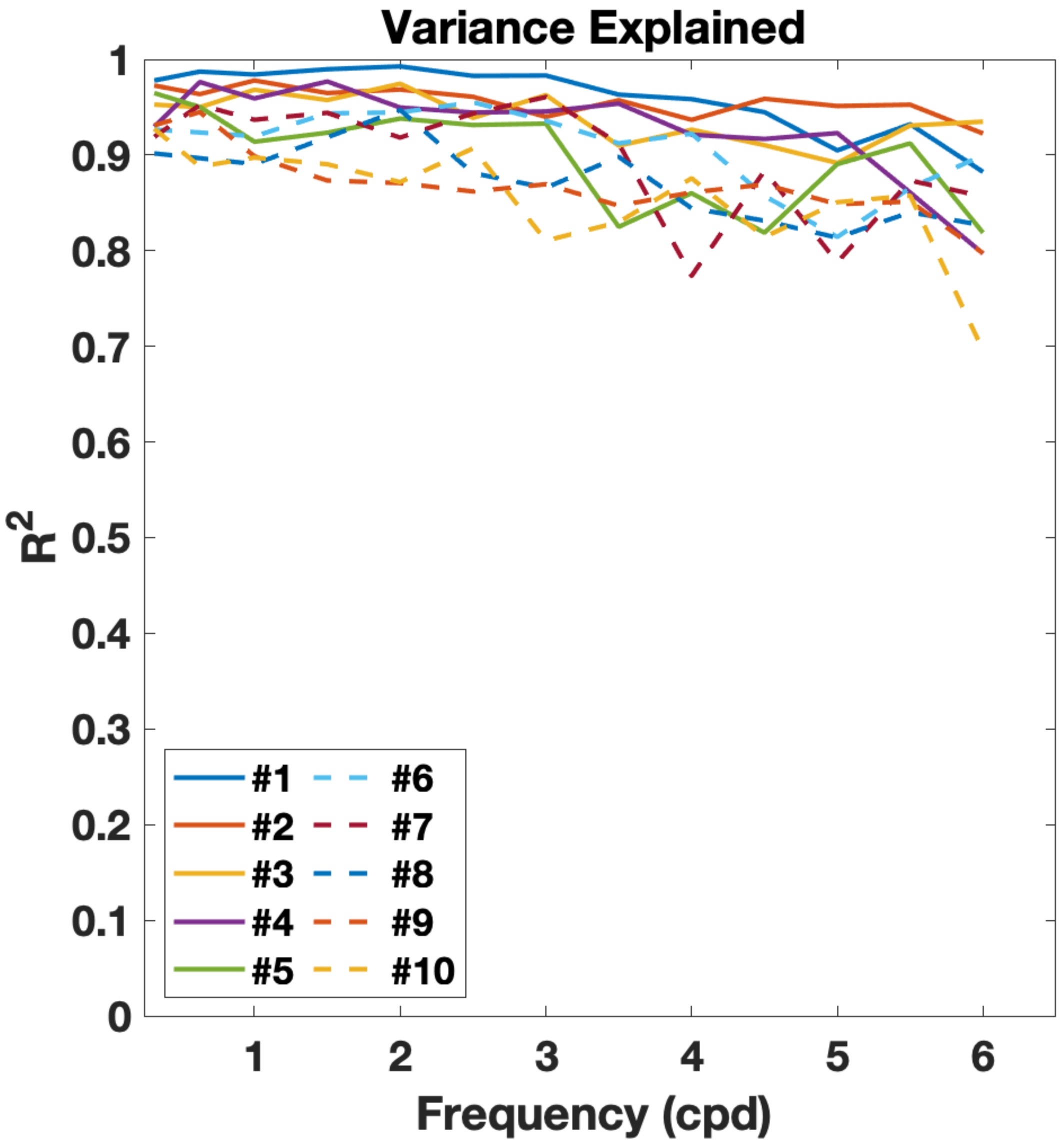
Can also represent source Estimates as equivalent current sheet (e.g., at 110 km altitude)





Most of the variance in first 20 data modes can be stably fit with smooth source current sheets

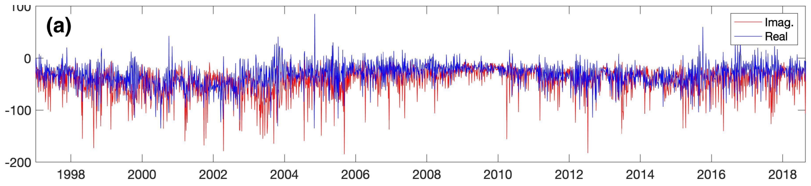
Fraction of Data Mode Variance Fit (R^2) by TIE basis functions: modes 1-10, all frequency bands



Step 3: convert back to time domain:

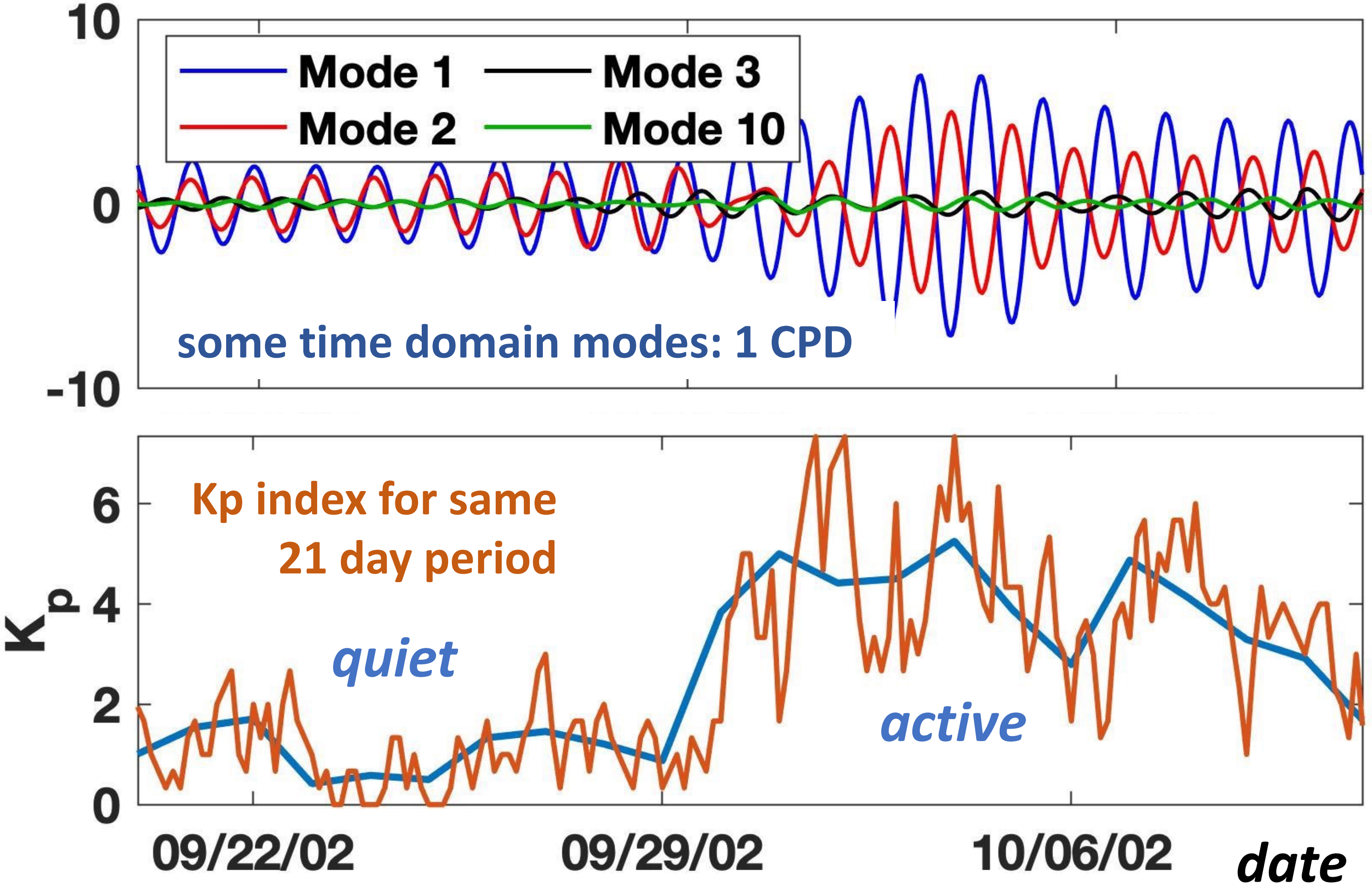
The sequence of Fourier coefficients from short-time FT w/overlapping windows can be easily inverted → back to time domain

Apply the same inversion to temporal modes computed from frequency domain PCA: for frequency band j, mode k



$$a_{kjn} \rightarrow \alpha_{kj}(t)$$

(complex – Re, Im are Hilbert transform pair)



some time domain modes: 1 CPD

Kp index for same 21 day period

quiet

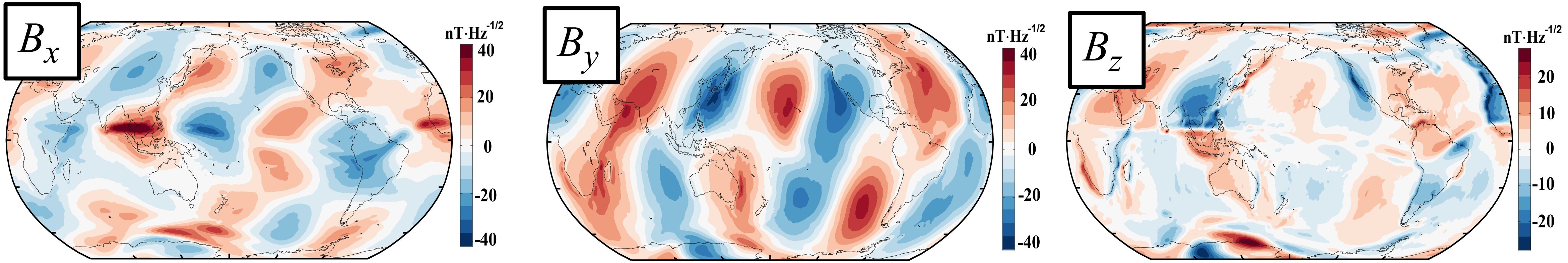
active

Model in Time Domain:

$$\mathbf{B}(\mathbf{r},t) = \mathrm{Re}\sum_{kl} \alpha_{kl}(t)\mathbf{B}_{kl}(\mathbf{r})$$

Temporal variations

Spatial variations: interpolated magnetic fields



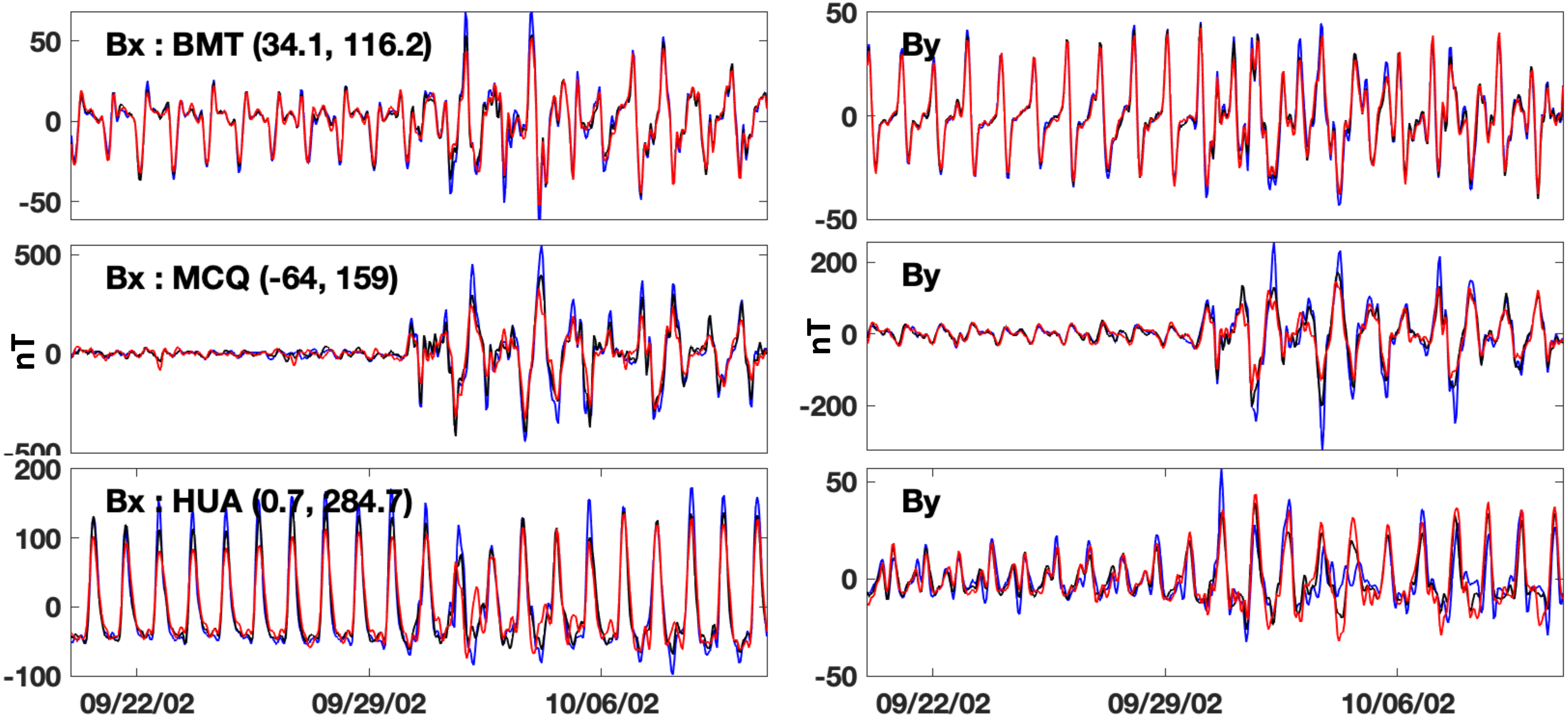
interpolated magnetic fields for one mode/band: mode 1, 3 cpd

Comparison between data, projection onto 20 PCA modes and fitted global model

mid-latitude

auroral zone

equatorial electrojet



note different ranges (nT)

Time interval is same as on previous slide – quiet conditions, then active all time series are high-passed, hourly samples

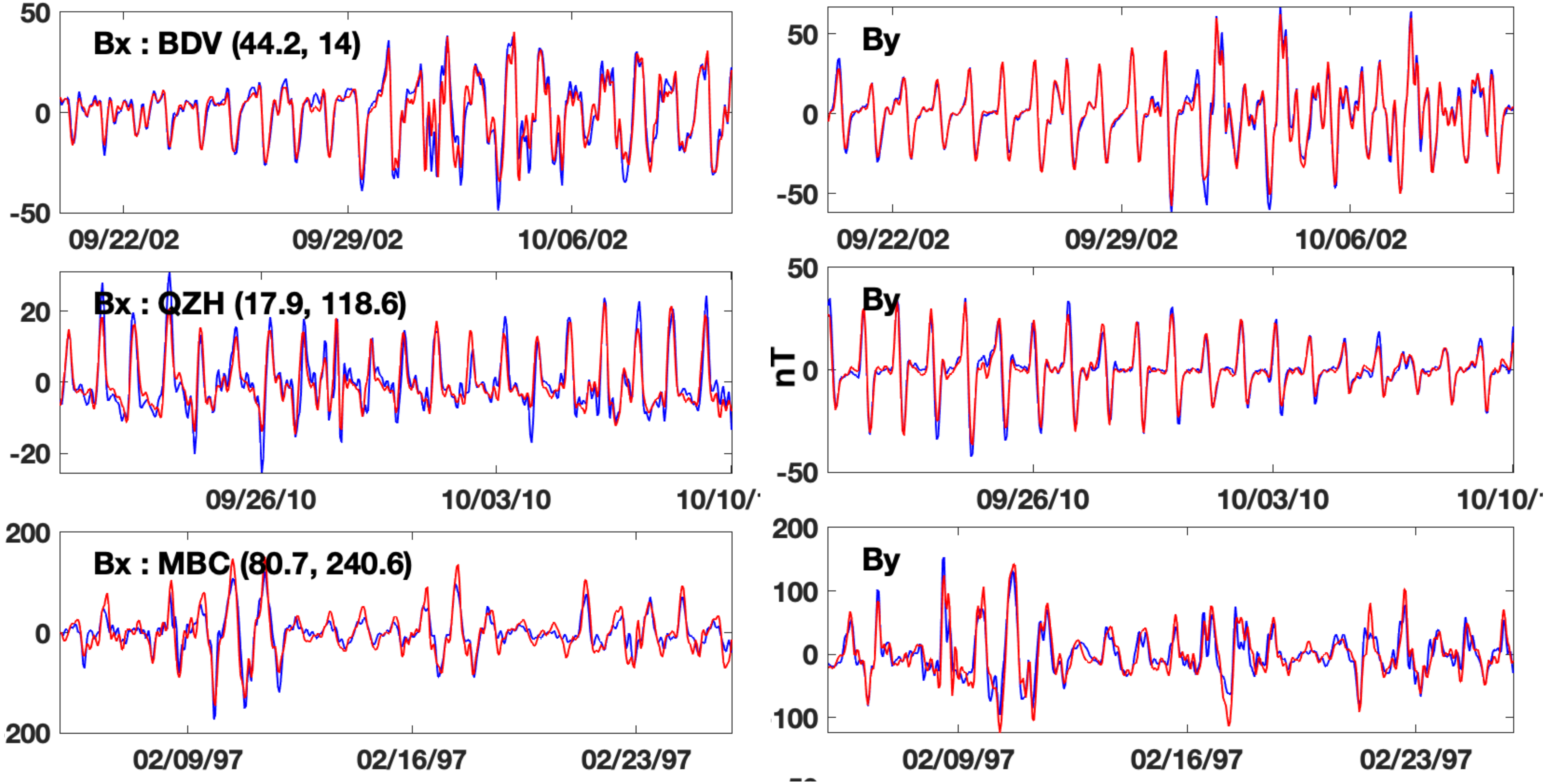
Comparison between data and fitted global model

Validation sites, not used for model construction

mid-latitude

mid-low latitude

high latitude



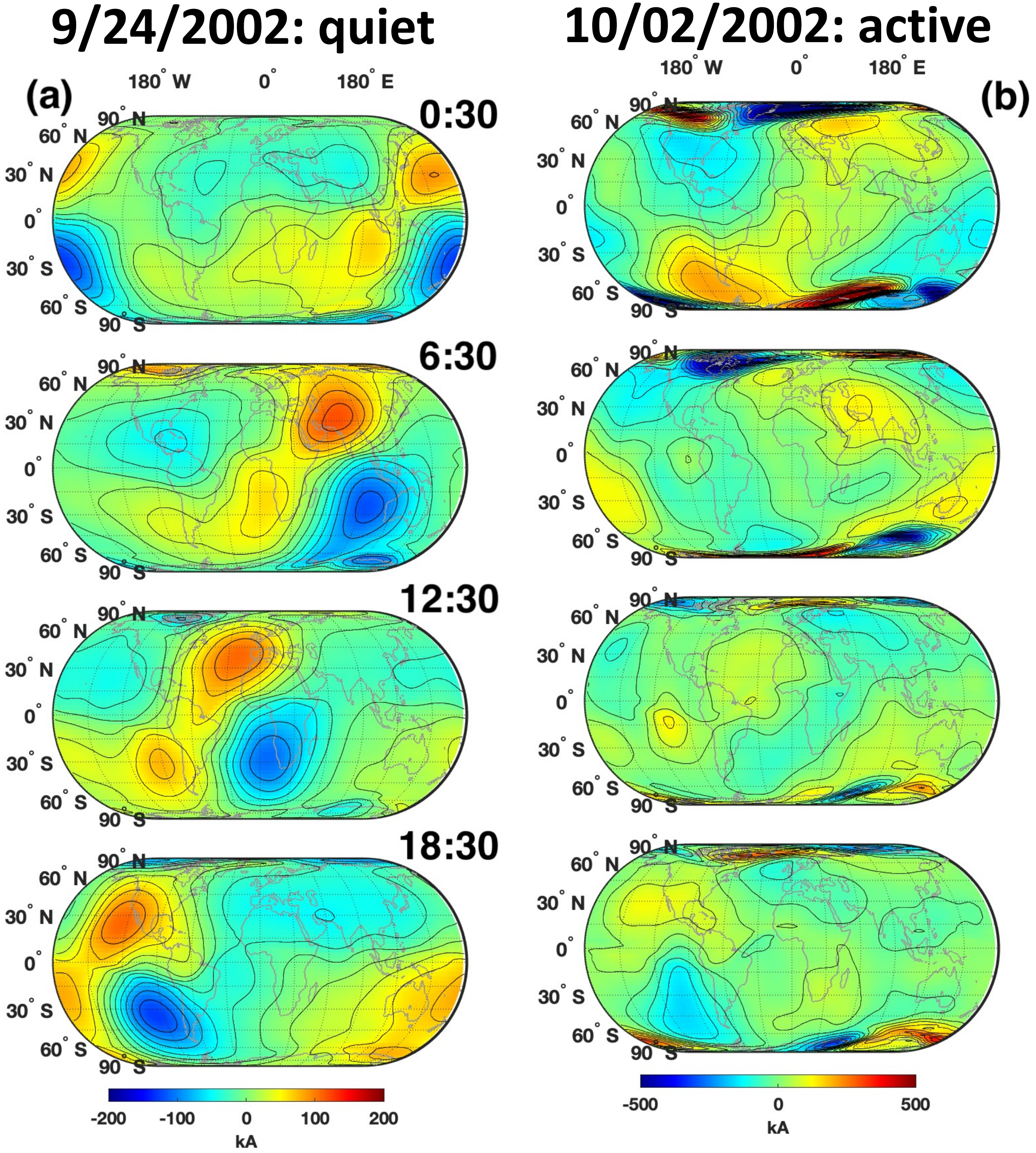
Different time intervals (not all sites operating at same time)

Snapshots of model:

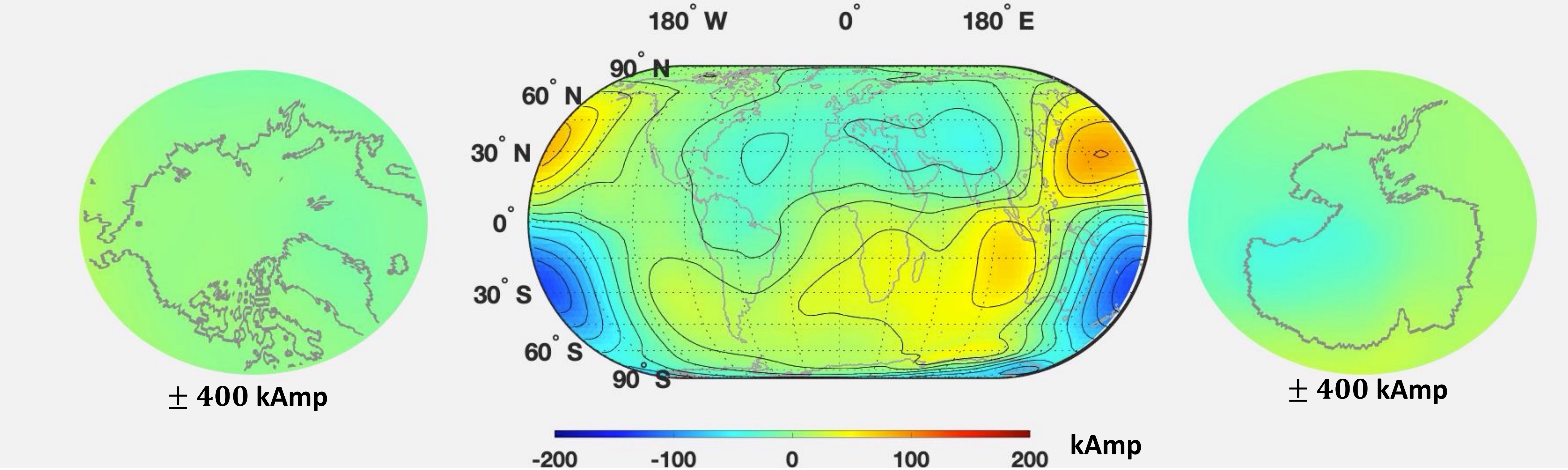
two days from the 21 day time window—first quiet, then active

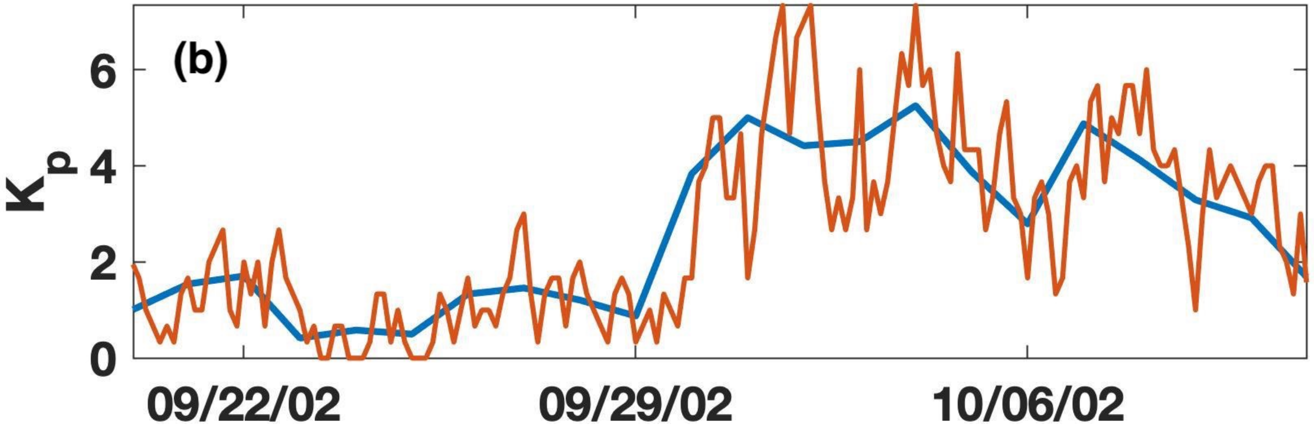
Similar pictures can be constructed for any times in the modeled interval – 1997-2018

stream functions: note different plotting ranges (kAmp)

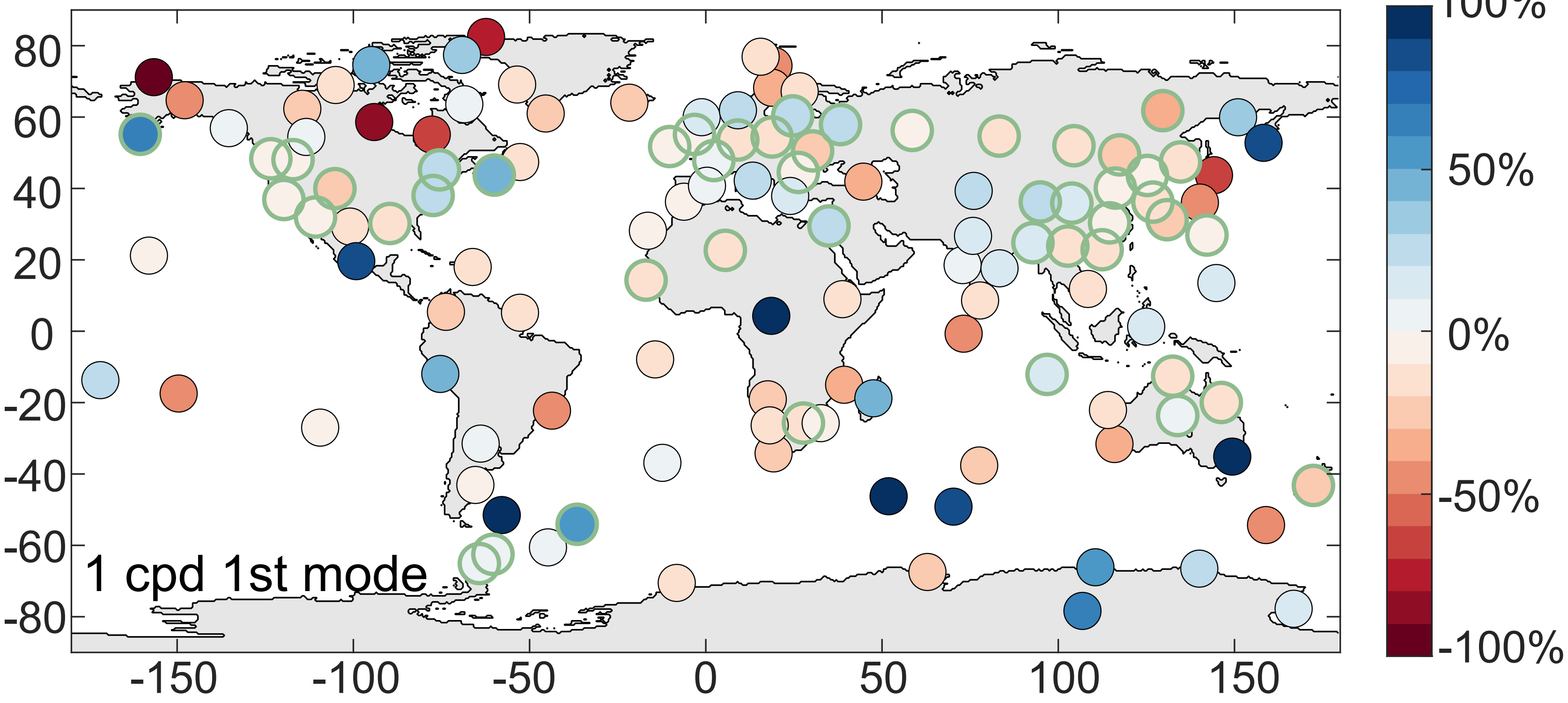


Stream Function for equivalent sheet current: 9/24/2002 – 10/6/2002





Initial mantle conductivity results: “*A relatively dry mantle transition zone revealed by geomagnetic diurnal variations*” *(H. Zhang, G. Egbert, Q. Huang; in revison)*



Sites used, with misfit to B_z amplitude shown by color

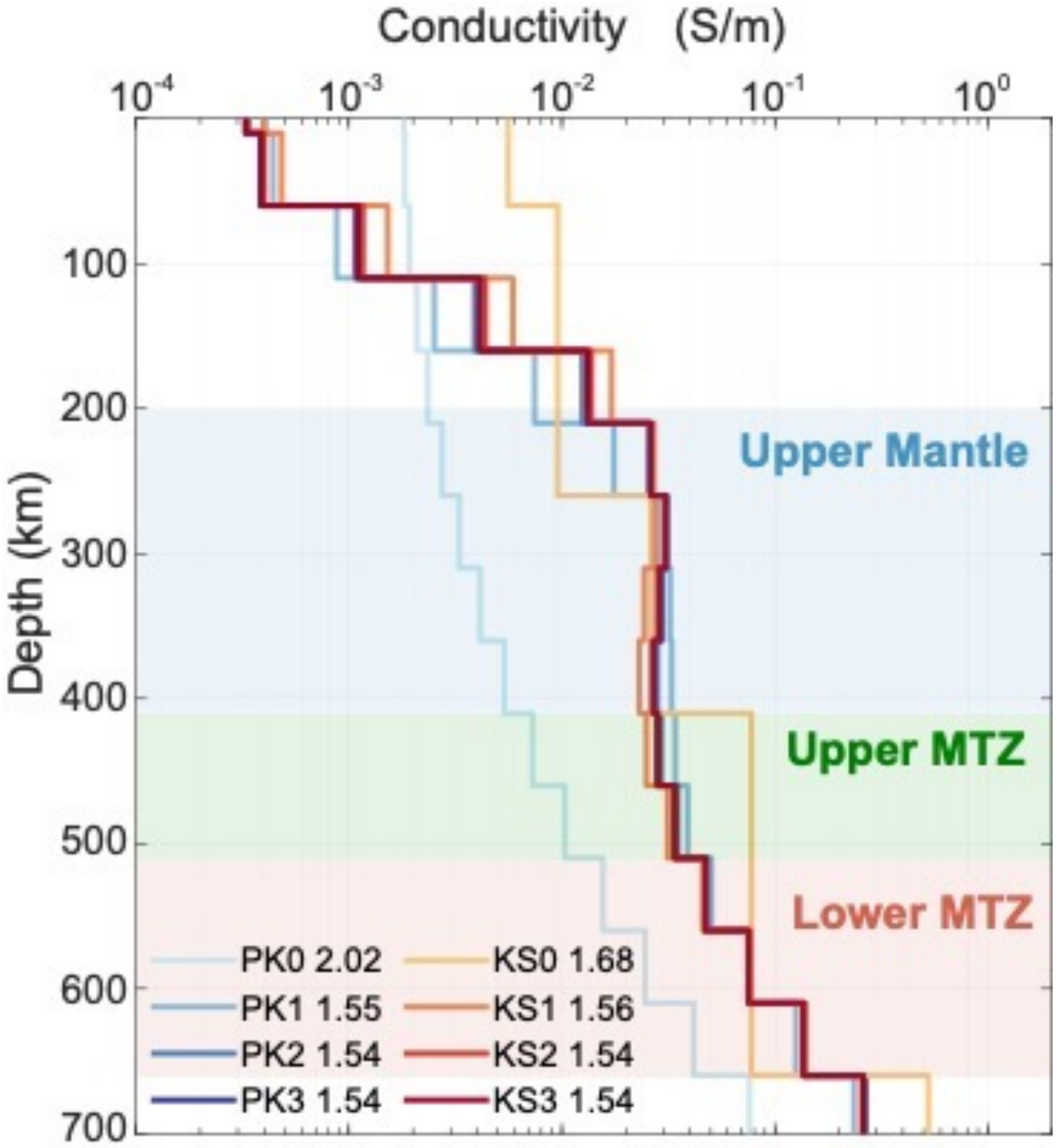
Often poor fits near coastlines, and near electojets

Omit these sites from global inversion

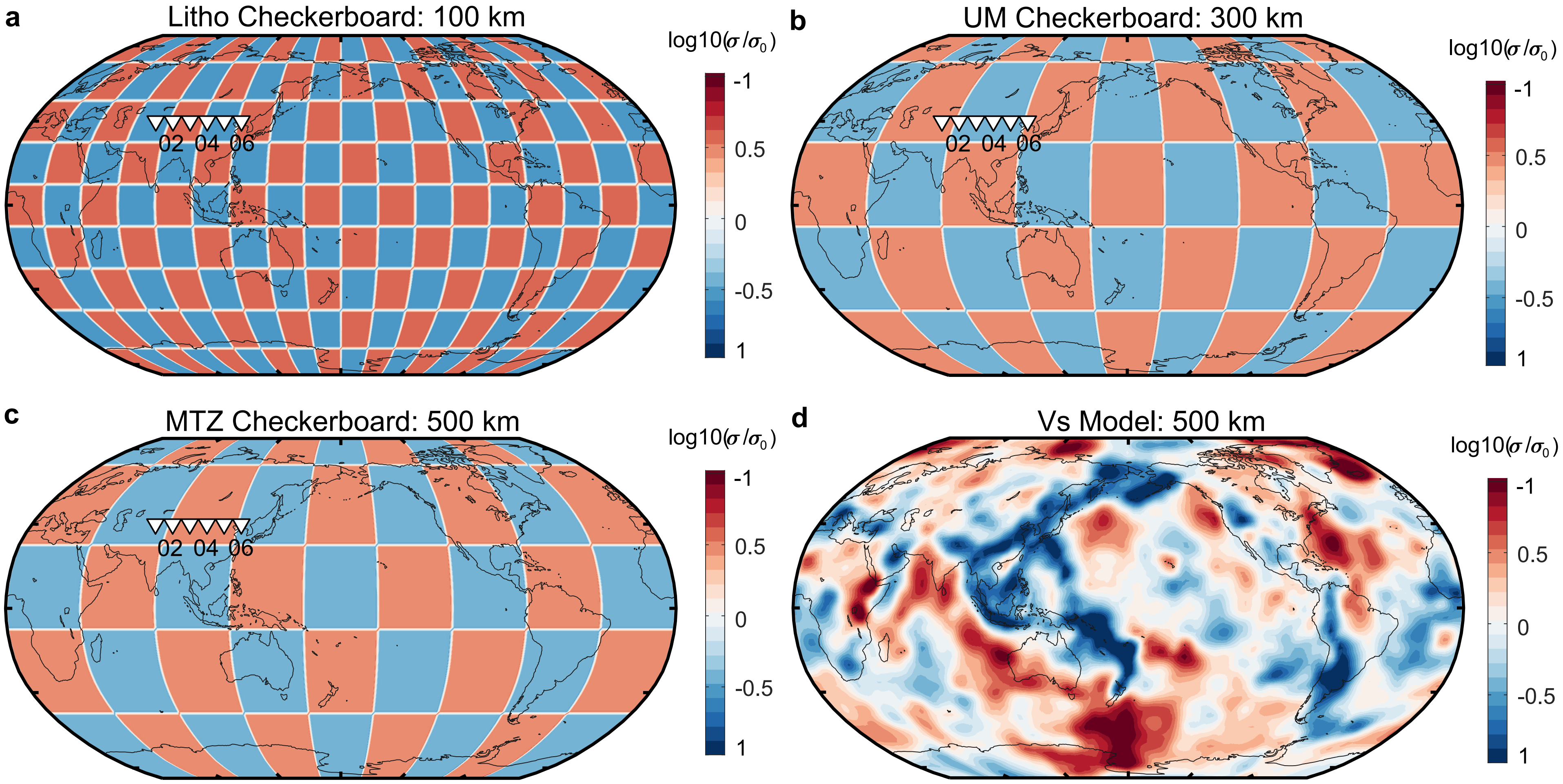
Global 1D inversion – using fixed thin-sheet model

- Use horizontal fields B_x, B_y to estimate source (1-4 cpd)
- Invert vertical fields B_z for global 1D conductivity

- Invert data for modes 1-5, 4 periods (1-4 cpd)
- Occam Inversion
- started from two different 1D profiles (PK = Puthe et al., 2015; KS – Kelbert et al., 2009)
- Resolution is best in mantle transition zone (MTZ)—resisitivty 30-50 ohm-m



Effect of 3D structure: tests with synthetic data



Checkerboards models: $\pm\frac{1}{2}$ order of magnitude deviation from 1D inversion result; individual layers, mixed

Derived by scaling global dVs model of Hosseini et al., 2018

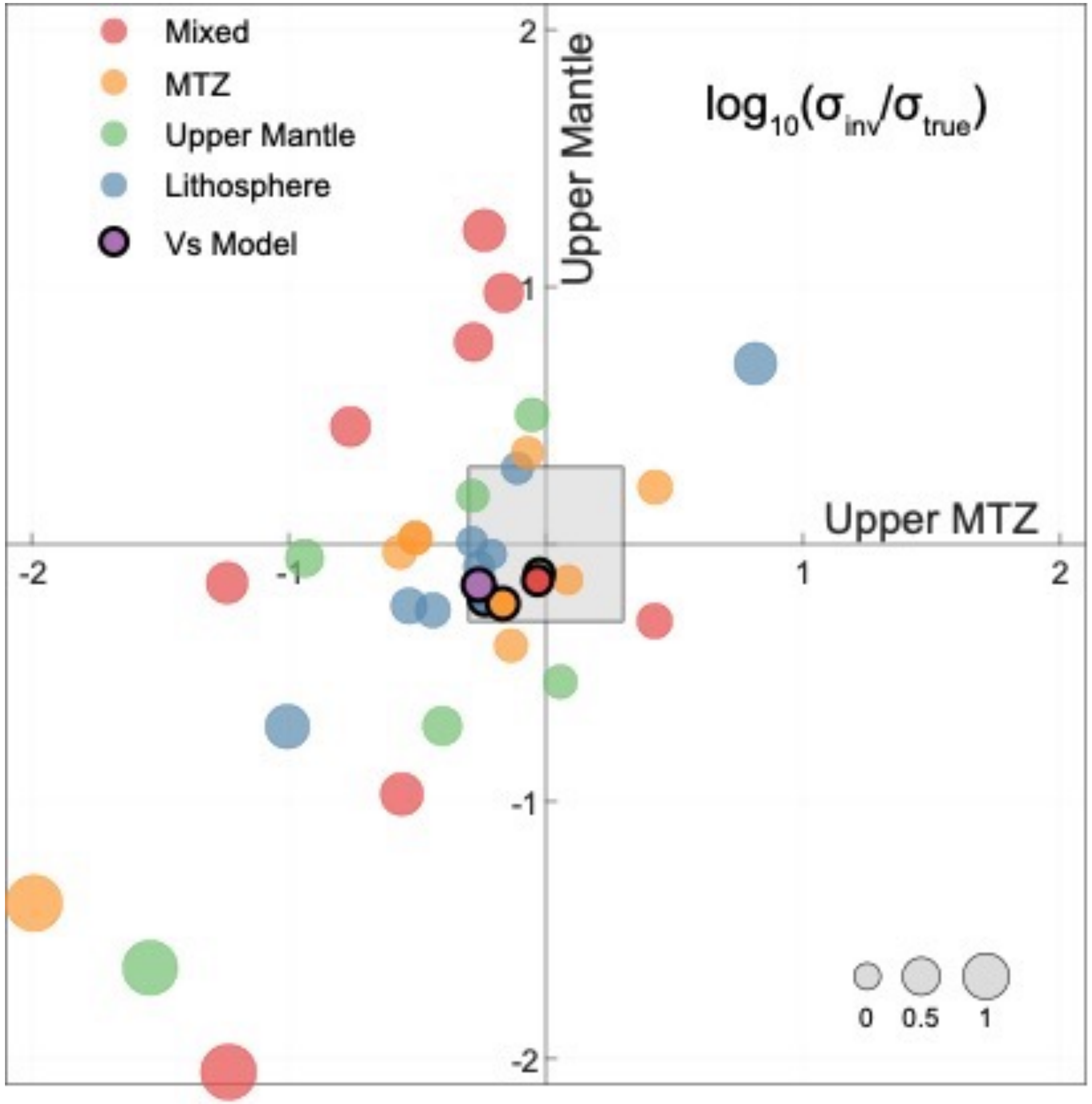
Some 3D models used to generate synthetic data, using estimated source fields

Effect of 3D structure: tests with synthetic data

Results: $\log_{10}(\sigma_{inv}/\sigma_{true})$ plotted for various test cases

- Color indicates synthetic model
- Circles with black outlines are ”global” (fit all sites simultaneously)
- Circles with no outline are single site inversions
- “true” conductivity is average beneath all sites used

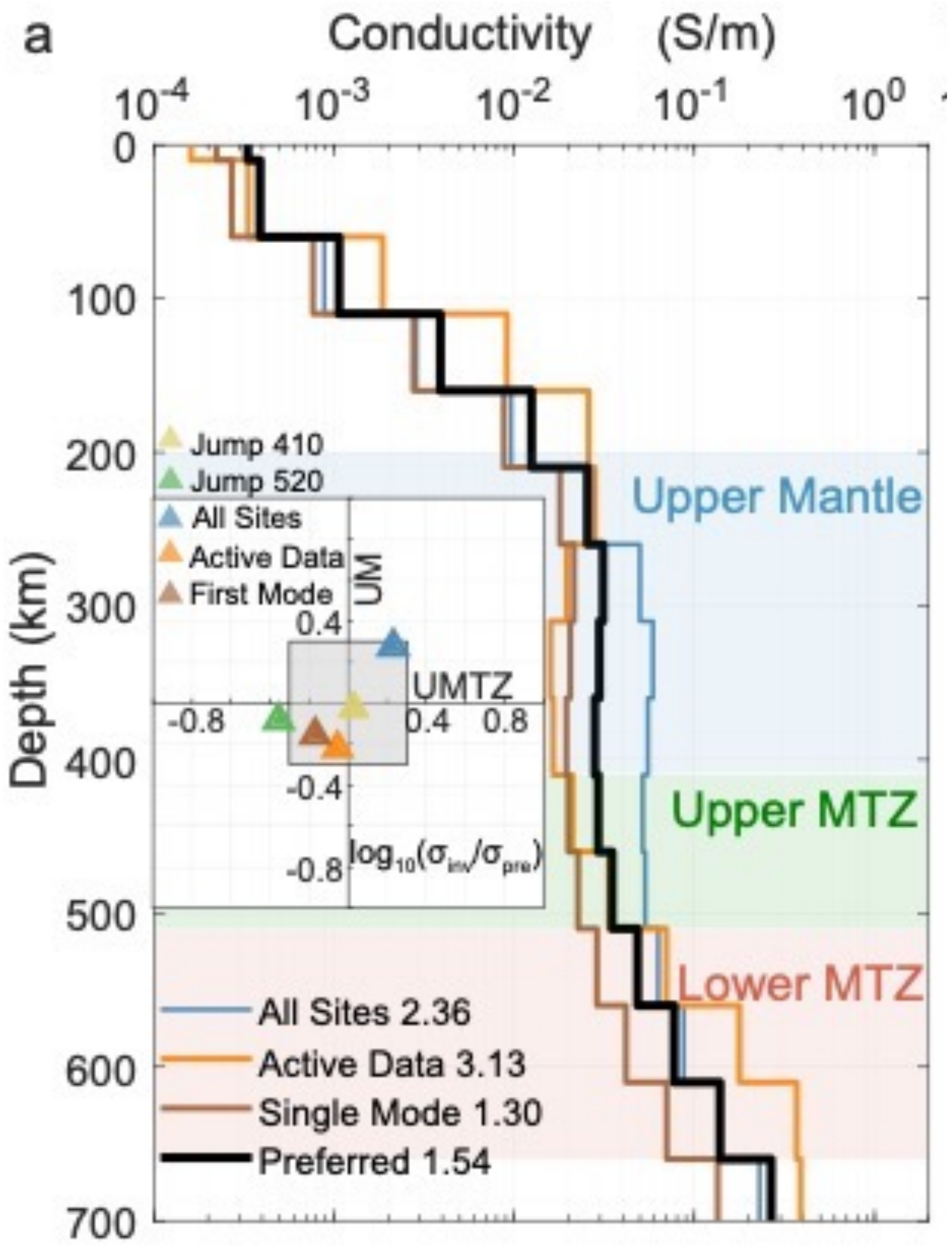
Single site results are not reliable, but global inversions produce correct result, within a factor of two or better



Other tests

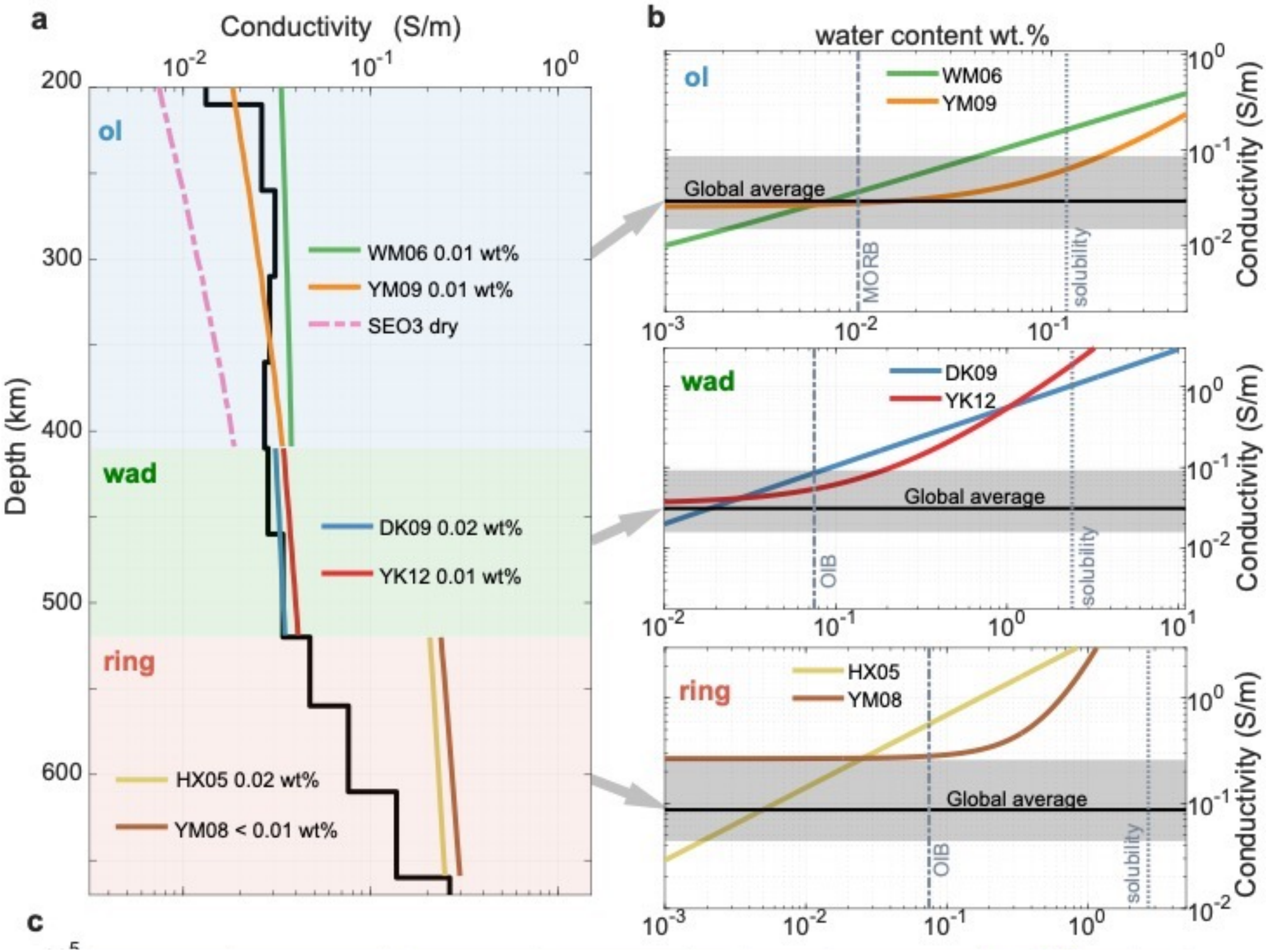
- Use all sites
- Include geomagnetically active vs. quite times only
- Fit only the best determined first mode

All of these variants yield similar results

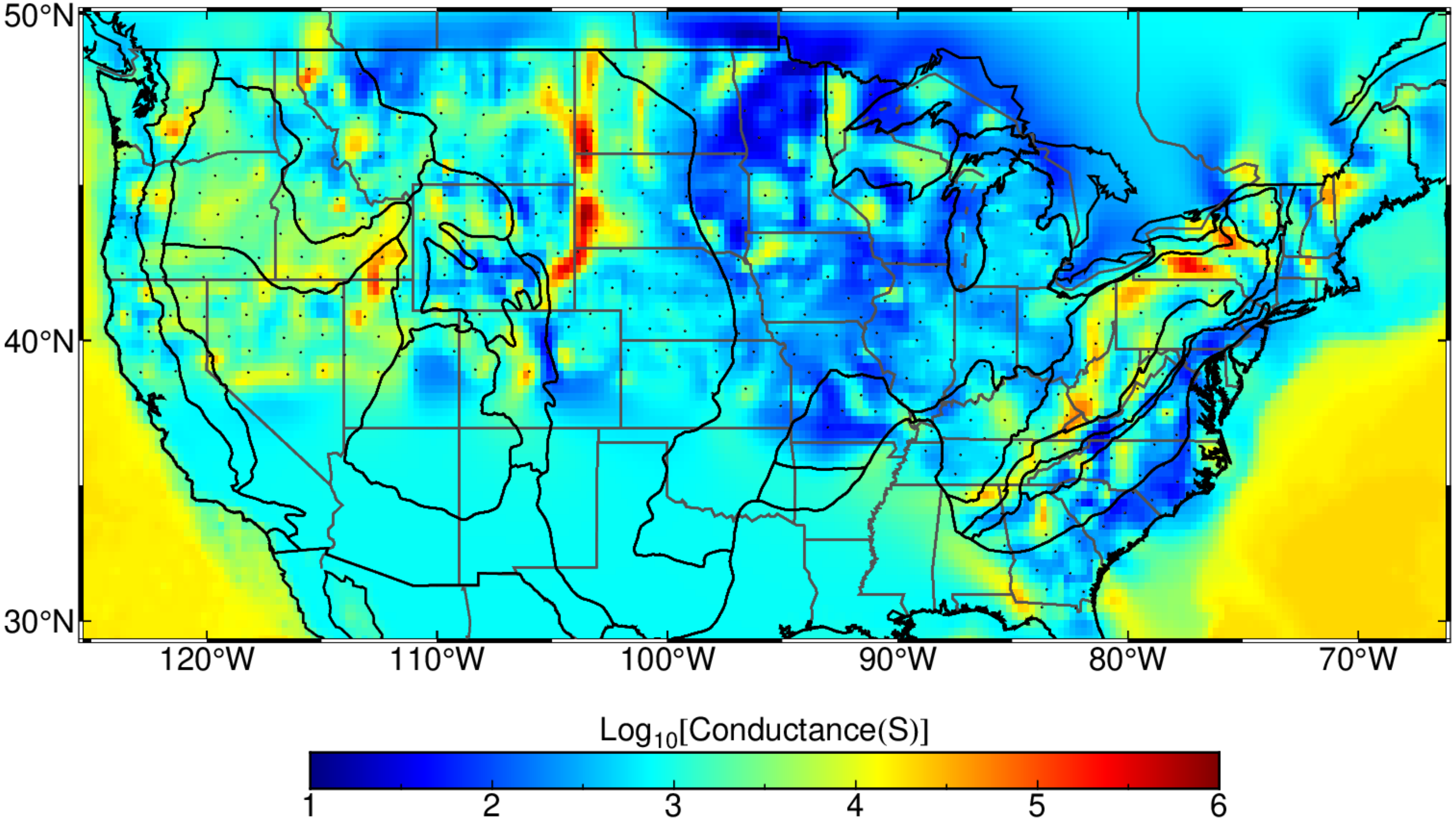


Combine with lab data to constrain global average water content

- Upper mantle: .01 wt% consistent with MORB
- MTZ : best estimate 0.02 wt%
- Considering uncertainties, could be as high as 0.1 wt%—still well below saturation (1-3wt%)



Conductance (0-150 km) from 3D inversion of EarthScope data: (Yang et al, 2021   )



Variations in lithosphere conductance in continental areas can be significant-- thin sheet model based on oceans and sediments misses a lot of variability in surface!

Need to do 3D (regional) inversion–and incorporate MT?

Classical terminology: B_z has two components

- ***Anomalous:*** due to lateral conductivity variation (tipper)

$$B_z = T_x B_x + T_y B_y$$

- ***Normal*** **:** due to non-uniform source (Horizontal Spatial Gradient or HSG)

$$B_z = C(\omega)\left[\partial_x B_x + \partial_y B_y\right] \qquad C(\omega) = i\omega Z(\omega)$$

Vertical Field TF (Tipper)

$$H_z = T_{zx}H_x + T_{zy}H_y$$

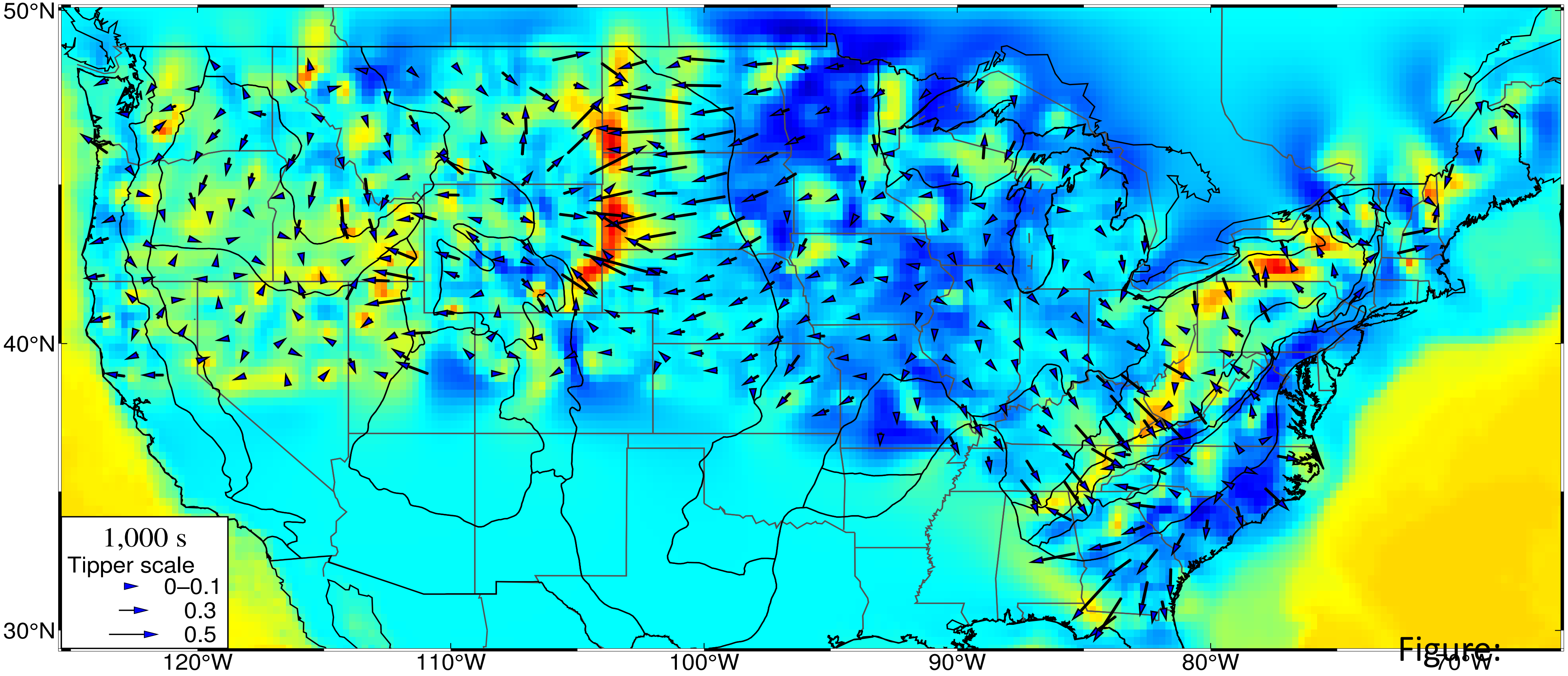


Figure:
Bo Yang

Parkinson vectors $(-T_{zx},-T_{zy})$ plotted for a reduced set of EarthScope MT sites, overlying conductance of lithosphere derived from LPMT

1D local inversions failed due to contamination of normal B_z by the anomalous components

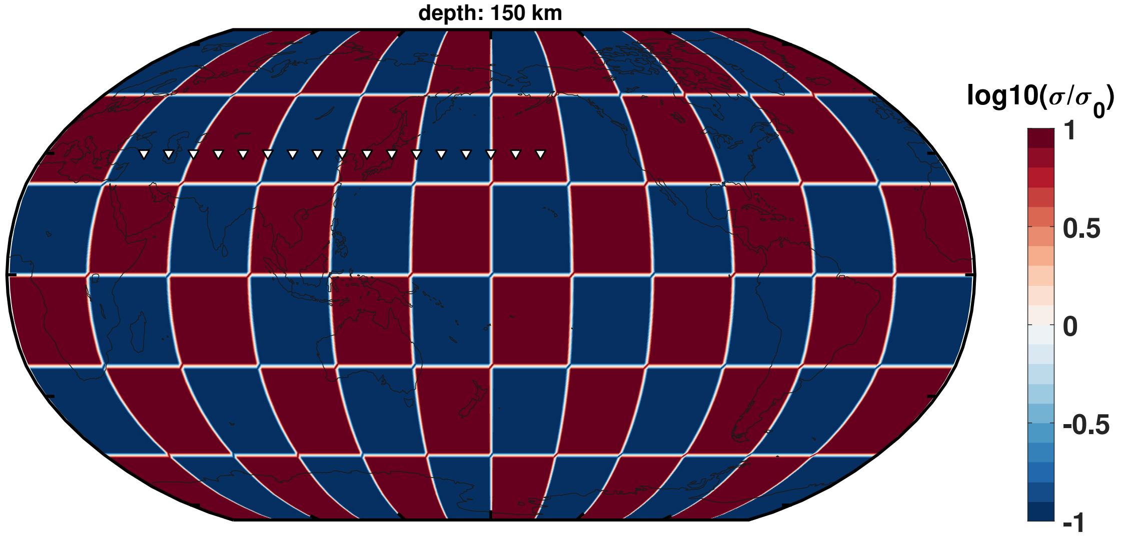
Can we use a good source model allow us to separate these components?

More broadly, how can we use transfer function ideas (both tipper and HSG) together with realistic source models?

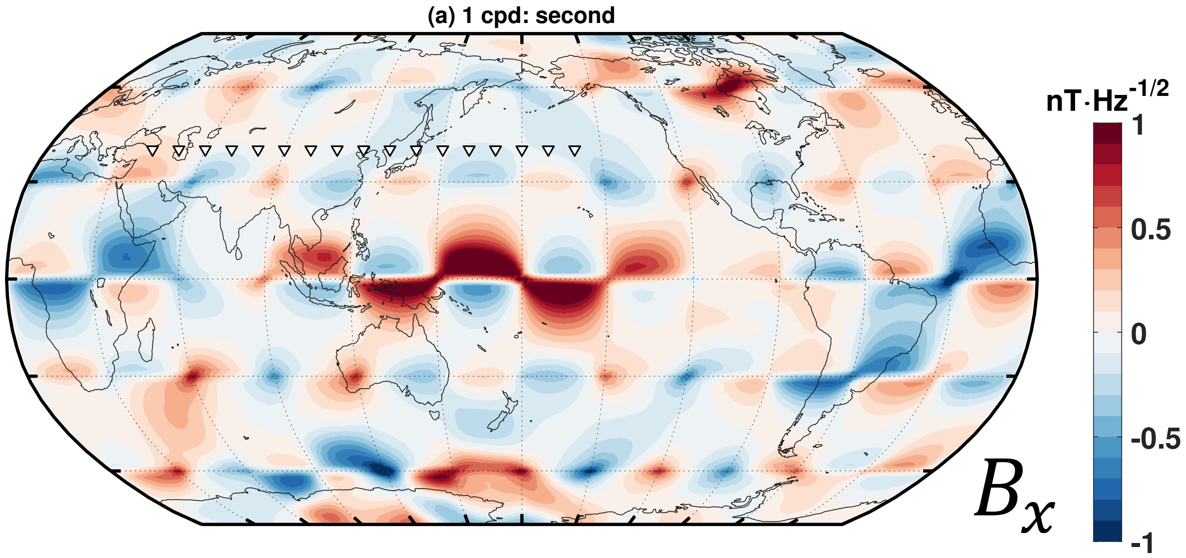
Some preliminary modeling results …

Synthetic modeling tests:

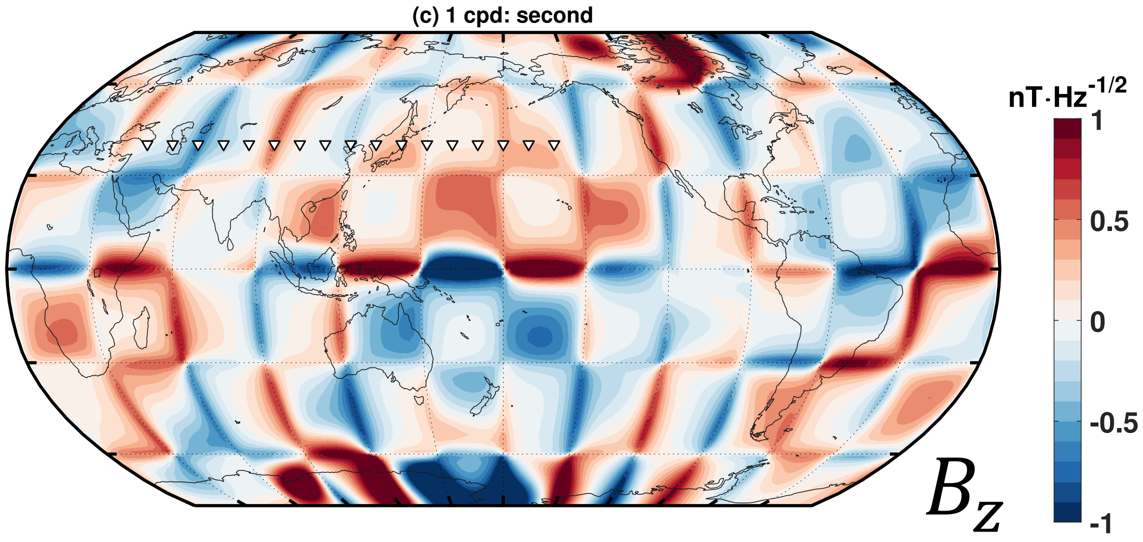
- 30˚checkerboard (large variations) 0-250 km depth
- 3D modeling with **realistic source fields**
- here: ***secondary fields*** (total fields computed for 1D reference resistivity subtracted), 1st mode, 1 cpd



depth: 150 km



(a) 1 cpd: second



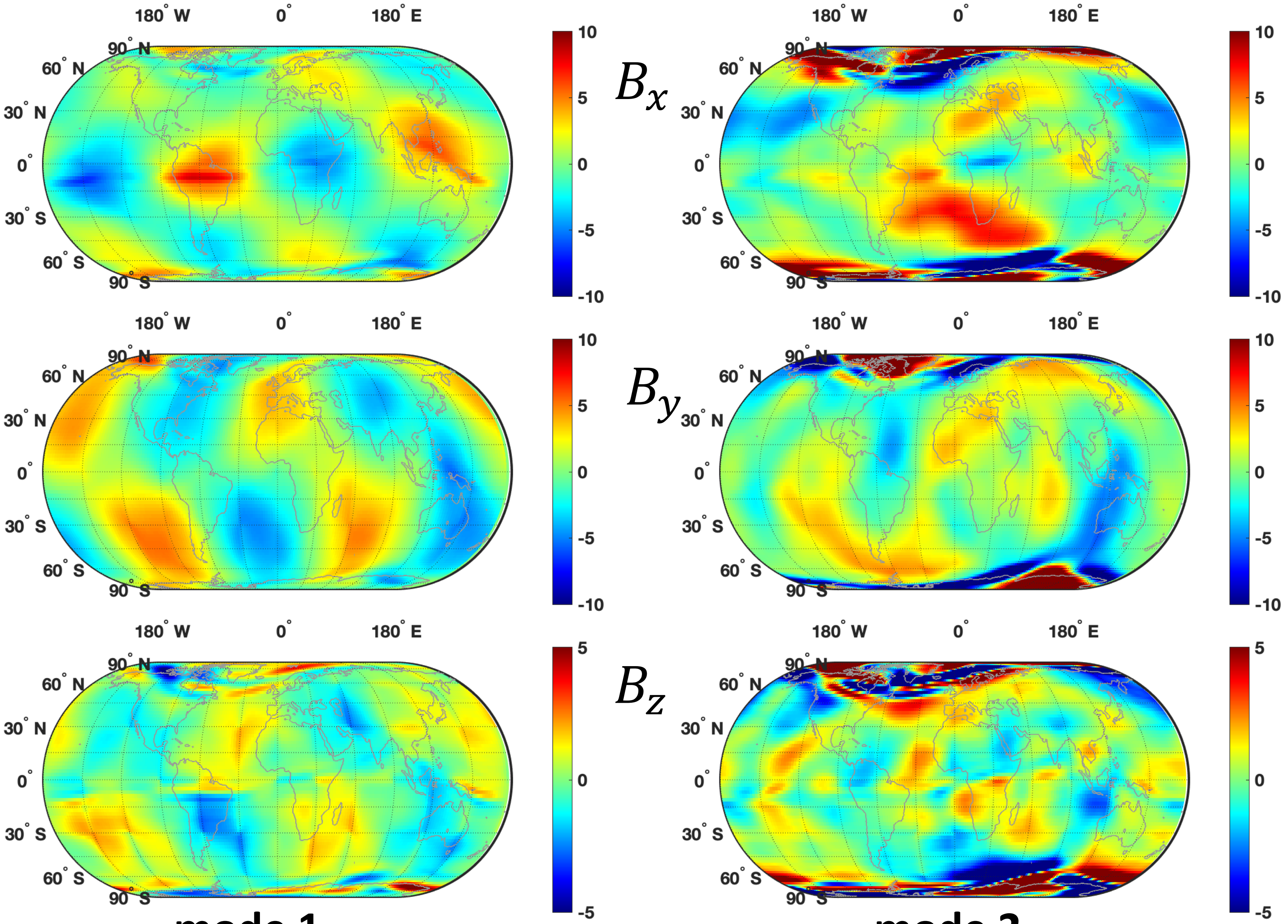
(c) 1 cpd: second

Can see both normal and anomalous B_z in these secondary field plots

Synthetic modeling tests:

- 30˚checkerboard (large variations) 0-250 km depth
- total fields plotted
- **do same thing for all 20 modes used for DV model**

can’t easily see conductivity in these total field plots

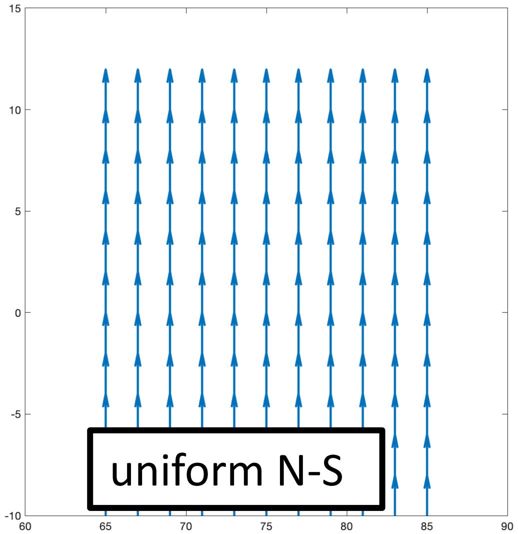


mode 1

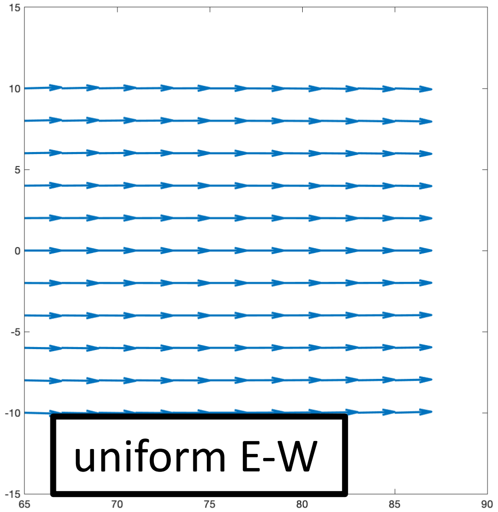
mode 2

2 cpd

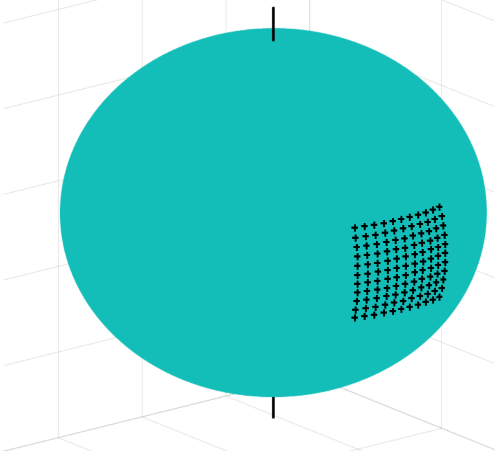
Idealized horizontal magnetic field patterns for transfer functions



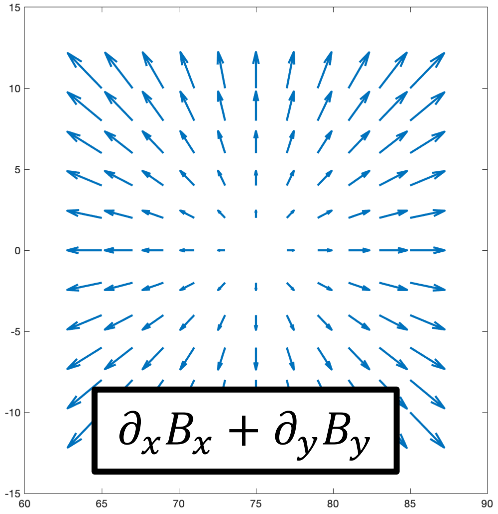
Uniform (MT) source fields

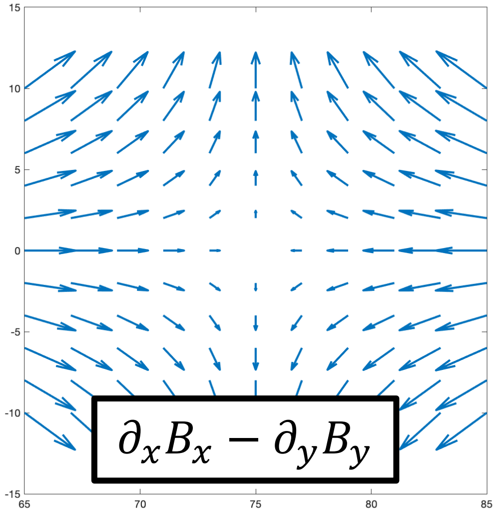


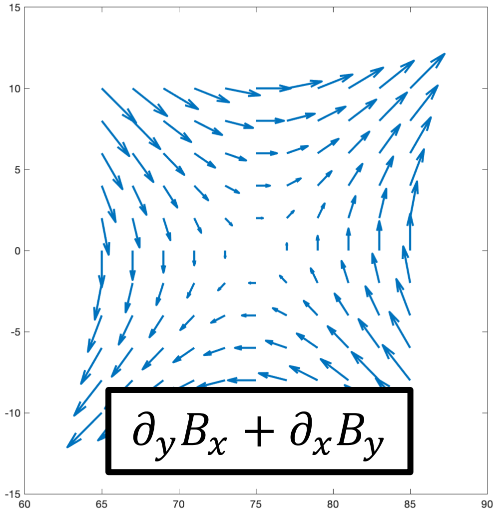
rectangular patch on equator



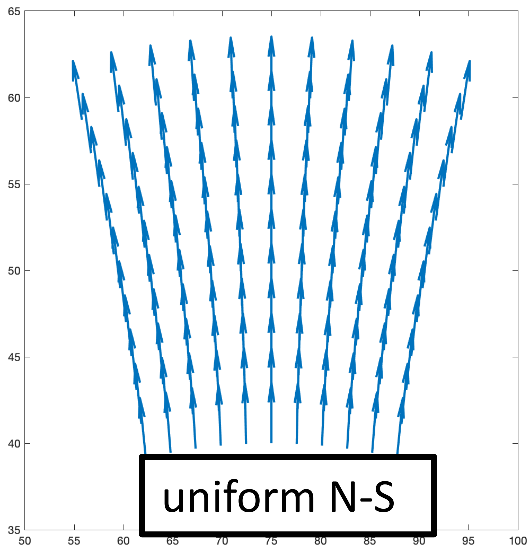
Three canonical curl-free gradients





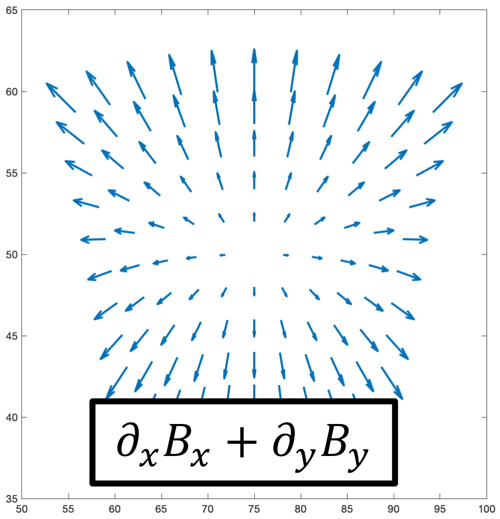


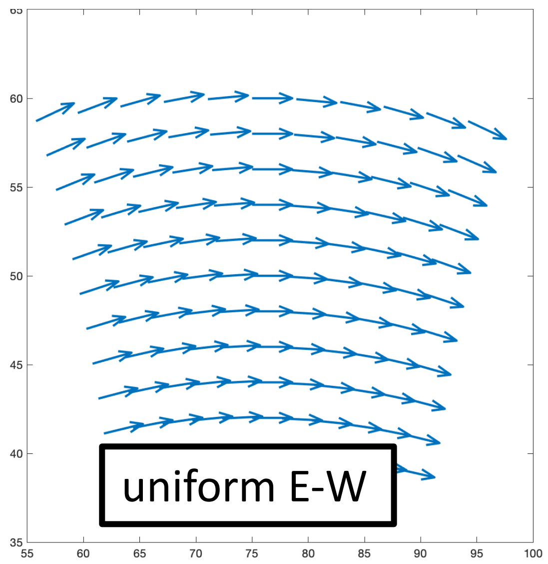
Idealized horizontal magnetic field patterns for transfer functions



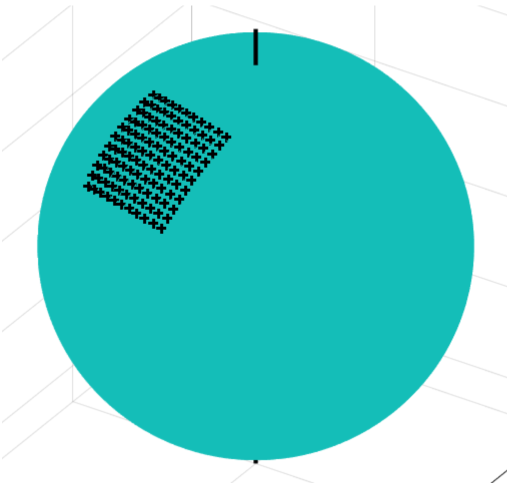
Uniform (MT) source fields

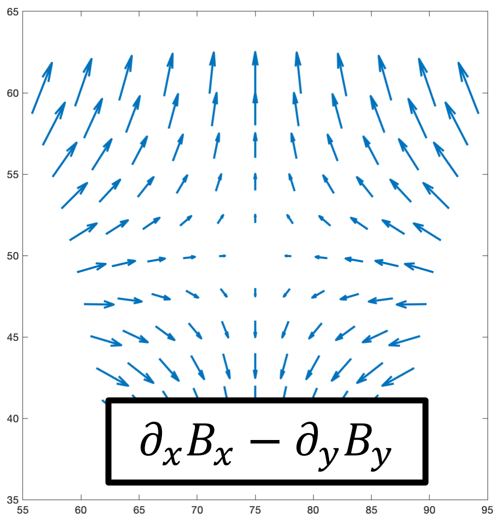
Three canonical curl-free gradients

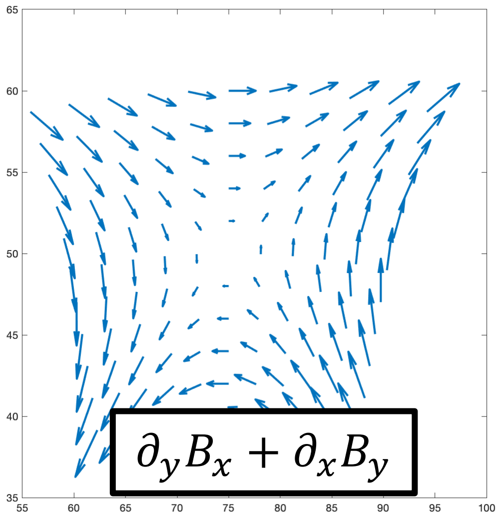




rectangular patch at higher latitude



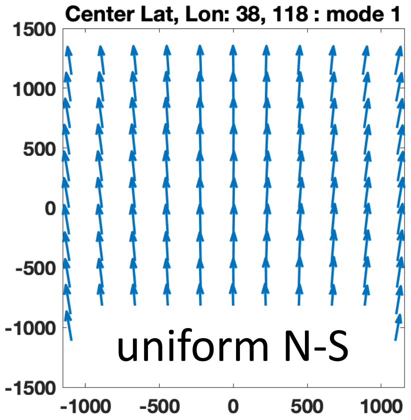




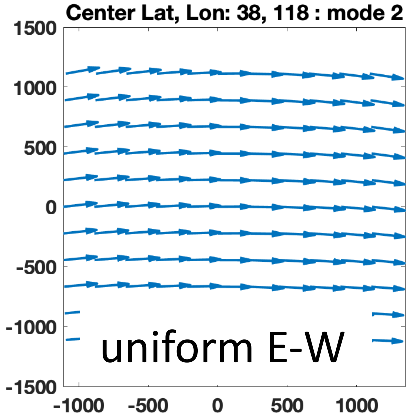
Sorting fitted modes locally, using TF ideas

At each location

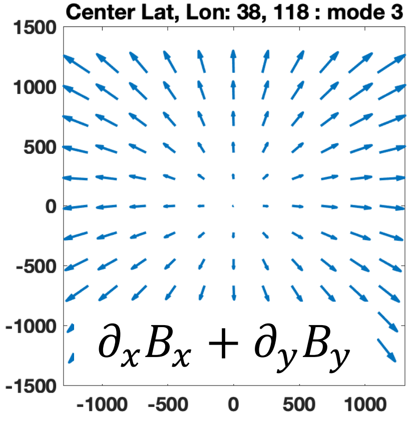
- form linear combination of all 20 modes (B_x,B_y only, total fields) that best approximate idealized uniform N-S, E-W + canonical gradients within a 20˚× 20˚ patch
- for each linear combination plot B_z at center of local patch



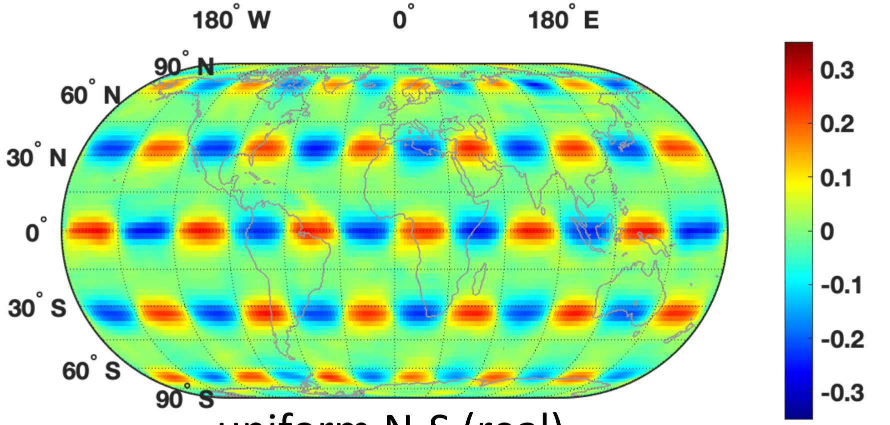
uniform N-S



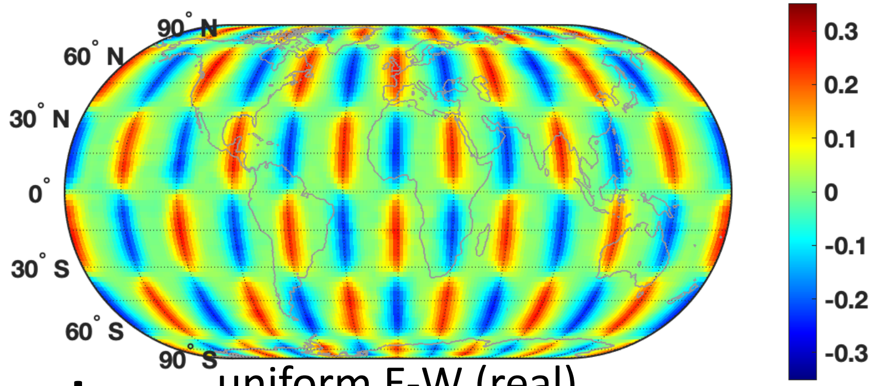
uniform E-W



$\partial_x B_x + \partial_y B_y$

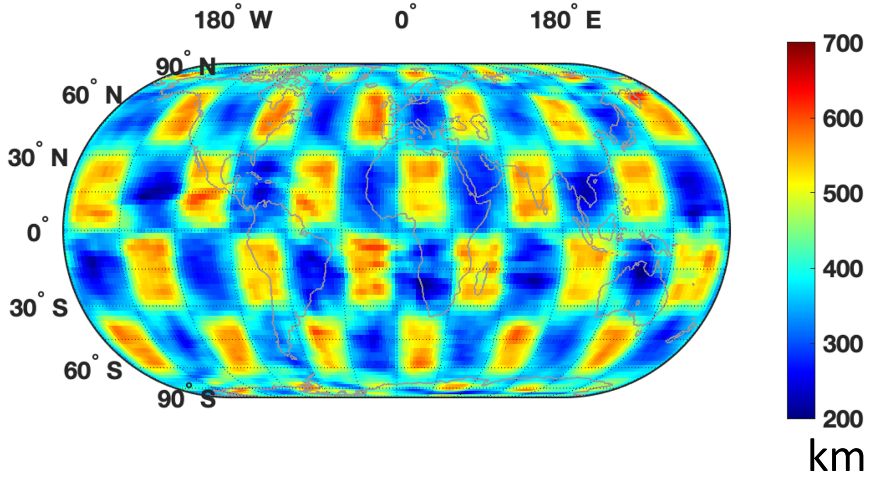


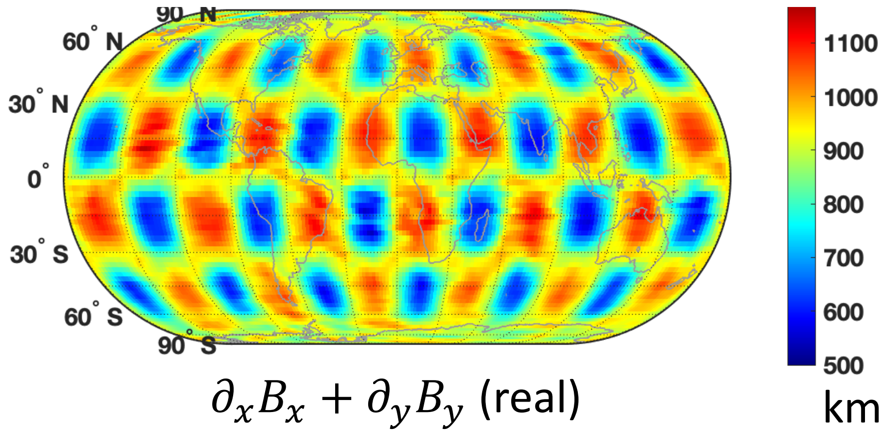
uniform N-S (real)



uniform E-W (real)

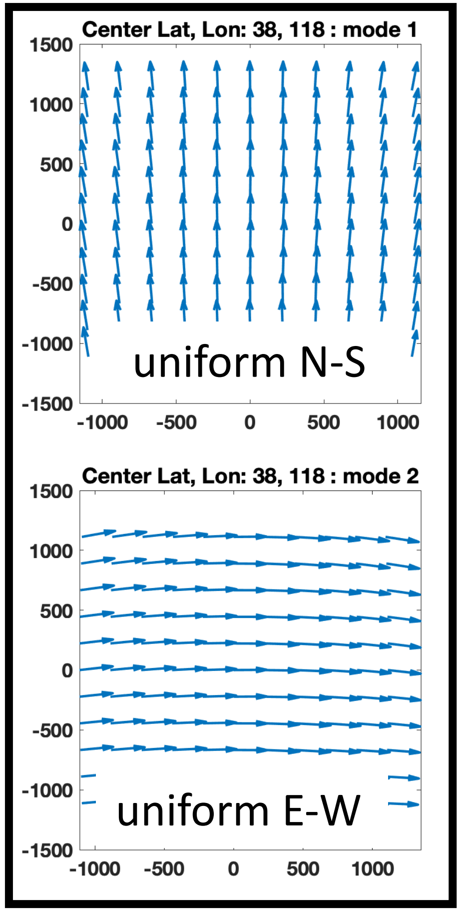
2 cpd

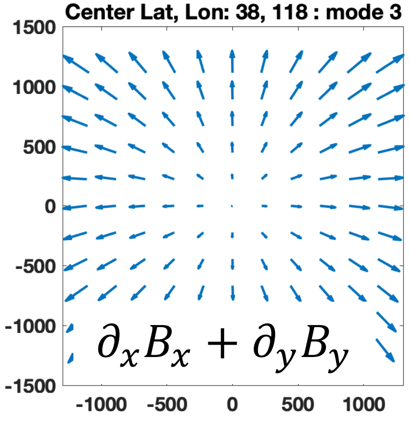




$\partial_x B_x + \partial_y B_y$ (real)

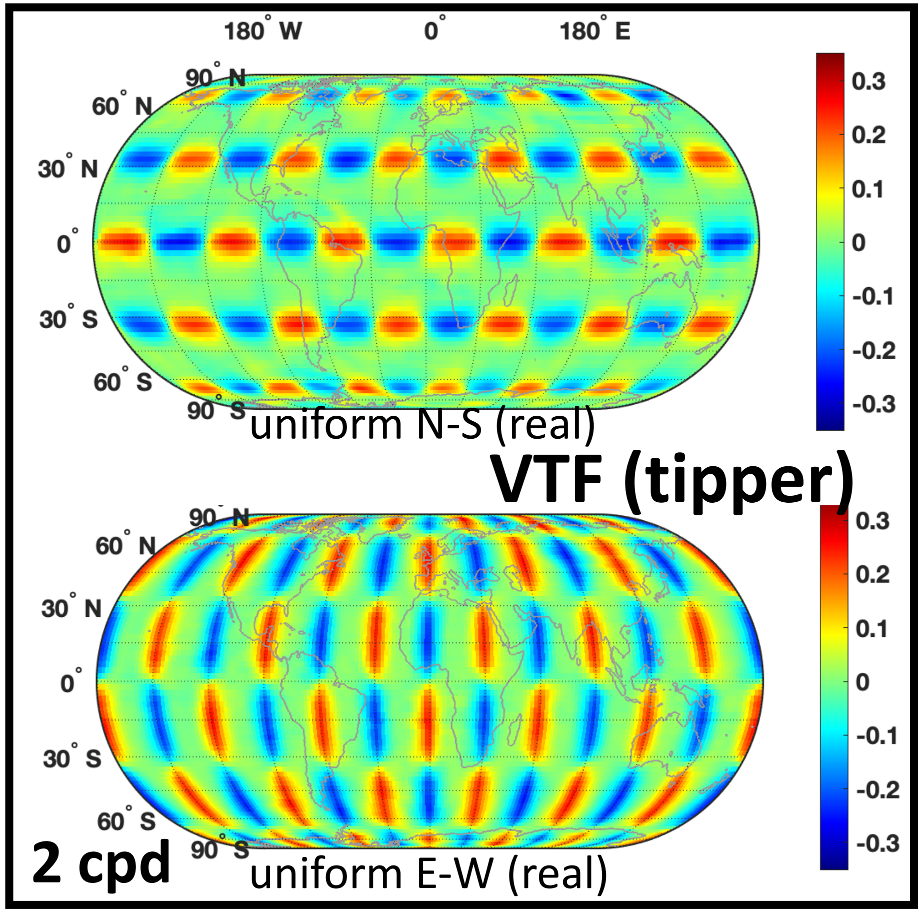
Sorting fitted modes locally, using TF ideas

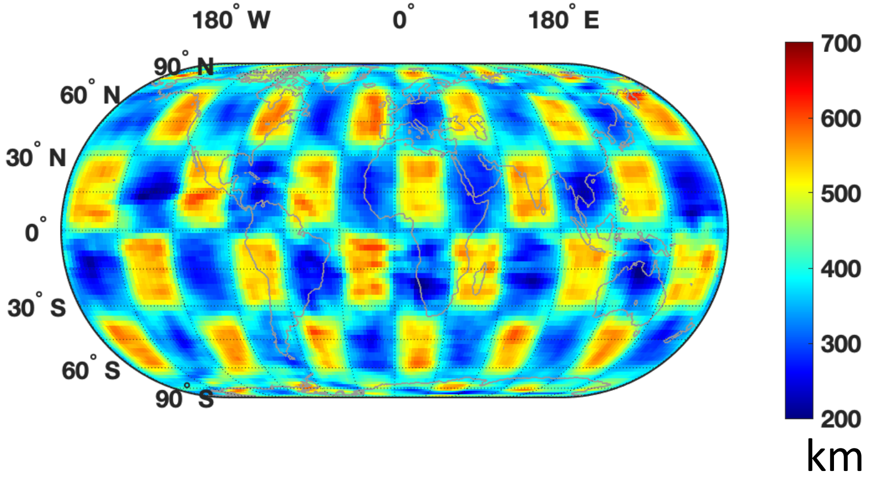


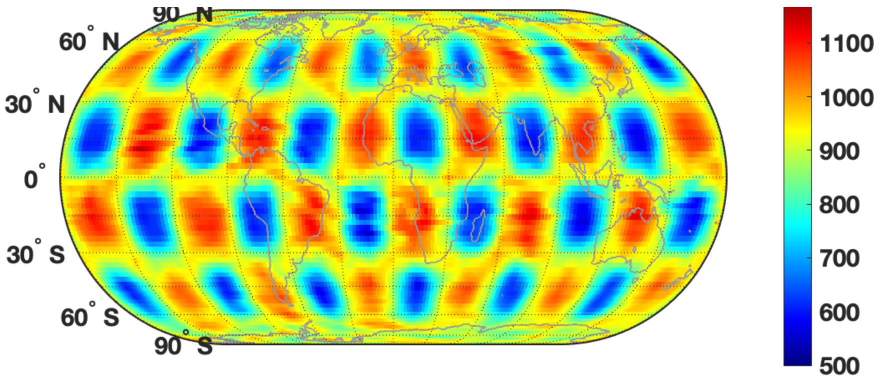


At each location

- form linear combination of all 20 modes (B_x,B_y only, total fields) that best approximate idealized uniform N-S, E-W + canonical gradients within a 20°× 20° patch
- for each linear combination plot B_z at center of local patch

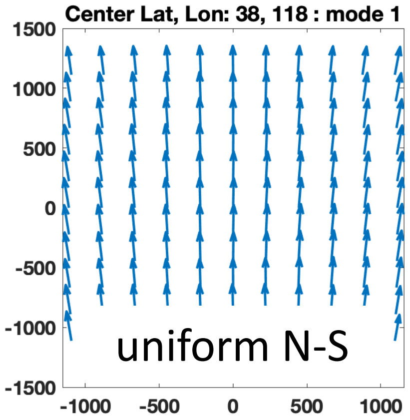




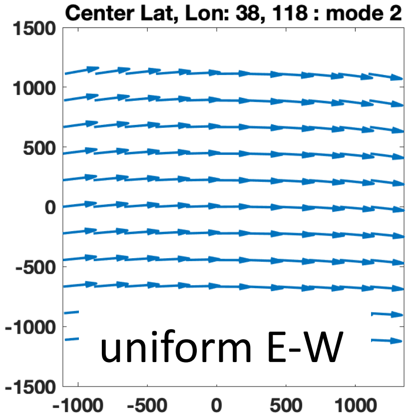


$\partial_x B_x + \partial_y B_y$ (real)

Sorting fitted modes locally, using TF ideas



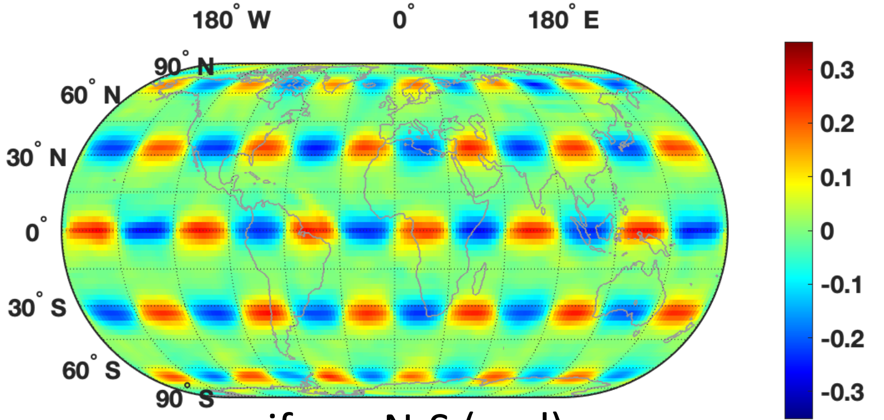
uniform N-S



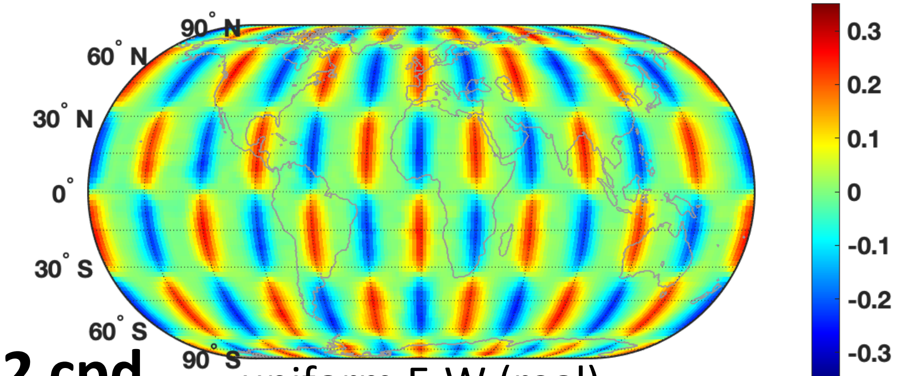
uniform E-W

At each location

- form linear combination of all 20 modes (B_x,B_y only, total fields) that best approximate idealized uniform N-S, E-W + canonical gradients within a 20˚× 20˚ patch
- for each linear combination plot B_z at center of local patch

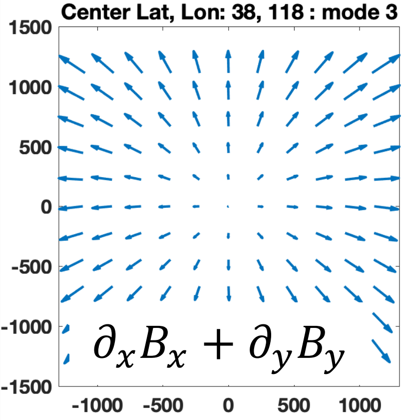


uniform N-S (real)



2 cpd

uniform E-W (real)

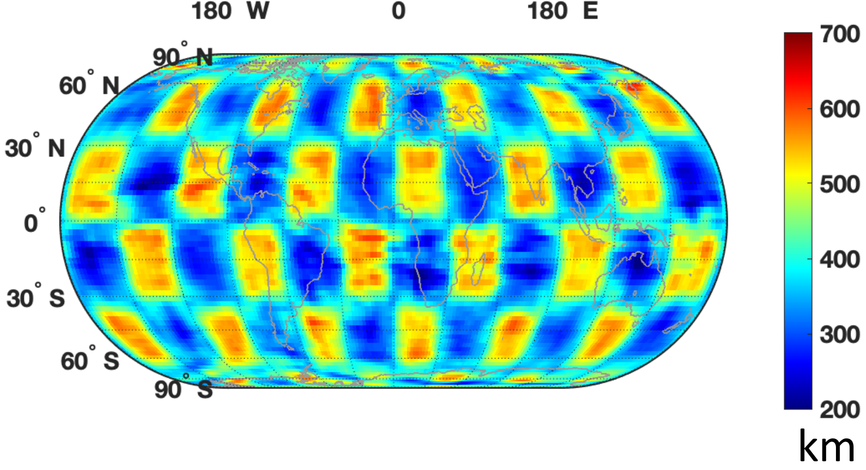


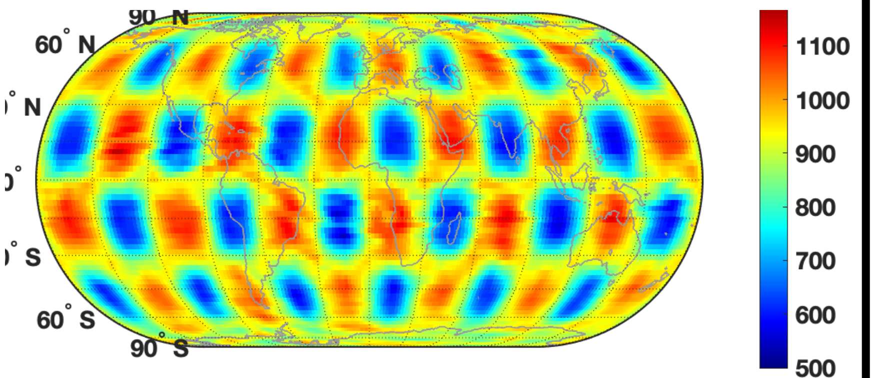
$\partial_x B_x + \partial_y B_y$

HSG

TF

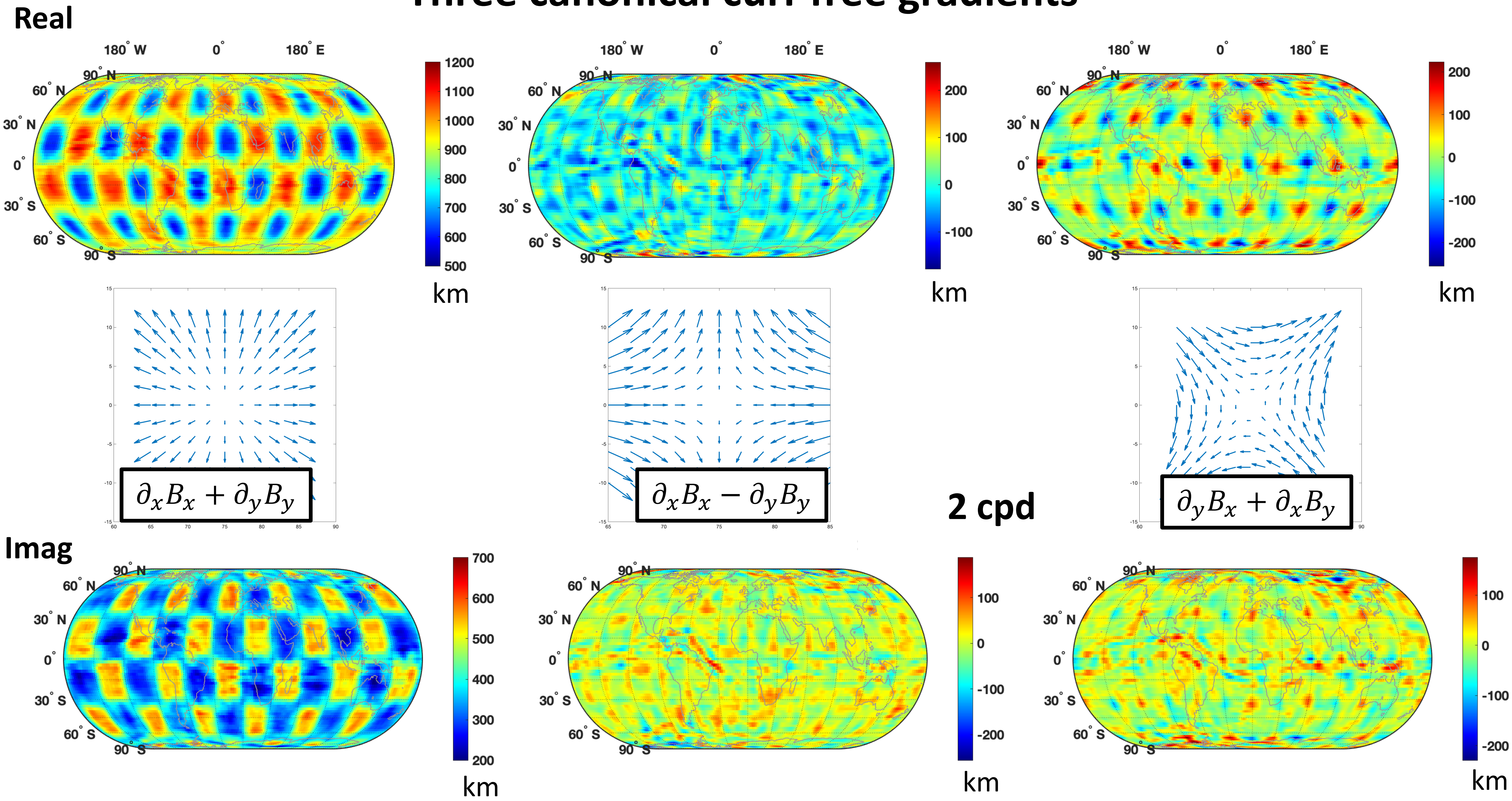
$\boldsymbol{C(\omega)}$



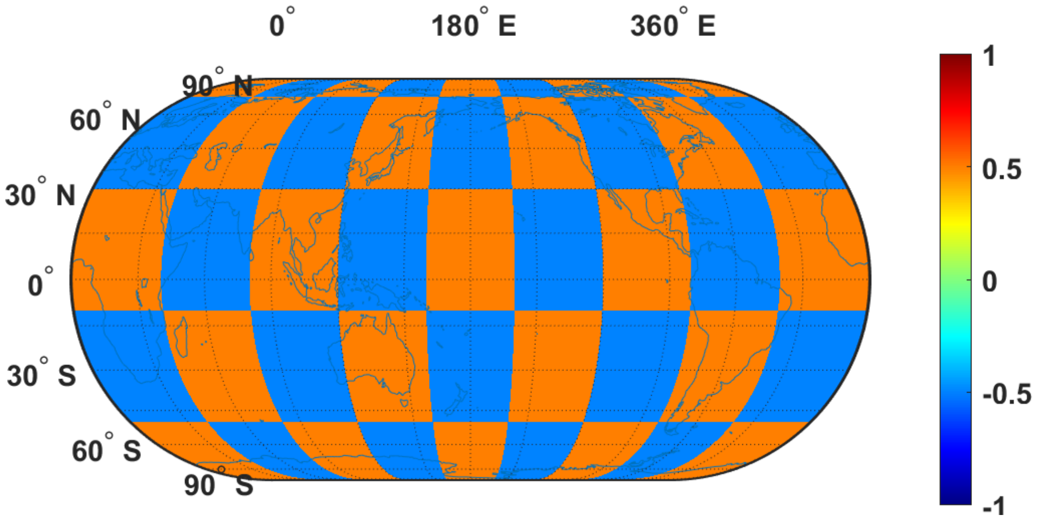


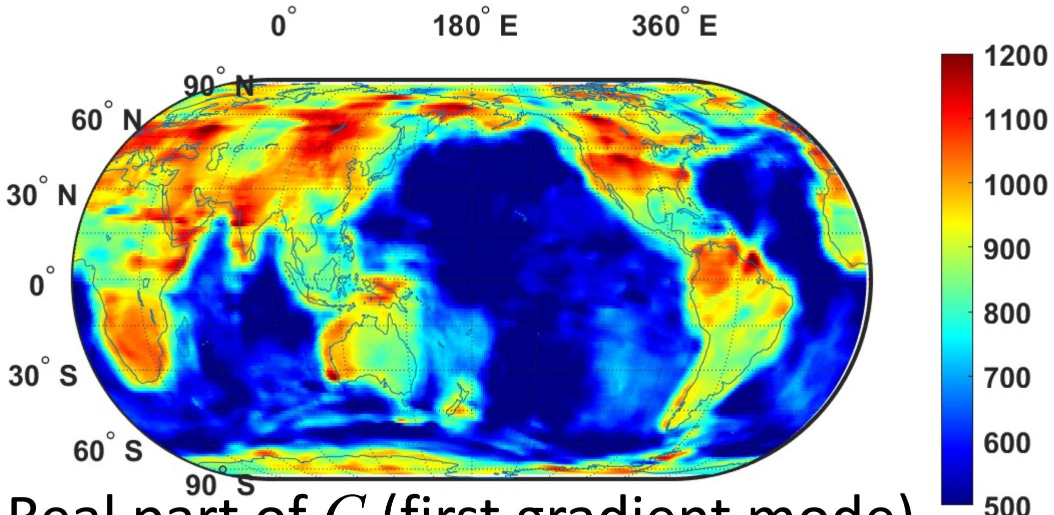
$\partial_x B_x + \partial_y B_y$ (real)

Three canonical curl-free gradients



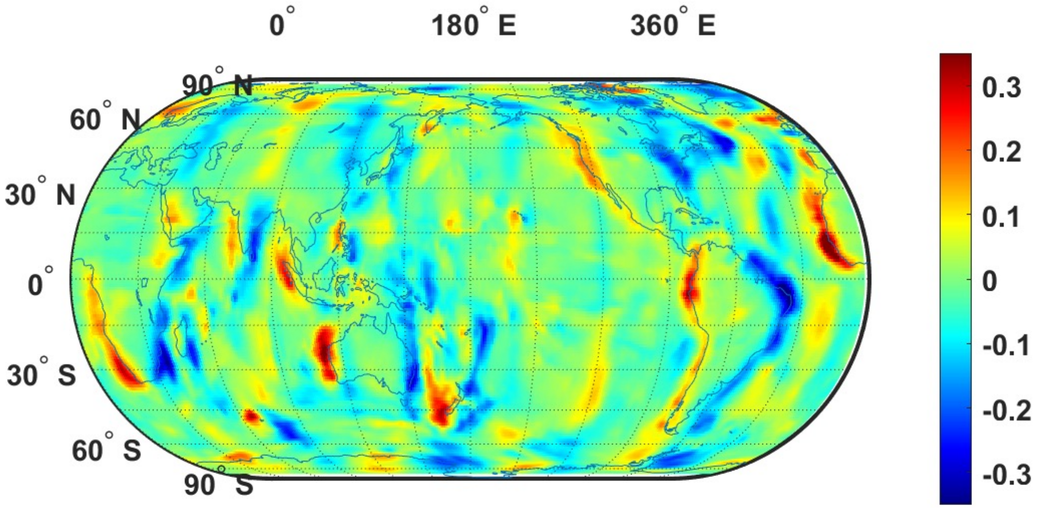
Another synthetic test – checkerboard layer ± half order of magnitude variation at 250-410 km depth, ocean/continent layer included



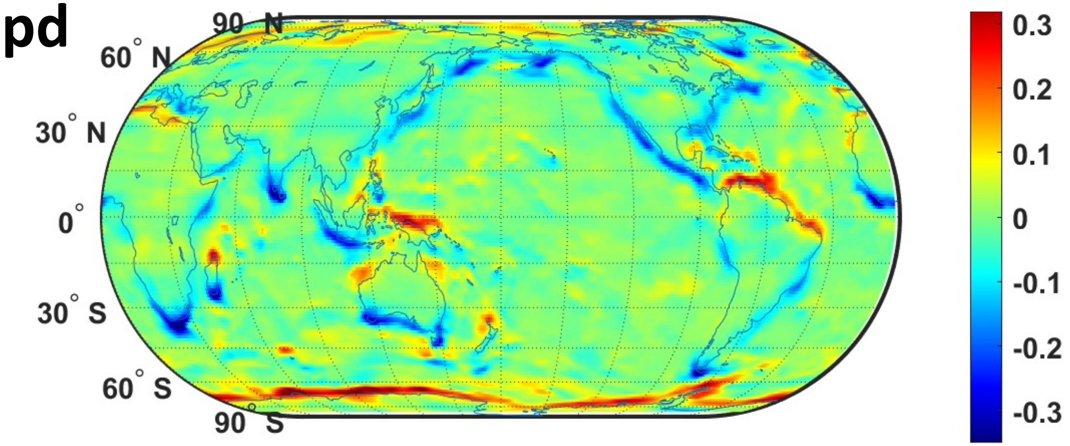


Real part of C (first gradient mode)

1 cpd



Real part of T_y (uniform E-W B_y)



Imag part of T_x (uniform N-S B_x)

Note – this PW + G separation may make 1D inversion of GDS more useful

But I would still advocate full 3D inversion (why not, we can)

Source Models at Continental Scale

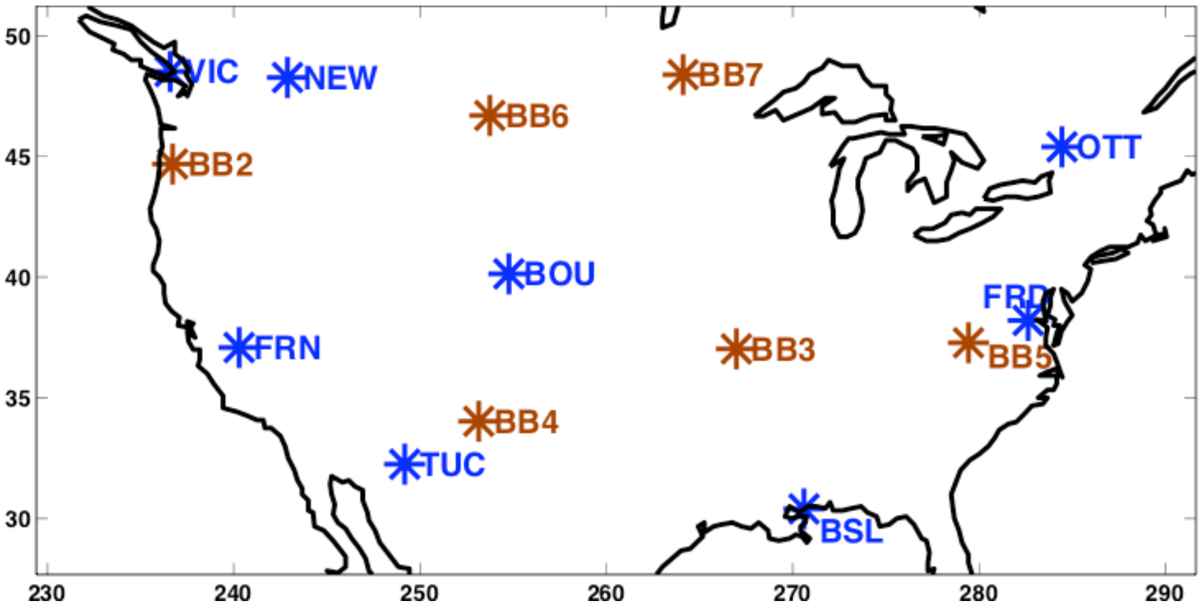
- at mid-latitudes, simpler models likely are sufficient:
  Uniform + Gradient (+ field aligned currents)
  captures most of the variance

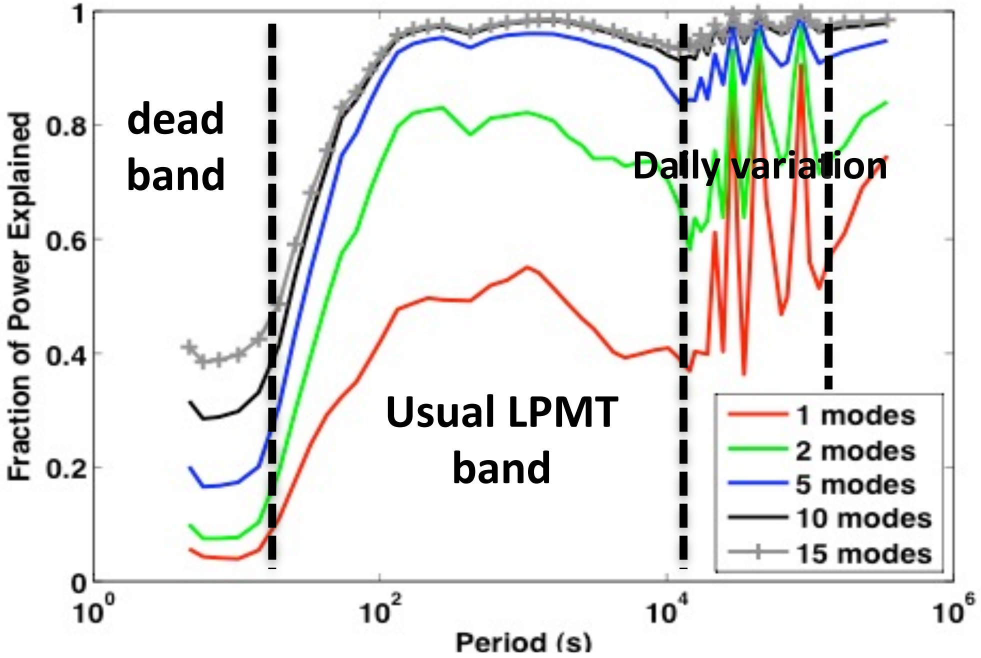
- Embed in global source model to reduce boundary effects …

Example: 14 site array in CONUS:

→ **Geomagnetic Observatories** (nearly continuous sampling)

→ **Earthscope "backbone" MT** (multi-year occupations, but many large gaps)



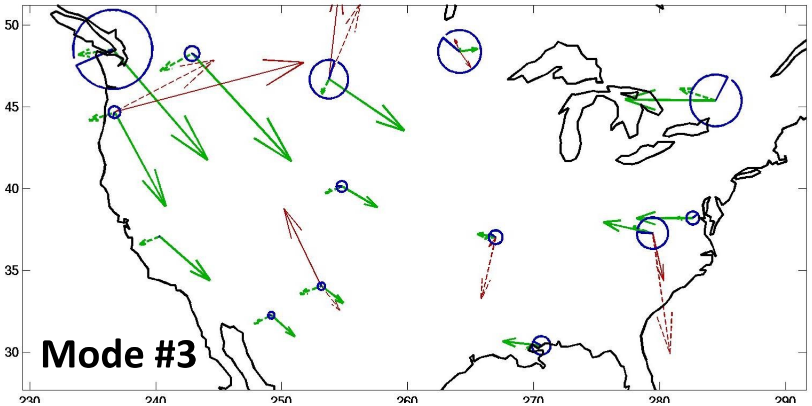


5 modes explain most variance (many more required for global)

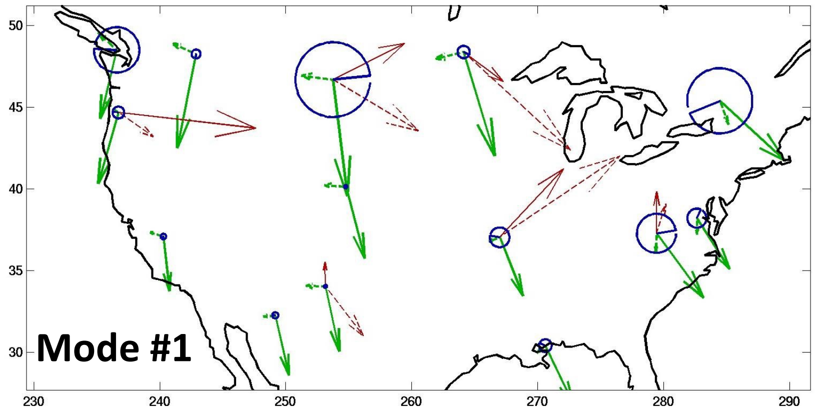
First 5 spatial modes:
T = 1000 s

(H_x, H_y) H_z (E_x, E_y)

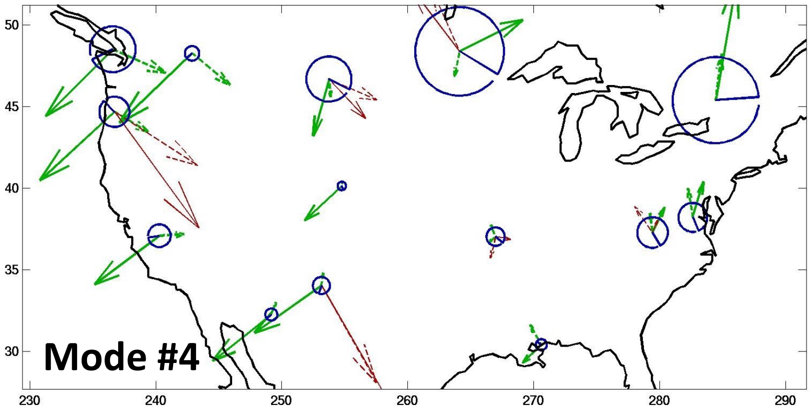
OBS$_B$BY Mode 3 : Period 1059 sec



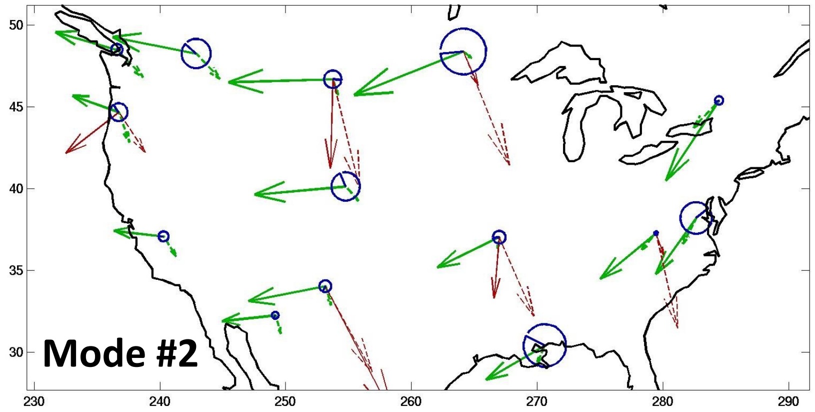
OBS$_B$BY Mode 1 : Period 1059 sec



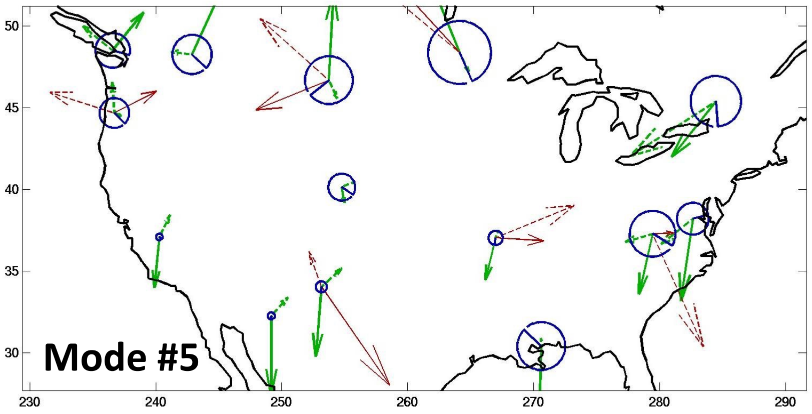
OBS$_B$BY Mode 4 : Period 1059 sec



OBS$_B$BY Mode 2 : Period 1059 sec



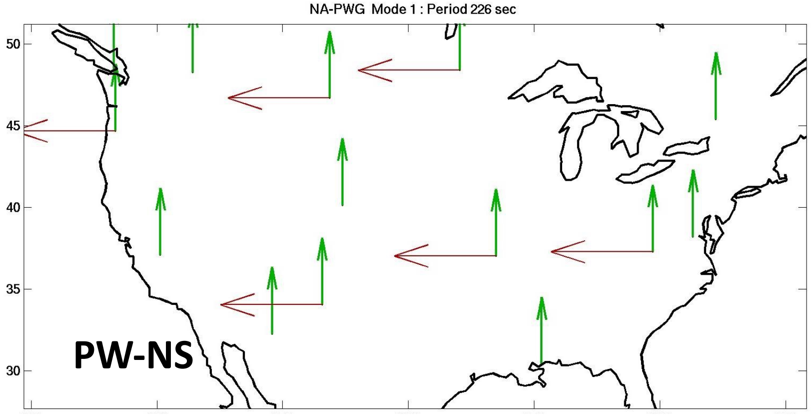
OBS$_B$BY Mode 5 : Period 1059 sec

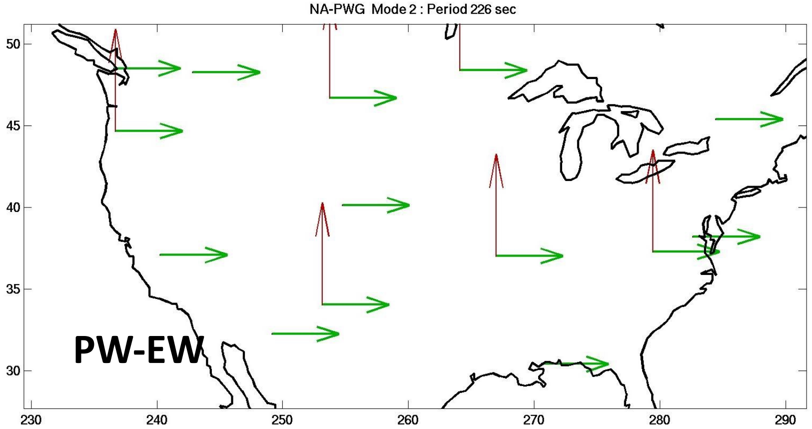


Taylor series approximation of magnetic fields:

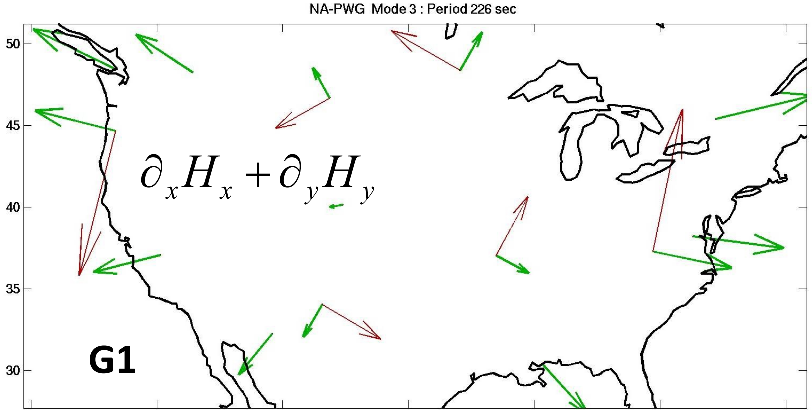
idealized spatial basis functions over a 1D Earth

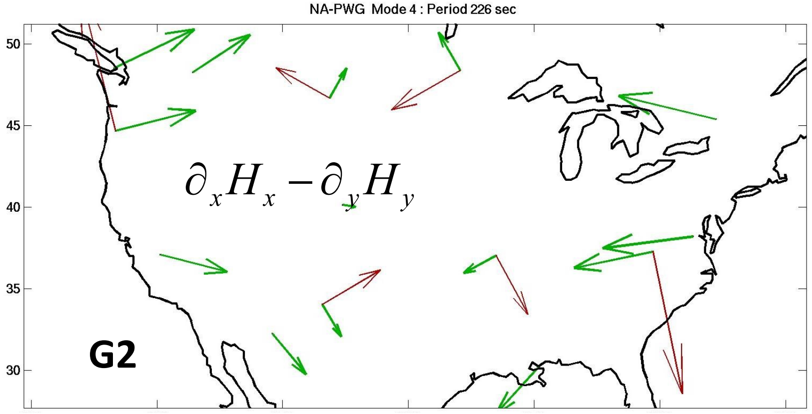
Uniform (plane-wave)

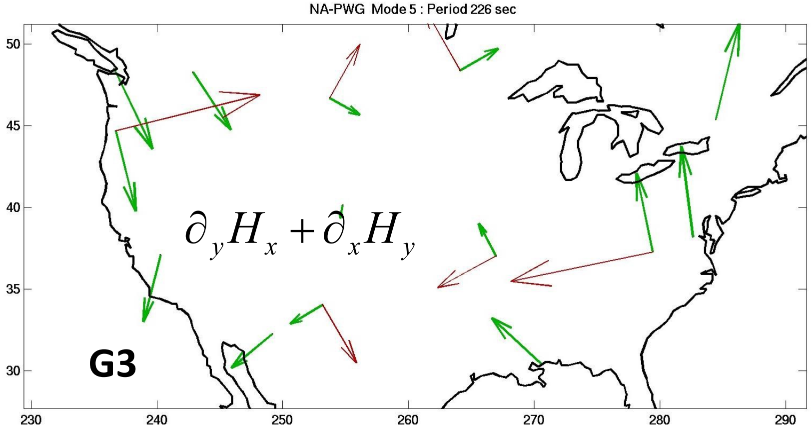




Curl-free gradients



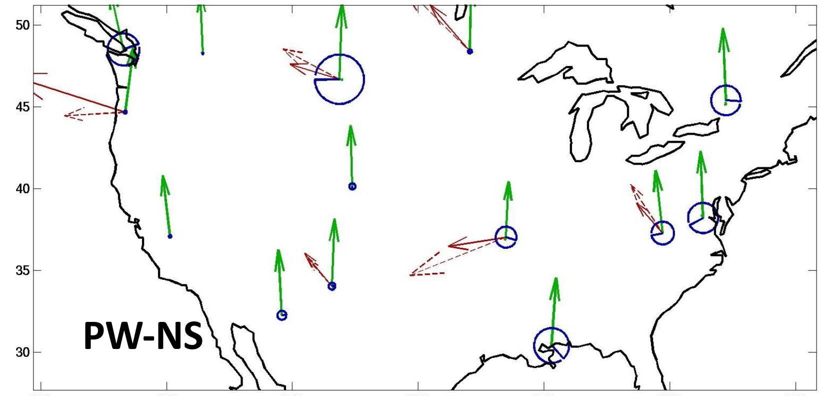




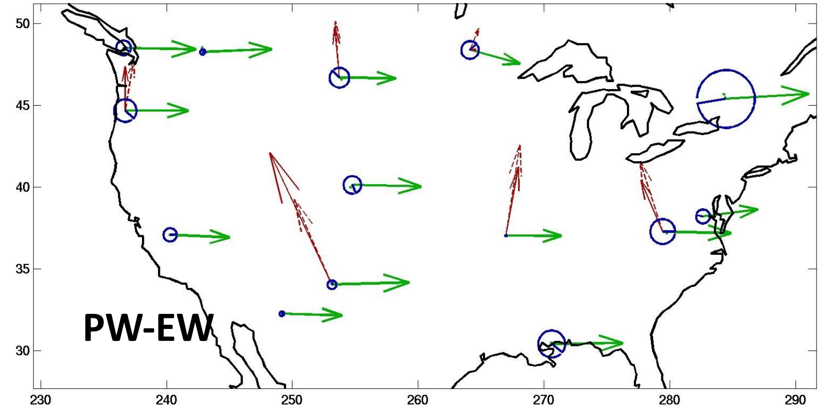
“Sort” estimated modes: find linear combinations of leading modes (8 used here) that best resemble the idealized form

Uniform (plane-wave)

OBS-BB-PW Mode 1 : Period 1059 sec

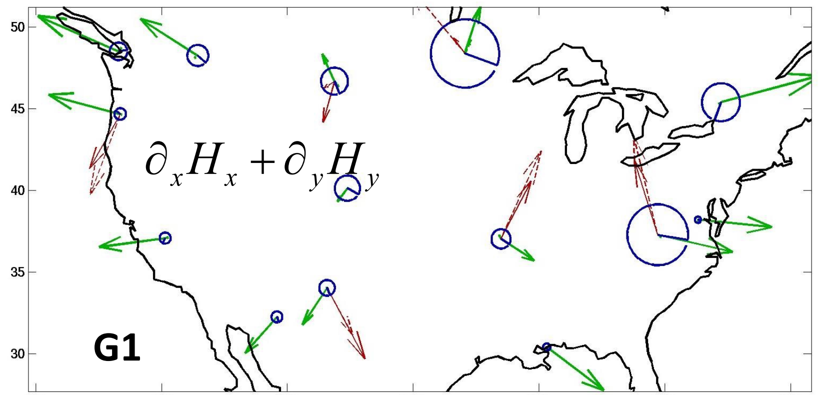


OBS-BB-PW Mode 2 : Period 1059 sec

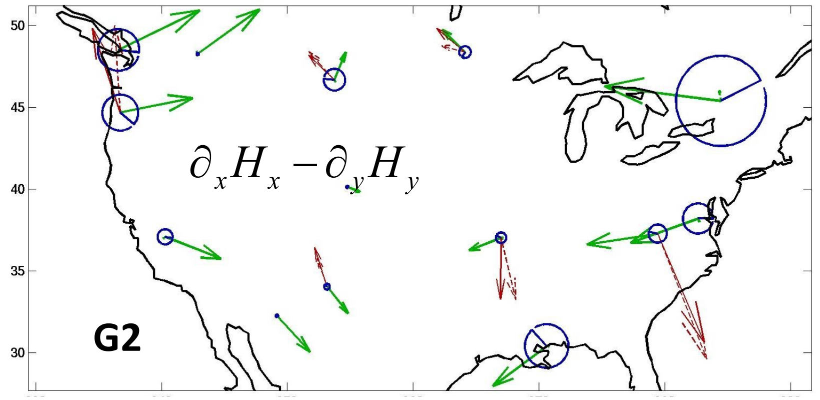


Curl-free gradients

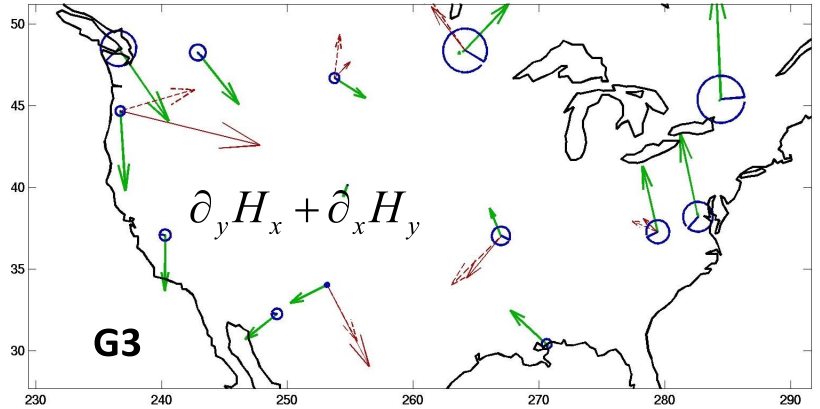
OBS-BB-PW Mode 3 : Period 1059 sec



OBS-BB-PW Mode 4 : Period 1059 sec



OBS-BB-PW Mode 5 : Period 1059 sec

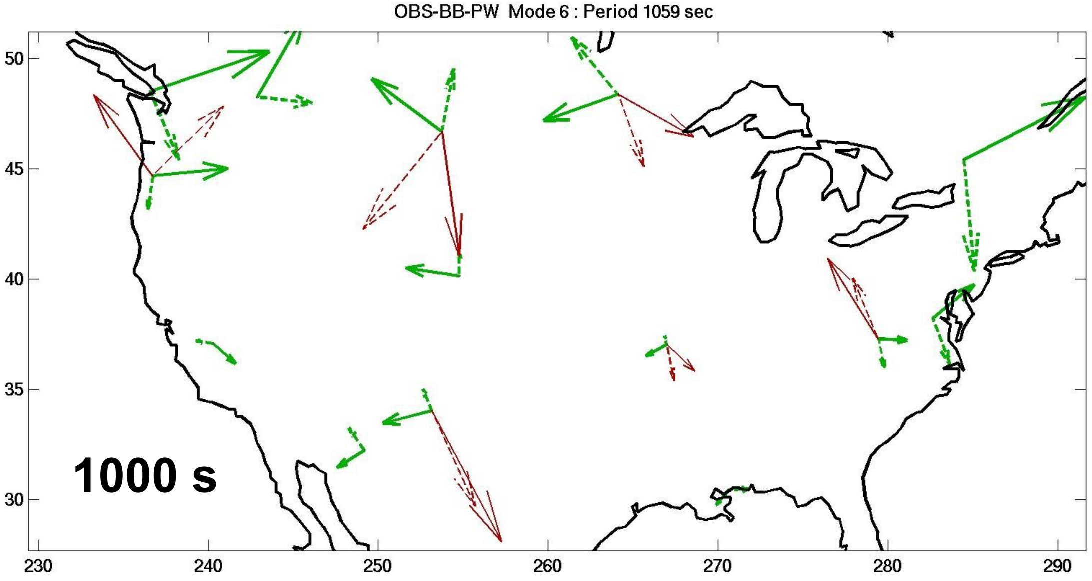


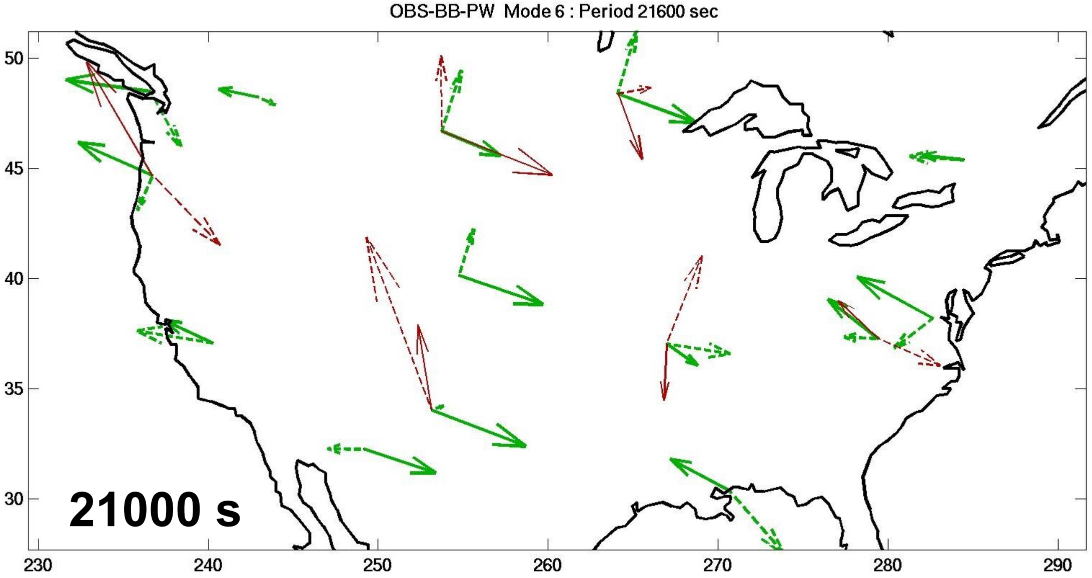
"Mode 6" (and 7)

Spatial pattern with maximum variance after fitting PW and gradient components:

Shorter meridional wavelength FAC components

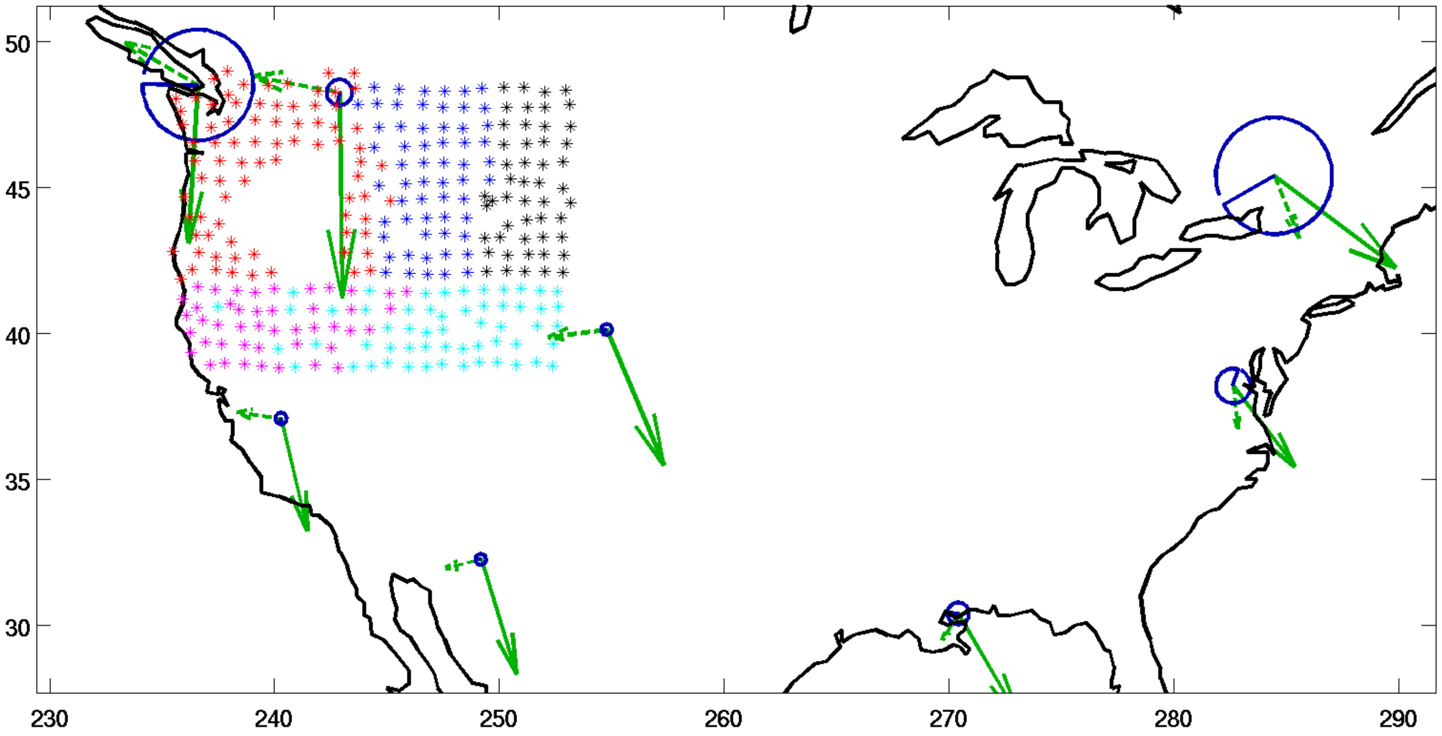
Similar for most periods





Thus: Use observatory data to define dominant modes of source variability in CONUS:

- → 8 observatories, 2007-2014
- → 1 hz data (subsampled at 10 s)
- → model source as uniform + gradients (+field aligned currents?)



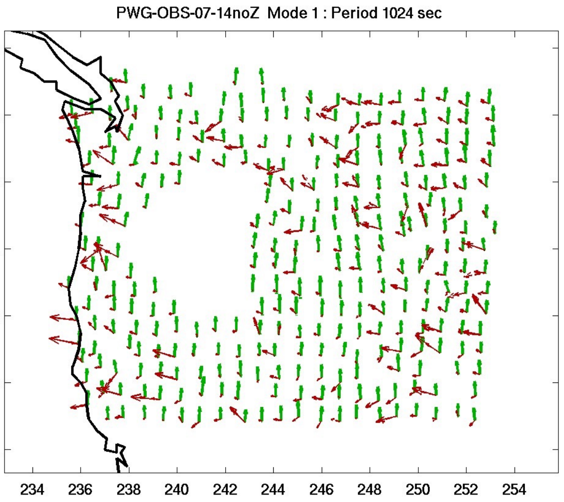
Can combine with EarthScope data (shown here for 5 field seasons: 2007-2011)

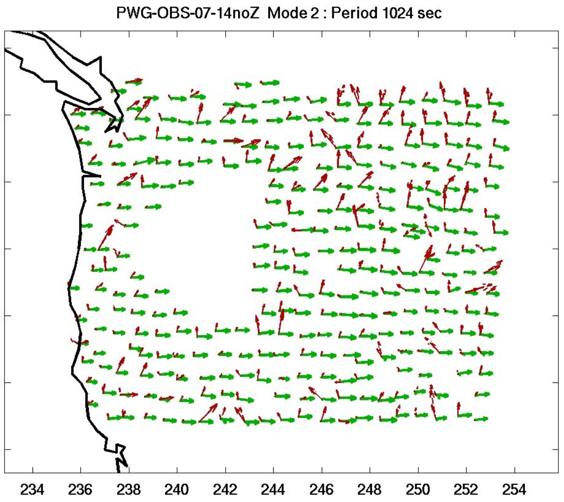
Using this source model, estimate “hypothetical events” – magnetic fields that would be seen for each of the source spatial modes

Green: Horizontal Magnetics

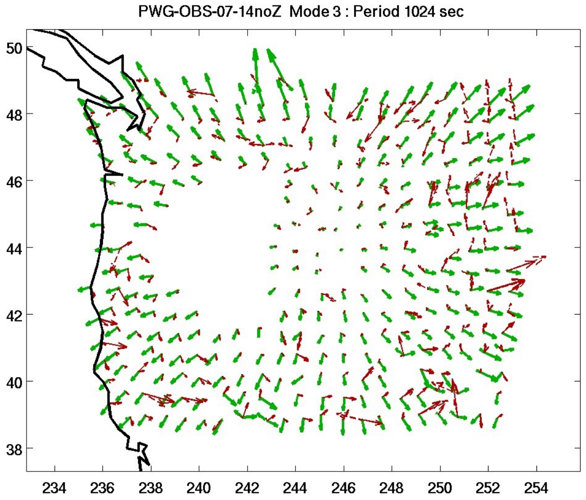
Red Electrics

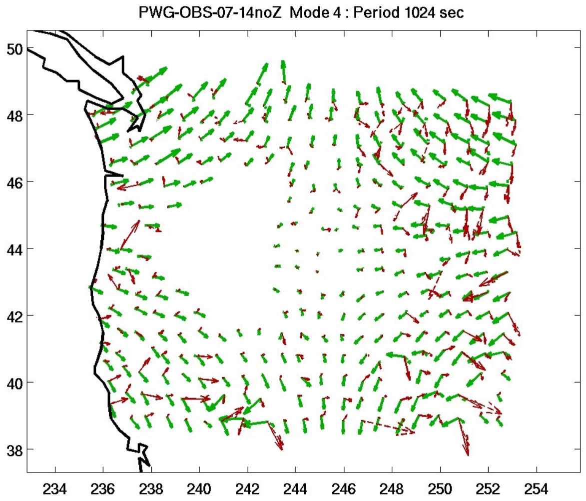
Solid: in phase
Dash: quadrature

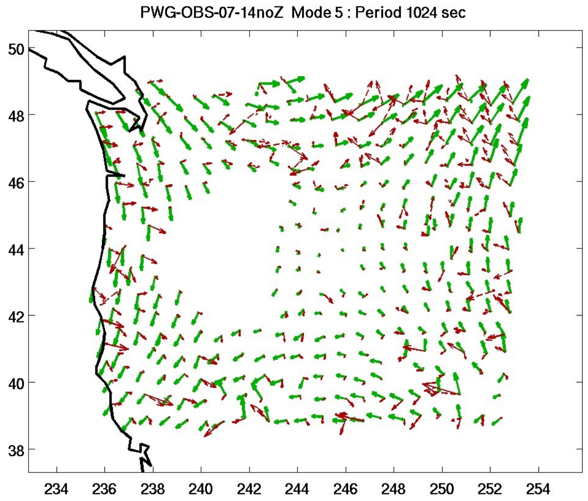

PWG-OBS-07-14noZ Mode 1 : Period 1024 sec


PWG-OBS-07-14noZ Mode 2 : Period 1024 sec

Estimated plane wave (upper) and gradient modes (lower)


PWG-OBS-07-14noZ Mode 3 : Period 1024 sec


PWG-OBS-07-14noZ Mode 4 : Period 1024 sec


PWG-OBS-07-14noZ Mode 5 : Period 1024 sec

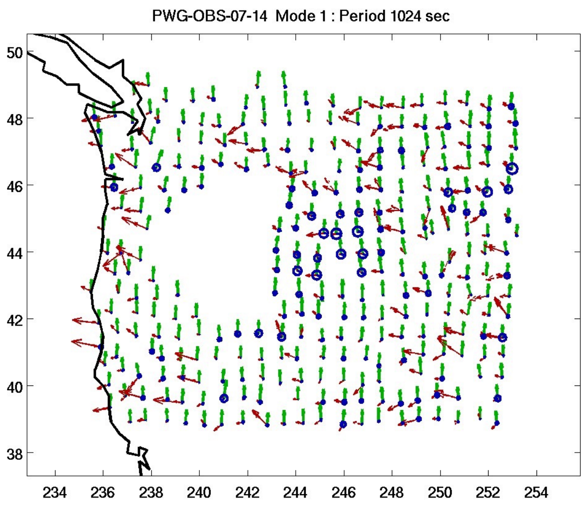
Using this source model, estimate “hypothetical events” – magnetic fields that would be seen for each of the source spatial modes

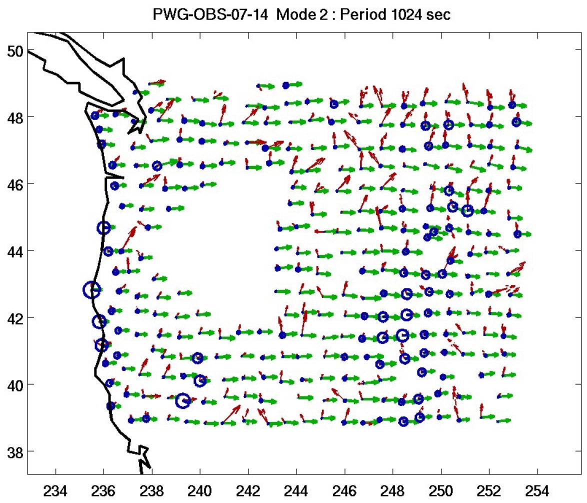
Green: Horizontal Magnetics

Red Electrics

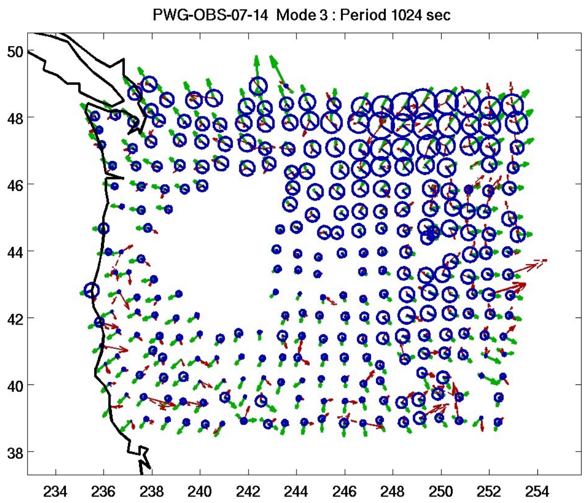
Blue: vertical magnetics

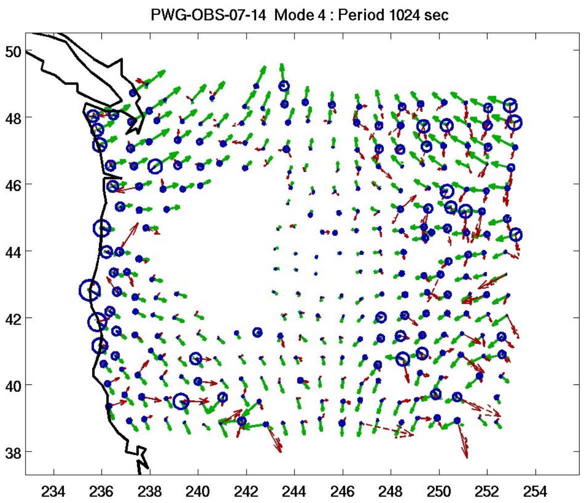
Solid: in phase
Dash: quadrature

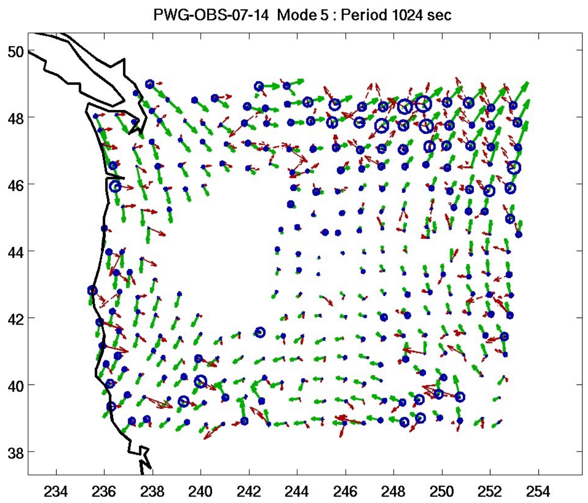

PWG-OBS-07-14 Mode 1 : Period 1024 sec


PWG-OBS-07-14 Mode 2 : Period 1024 sec

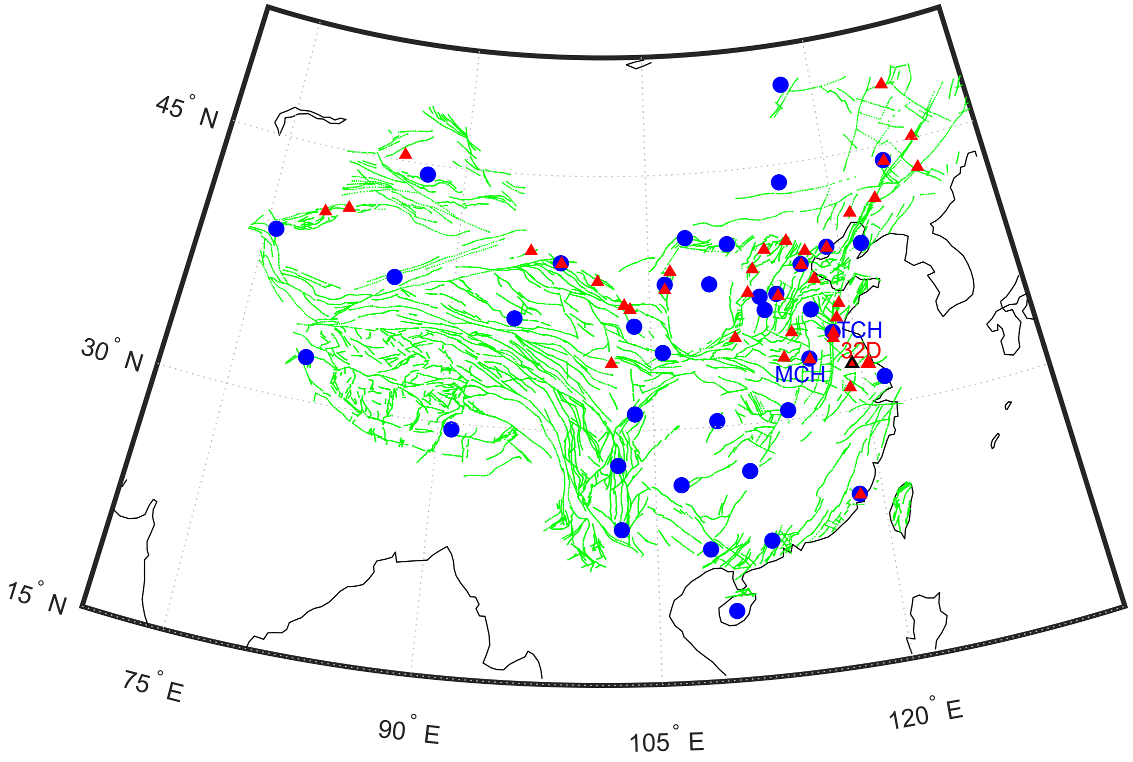
Estimated plane wave (upper) and gradient modes (lower)


PWG-OBS-07-14 Mode 3 : Period 1024 sec

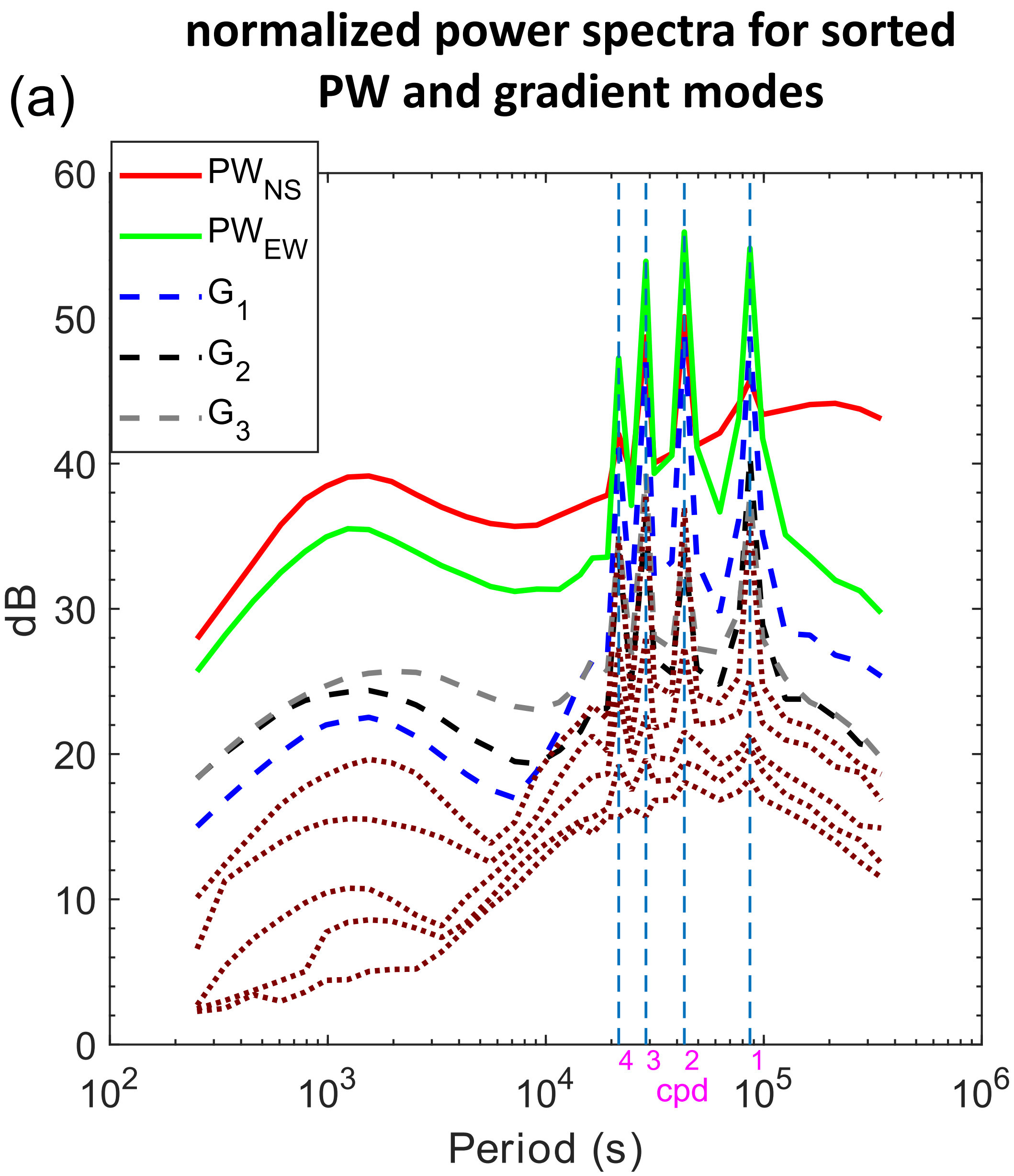

PWG-OBS-07-14 Mode 4 : Period 1024 sec


PWG-OBS-07-14 Mode 5 : Period 1024 sec

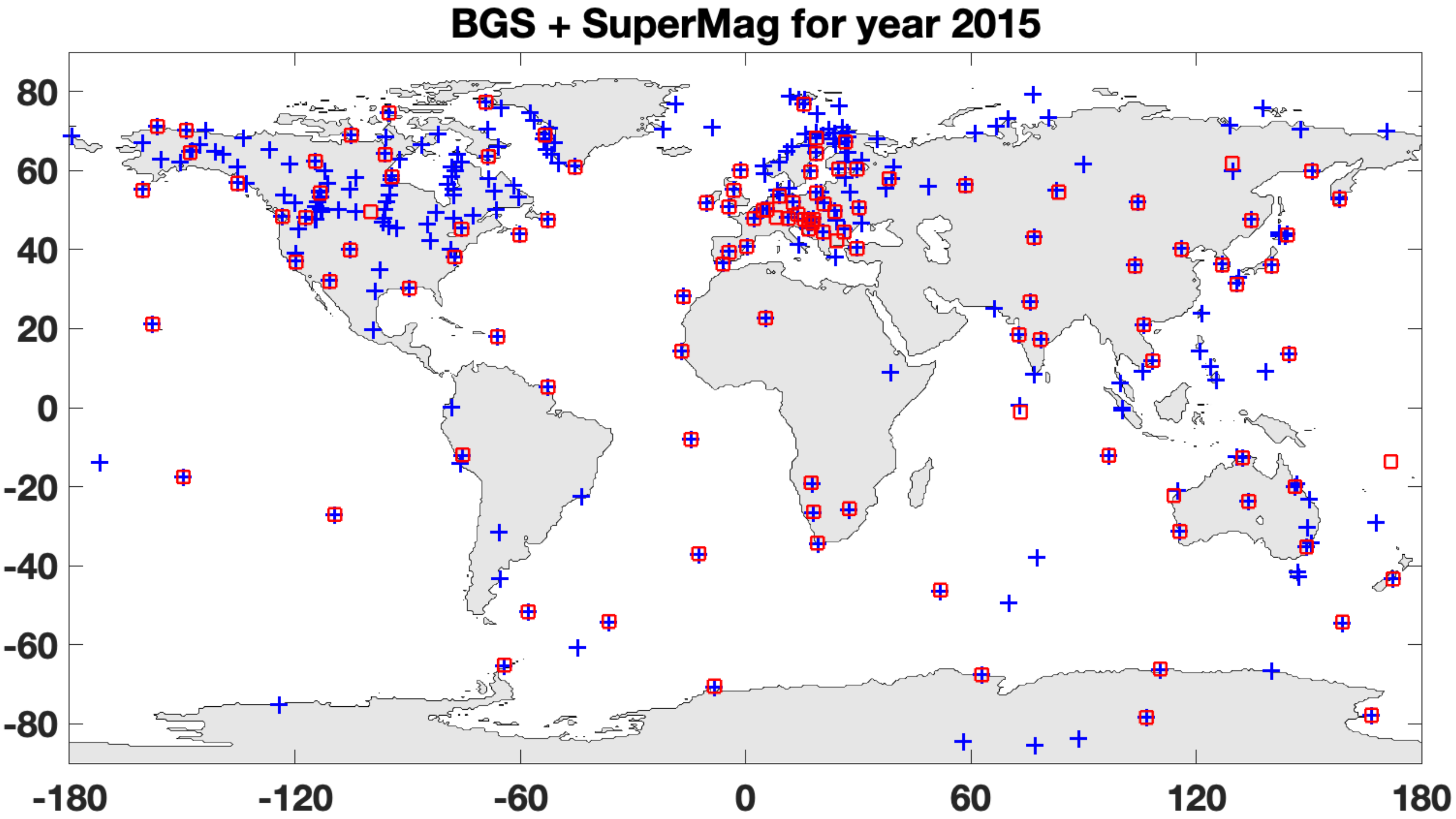
Similar results are obtained from an array in China: 10 years of magnetometer and electrometer data



(with Hui Wang; data from China Earthquake Administration)


normalized power spectra for sorted PW and gradient modes

Long period geomagnetic data collected for space physics research is now easily available from SuperMag (https::/supermag.jhuapl.edu)



sites with 1 min data available:

red circles geomagnetic observatories

blue crosses other sites (variometers)

Incorporating satellite data: DV Model in Time Domain:

$$\mathbf{B}(\mathbf{r},t) = \mathrm{Re}\sum_{kl}\alpha_{kl}(t)\sum_{i}\beta_{kli}\Phi_{li}(\mathbf{r})$$

Temporal variations for mode k, frequency band l

Expansion coefficients ($i = 1,I$) frequency band l, data mode k

Model spatial modes ($i = 1,I$) frequency band l

In this form the model is suitable for fitting time domain (Swarm, CHAMP) data directly

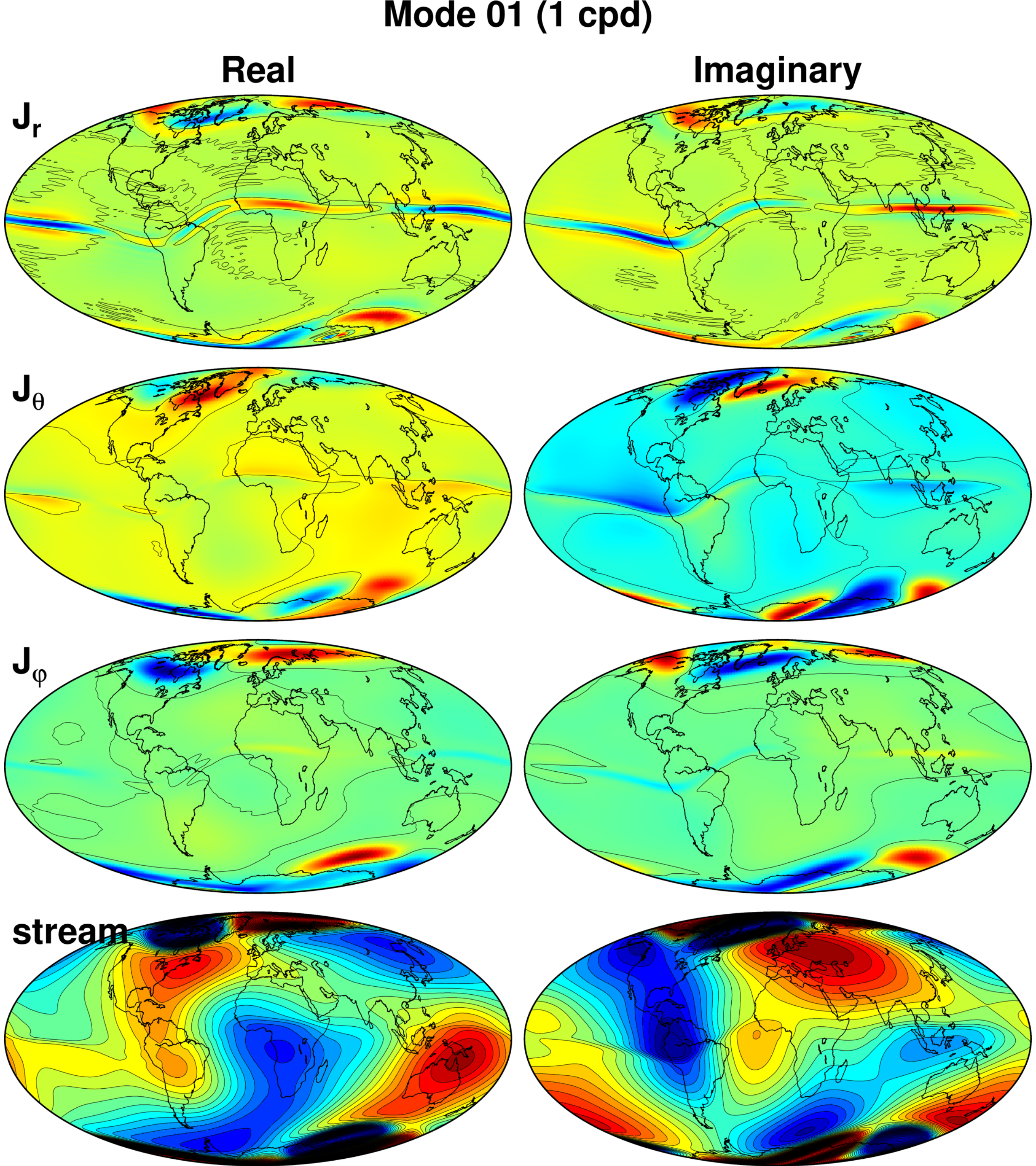
→ temporal modes $\alpha(t)$ from ground data

→ (3D) spatial modes $\Phi(\mathbf{r})$ from TIEGCM

→ estimate expansion coefficients β from satellite (and ground) data

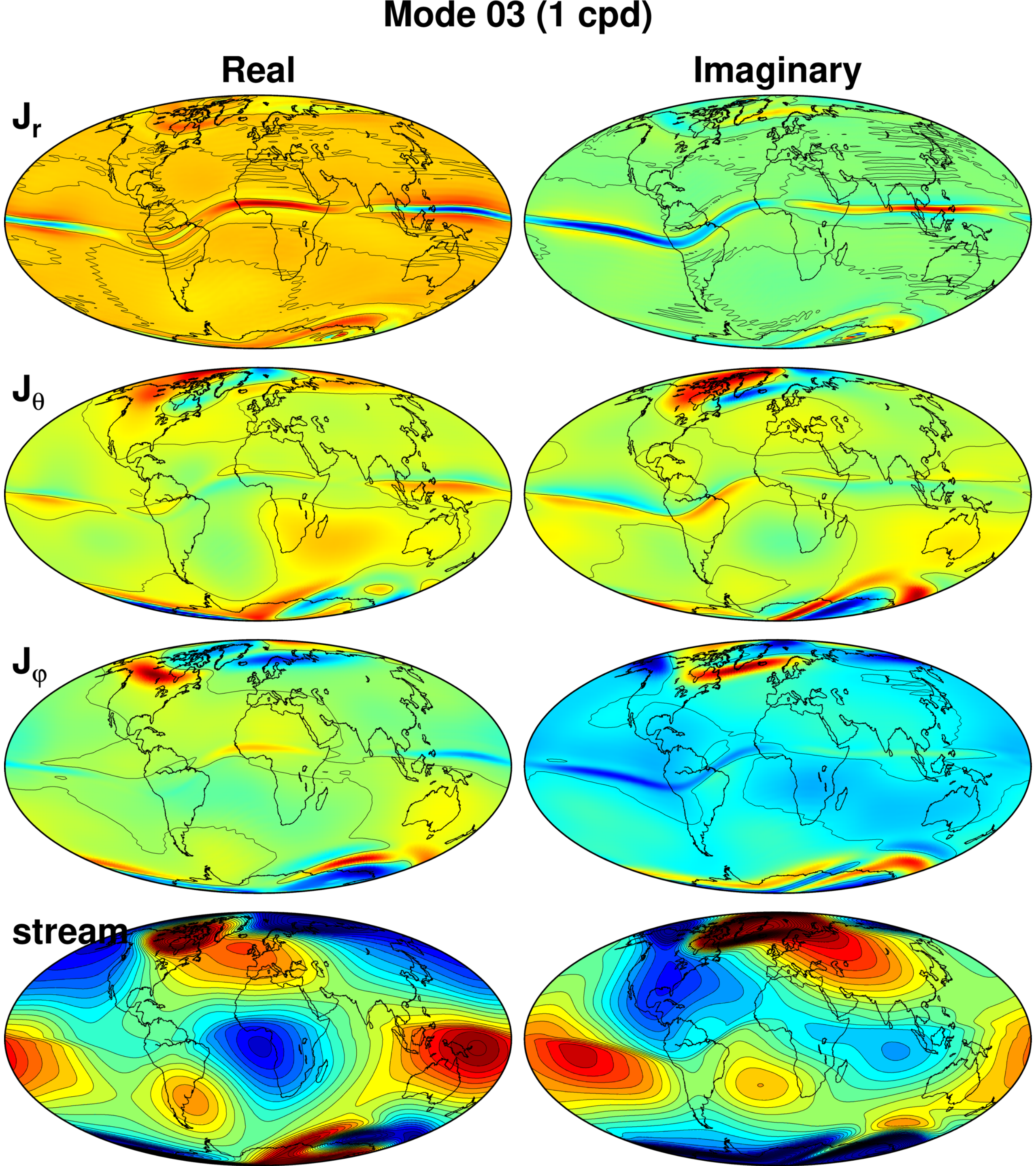
Incorporating satellite data: Requires PCA of 3D model outputs Yields a 3D model of ionospheric currents (figures from P. Alken)

mode 1 at 1 cpd (110km)



Incorporating satellite data: Requires PCA of 3D model outputs Yields a 3D model of ionospheric currents (figures from P. Alken)

mode 3 at 1 cpd (110km)



Ongoing Efforts:

- → Incorporating satellite data (w/ Patrick Alken, A. Maute)
- → Modeling storms for GIC (w/ P. Alken, A. Maute, G. Lu, A. Kelbert, E. Riegler)
- → 3D inversion in DV band (w/ H. Zhang)
- → Modeling long-period variations using TIEGCM for long period ionosphere

Summary and Conclusions

- We can build useful models of external source fields, at both global and regional scales
- These will allow improved use of GDS methods – capable of seeing deeper, and also perhaps providing "distortion free" (TE mode only) constraints on conductivity at large scales
- Combining good source models with TF methods may be fruitful
- Likely many applications of good models of time varying magnetic fields
- Huge potential benefit in working with space physics modelers—but the induction community also has a role to play