

# Surface geometry inversion of geophysical electromagnetic data

Xushan Lu<sup>1</sup>, Chris Galley<sup>2</sup>, Colin Farquharson<sup>1</sup>, Peter Lelièvre<sup>3</sup>

<sup>1</sup>Department of Earth Sciences, Memorial University of Newfoundland, St. John's, NL, Canada

<sup>2</sup>Department of Earth and Environmental Sciences, University of Ottawa, Ottawa, ON, Canada

<sup>3</sup>Department of Mathematics and Computer Science, Mount Allison University, Sackville, NB, Canada

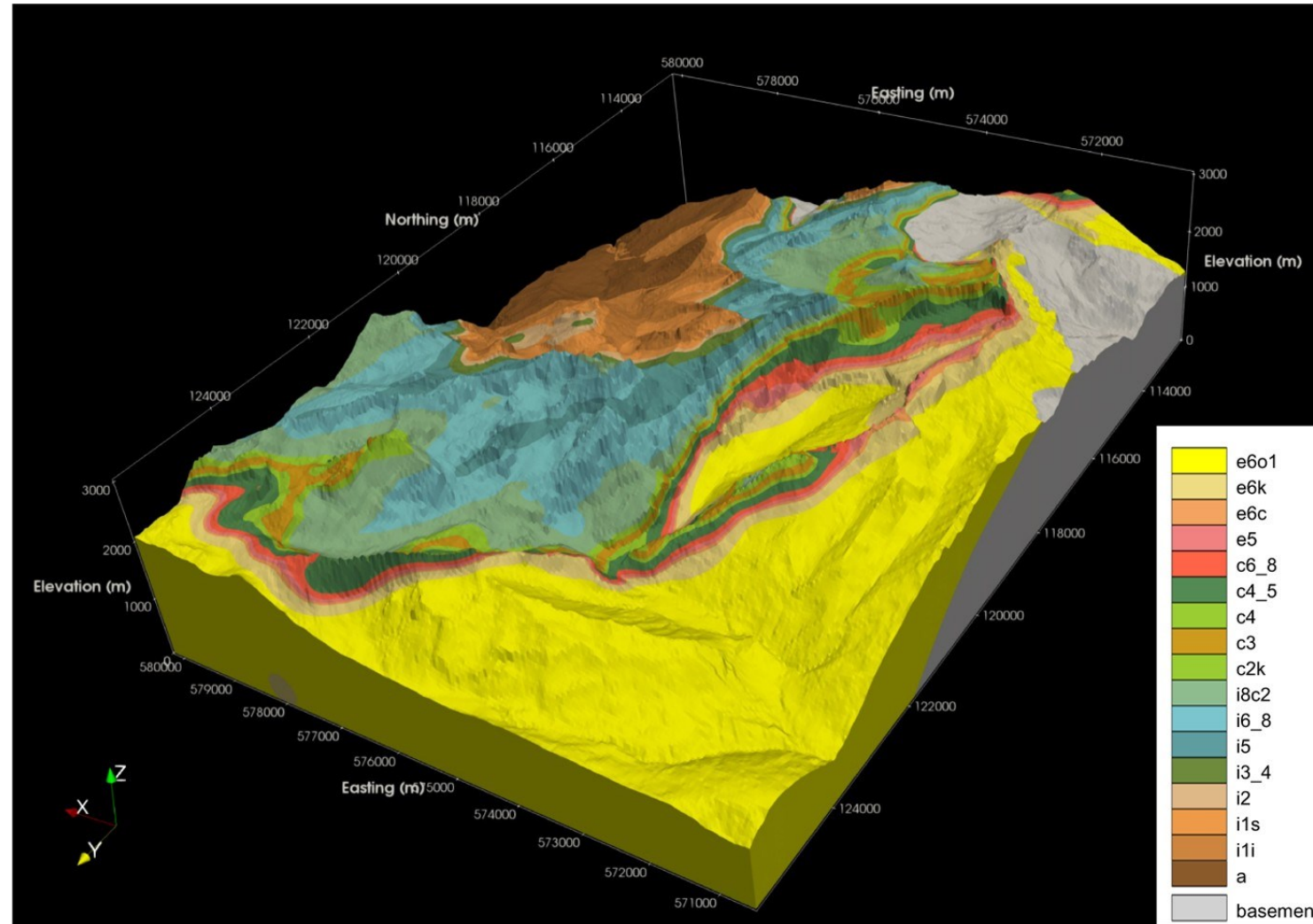
# Outline

- Motivation
- Surface geometry inversion
- Marine CSEM examples
- TEM examples
- Conclusions

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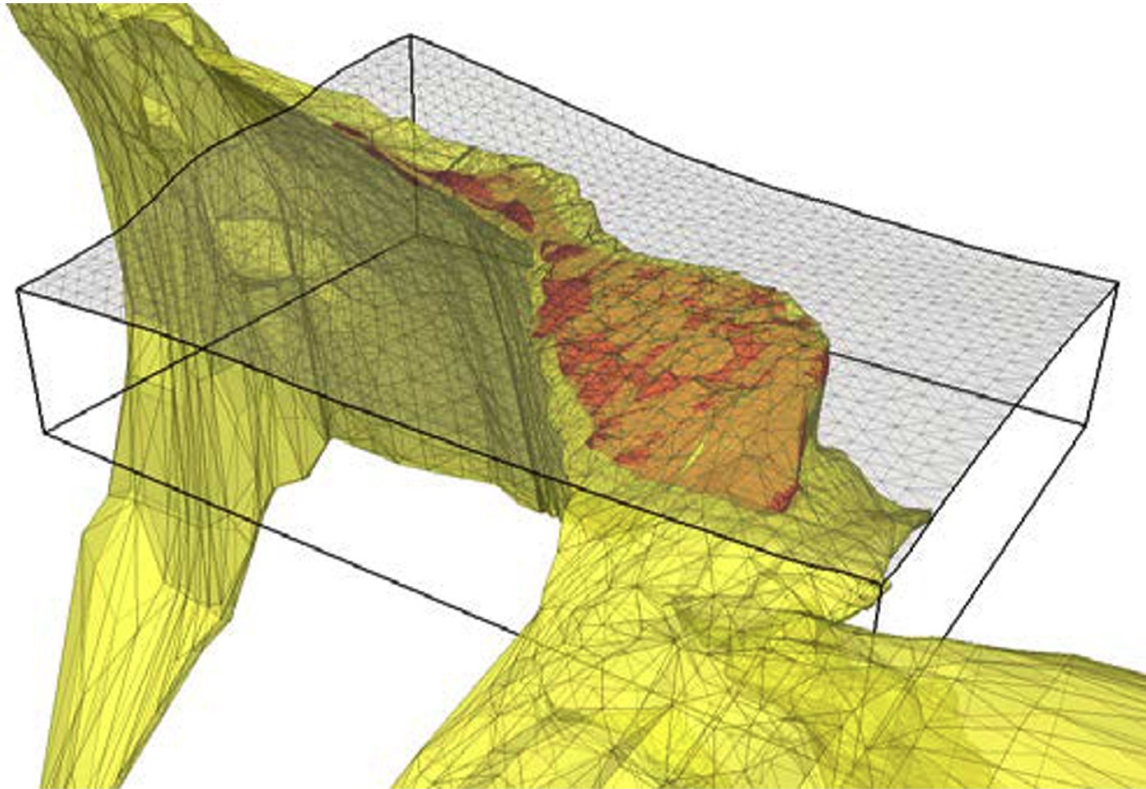
- **Motivation**
- Surface geometry inversion
- Marine CSEM examples
- TEM examples
- Conclusions

# Geological models

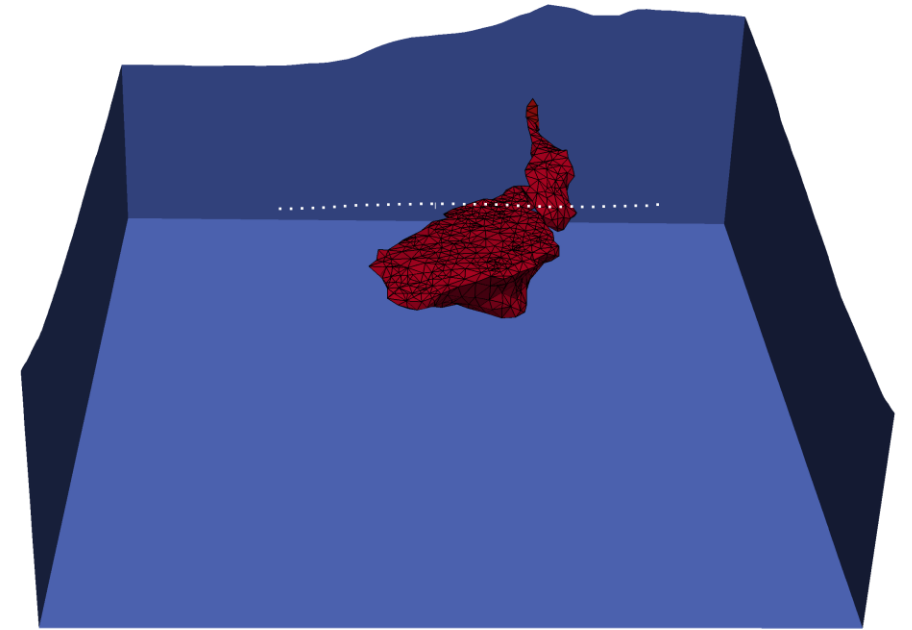


(Thornton et. al., Scientific Data, 2018)

# Geological models

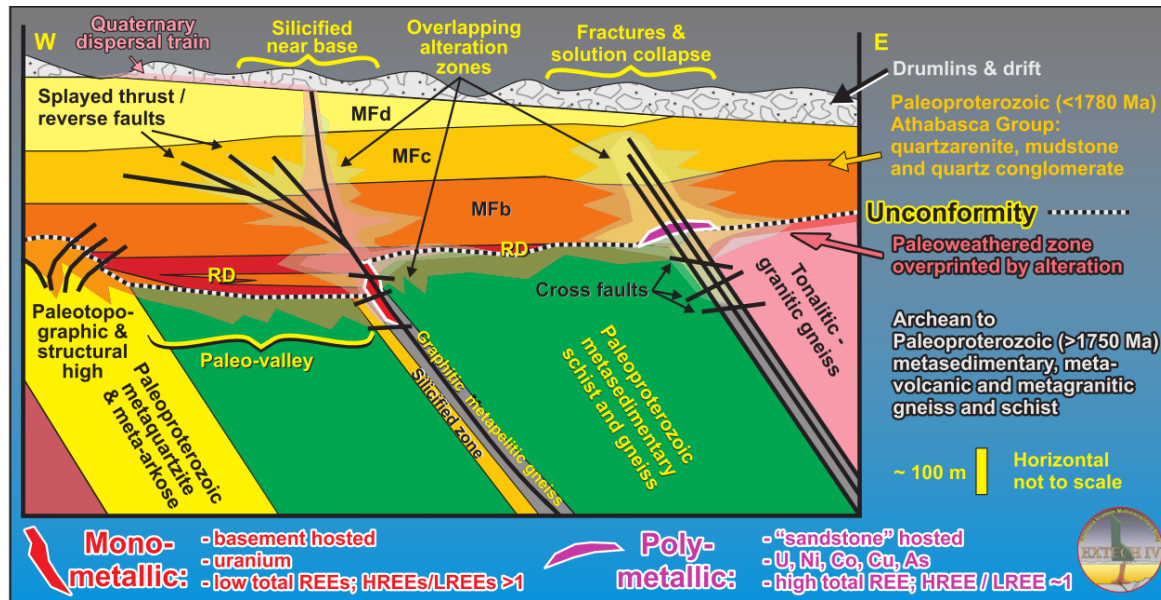


Ovoid massive sulfide ore deposit (Lelièvre et. al., TLE, 2012)

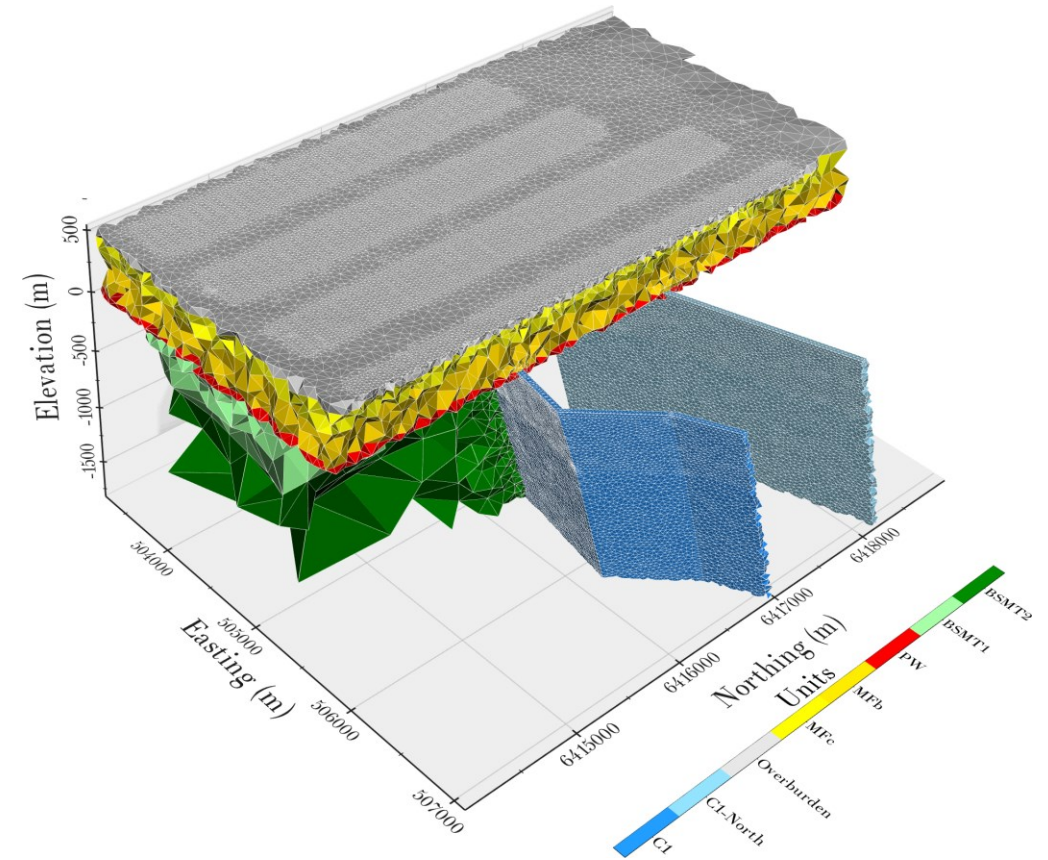


Jahandari & Farquharson (Geophysics, 2014)

# Geological models

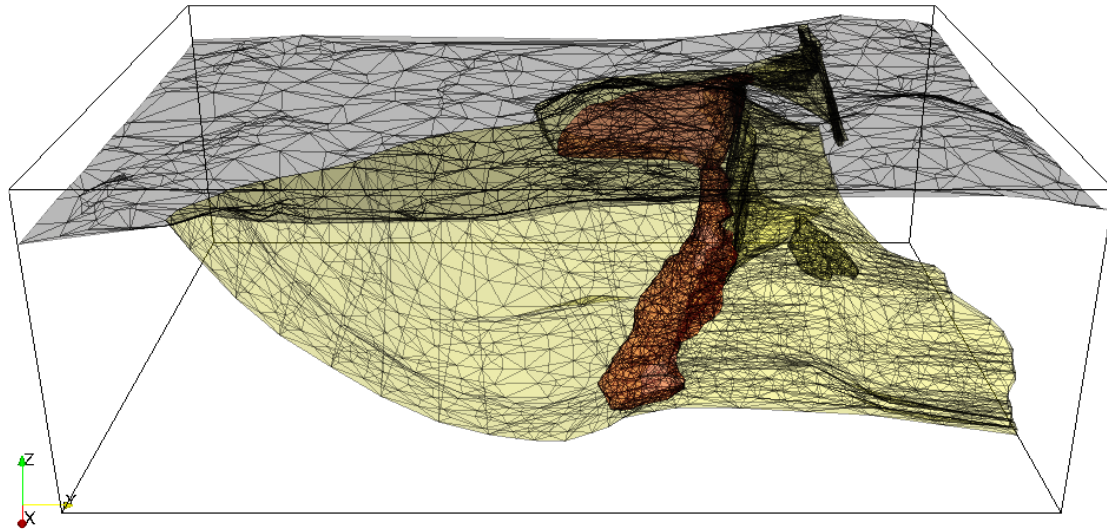


(Jefferson et. al., 2007)

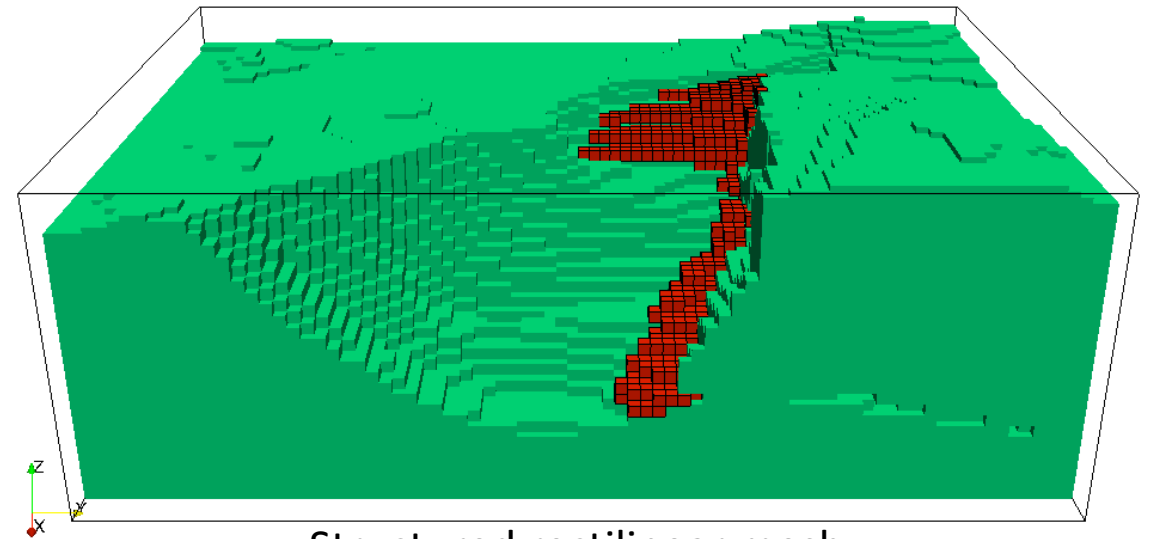


Lu et al. (Geophysics, 2021)

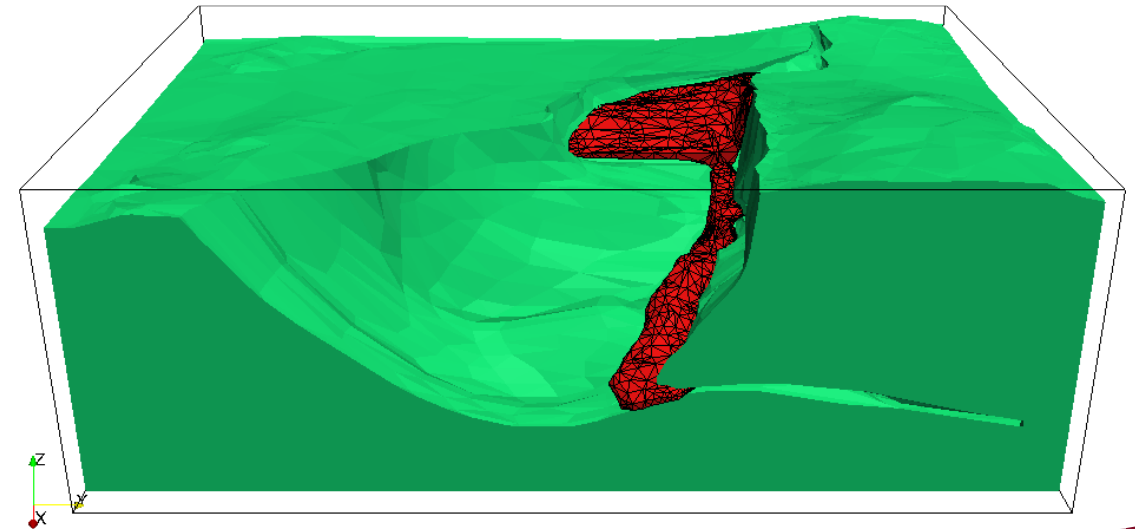
# Geophysical modelling



Geological wireframe model



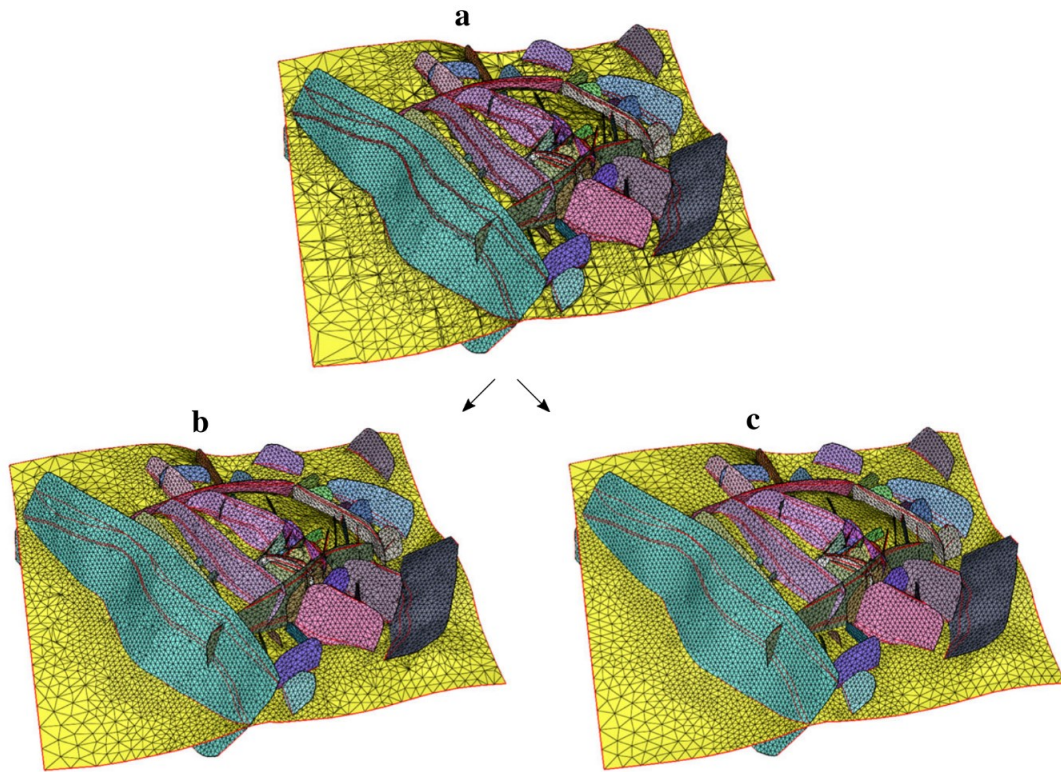
Structured rectilinear mesh



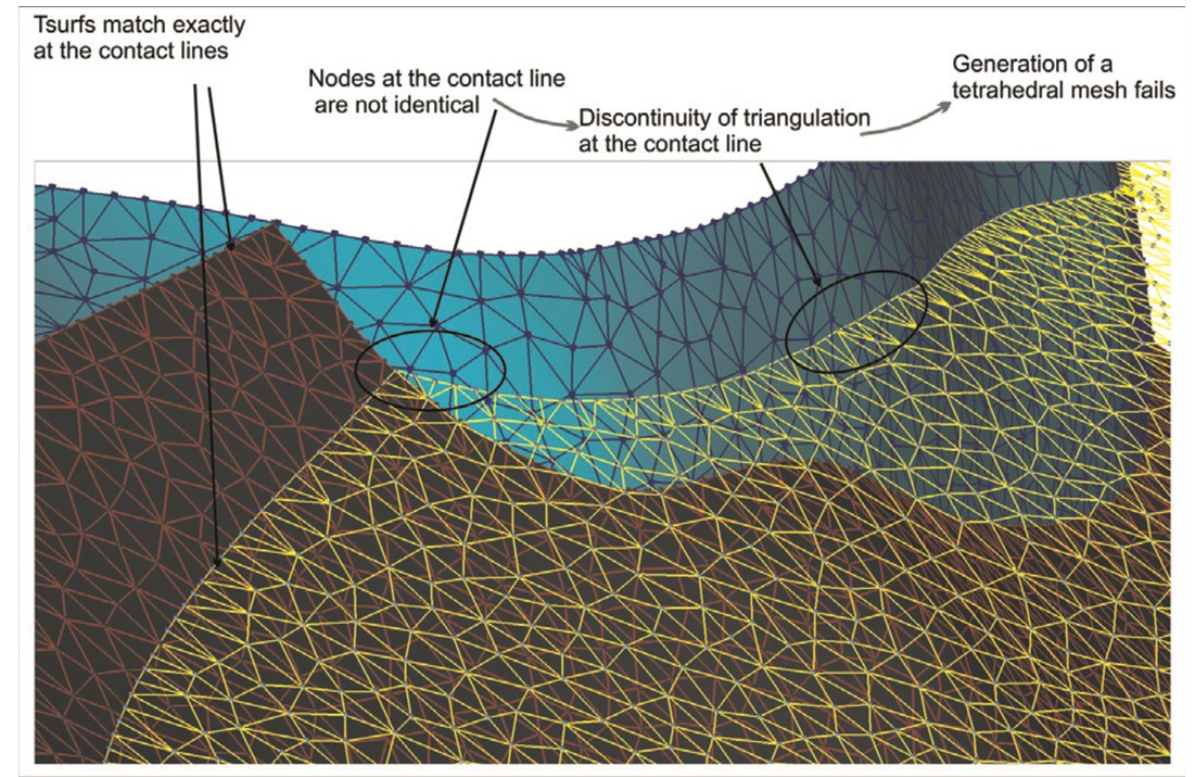
Unstructured tetrahedral mesh

# Geophysical modelling

## Quality mesh from geological models



Surface mesh generation (Irakarama1 M.,2022)

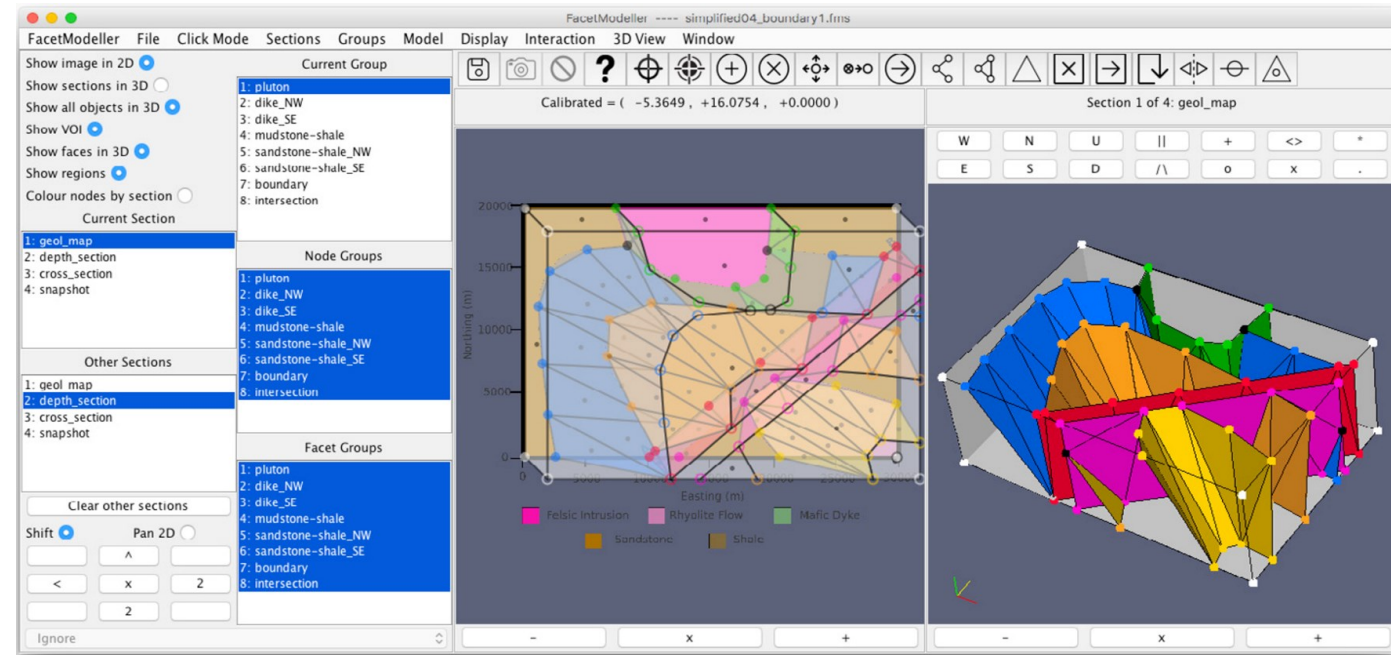
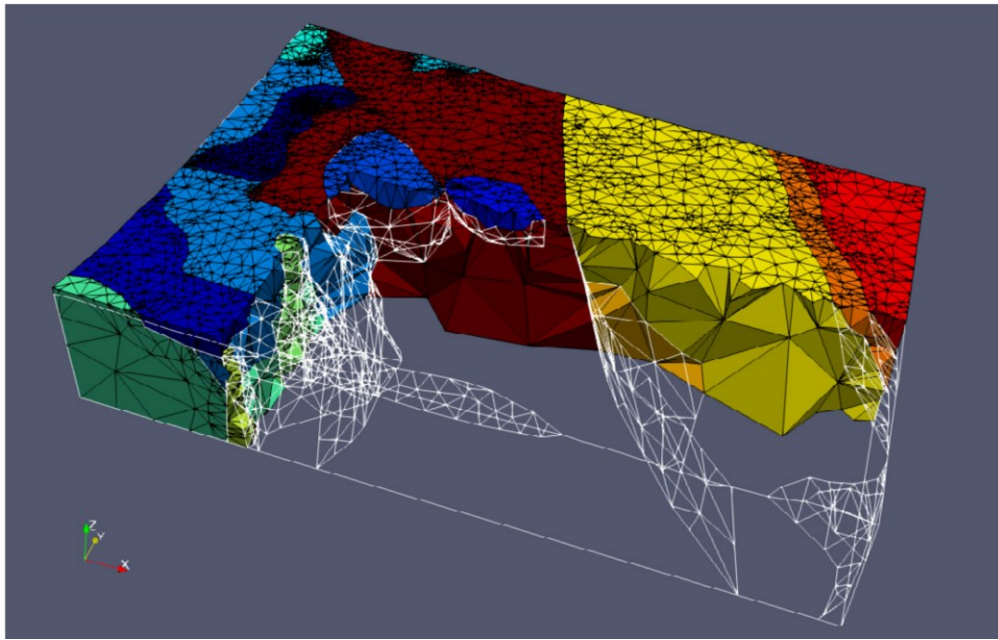


Geologic boundaries generated with Gocad (Zehner et. al., 2015)



# Geophysical model building

## Quality mesh from geological models



FacetModeller (<https://github.com/pglelievre/facetmodeller>)

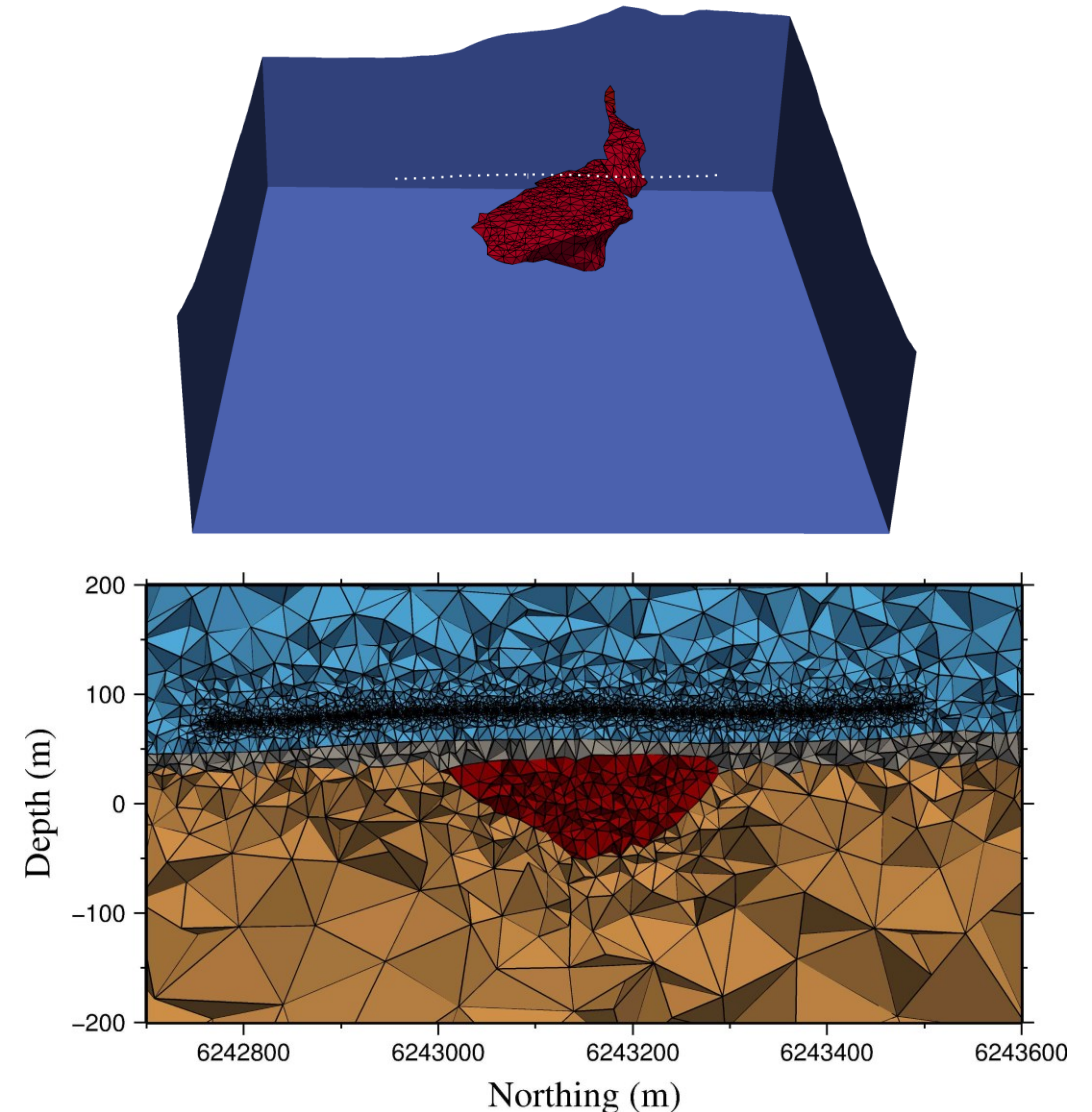
# Geophysical modelling

## ➤ Numerical methods

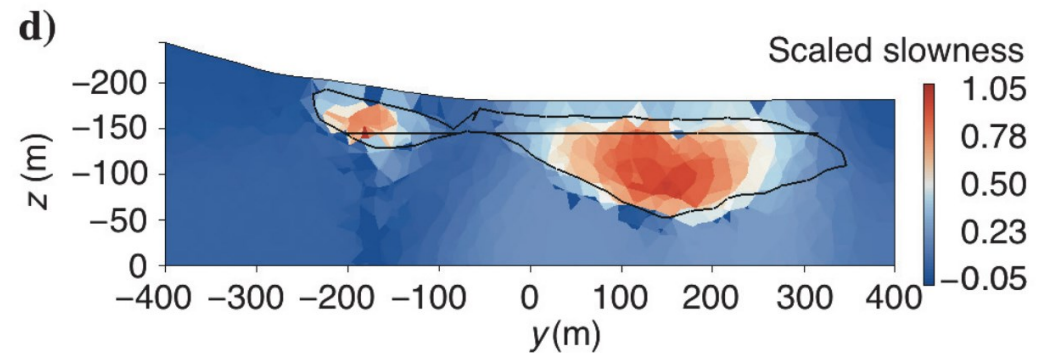
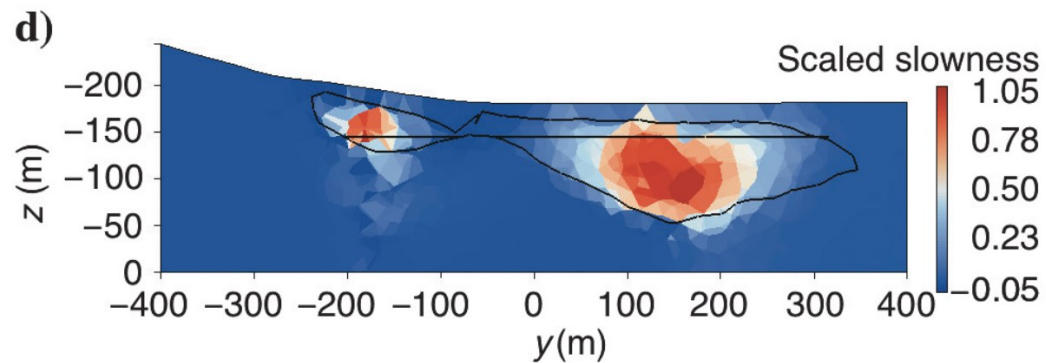
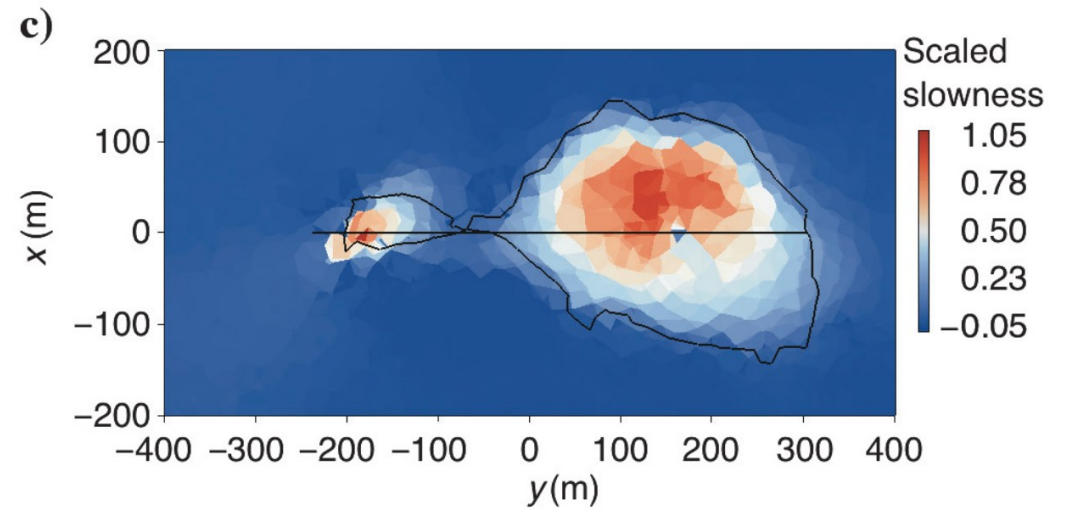
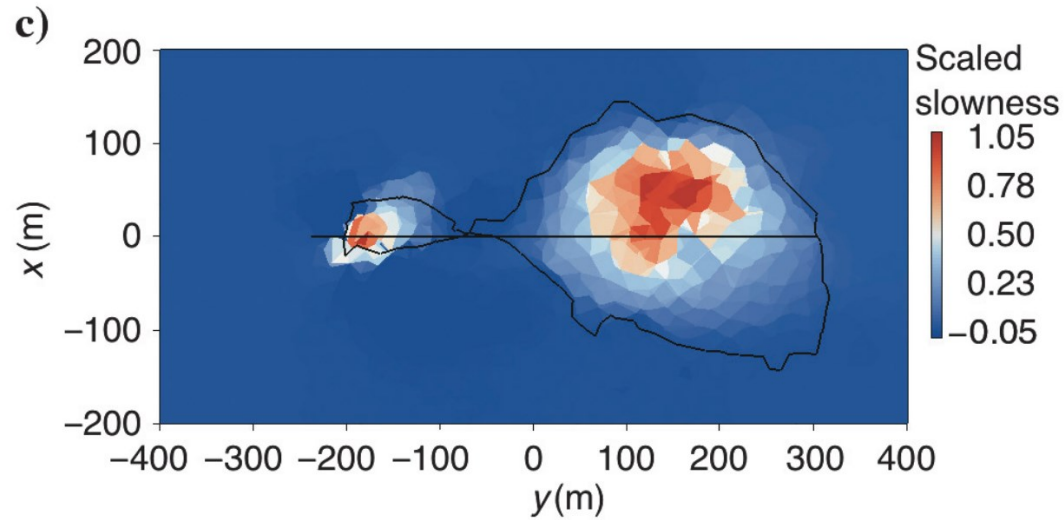
- Finite element
- Finite volume
- Mimetic finite difference
- Mesh free

## ➤ Geophysical data types

- Gravity & magnetic
- CSEM, TEM, MT, DC/IP
- Seismic travel time



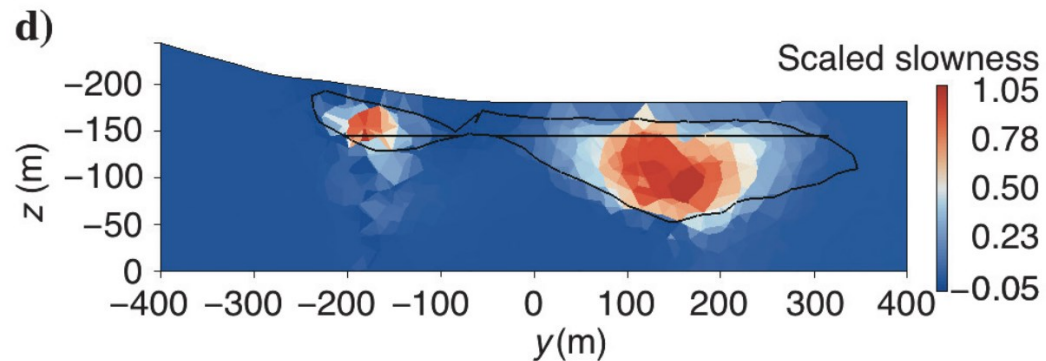
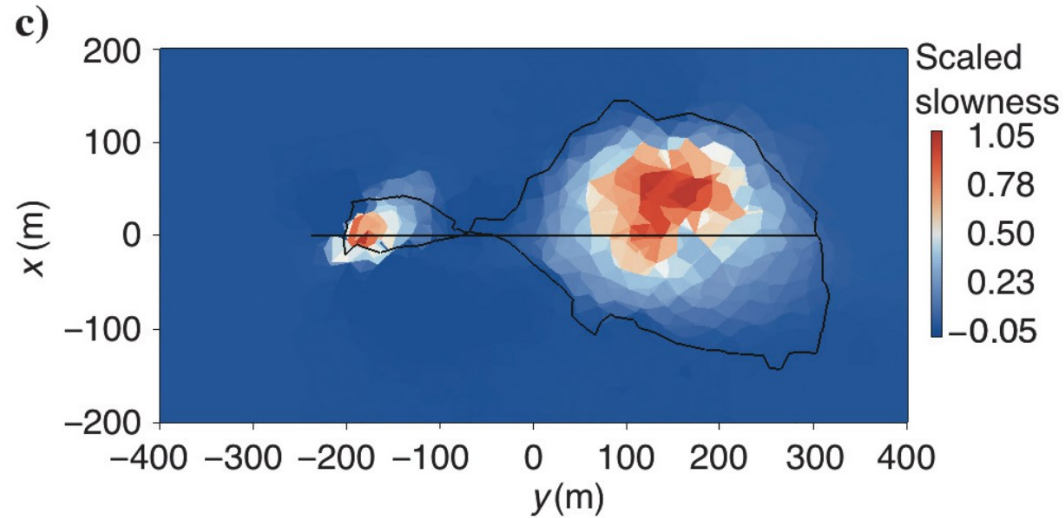
# Occam-style, minimum-structure inversion



Seismic travel time inversion

Gravity + seismic joint inversion

# Occam-style, minimum-structure inversion



Seismic travel time inversion

## Minimum-structure inversion objective function:

$$\phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta \phi_m(\mathbf{m}),$$

➤ Data misfit:

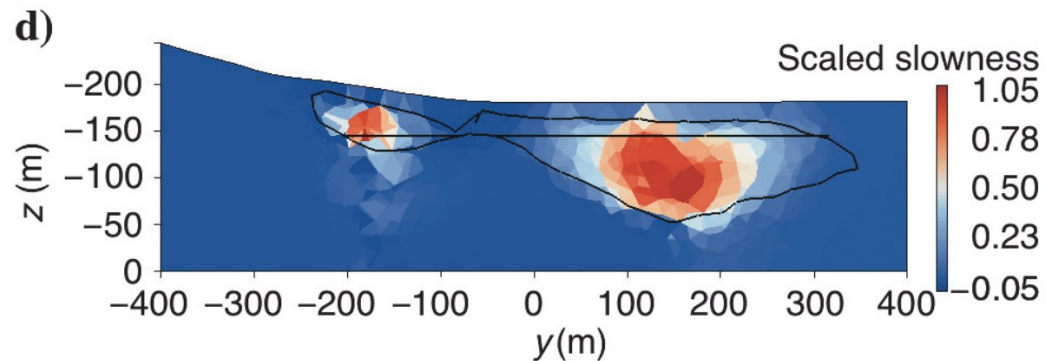
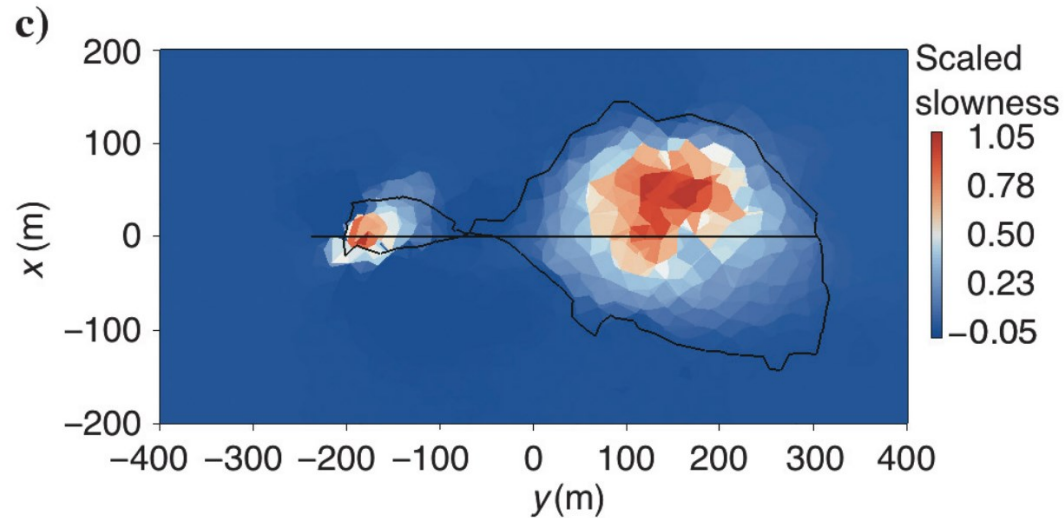
$$\phi_d(\mathbf{m}) = \| \mathbf{W}_d [\mathbf{d}^{obs} - \mathbf{d}(\mathbf{m})] \|^2,$$

➤ Model structure (smoothness):

$$\phi_m(\mathbf{m}) = \sum_k \| \mathbf{W}_k (\mathbf{m} - \mathbf{m}^{ref}) \|^2.$$

(Lelièvre et. al. GJI, 2012)

# Occam-style, minimum-structure inversion



Seismic travel time inversion

## Minimum-structure inversion objective function:

$$\phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta \phi_m(\mathbf{m}),$$

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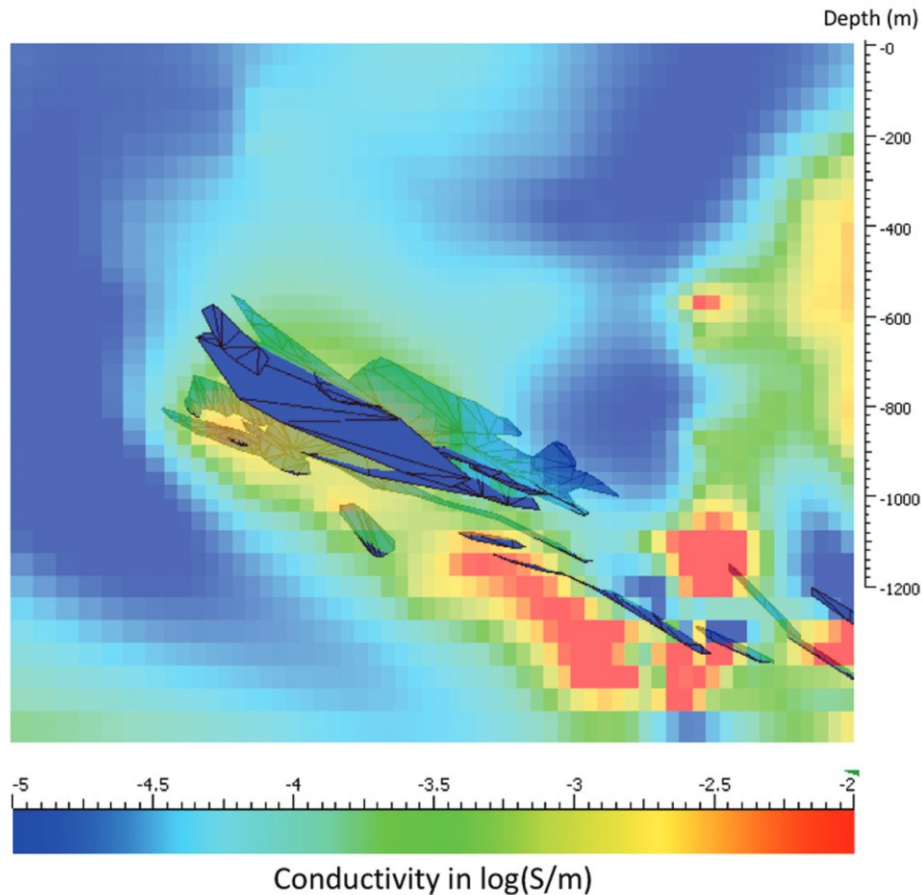
$$\phi_d(\mathbf{m}) = \| \mathbf{W}_d [\mathbf{d}^{obs} - \mathbf{d}(\mathbf{m})] \|^2,$$

➤ Model structure (**smoothness**):

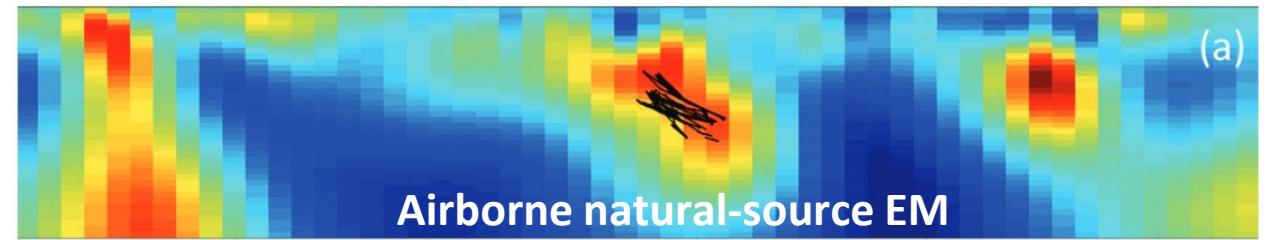
$$\phi_m(\mathbf{m}) = \sum_k \| \mathbf{W}_k (\mathbf{m} - \mathbf{m}^{ref}) \|^2.$$

(Lelièvre et. al. GJI, 2012)

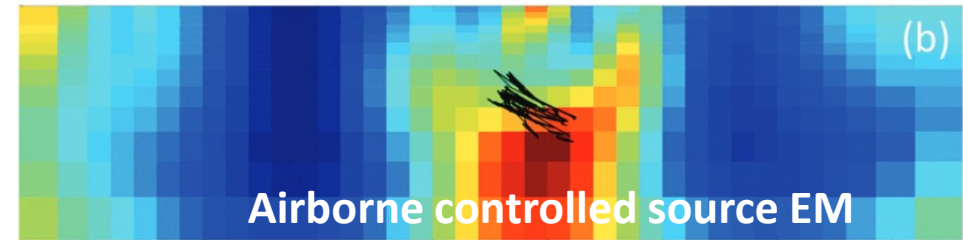
# Occam-style, minimum-structure inversion



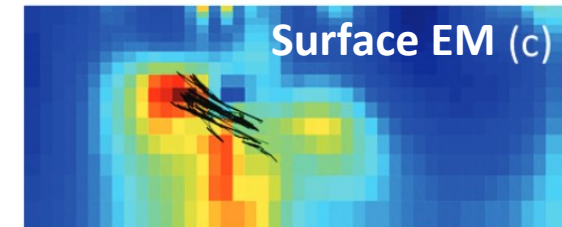
PULSE-EM surface-borehole TEM data inversion of the Lalor deposit



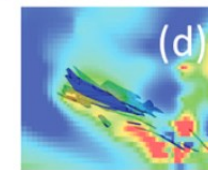
Airborne natural-source EM



Airborne controlled source EM



Surface EM (c)

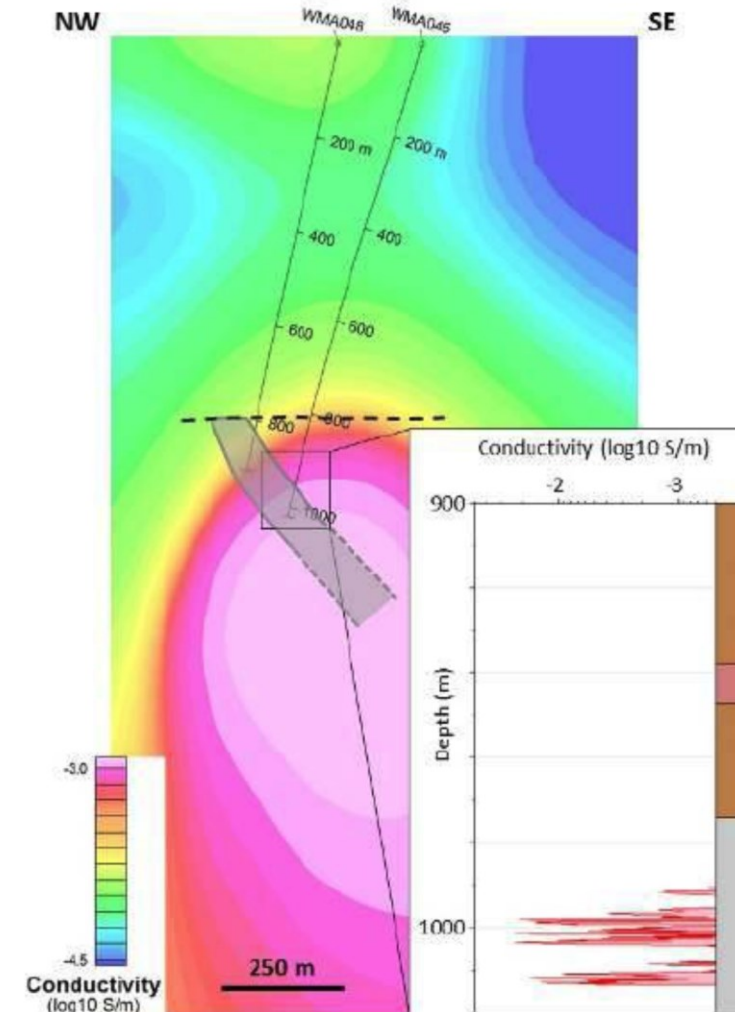
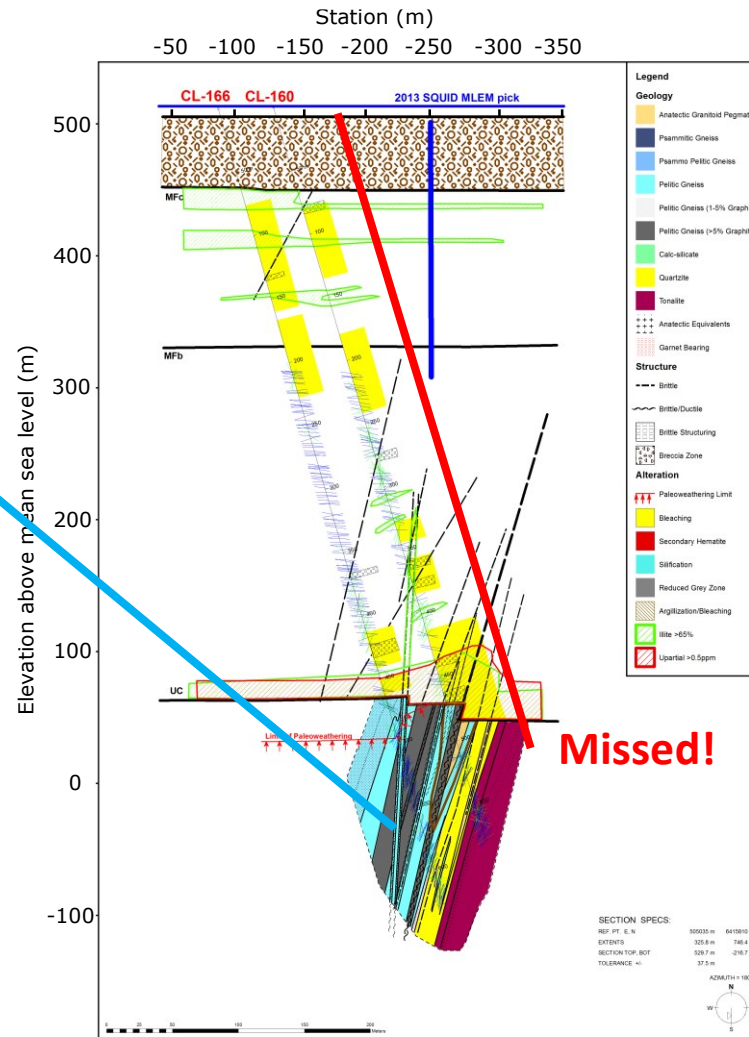


Borehole EM

Vertical sections of conductivity models from different EM surveys

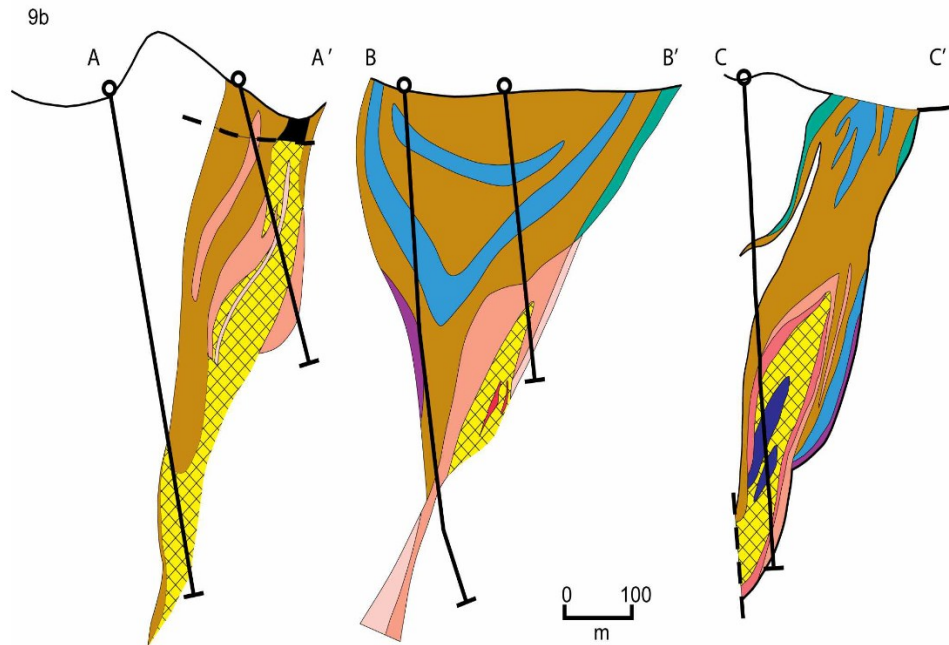
# Occam-style, minimum-structure inversion

- Constructed models are smooth
- Lack of boundary information for the anomaly
- Problematic for steeply dipping thin structures



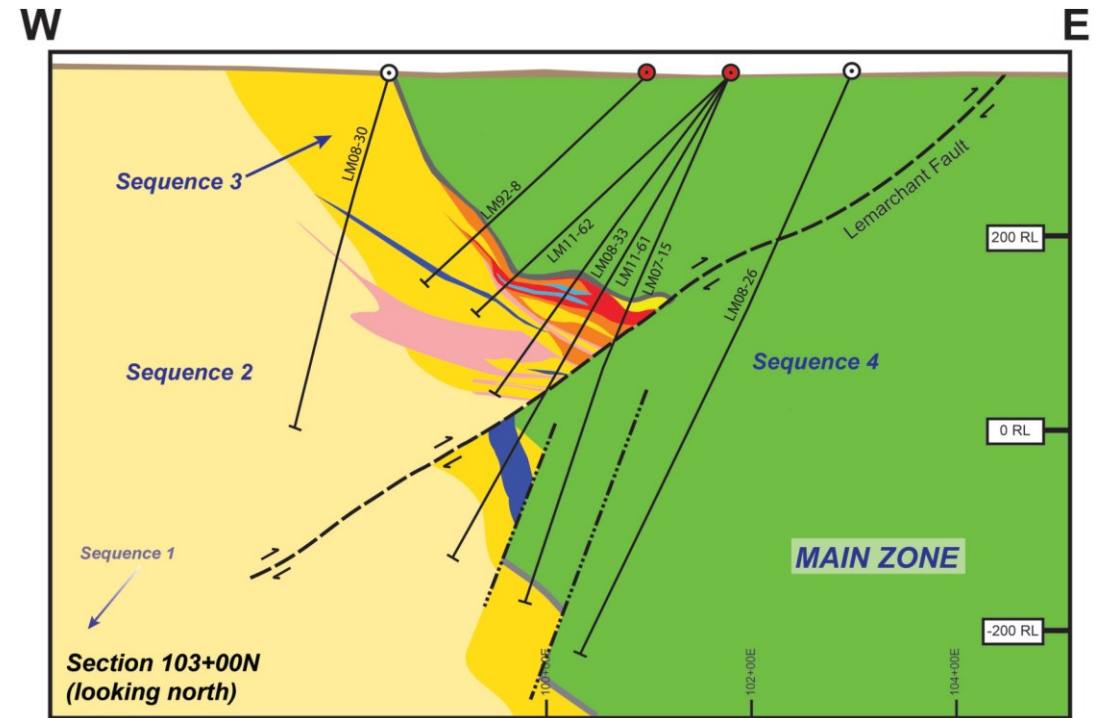
Keller (SEG, 2019)

# Thin, steeply dipping ore bodies



- |                              |                                   |
|------------------------------|-----------------------------------|
| ■ Lherzolite                 | ■ Oxidized sulphide               |
| ■ Plagioclase lherzolite     | ■ Metasomatic sulphide            |
| ■ Olivine websterite         | ■ Massive sulphide                |
| ■ Websterite                 | ■ Semi-net-textured-rich sulphide |
| ■ Disseminated sulphide      | ■ Platinum enriched zone          |
| ■ Net-textured-rich sulphide | — Faults                          |

Jinchuan nickel sulphide deposit (Lightfoot, proceedings of Exploration 07)



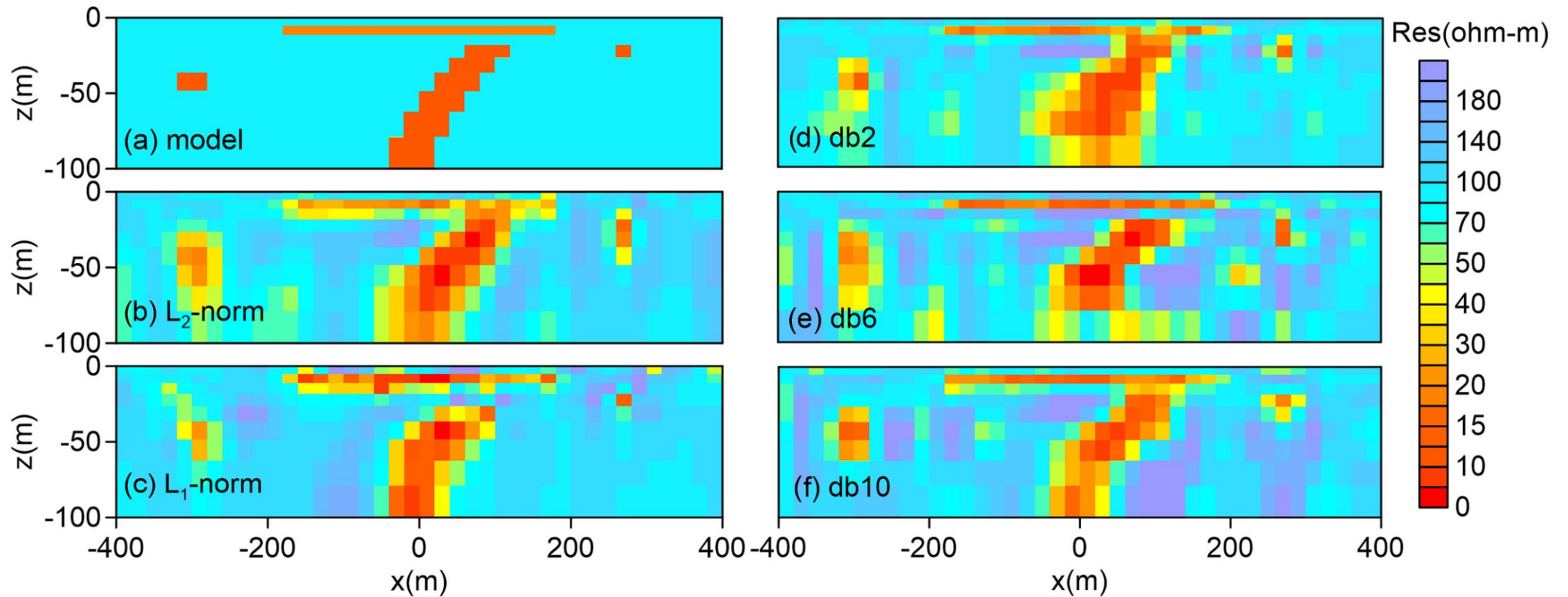
- |                          |   |                                     |
|--------------------------|---|-------------------------------------|
| ■ Felsic intrusive rocks | ■ Felsic volcanic rocks (altered, mineralized)        | ■ Massive barite                    |
| ■ Mafic intrusive rocks  | ■ Felsic volcanic (Ba-enriched, altered, mineralized) | ■ Massive to semi-massive sulphides |
| ■ Mafic volcanic rocks   | ■ Mudstone (pyrite, pyrrhotite)                       | ■ Stringer sulphide mineralization  |
- 100 m

Lemarchant Zn-Pb-Cu-Ag-Au-rich volcanogenic massive sulphide deposit, Newfoundland, Canada (Lajoie et al., 2018)



# Inversion techniques to get sharper boundary

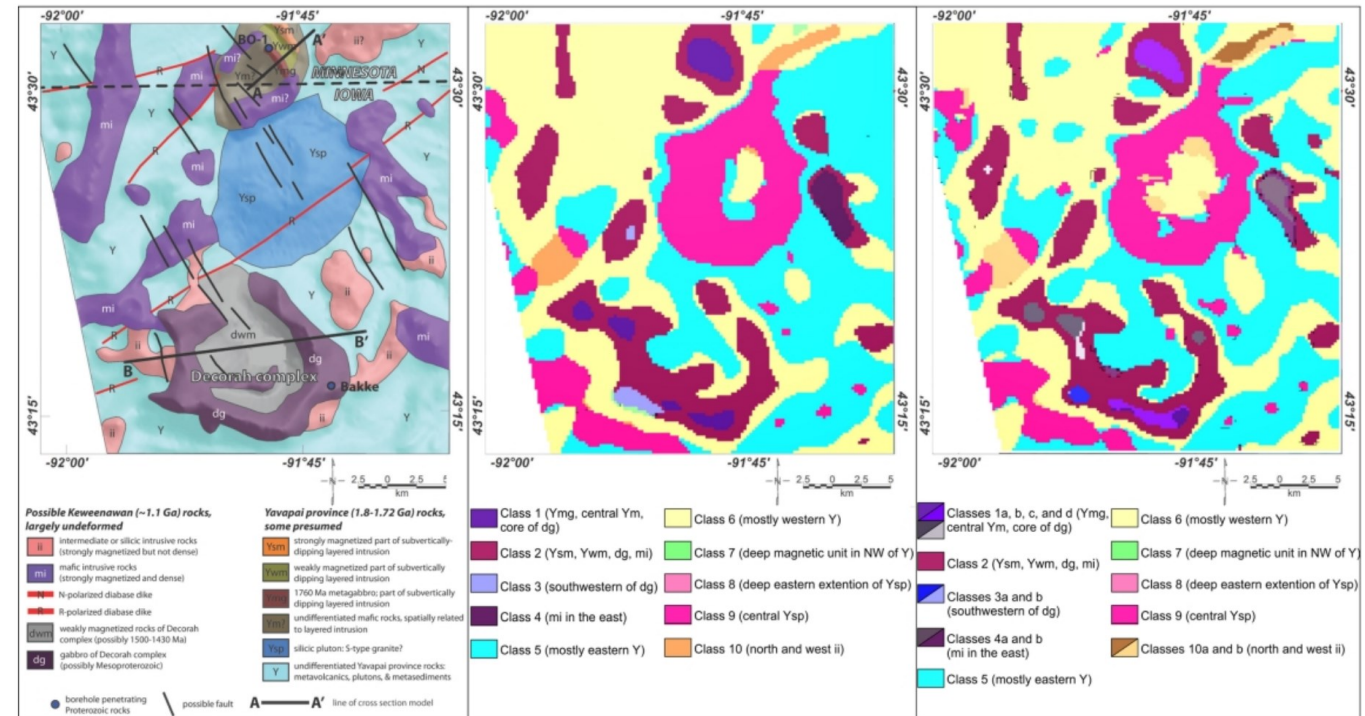
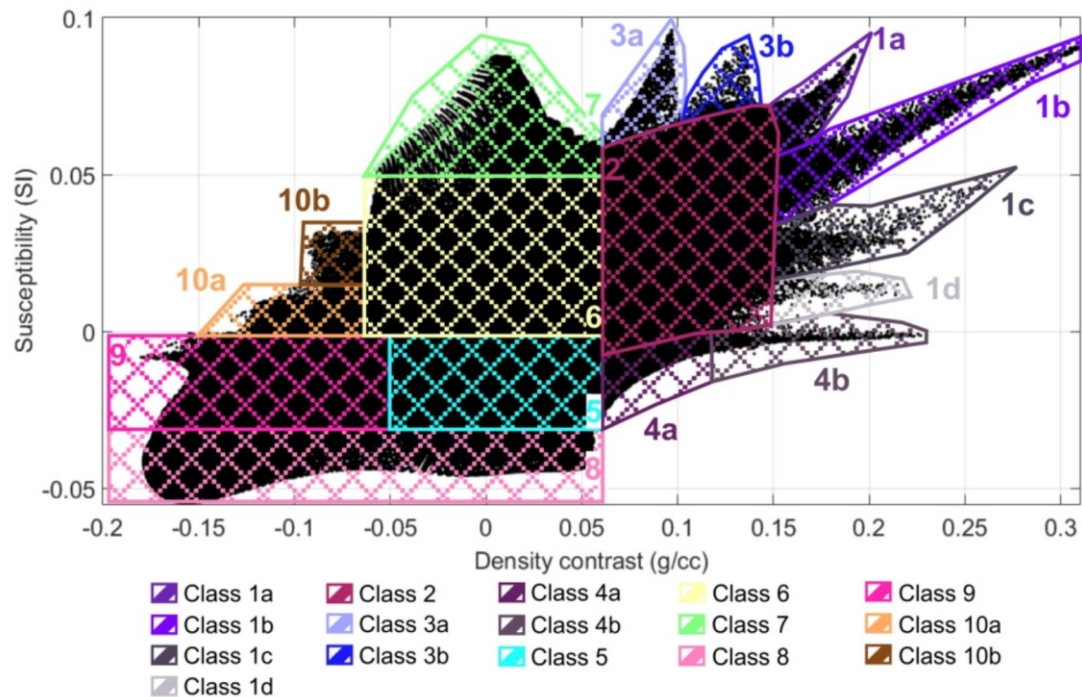
## L1-norms and wavelet-based methods



Liu et. al. (GJI, 2015)

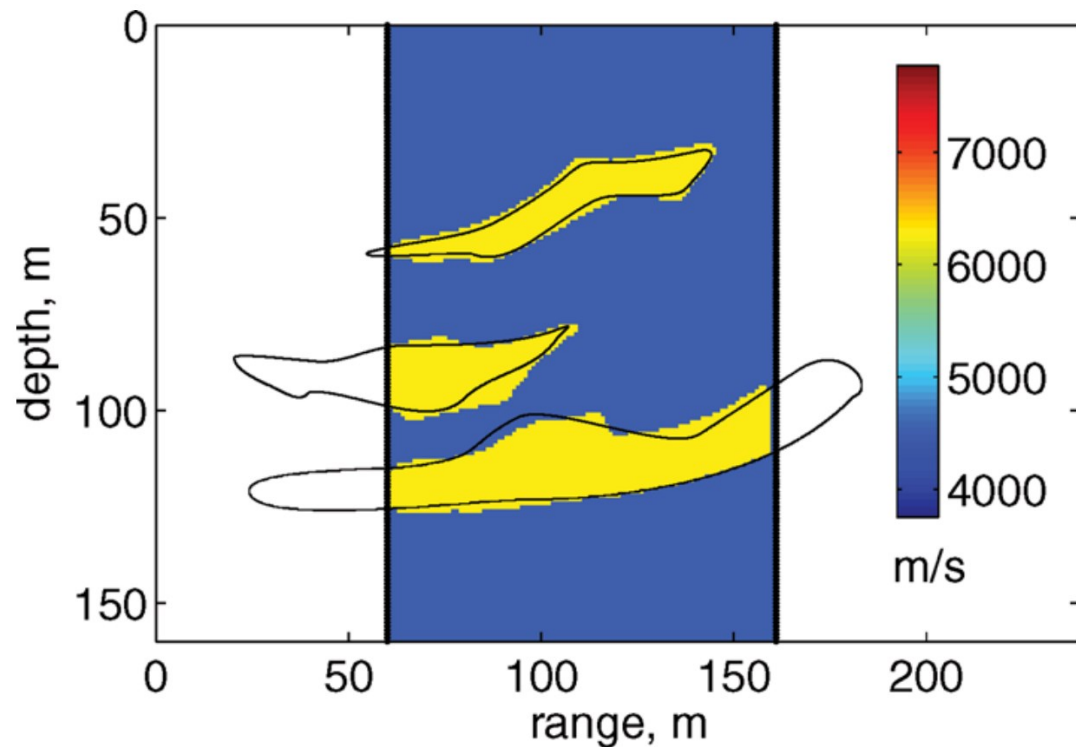
# Inversion techniques to get sharper boundary

## Clustering

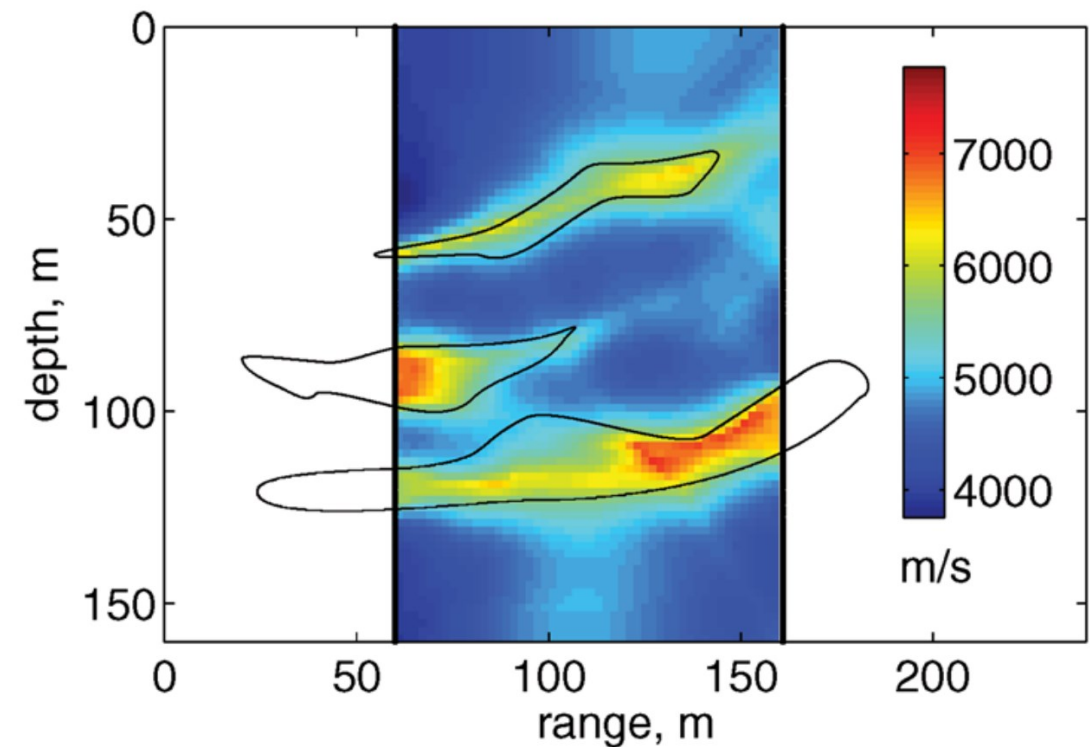


# Inversion techniques to get sharper boundary

- Level-set inversion



Level-set inversion

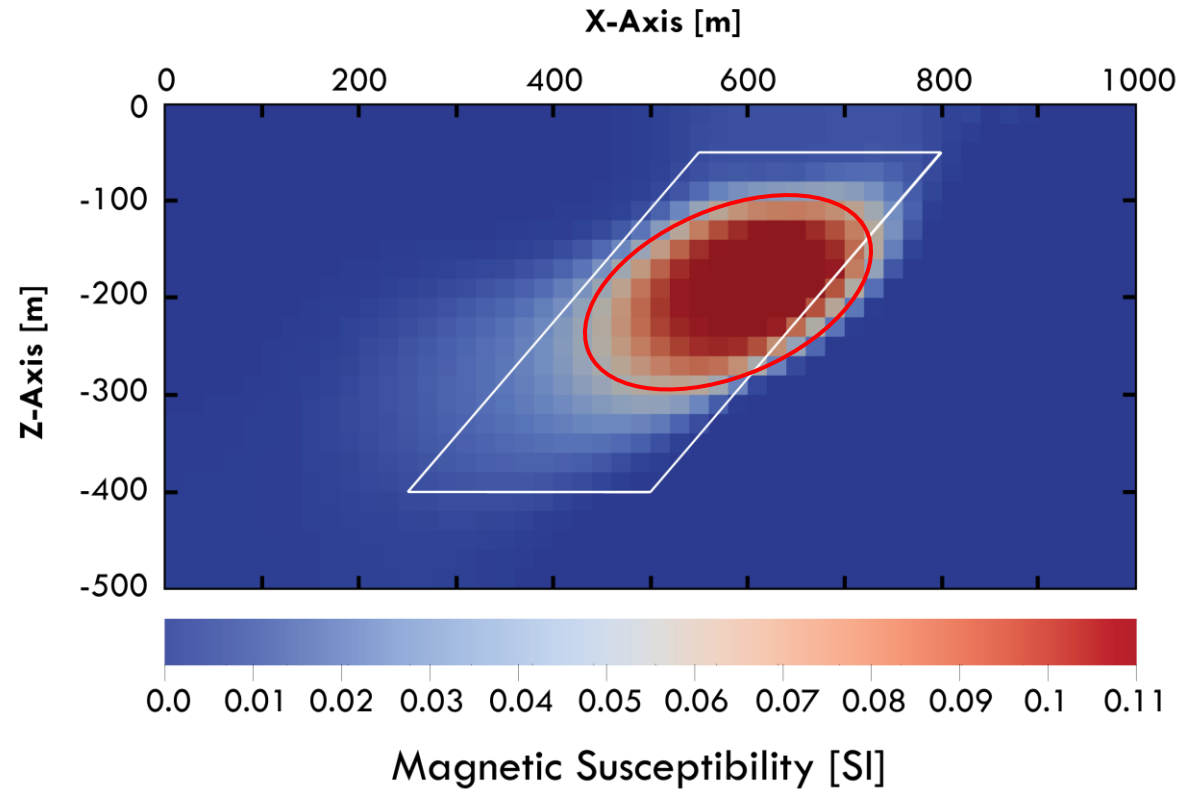


Minimum-structure inversion

(Zheglova et. al., GJI, 2013)

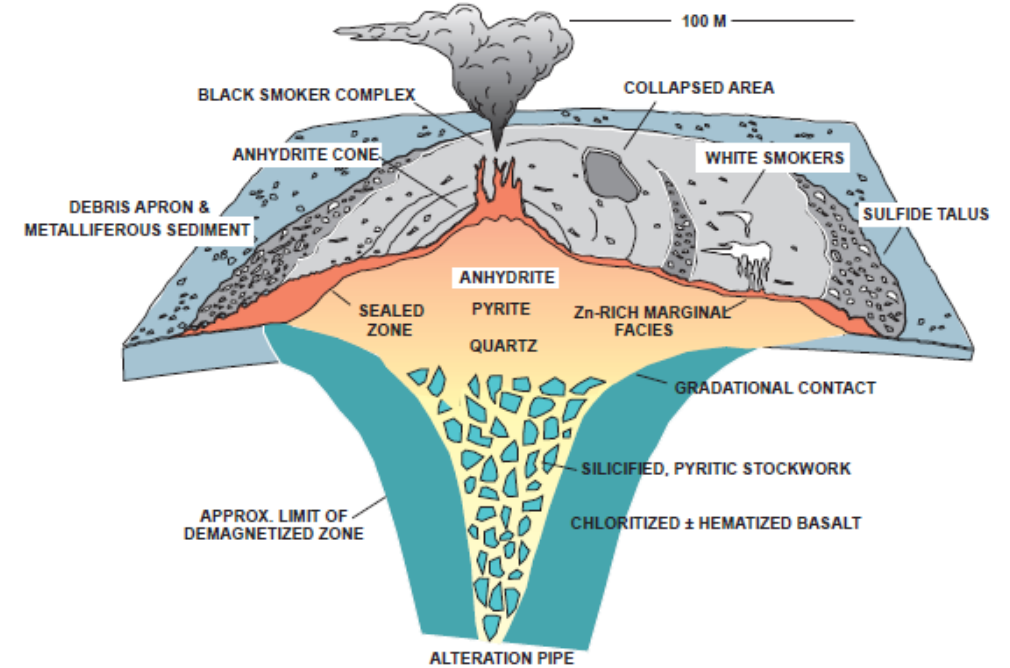
# Surface geometry inversion (SGI)

- Conventional inversion: physical properties inside a cell
- Boundaries: large physical property gradient

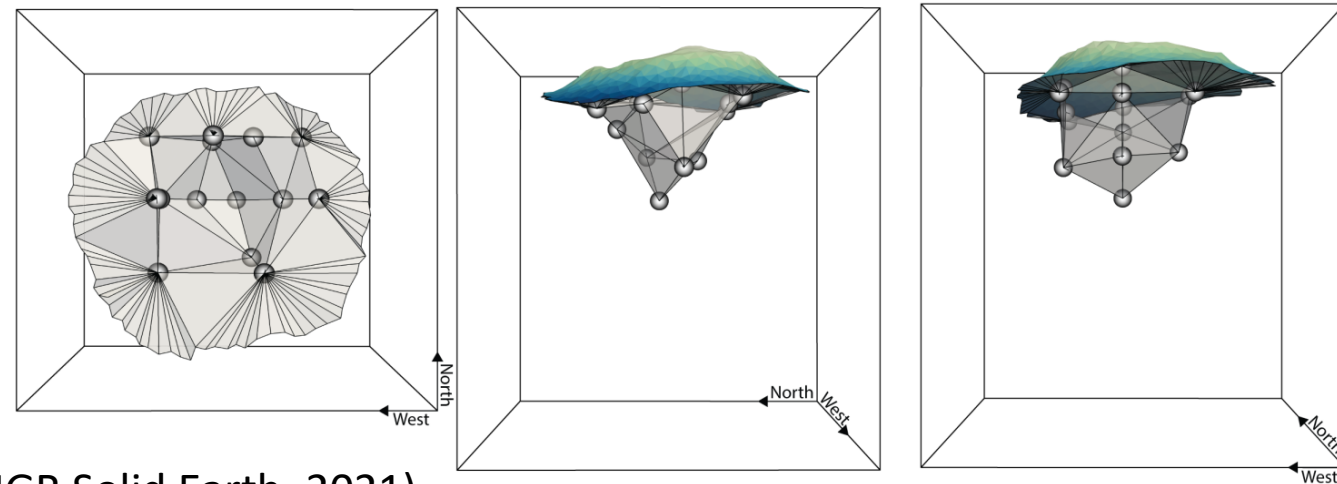


# Surface geometry inversion

- Conventional inversion: physical properties inside a cell
- Boundaries: large physical property gradient
- Surface geometry inversion: nodal coordinates
- Requires prior information of local geology
  - Anomaly type/shape
  - Typical physical property values
  - Late-stage interpretation

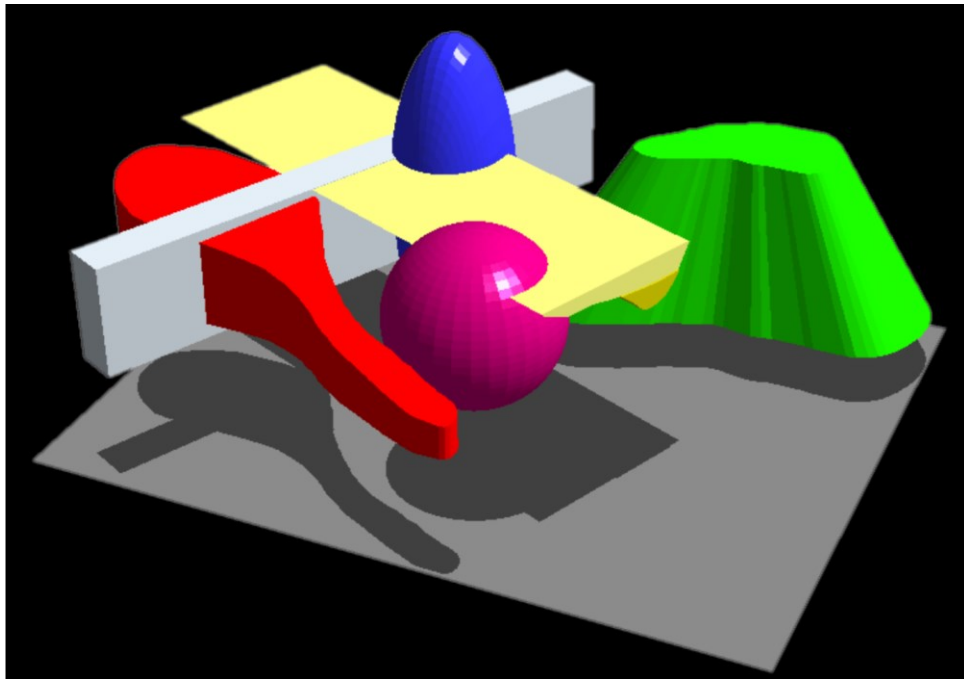


Hannington et al (1995)

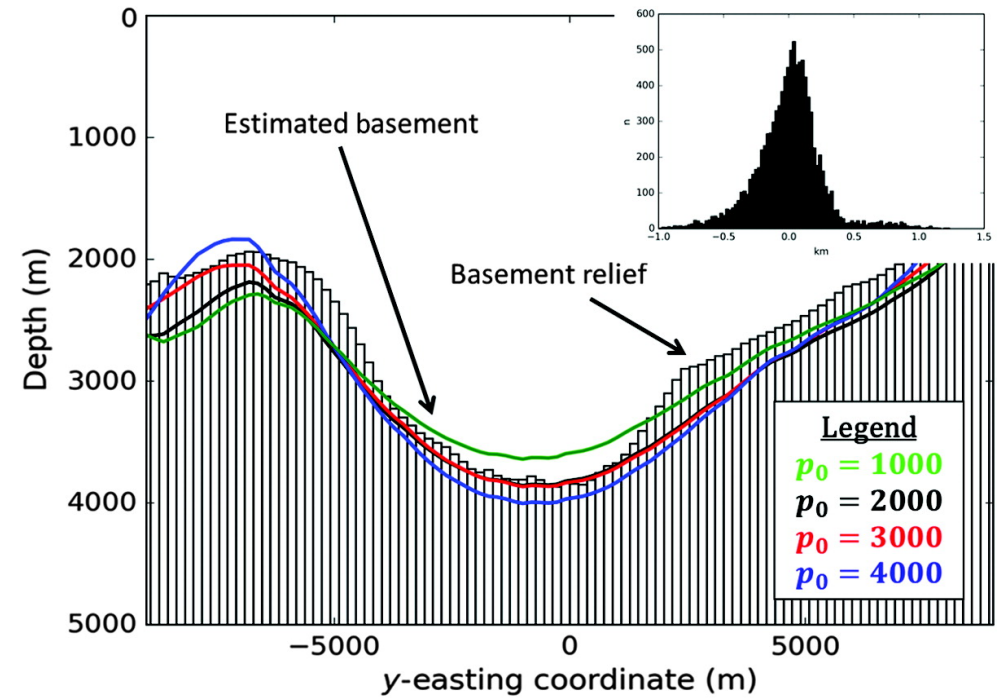


Galley et al. (JGR Solid Earth, 2021)

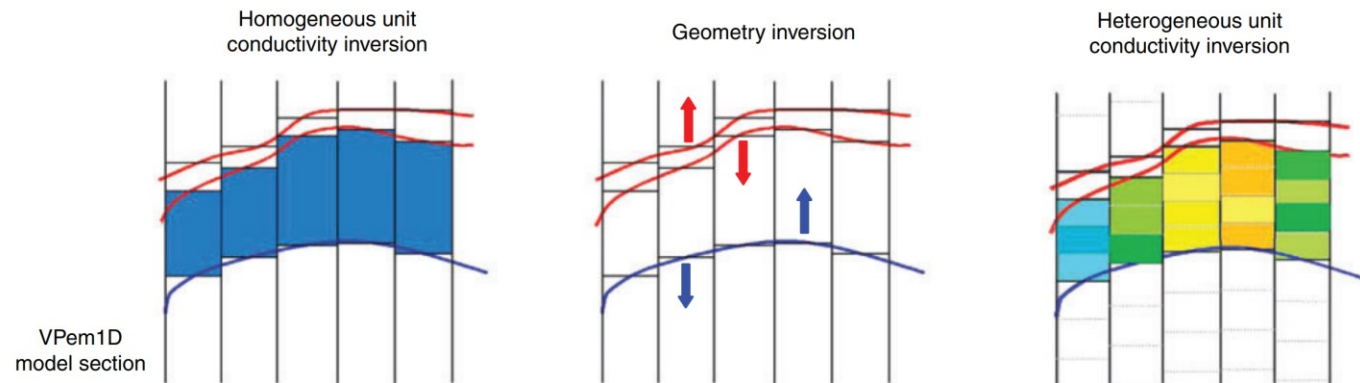
# SGL: parametric inversion



Discrete body inversion  
(Oldenburg & Pratt, 2007)



Hidalgo-Gato & Barbosa (Geophysics, 2019)



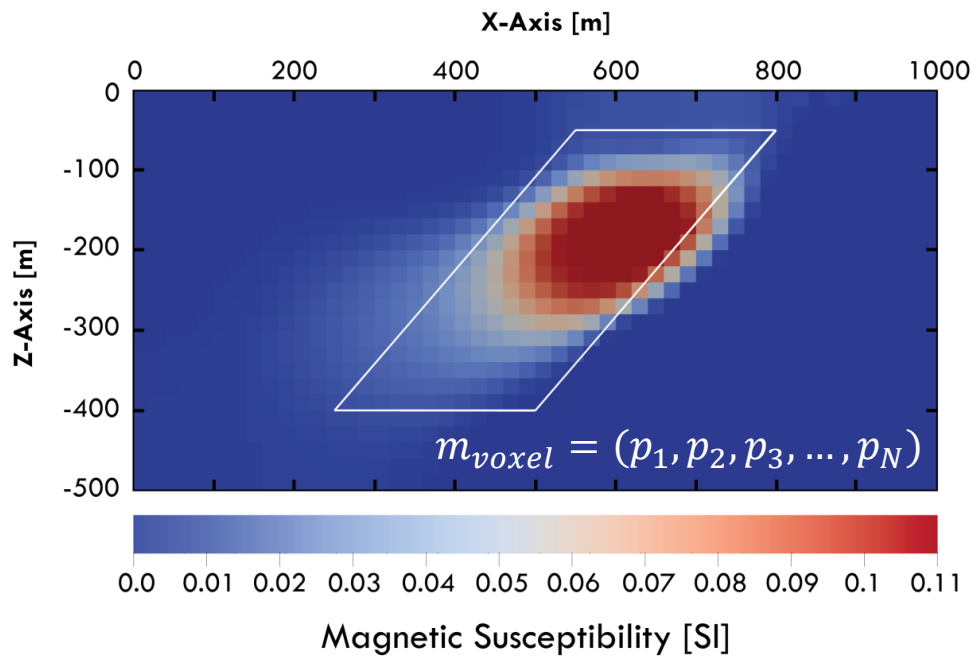
VPem inversion (Fullagar et al, 2015)

# Outline

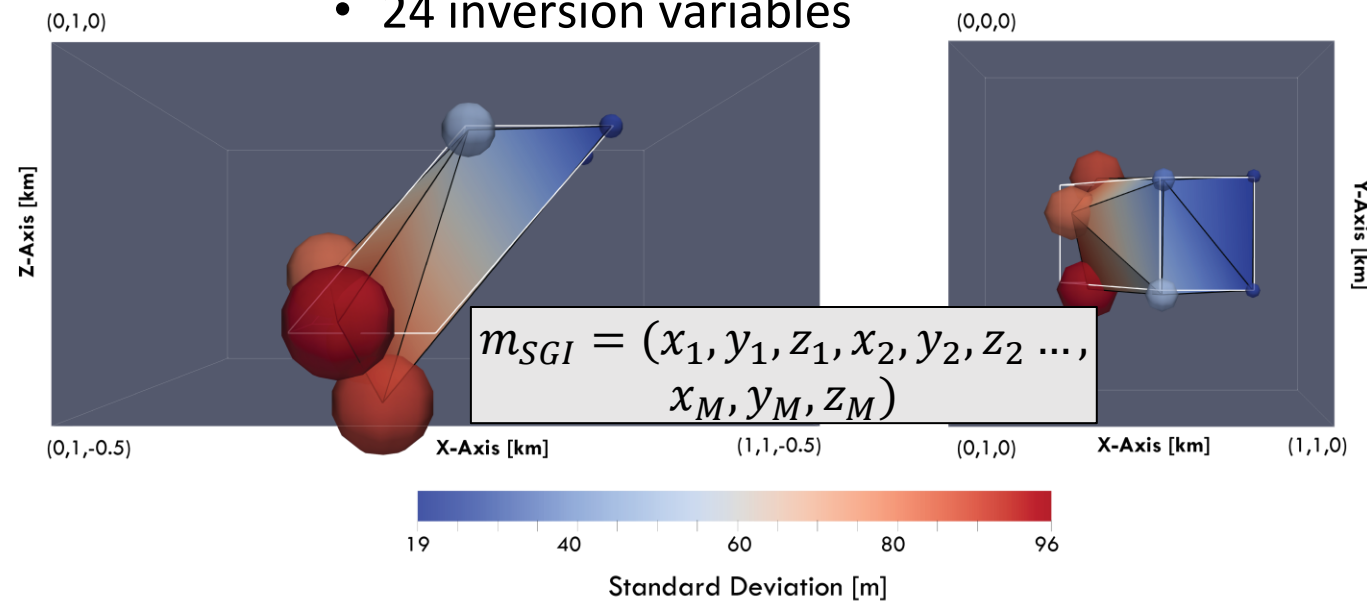
- Motivation
- **Surface geometry inversion**
- Marine CSEM examples
- TEM examples
- Conclusions

# Surface geometry inversion

- Minimum-structure magnetic inversion
  - Solves for the scalar effective Mag. Susc. in each cell.
  - 62500 inversion variables

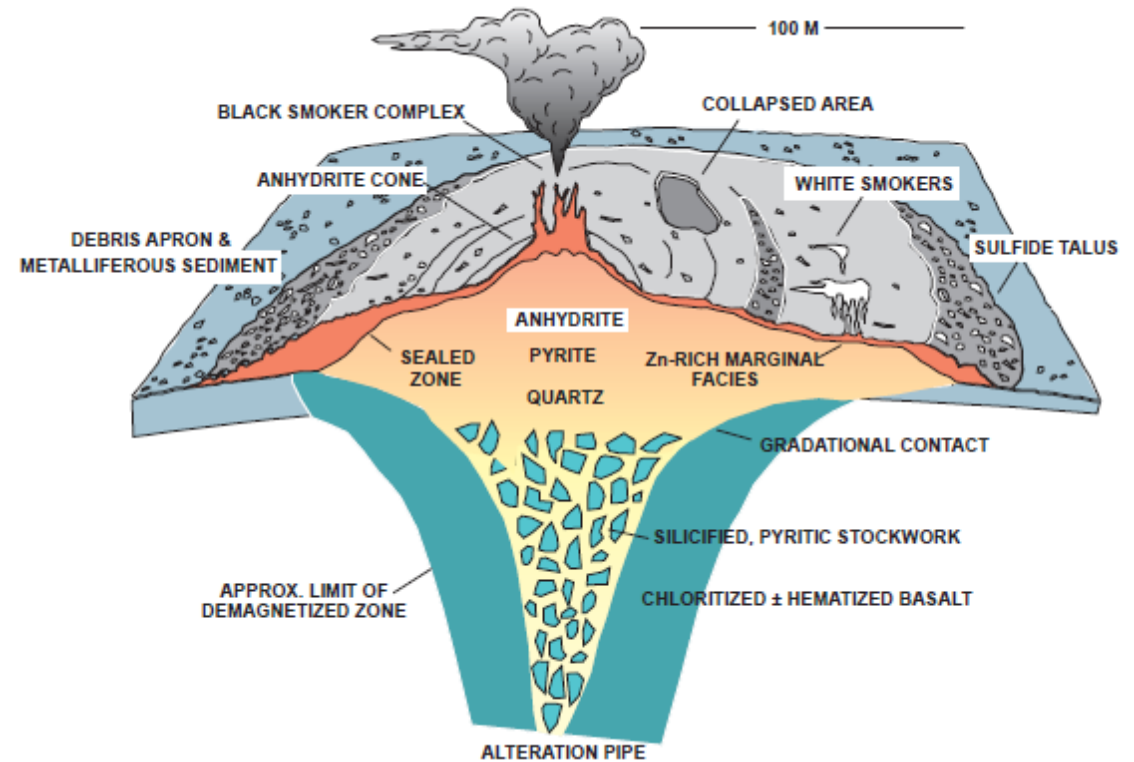
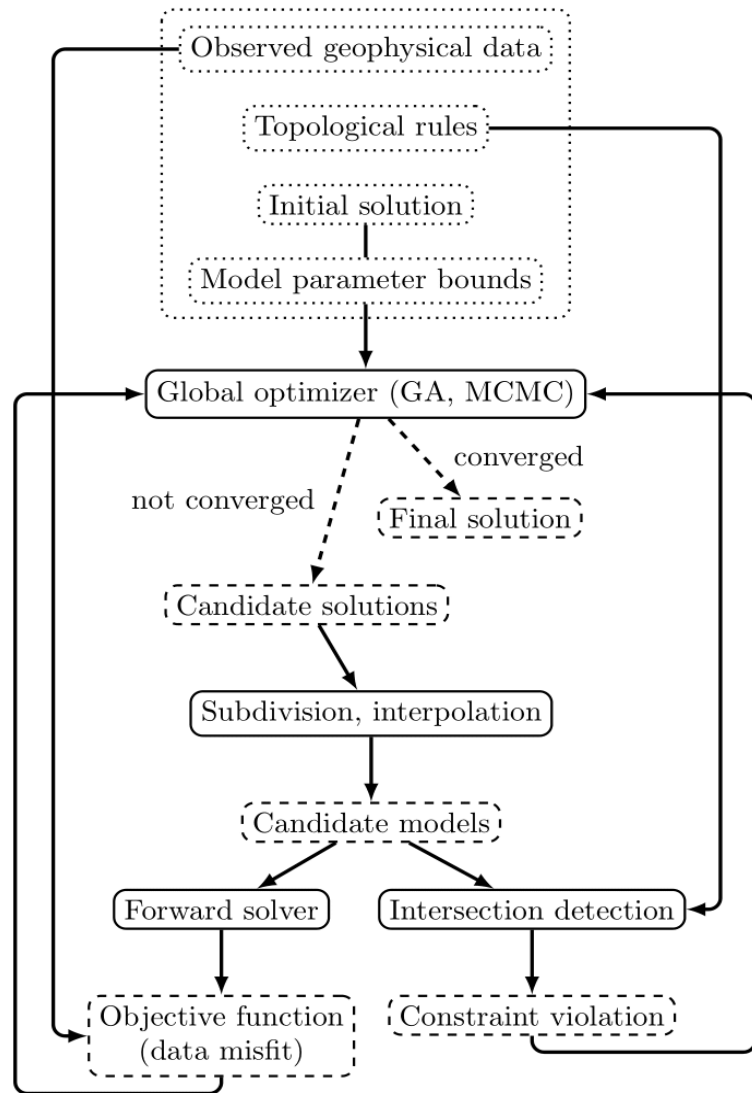


- Surface Geometry Inversion
  - Solves for the geometry of a wireframe model
  - Physical properties can be fixed or inverted
  - 24 inversion variables



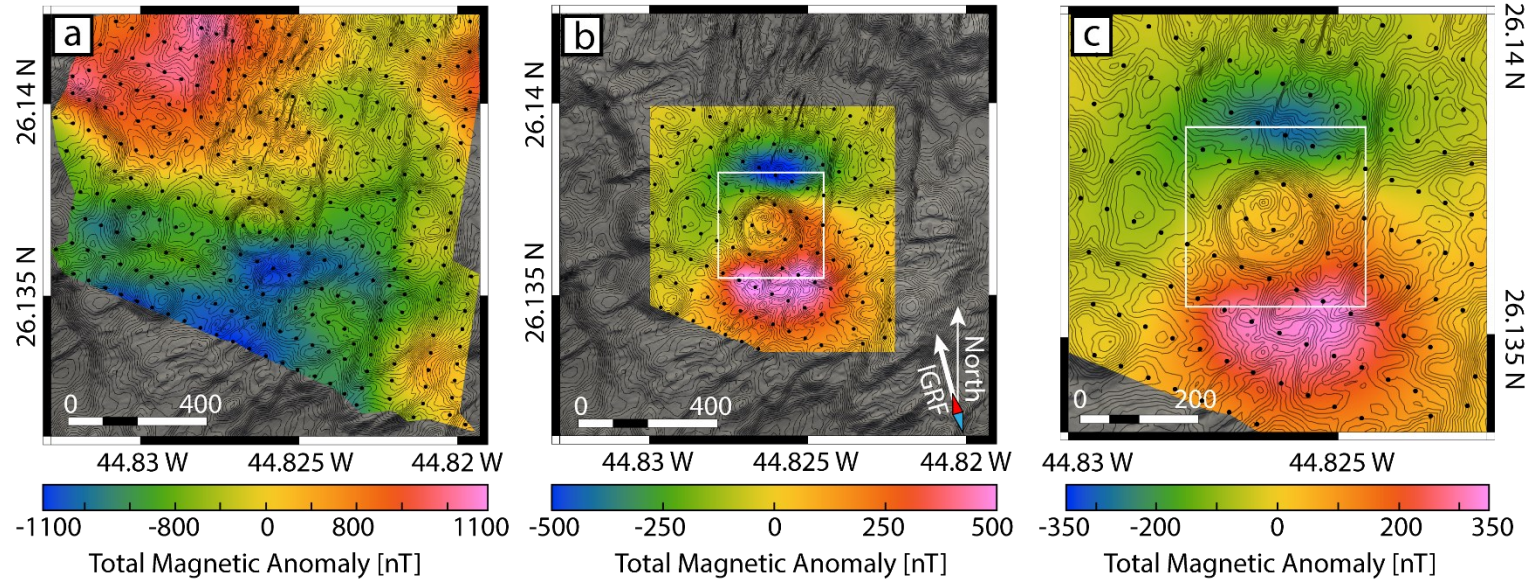
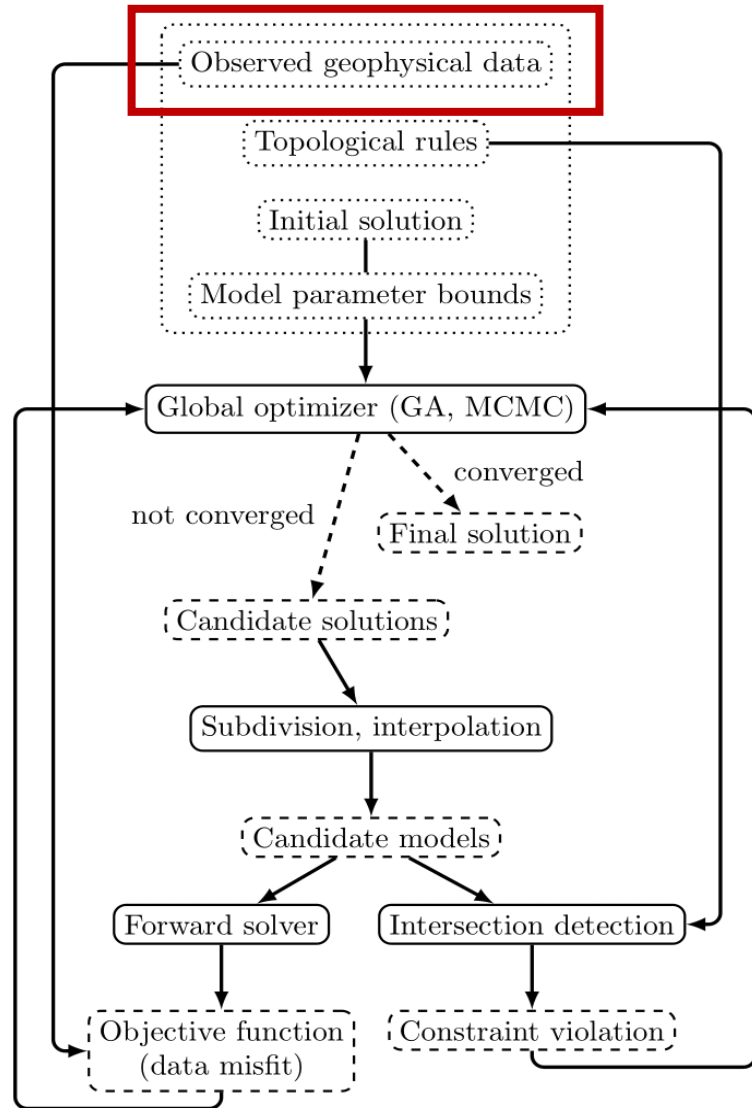


# Surface geometry inversion



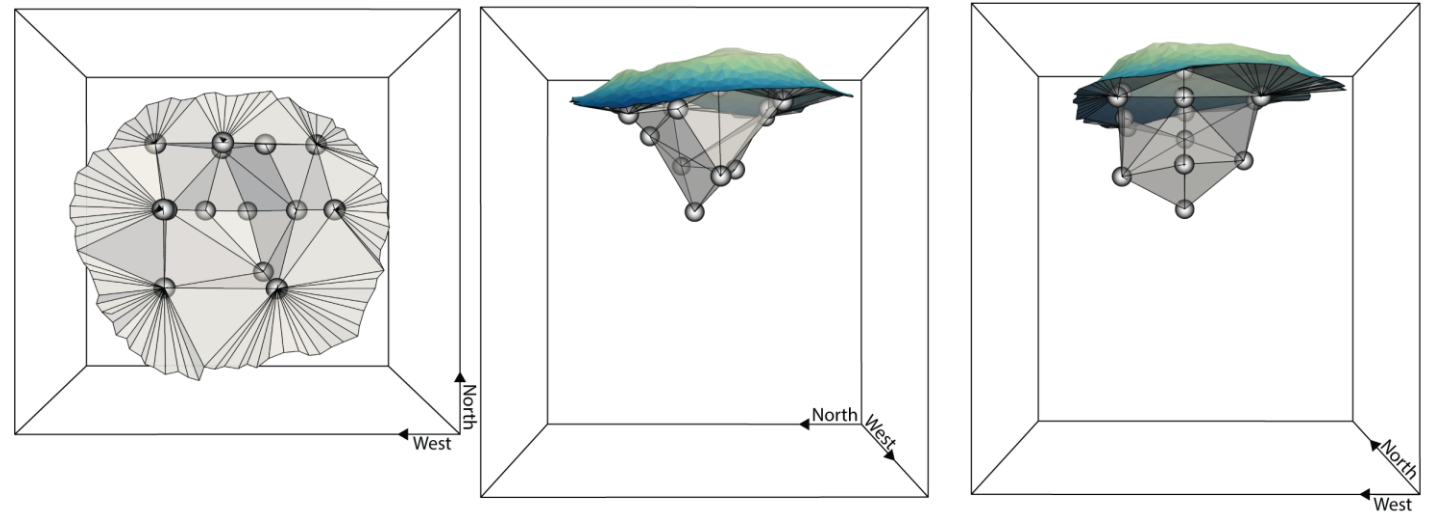
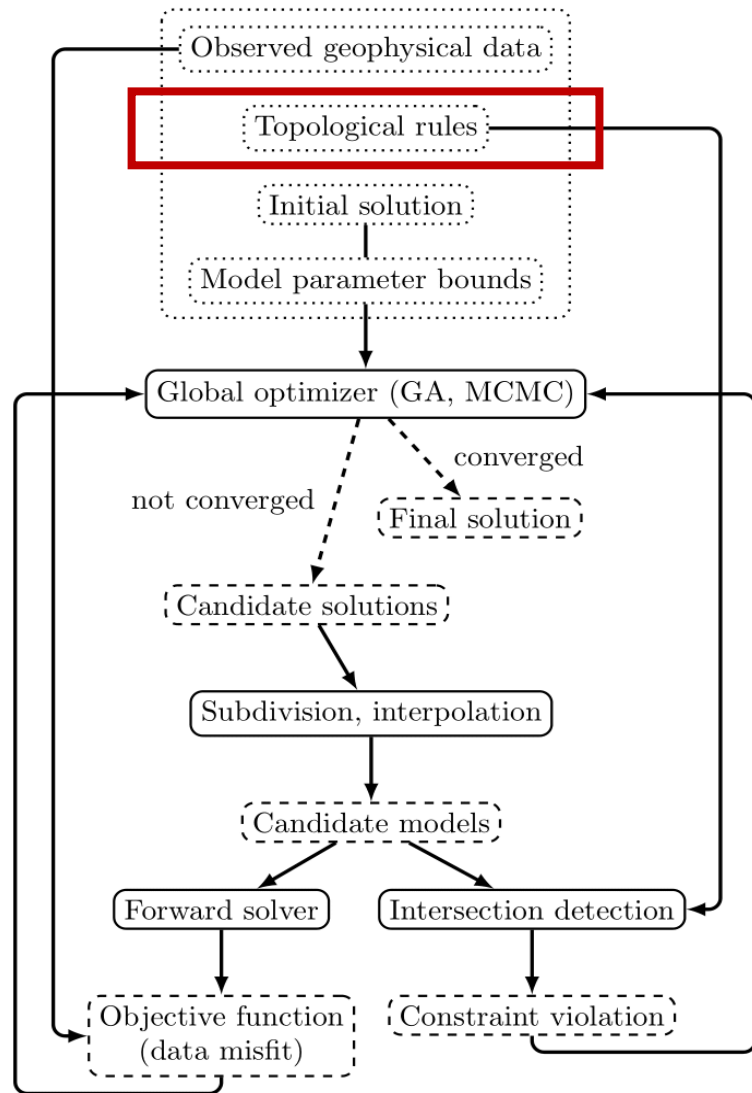
Hannington et al (1995)

# Surface geometry inversion



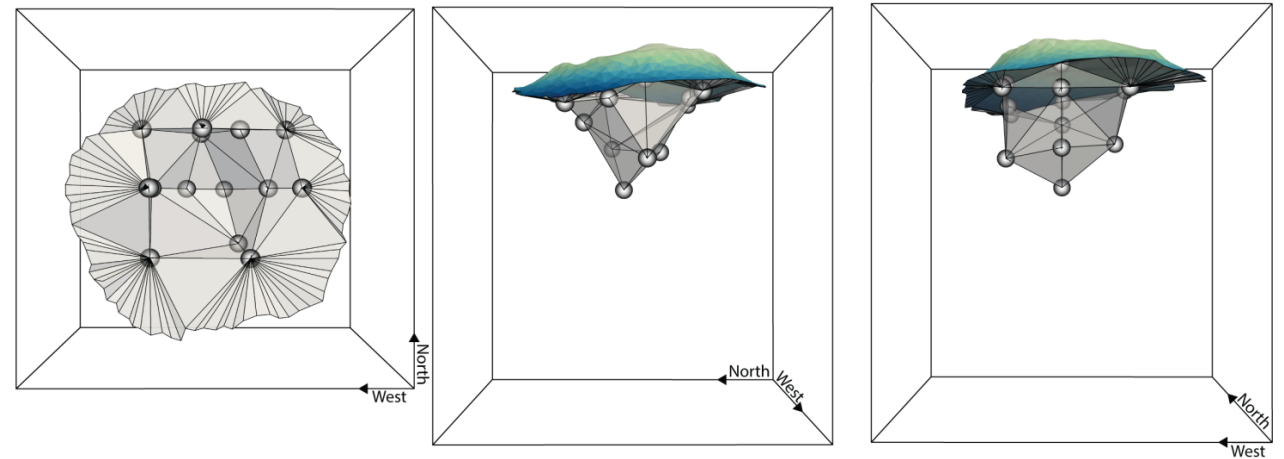
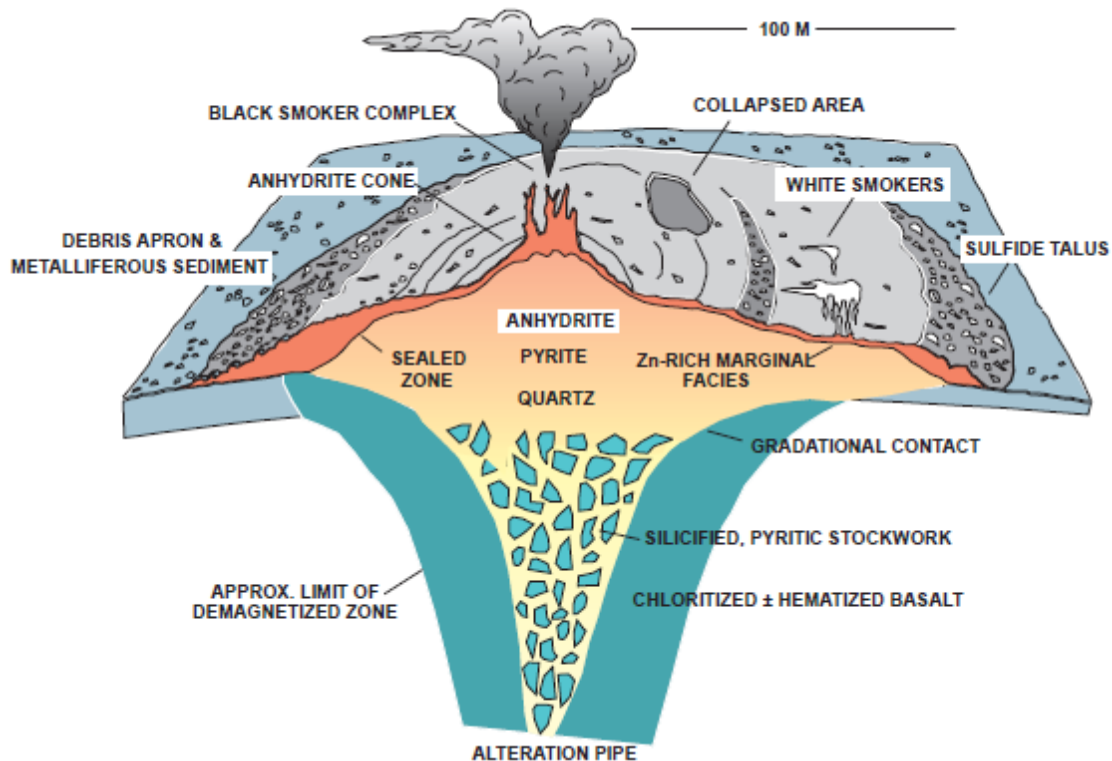
Observed geophysical data

# Surface geometry inversion



Topological rules

# Block parameterization: blocky models

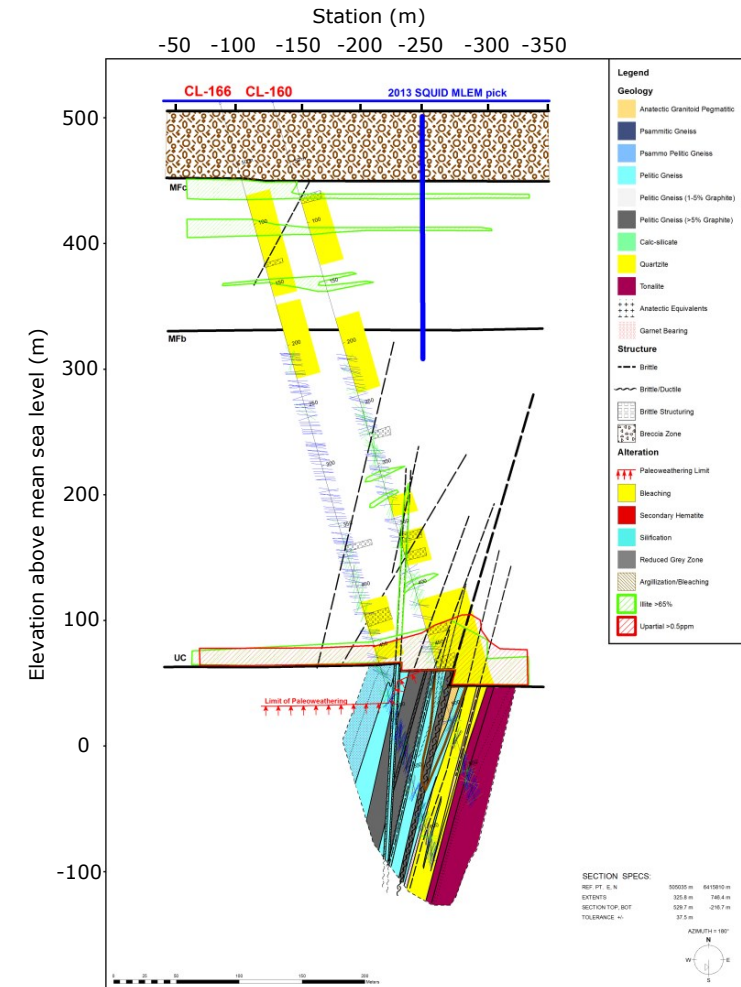
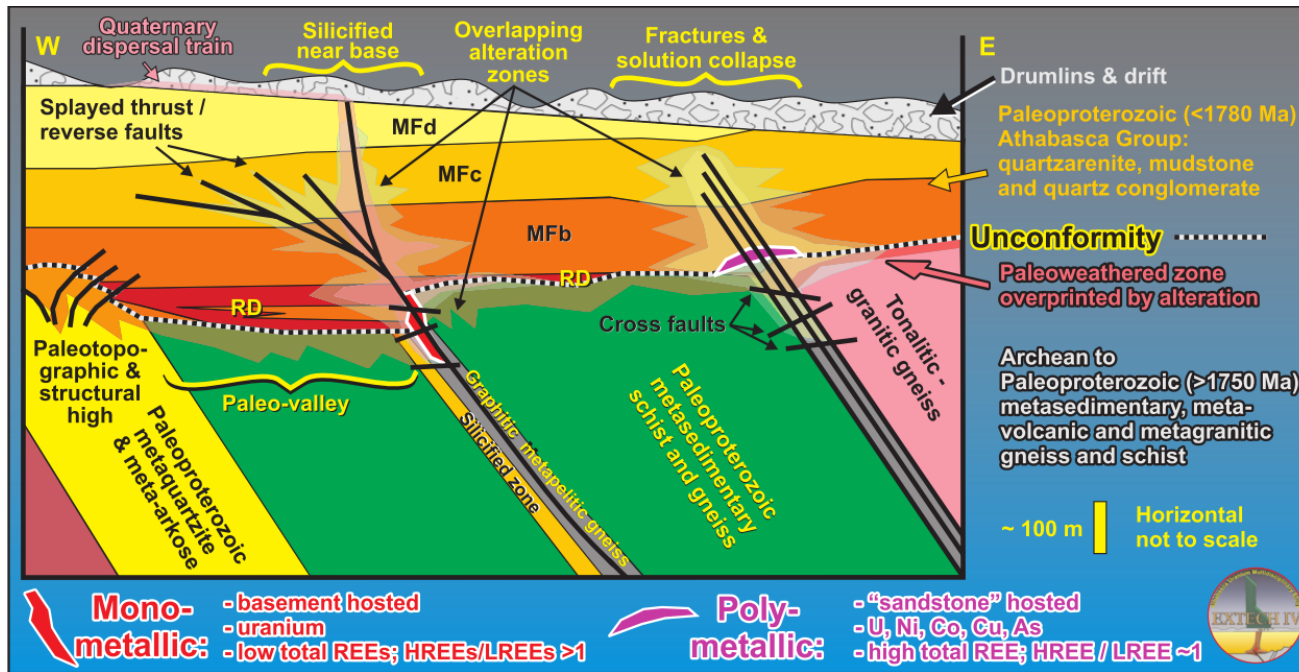


- Connect a small number of nodes into facets
- The facets comprise the wireframe representing the anomaly boundary
- The connections are fixed during the inversion

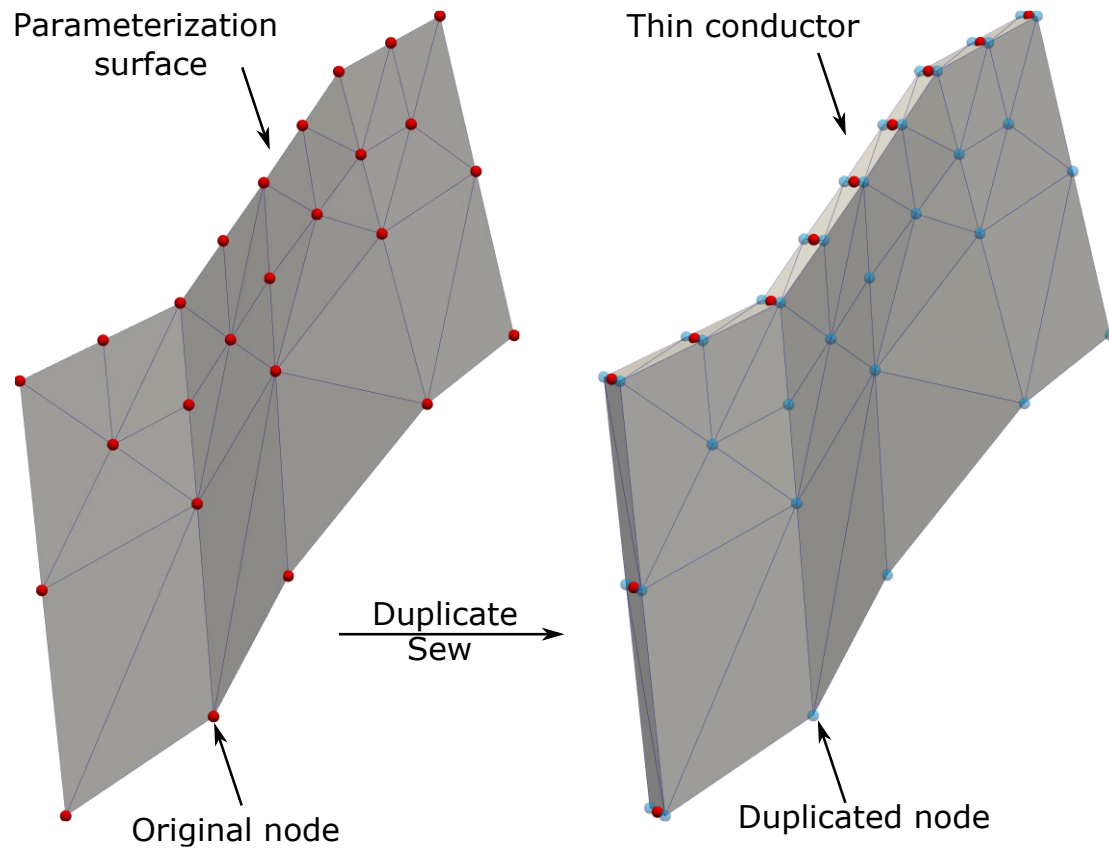
Hannington et al (1995)

Galley et al. (JGR Solid Earth, 2021)

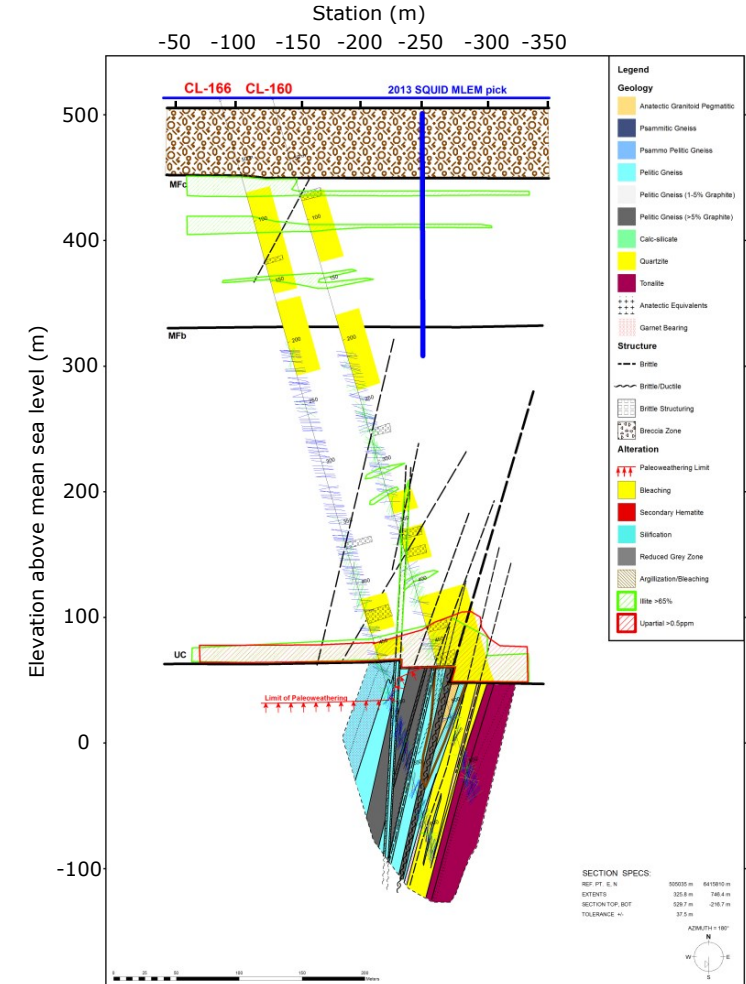
# Surface parameterization: thin, plate-like models



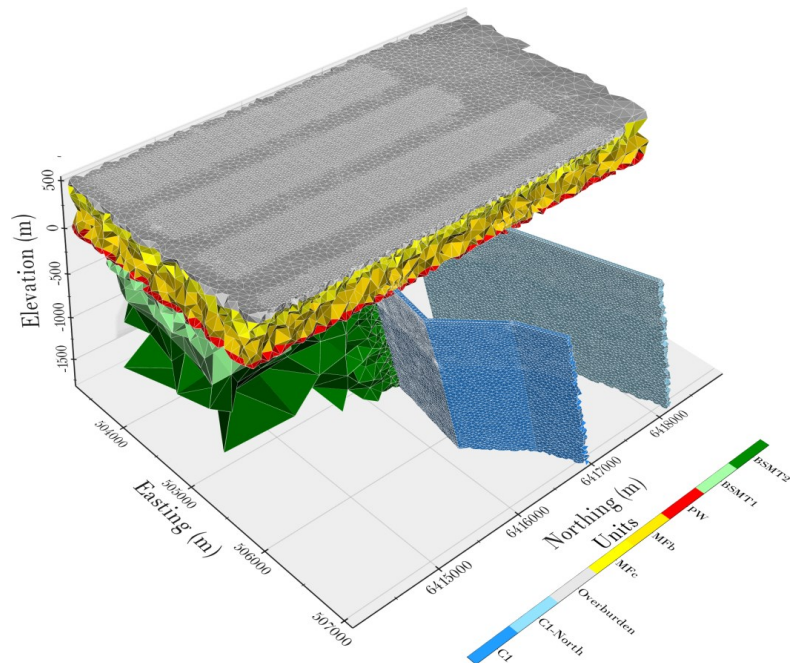
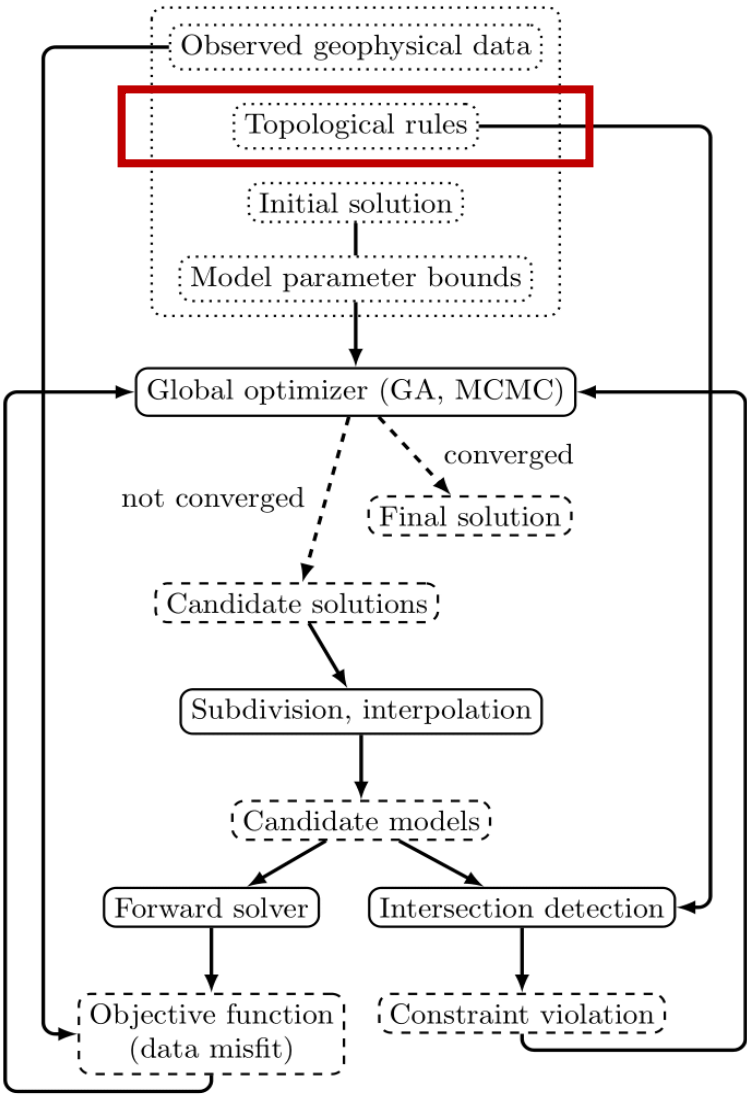
# Surface parameterization: thin, plate-like models



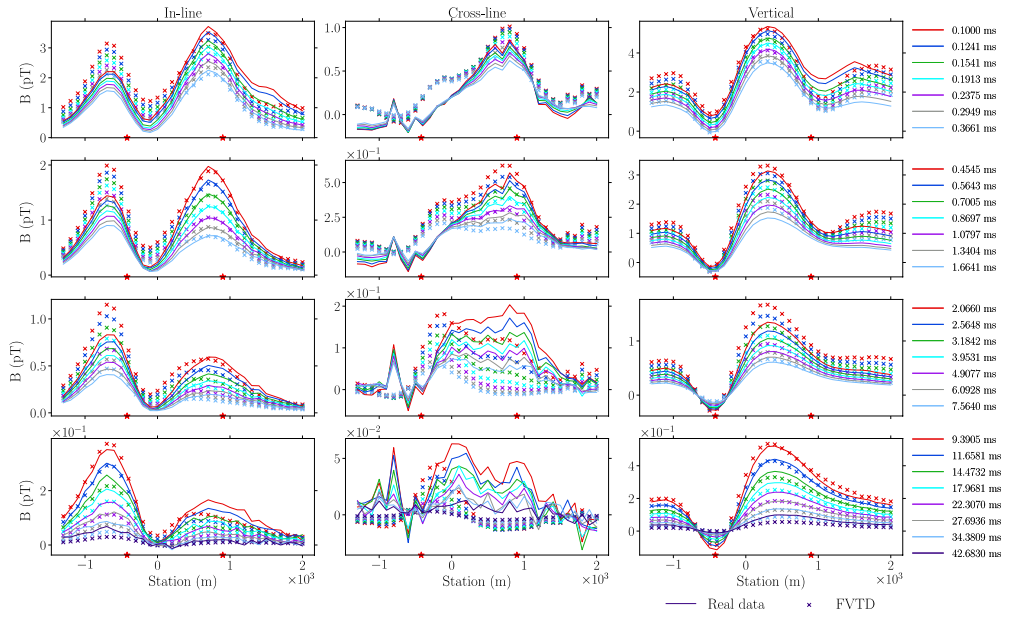
Surface parameterization of thin conductor



# Model estimation

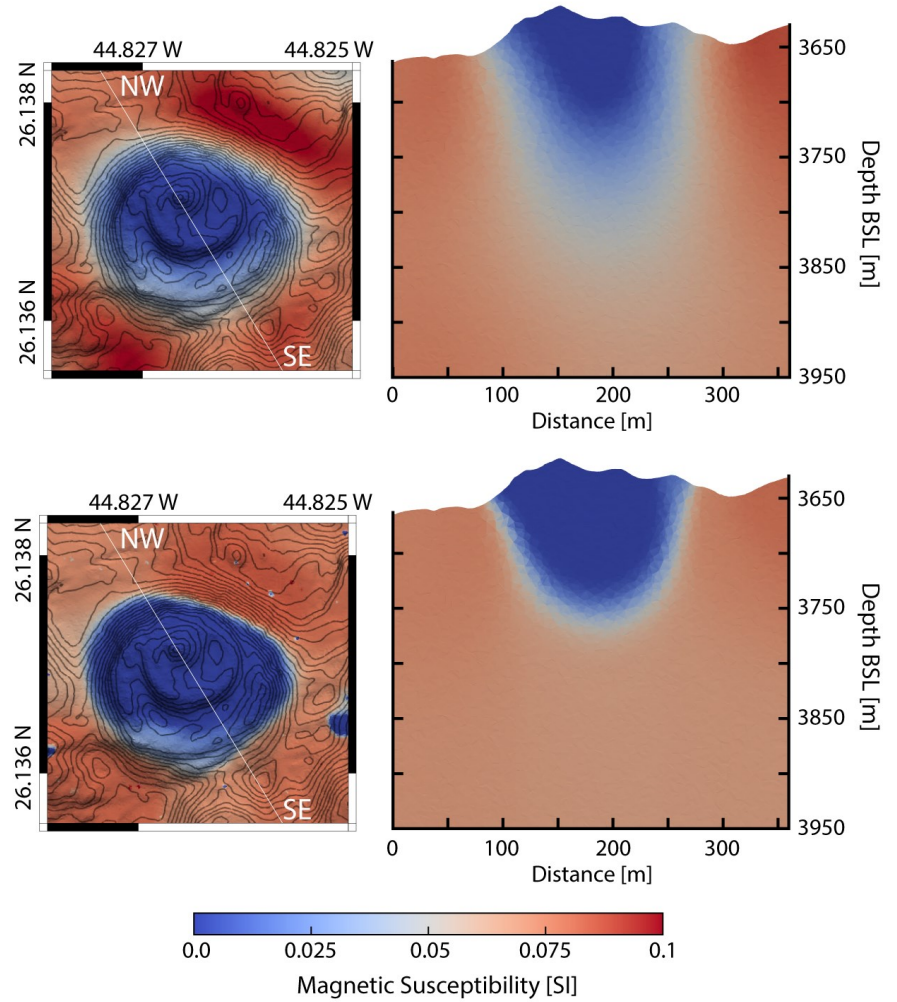
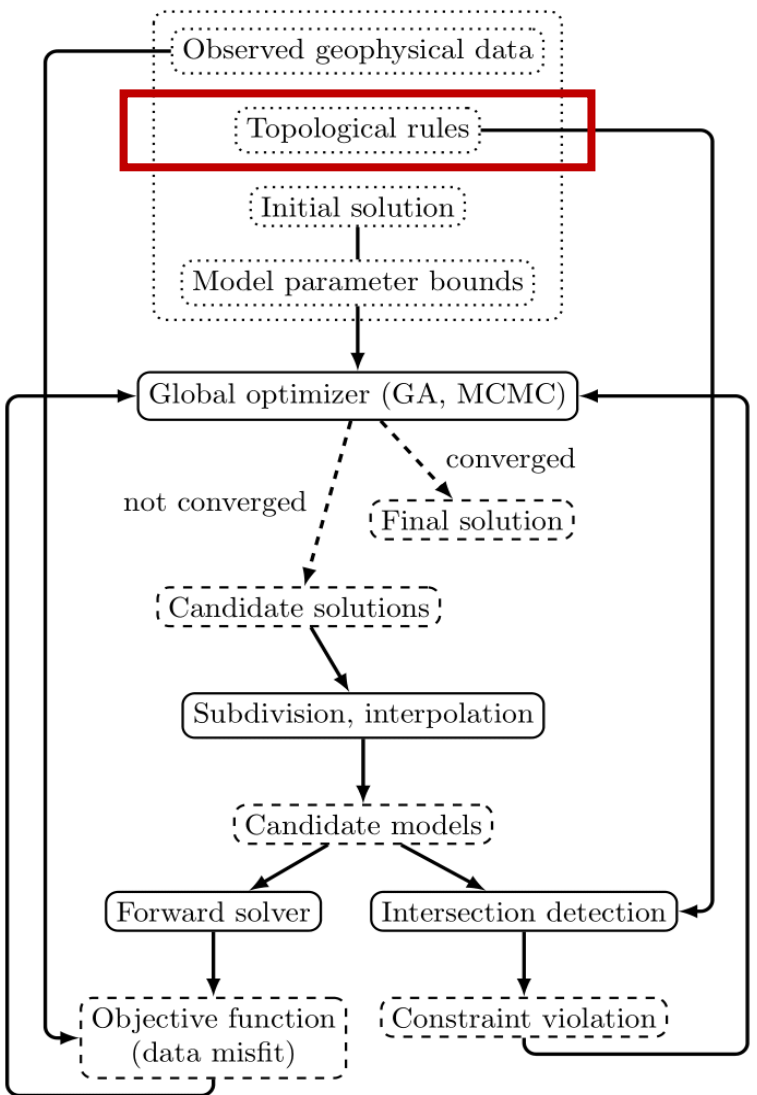


Lu et al. (Geophysics, 2021)



Trial-and-error modelling

# Model estimation



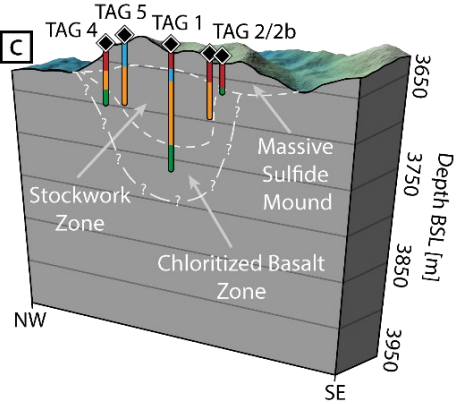
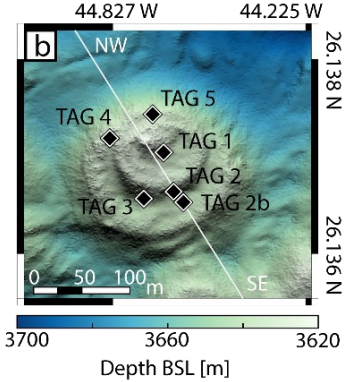
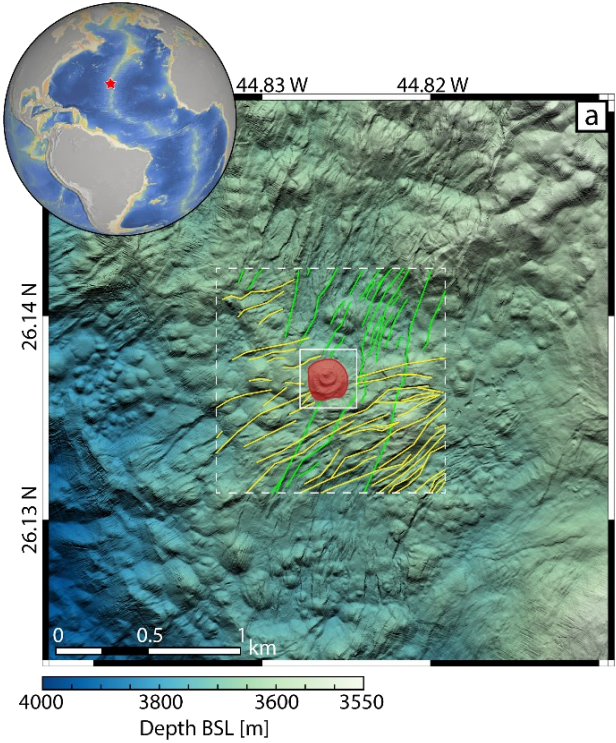
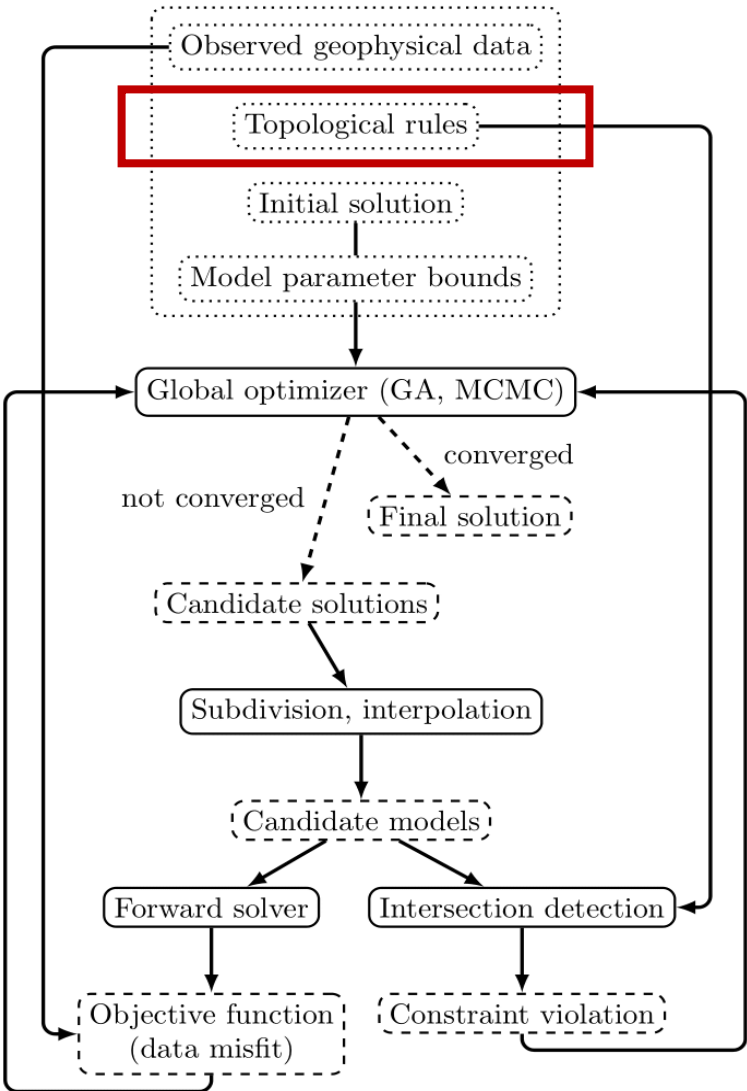
Constructed models from voxel inversion

Galley et al. (JGR Solid Earth, 2021)





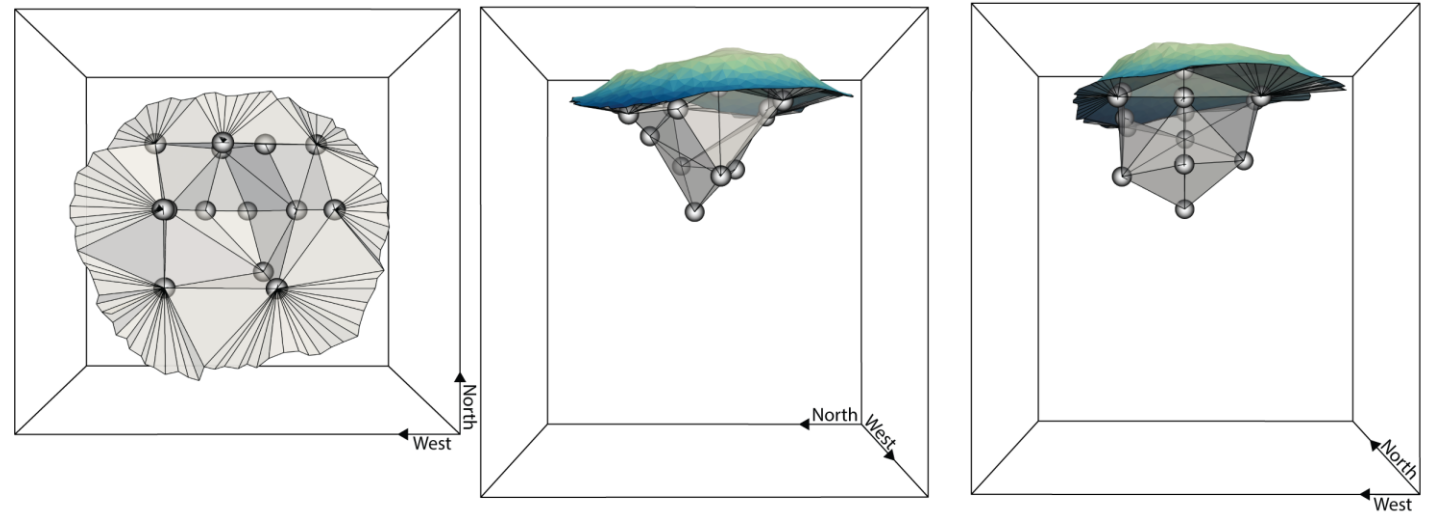
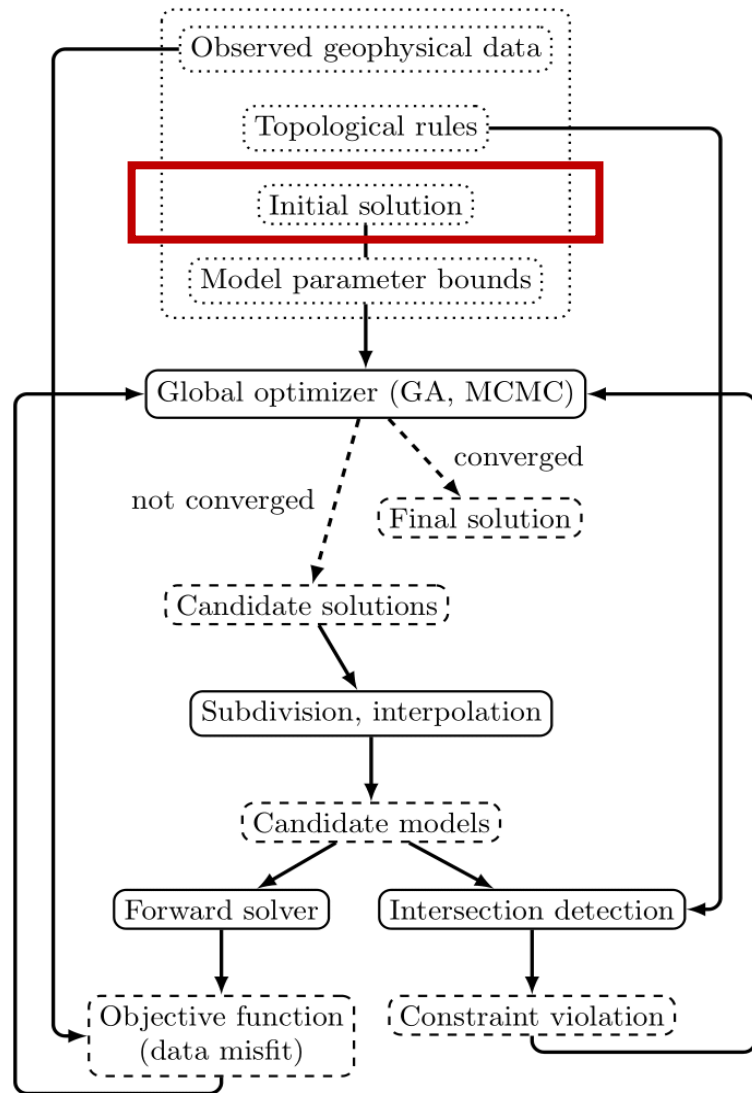
# Model estimation



- ◆ ODP Leg158 Drill Group Collar
- Massive Sulfide
- Silicified Wallrock Breccia
- Pyrite-Anhydrite
- Chloritized Basalt
- Axis-parallel faults
- Oblique faults and fissures

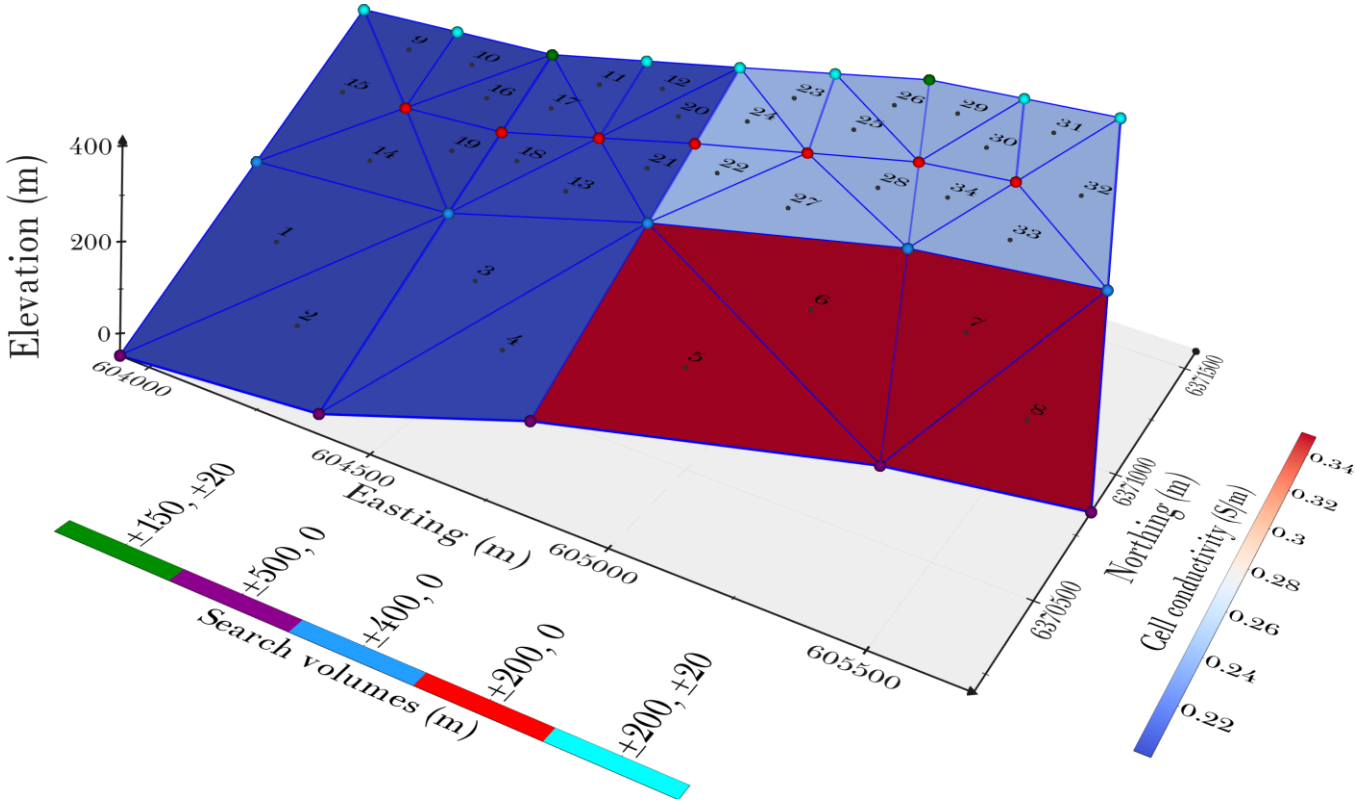
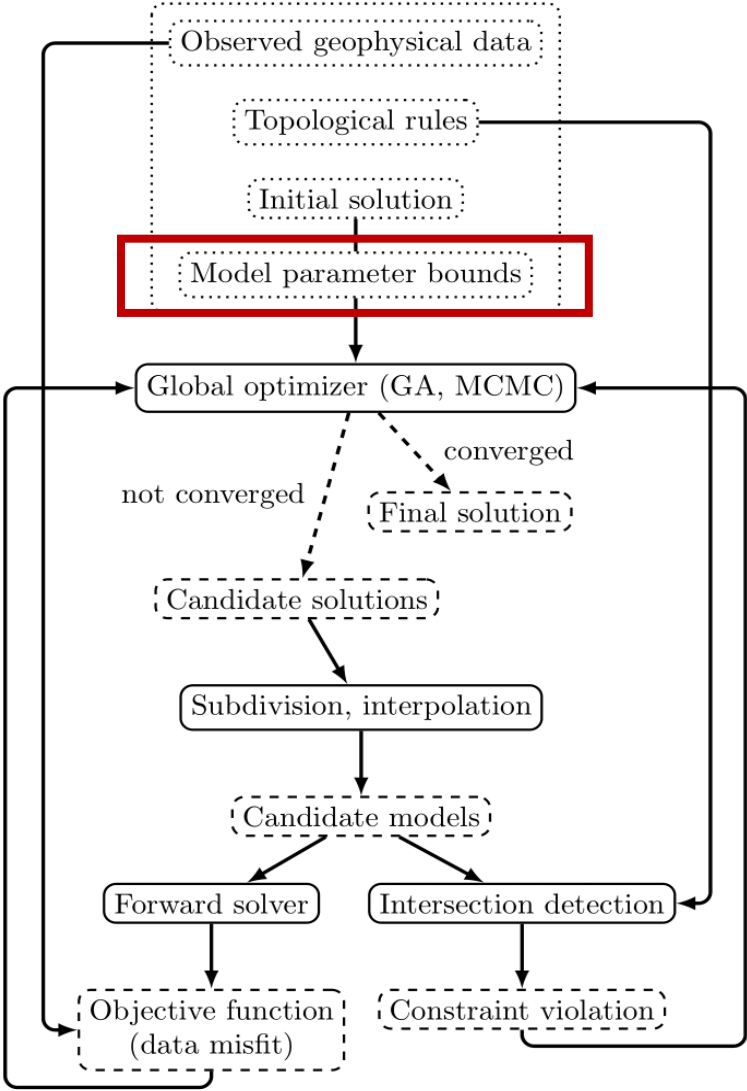
Drilling data

# Initial solution



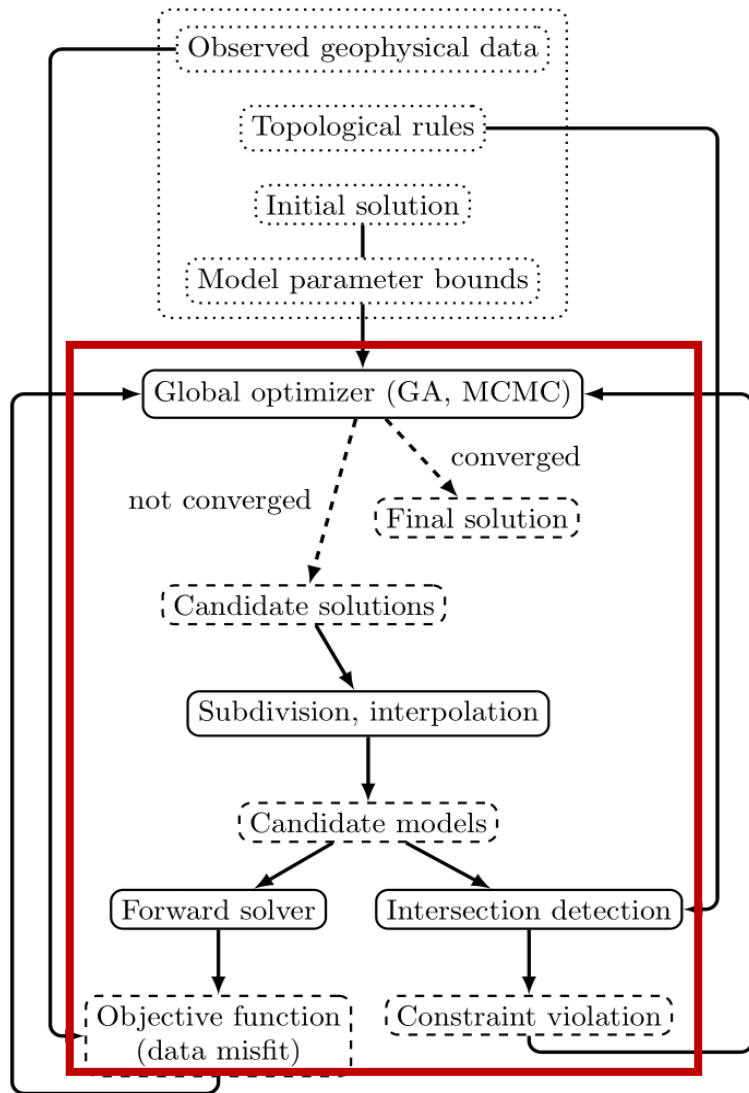
Topological rules

# Model parameter bounds (search volumes)



Different nodes have different bounds (search volumes)

# Surface geometry inversion



## Surface geometry inversion objective function:

$$\phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta \phi_m(\mathbf{m}),$$

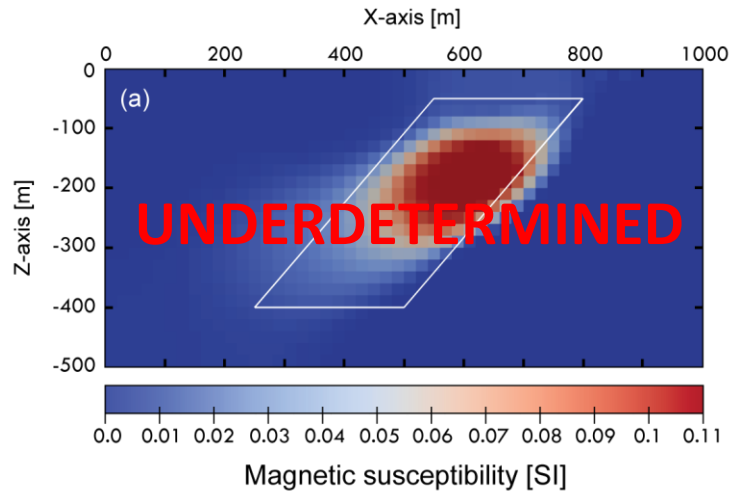
➤ Data misfit:

$$\phi_d(\mathbf{m}) = \| \mathbf{W}_d [\mathbf{d}^{obs} - \mathbf{d}(\mathbf{m})] \|^2,$$

➤ ~~Model structure (smoothness):~~

$$\phi_m(\mathbf{m}) = \sum_k \| \mathbf{W}_k (\mathbf{m} - \mathbf{m}^{ref}) \|^2.$$

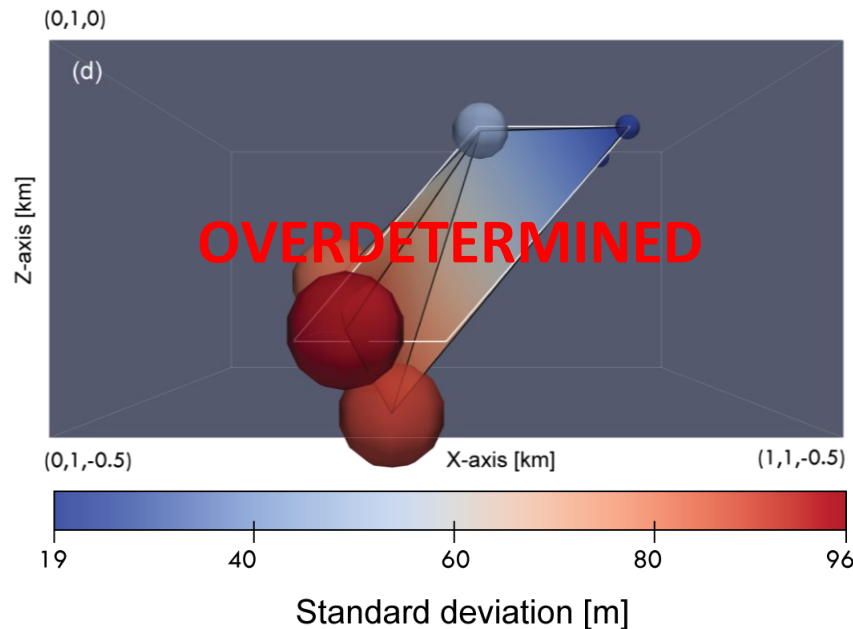
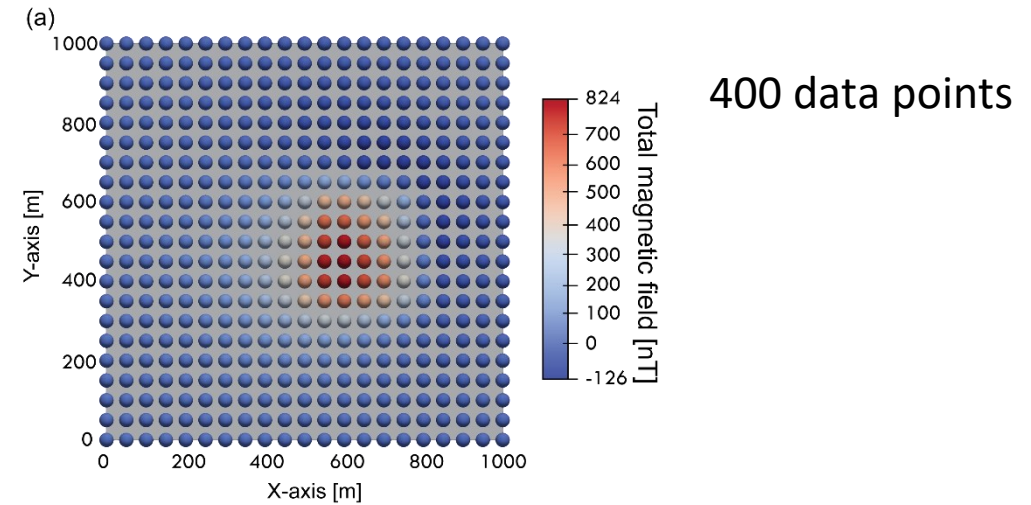
# Global optimization with genetic algorithm (GA)



$$\mathbf{m} = (\chi_1, \chi_2, \dots, \chi_M)$$

50x50x25 cells  
 → 62,500 cells

Requires Regularization



$$\mathbf{m} = (x_1, y_1, z_1, x_2, y_2, z_2, \dots, x_M, y_M, z_M)$$

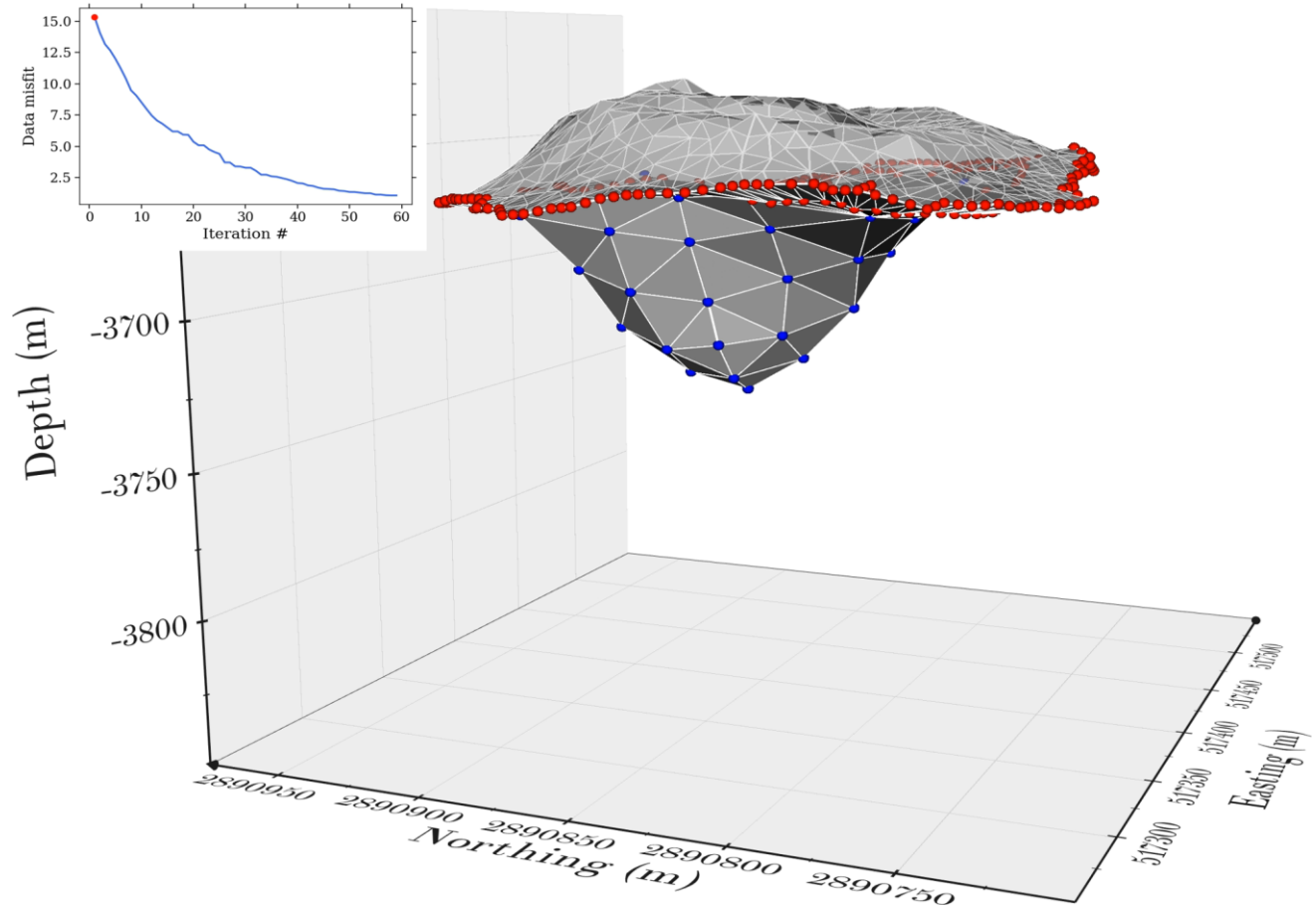
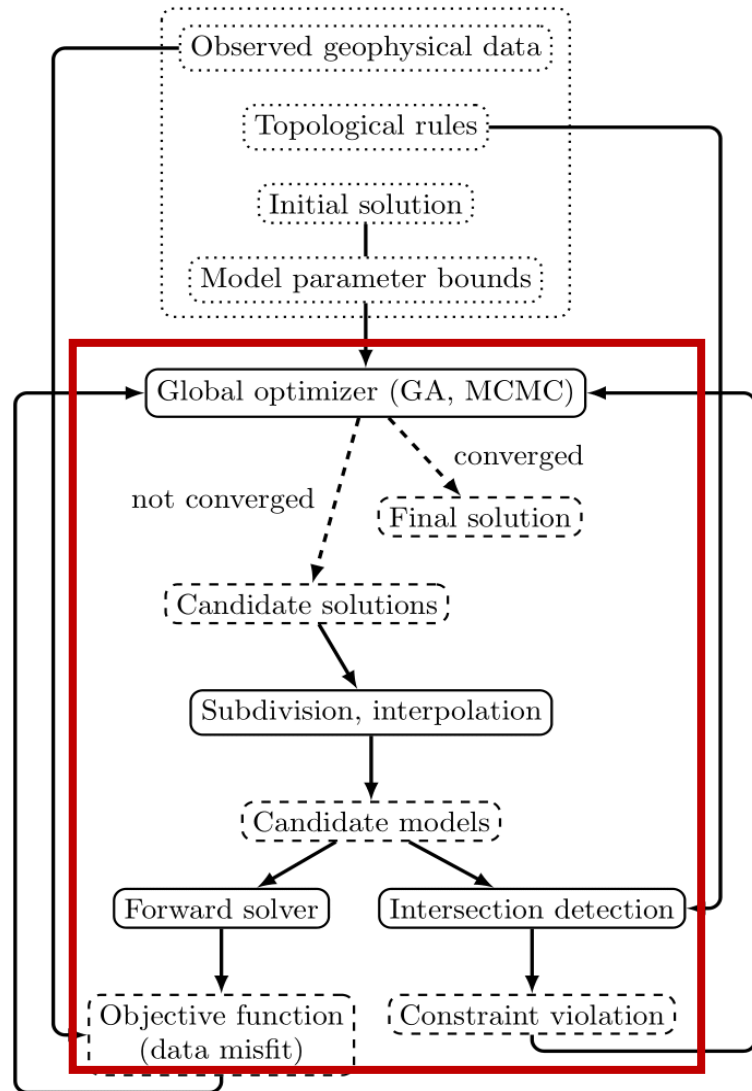
8 vertices  
 → 24 model parameters

Only the Data Misfit is Necessary

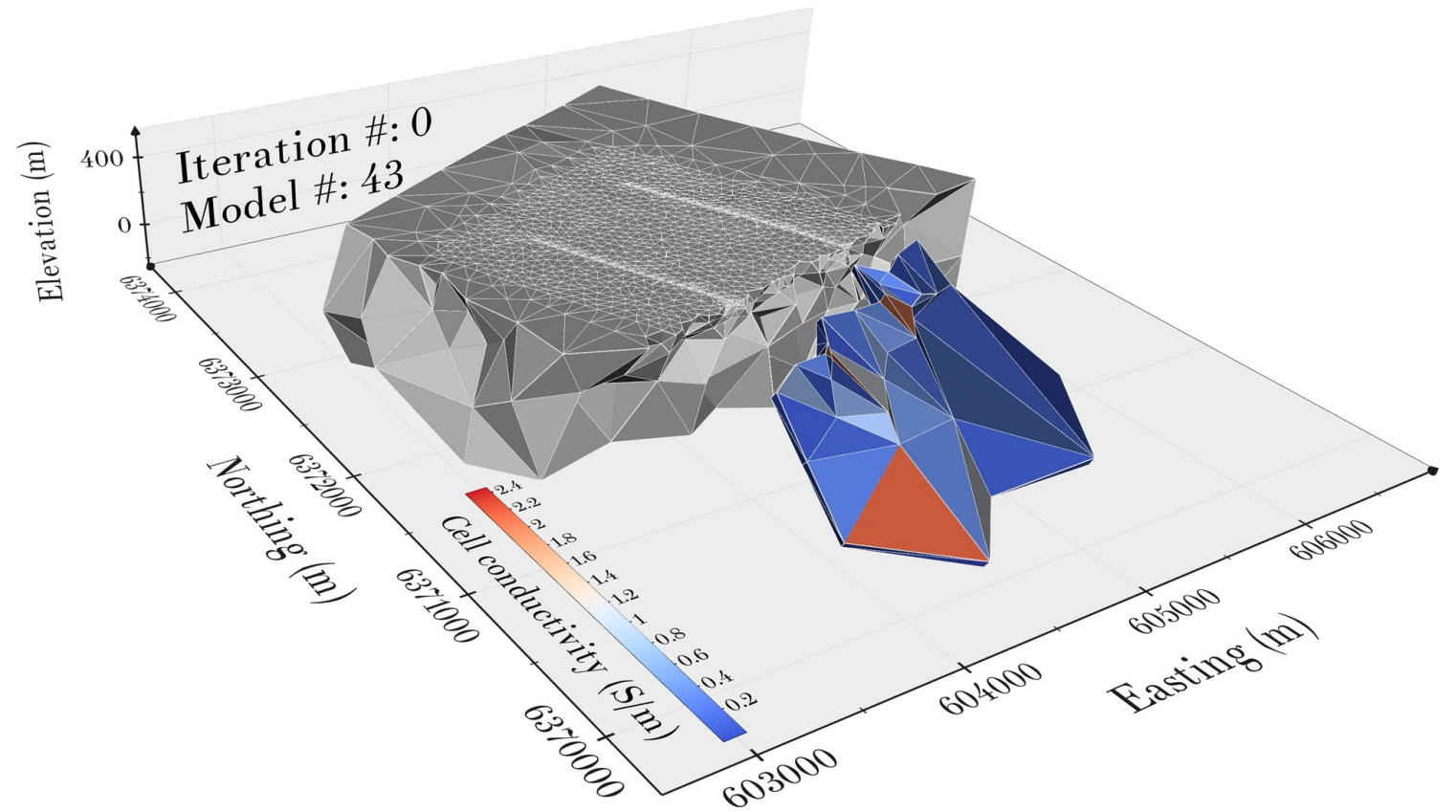
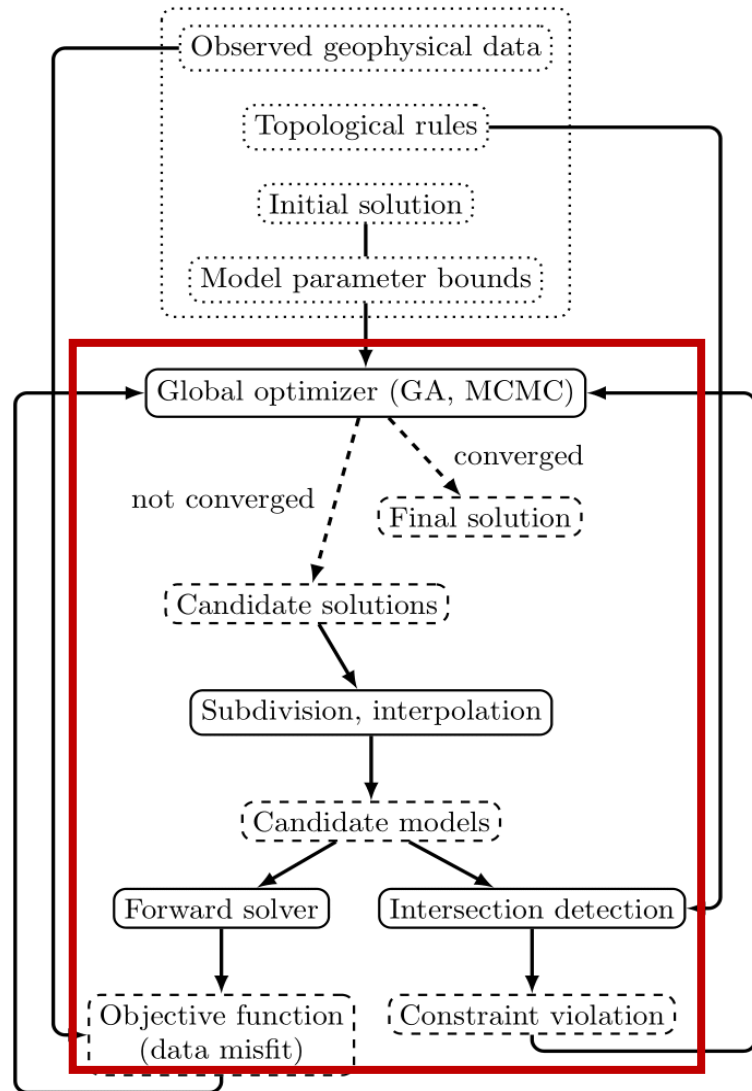
-> no extra regularization calculations

-> no solving for trade-off parameters

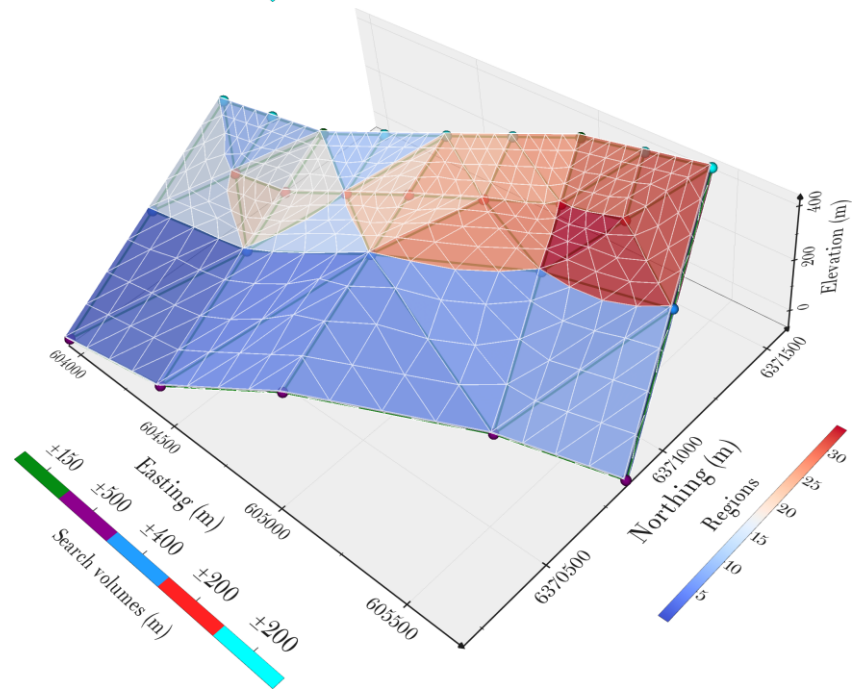
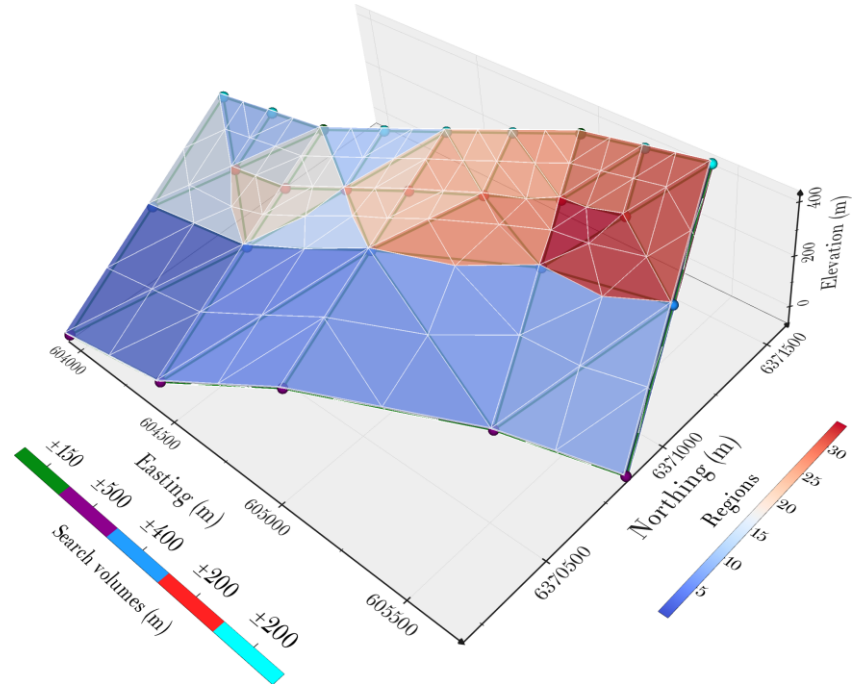
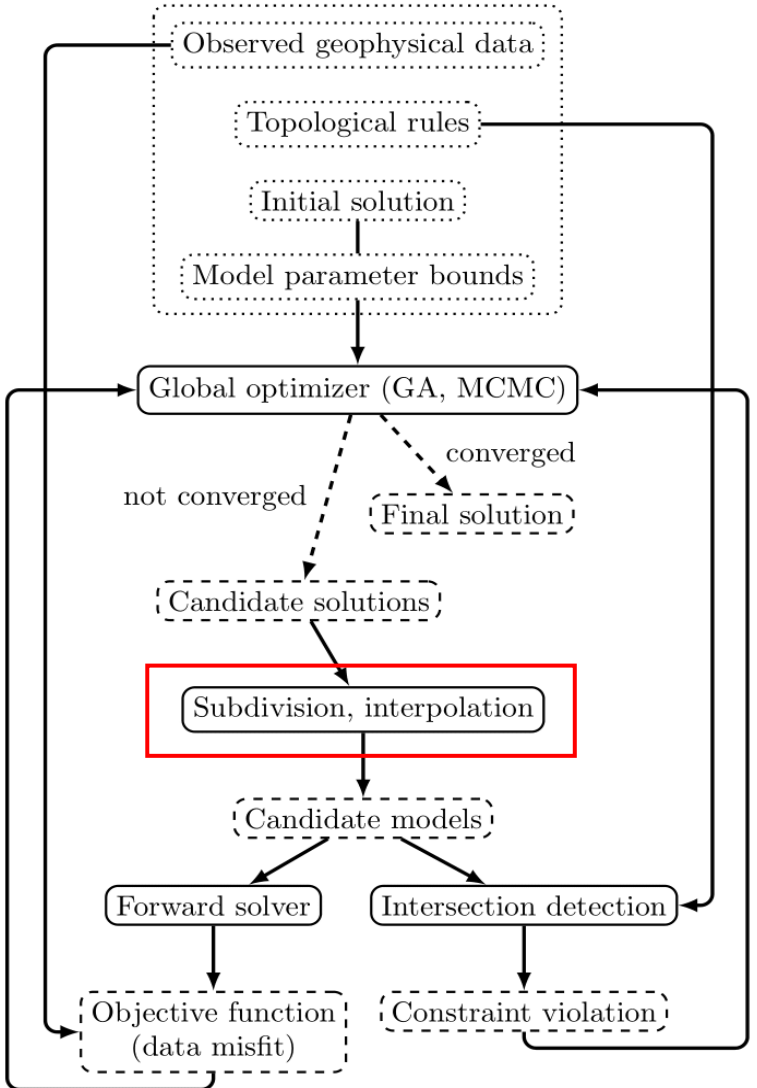
# Surface geometry inversion



# Surface geometry inversion

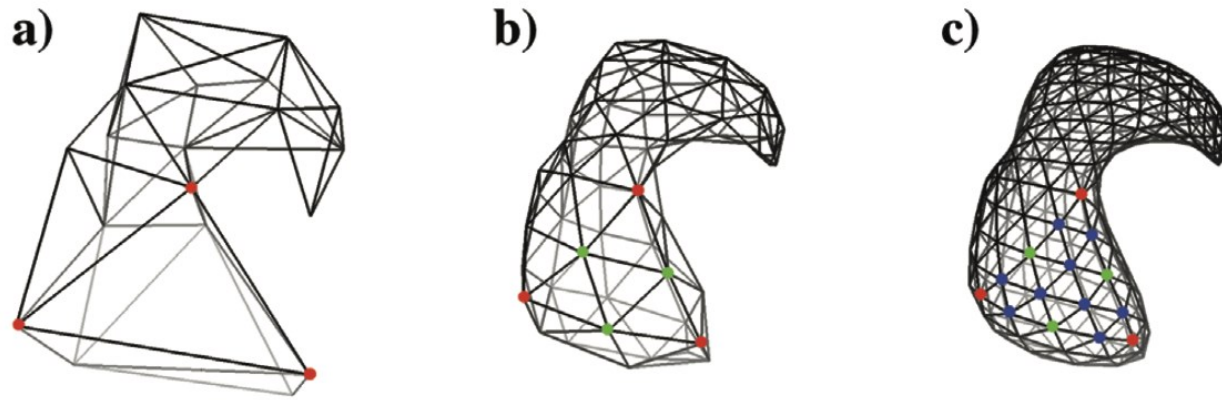


# Model subdivision

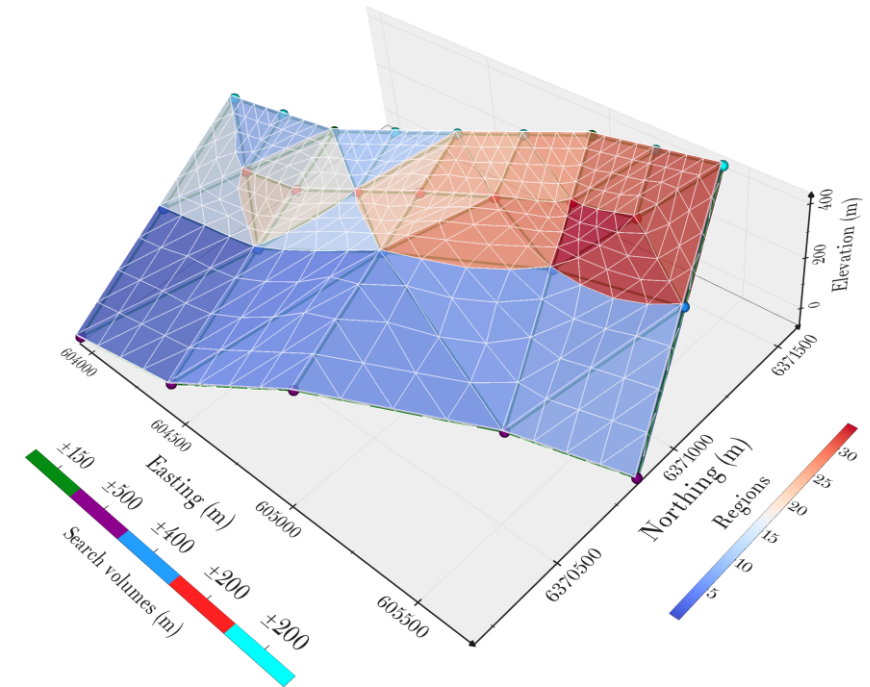
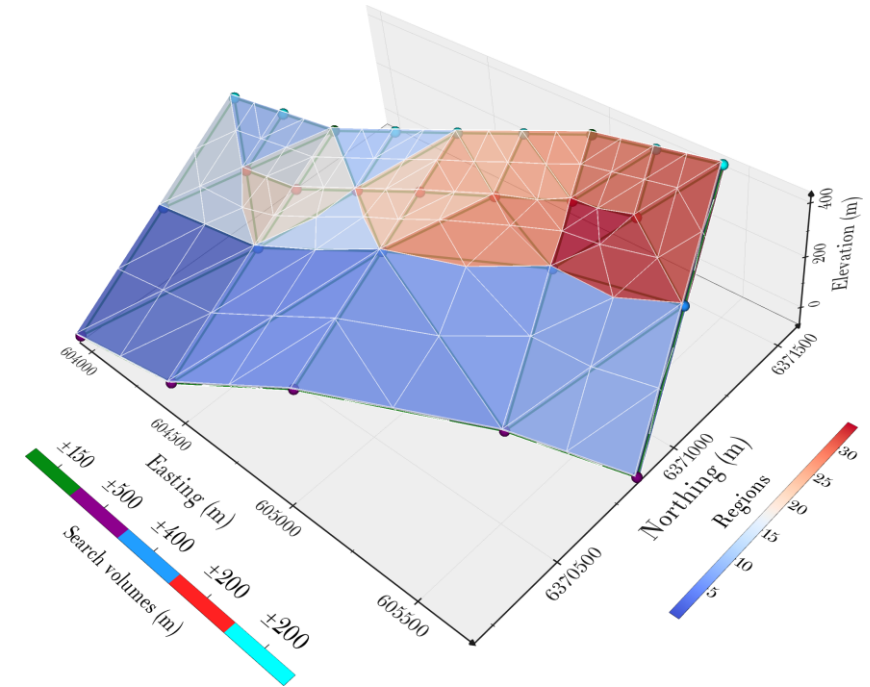




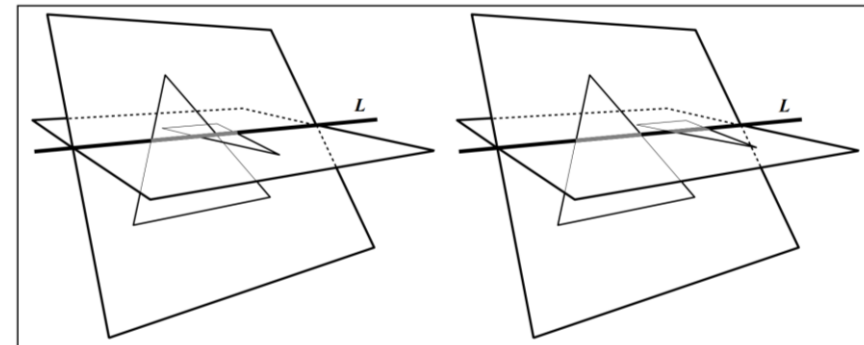
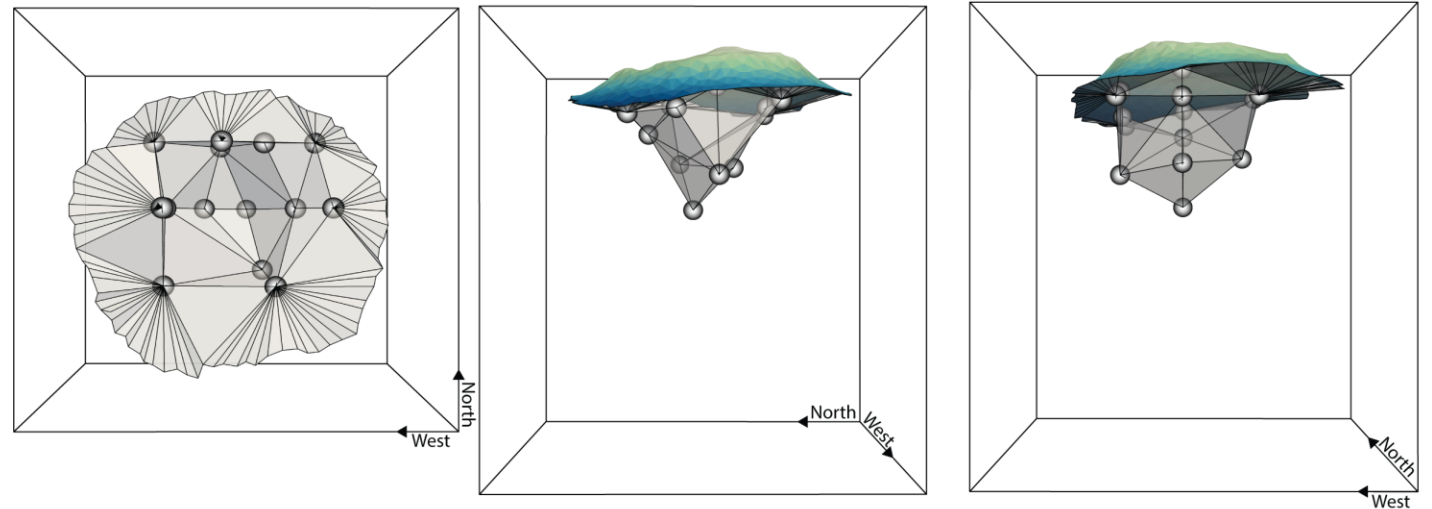
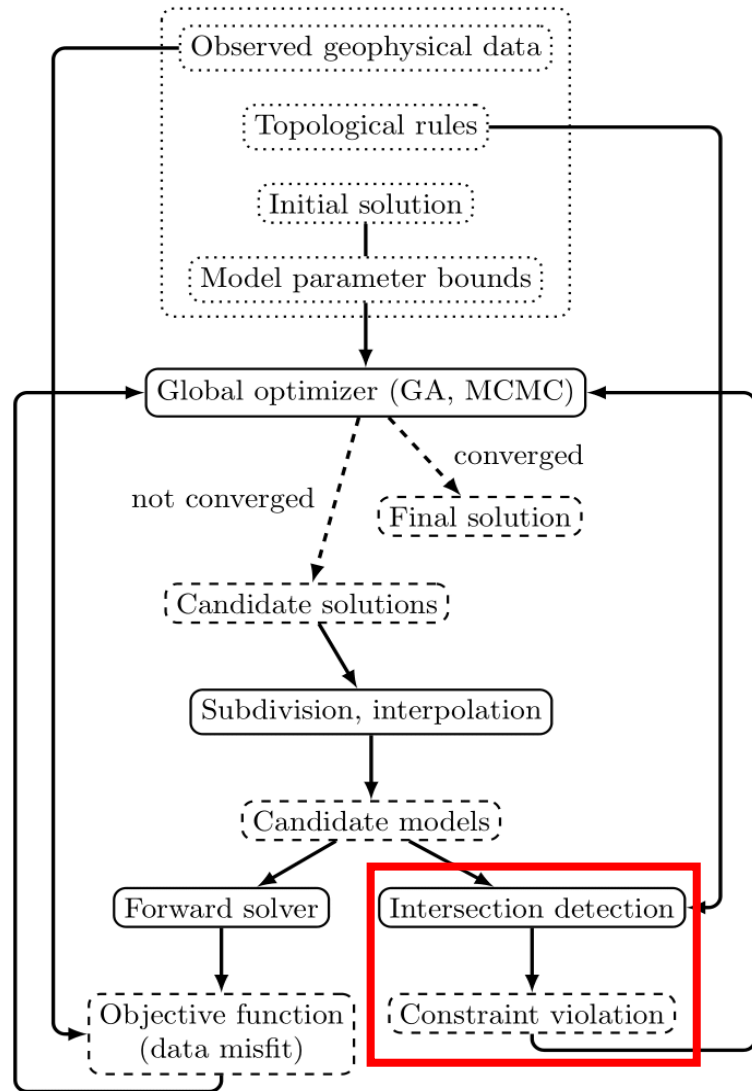
# Model subdivision



- Small # of nodes to reduce the # of inversion parameters
- Models can be subdivided up to two times
- 3D interpolation is performed to smooth the model

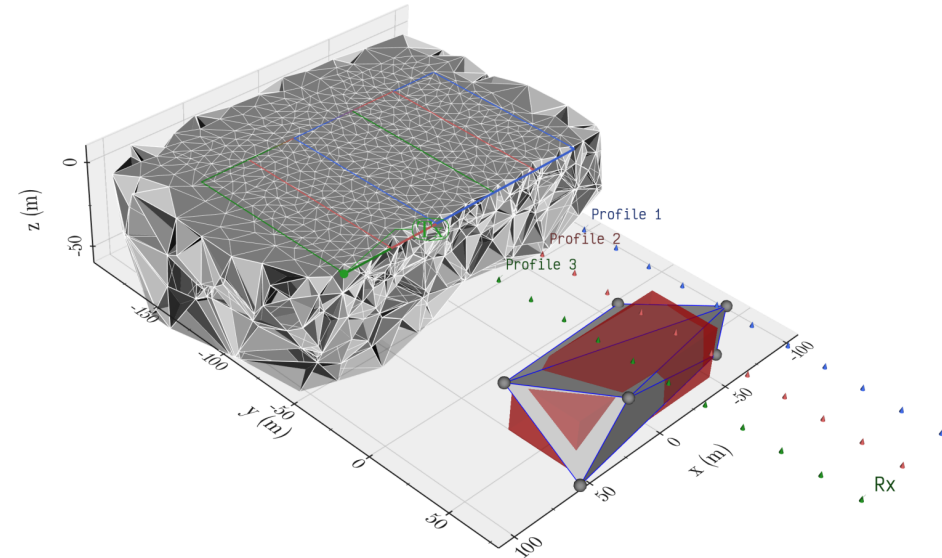
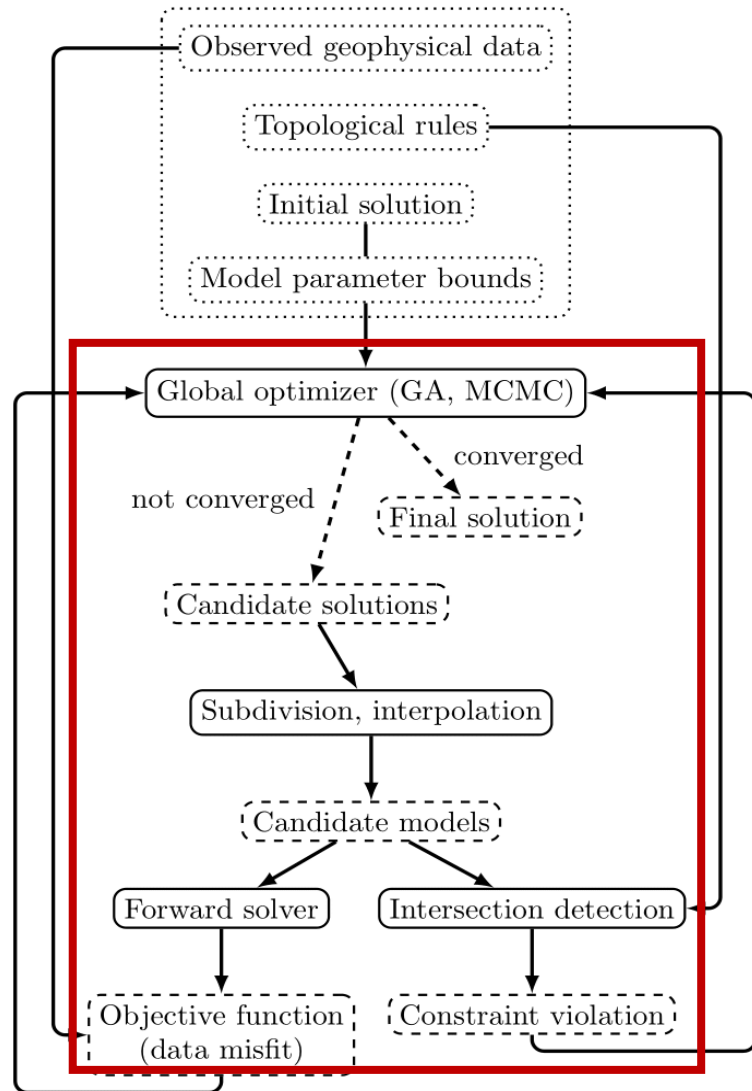


# Triangle-triangle intersection detection



Moller, 1997

# Surface geometry inversion for EM data

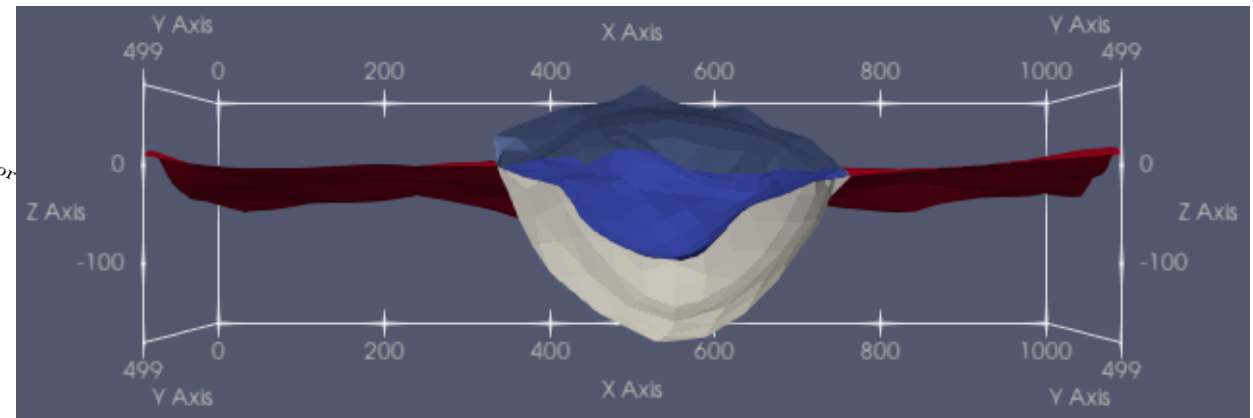
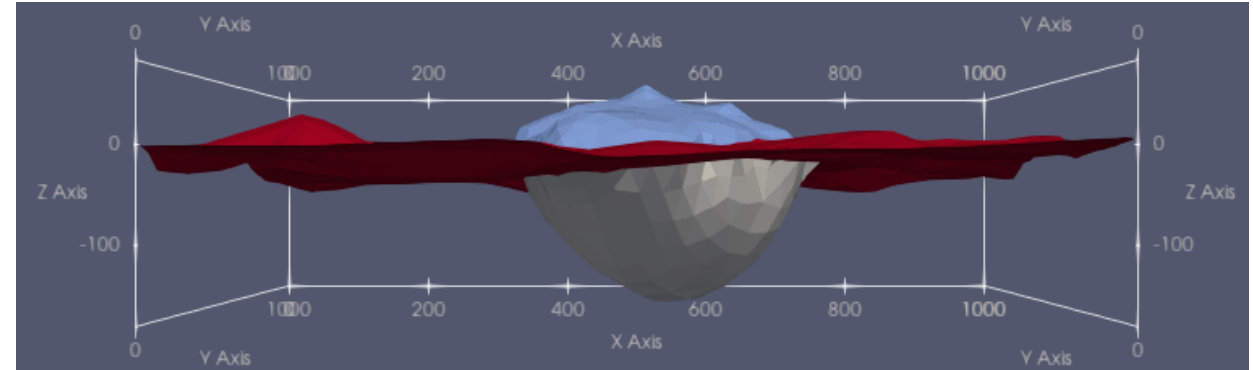
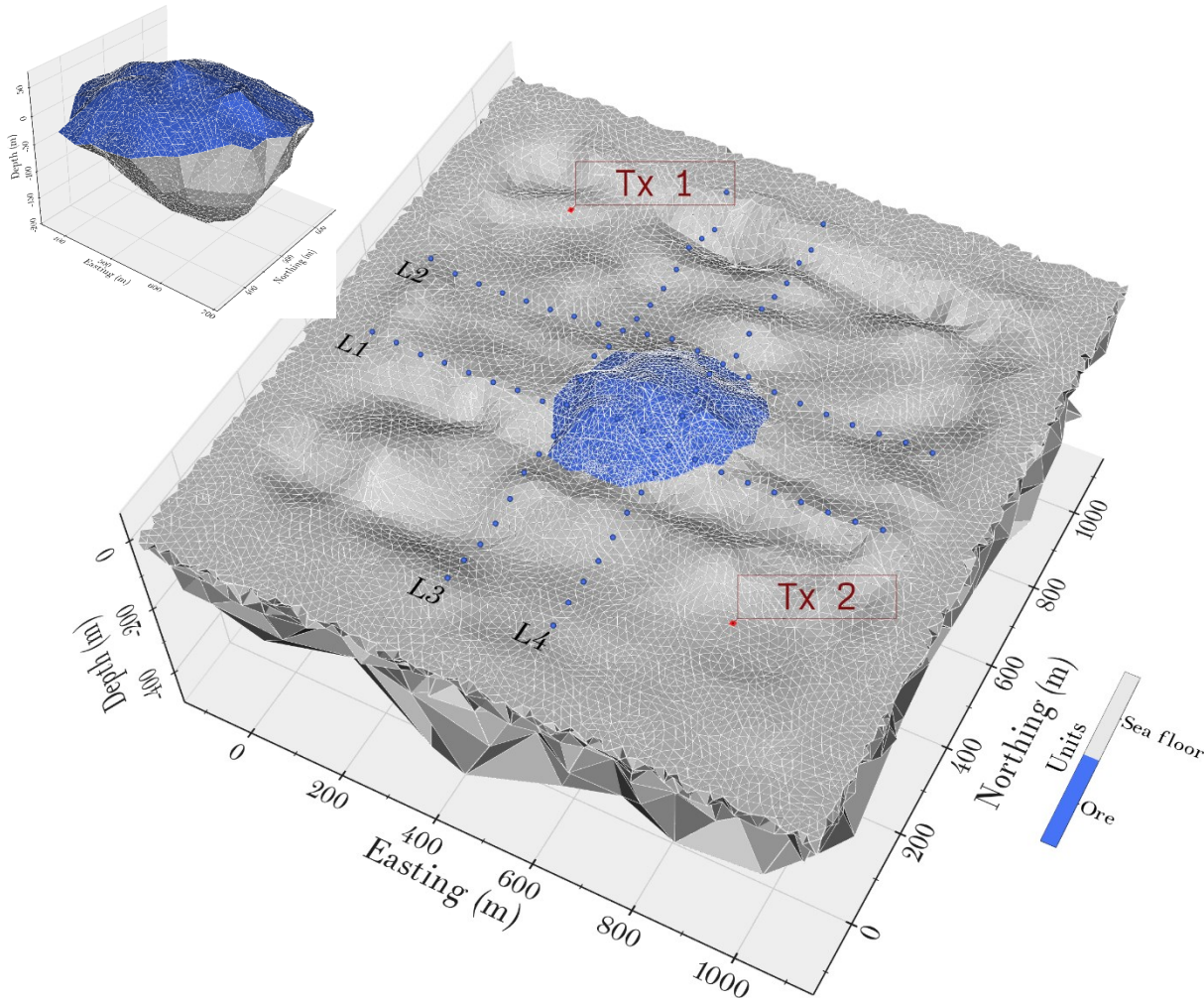


- To calculate the predicted data, the entire model needs to be discretized
- Automatic mesh generation for a given model (TetGen)
- Finite-element solver
- MPI + OpenMP parallelization

# Outline

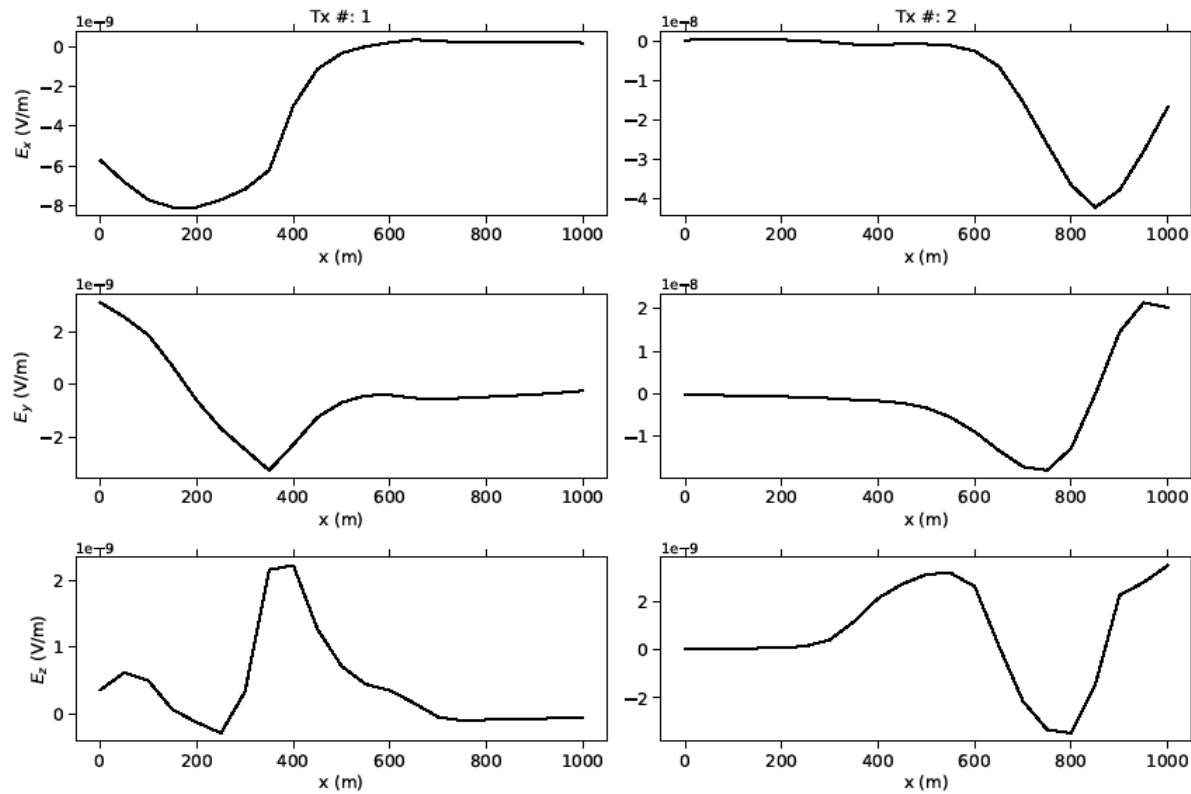
- Motivation
- Surface geometry inversion
- **Marine CSEM examples**
- TEM examples
- Conclusions

# Marine CSEM example

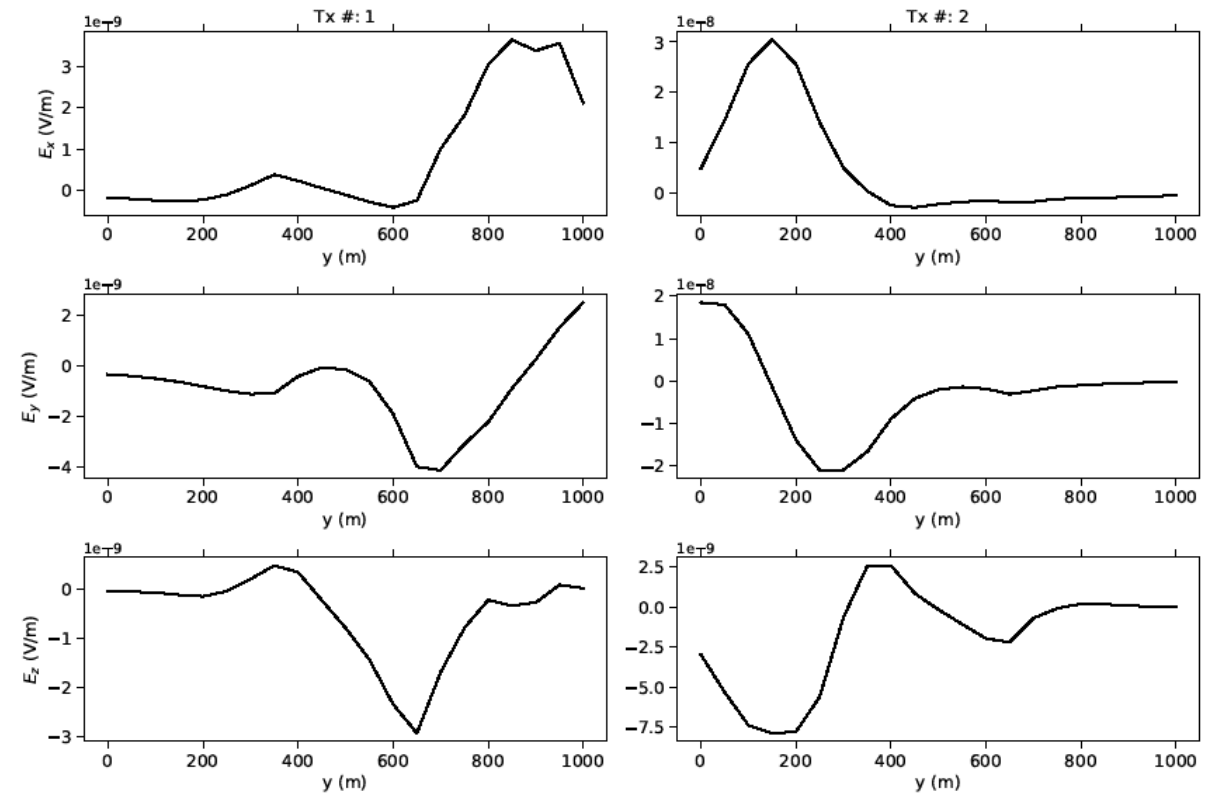


Tetrahedral meshes for the SMS deposits

# Marine CSEM example: synthetic data

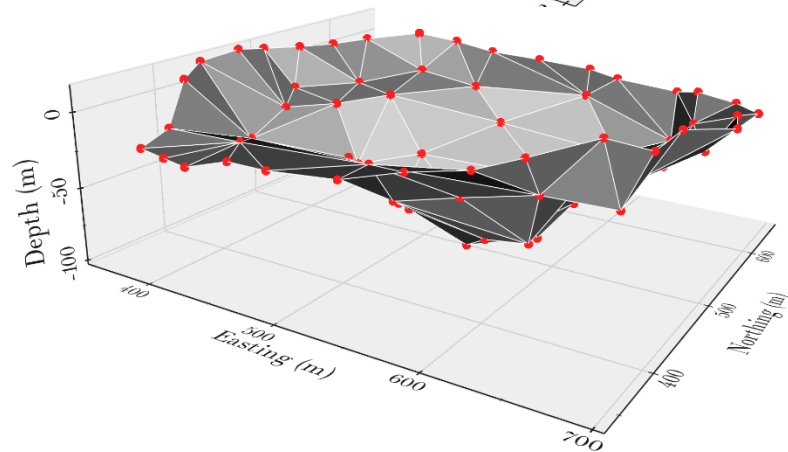
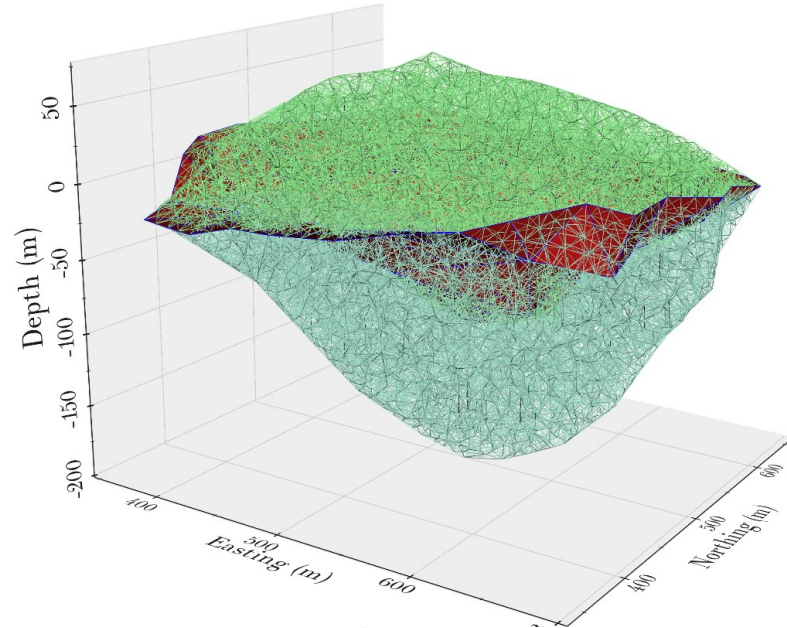


Electric field responses of profile L1



Electric field responses of profile L4

# Marine CSEM example: model setup



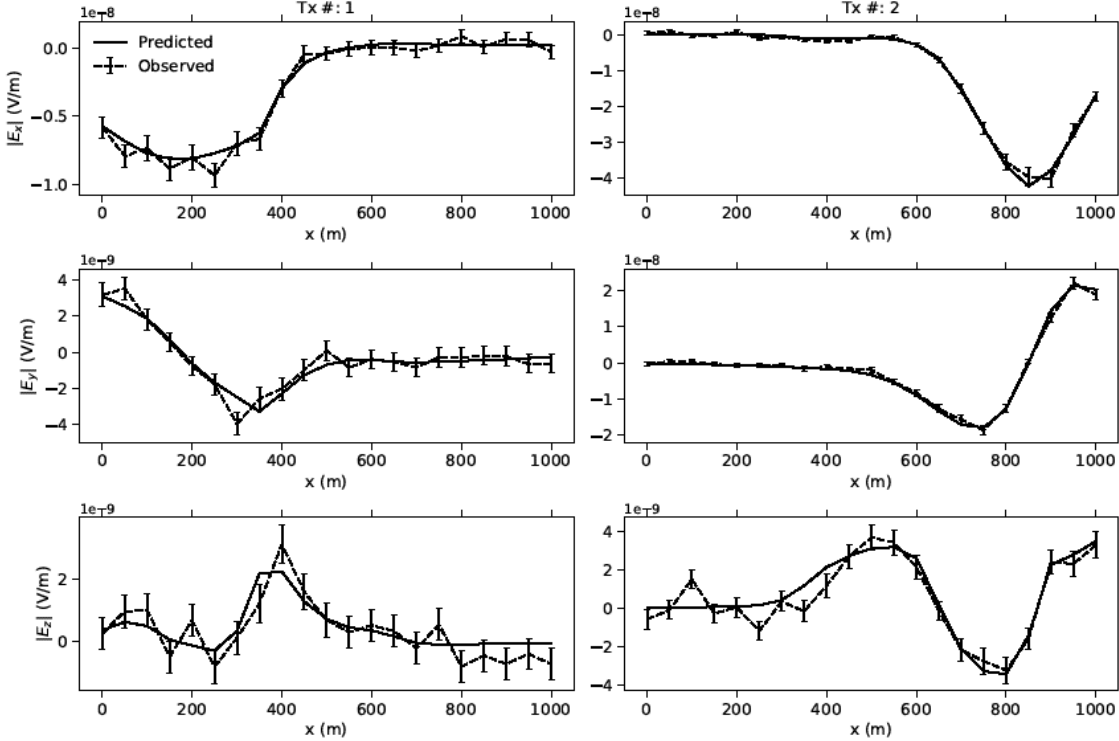
## Conductivities:

- Ore body: 10 S/m
- Sea water: 0.33 S/m
- Seafloor: 0.1 S/m
- True conductivity is used for inversion

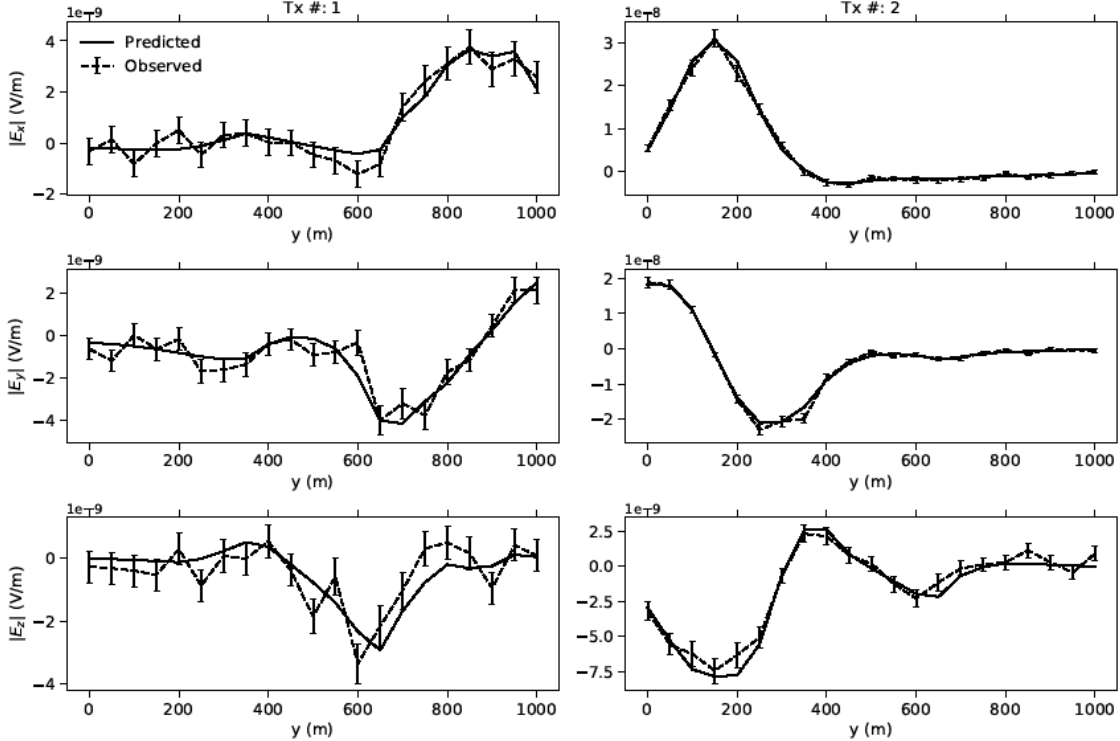
## Inversion parameters:

- 38 nodes in the surface model
- Each node is allowed to move vertically
- Moving range is (-100, 5) m
- 5% Gaussian noise
- GA population: 239

# Marine CSEM example: data fitting



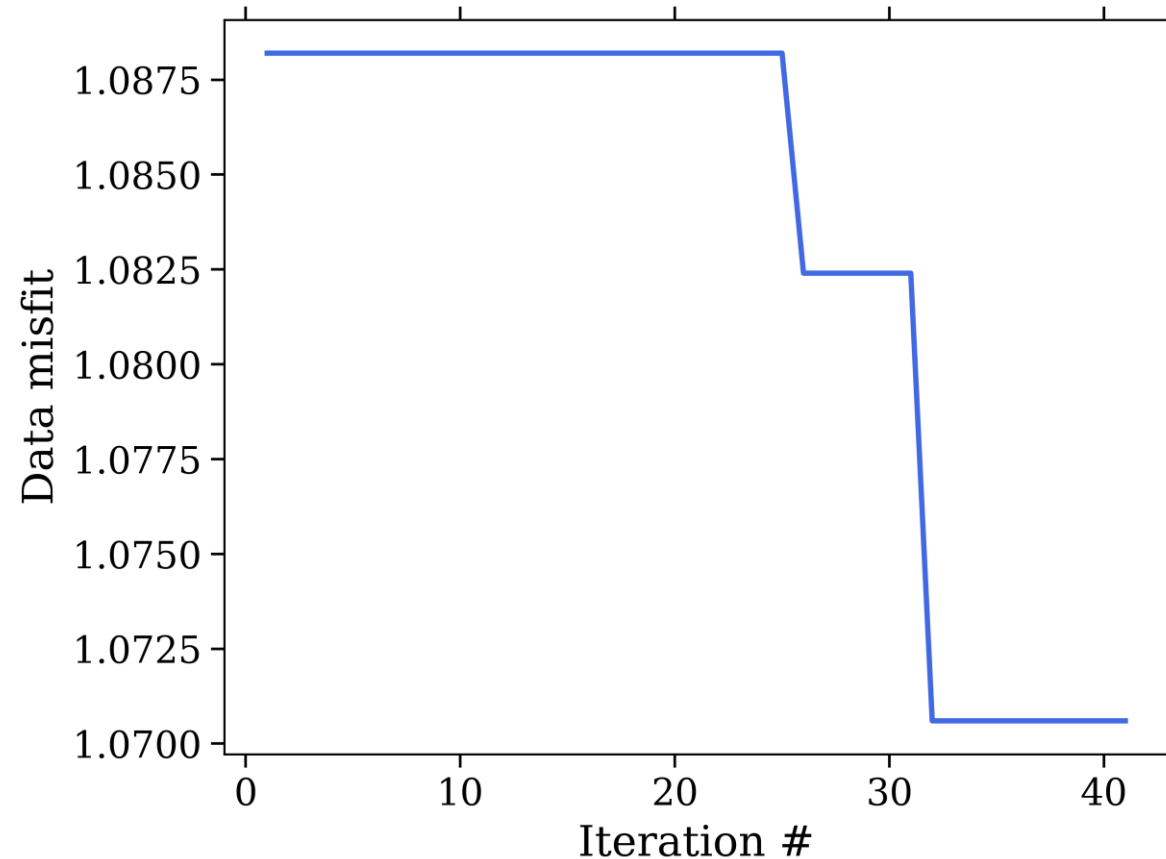
Electric field responses of profile L1



Electric field responses of profile L4

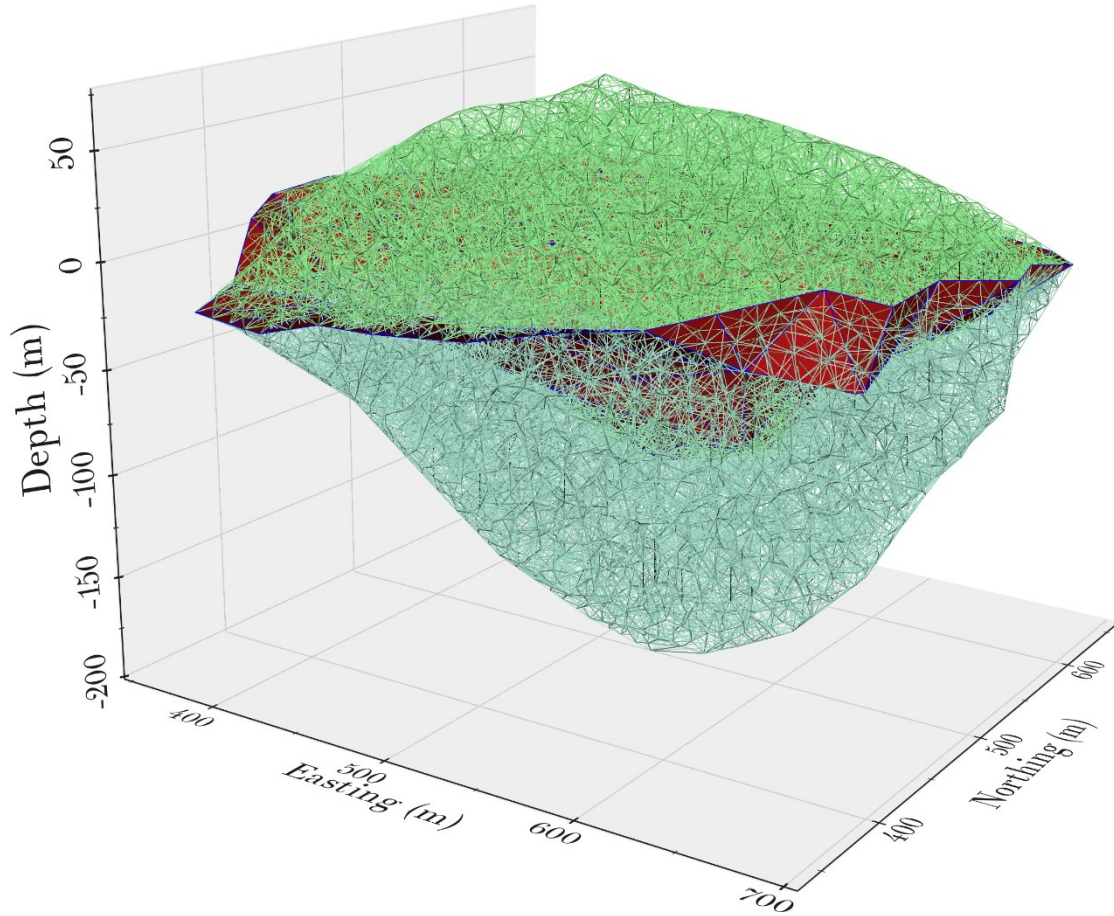


# Marine CSEM example: convergence

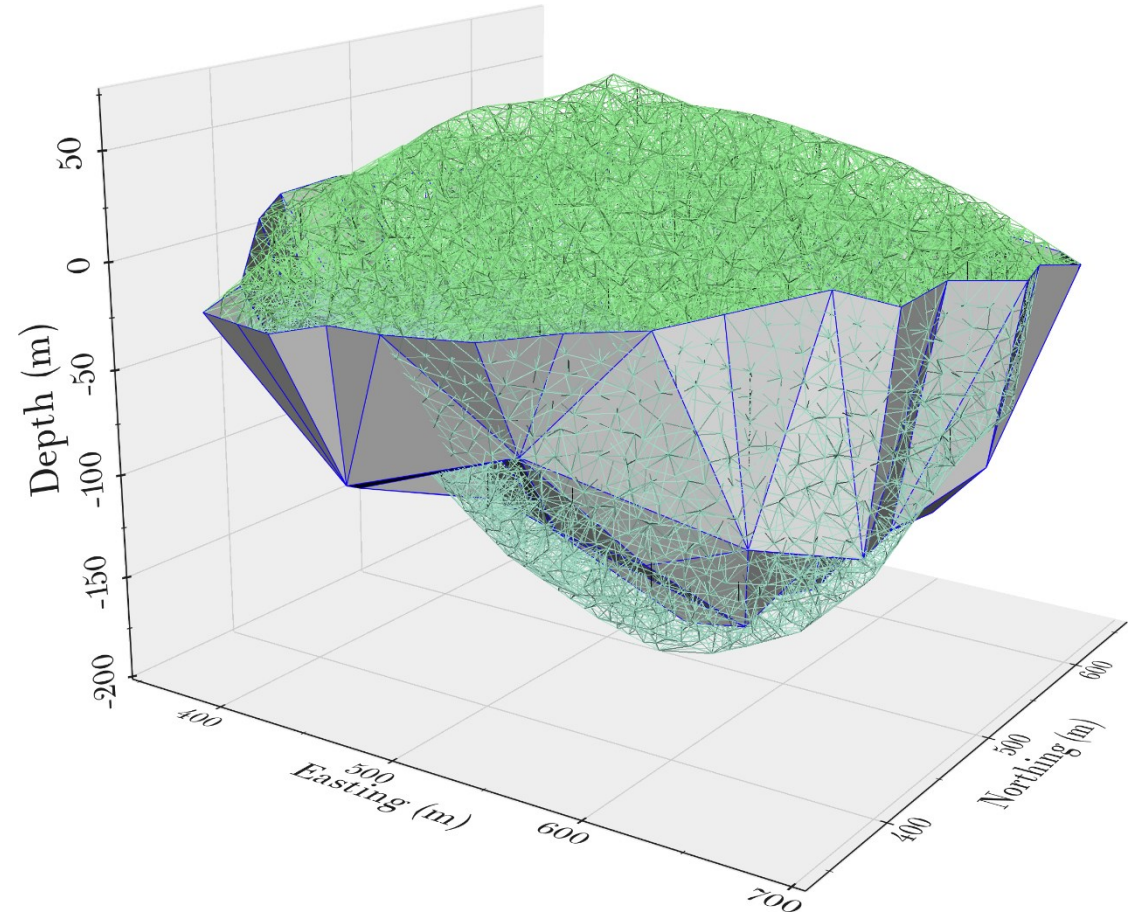


- # parameters: 38 (38 nodes each moving in one direction)
- GA population size: 239
- 240 CPU cores: Intel® Xeon® Gold 6248 Processor @ 2.5 GHz
- 1 CPU for each model (1 MPI process with 1 OMP thread)
- Computation time: 43 minutes
- Maximum RAM consumption: 656 GB

# Marine CSEM example: constructed model



Input (red) and true model (green wireframe)

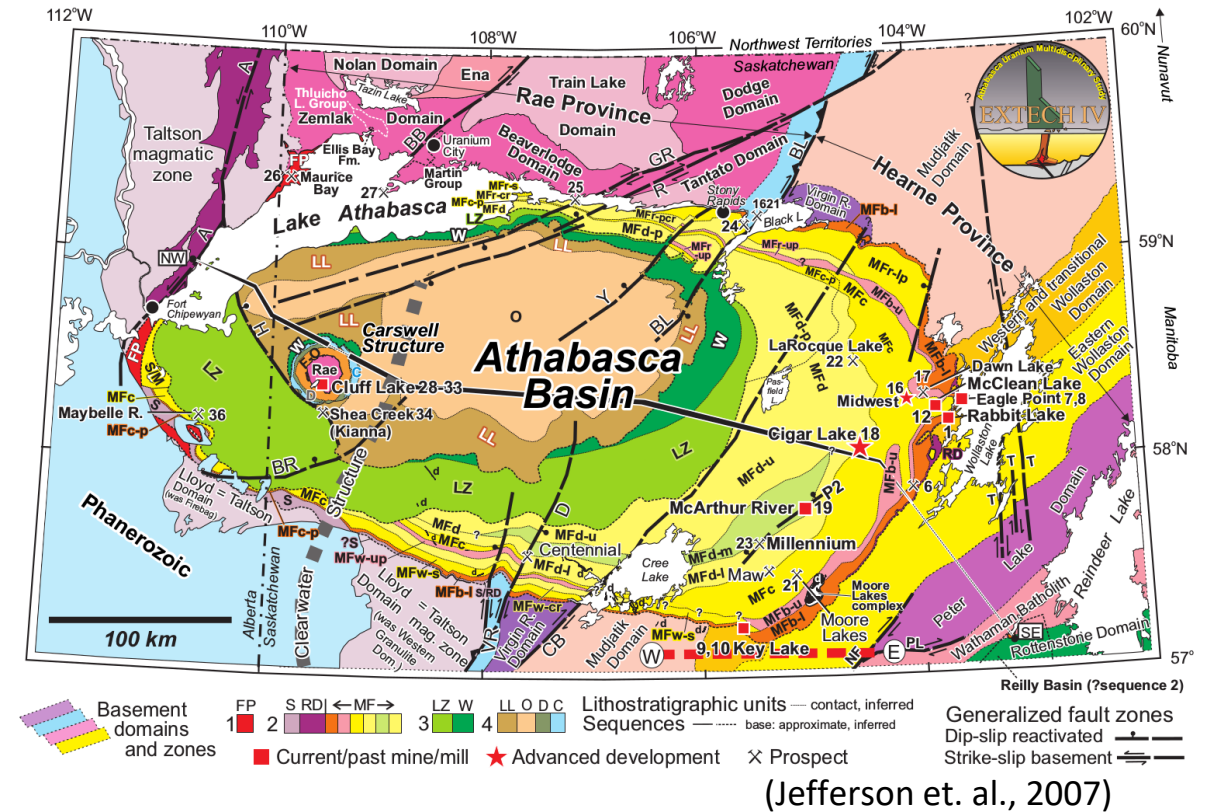
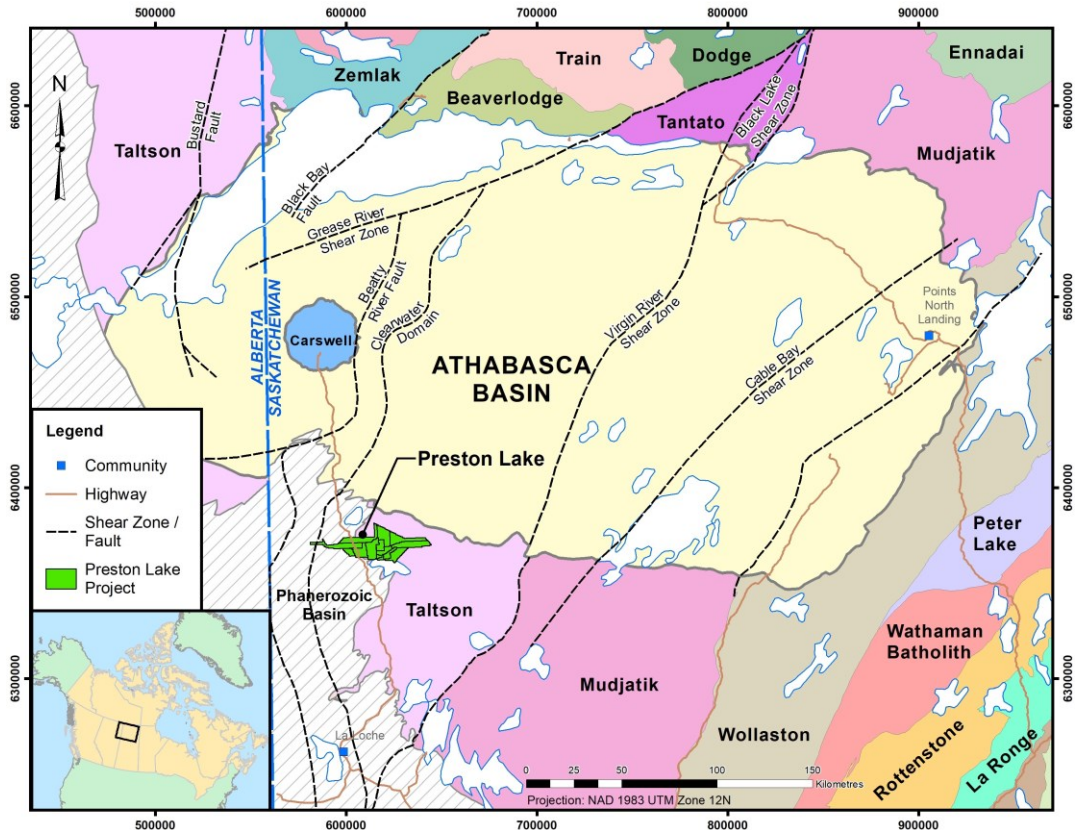


Constructed (gray) and true model (green wireframe)

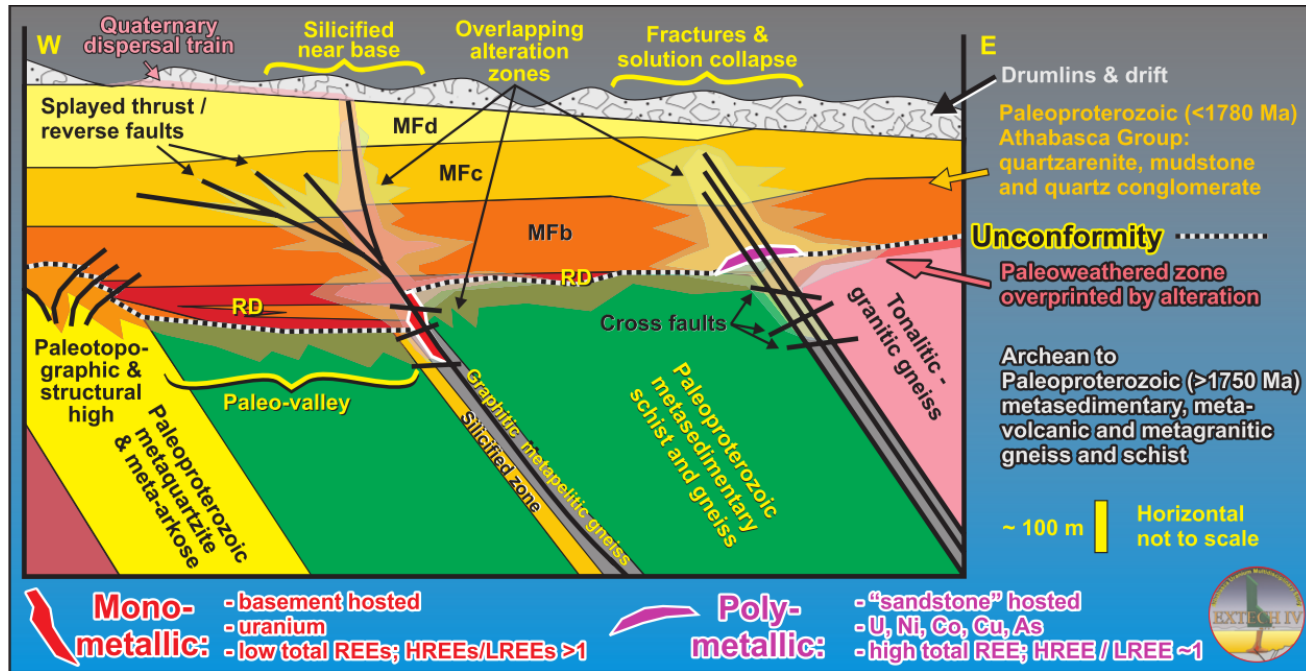
# Outline

- Motivation
- Surface geometry inversion
- Marine CSEM examples
- **TEM examples**
- Conclusions

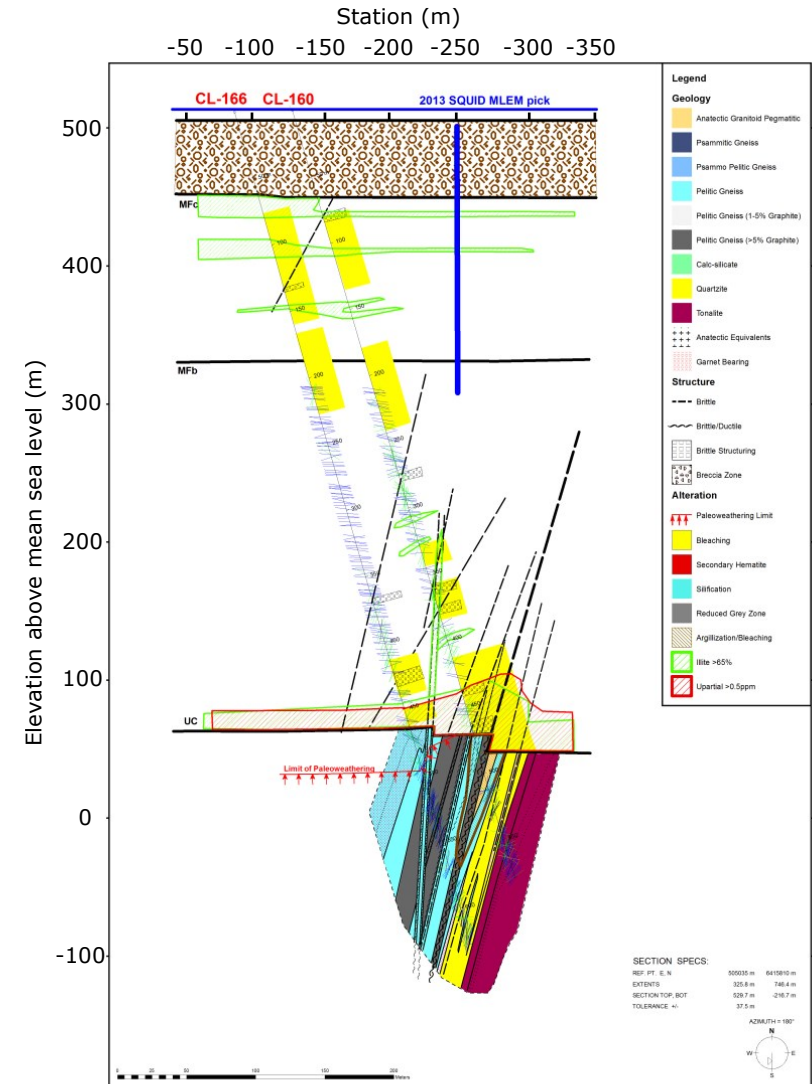
# Real-data example: uranium exploration



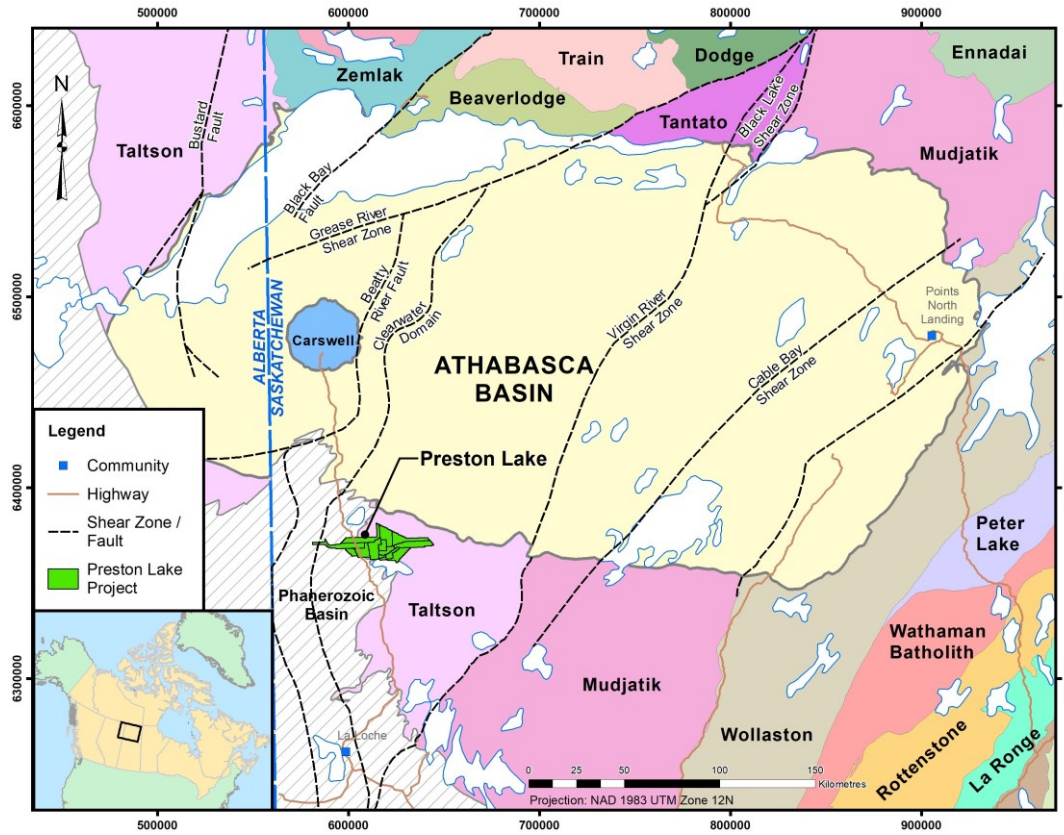
# TEM example: uranium exploration



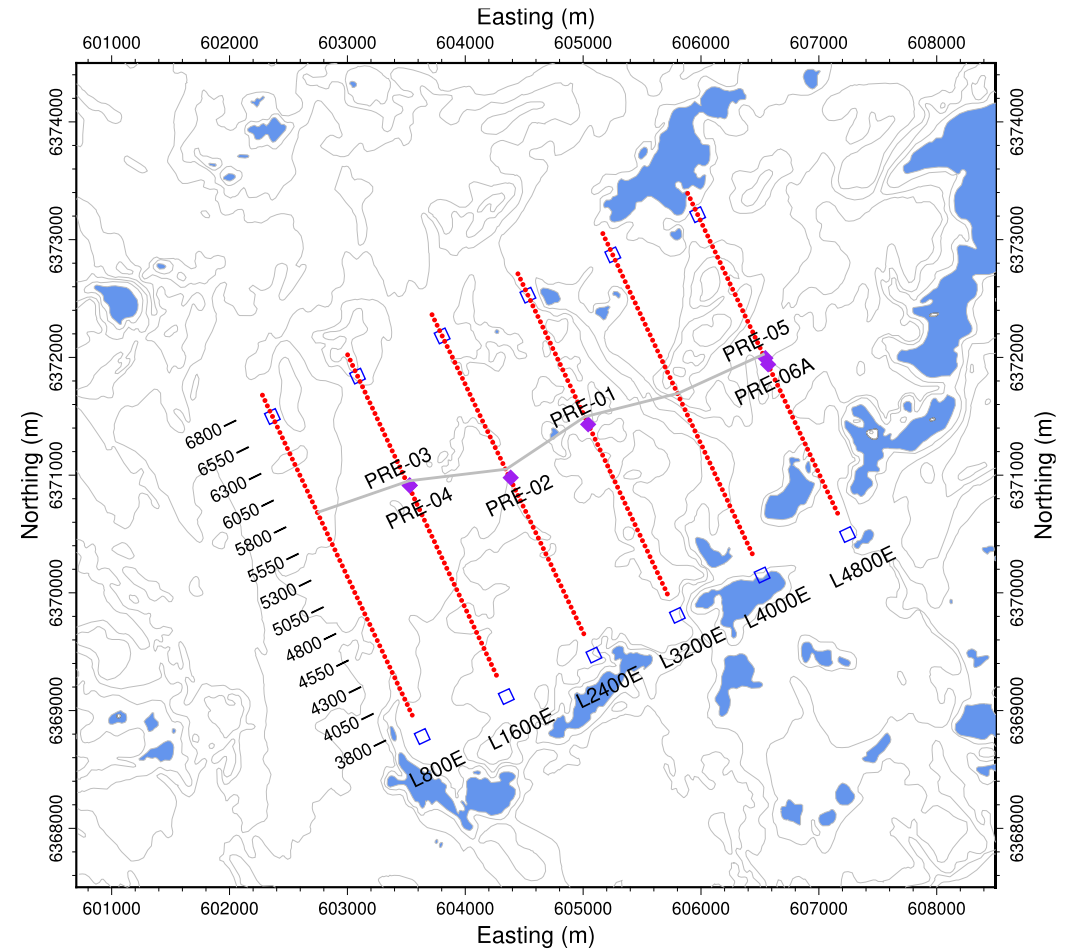
(Jefferson et. al., 2007)



# Preston Lake project

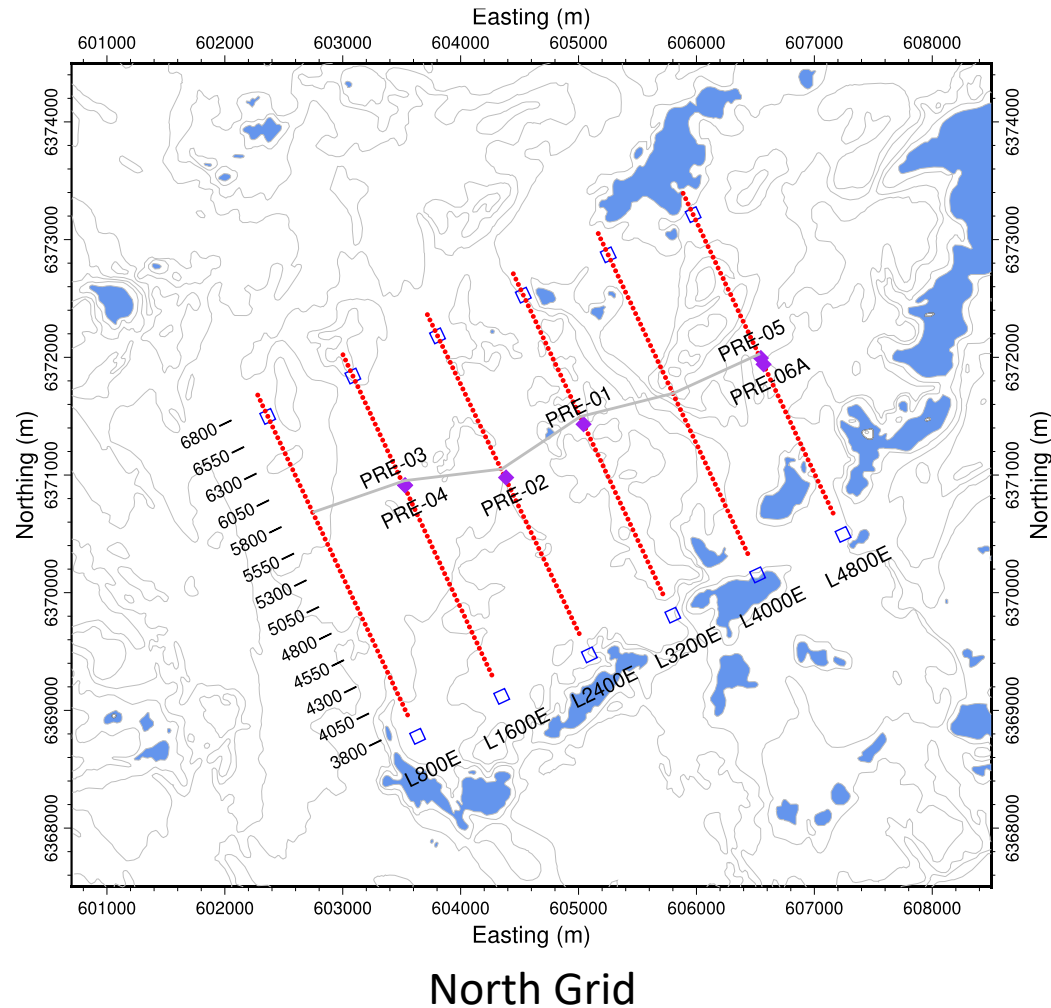


Project area



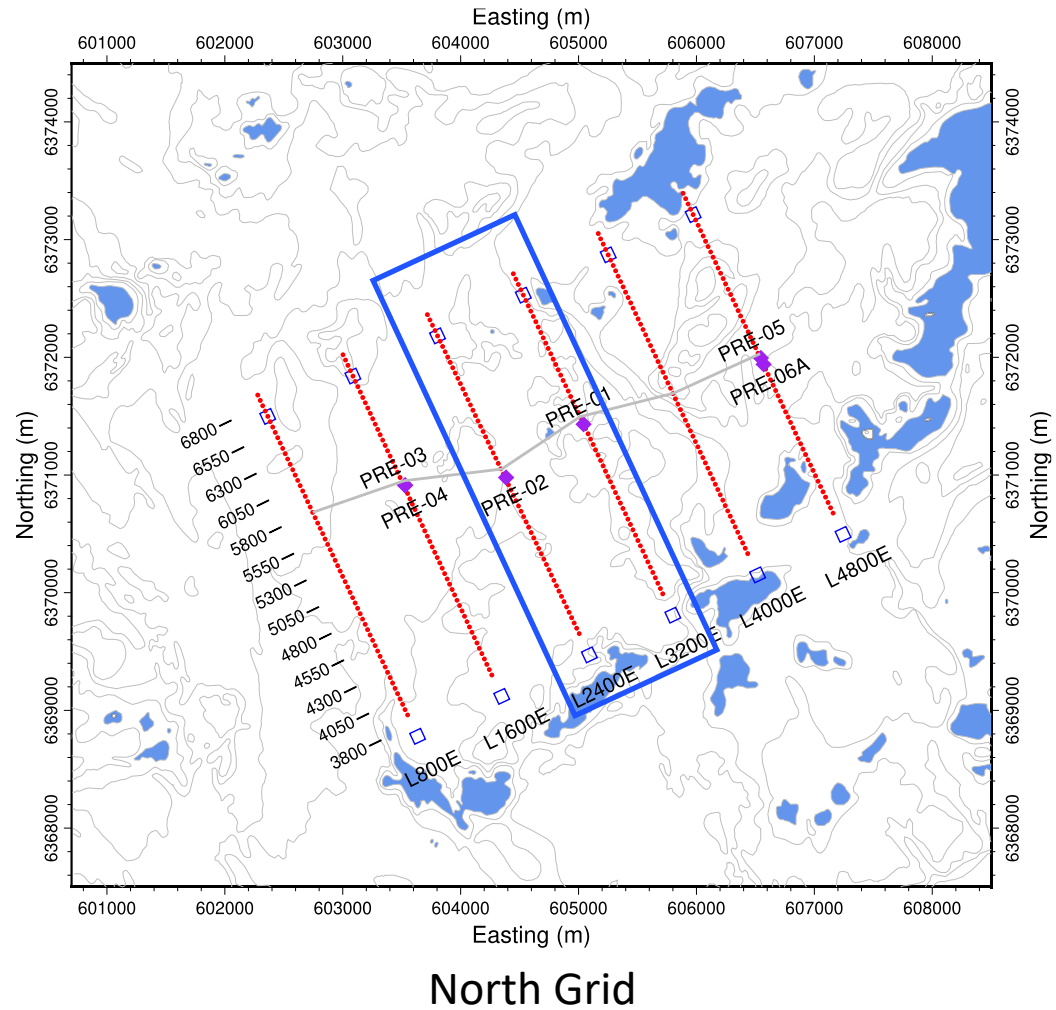
North Grid

# Preston Lake project: survey configuration



- 100 by 100 m loop source
- Station spacing: 50 m
- Rx located 200 m to the grid north of the center of Tx
- 61 stations: 3 km each profile
- Abitibi Geophysics ARMIT MK2 dB/dt & B sensor
- 20 channels from 0.1042 ms to 6.0928 ms

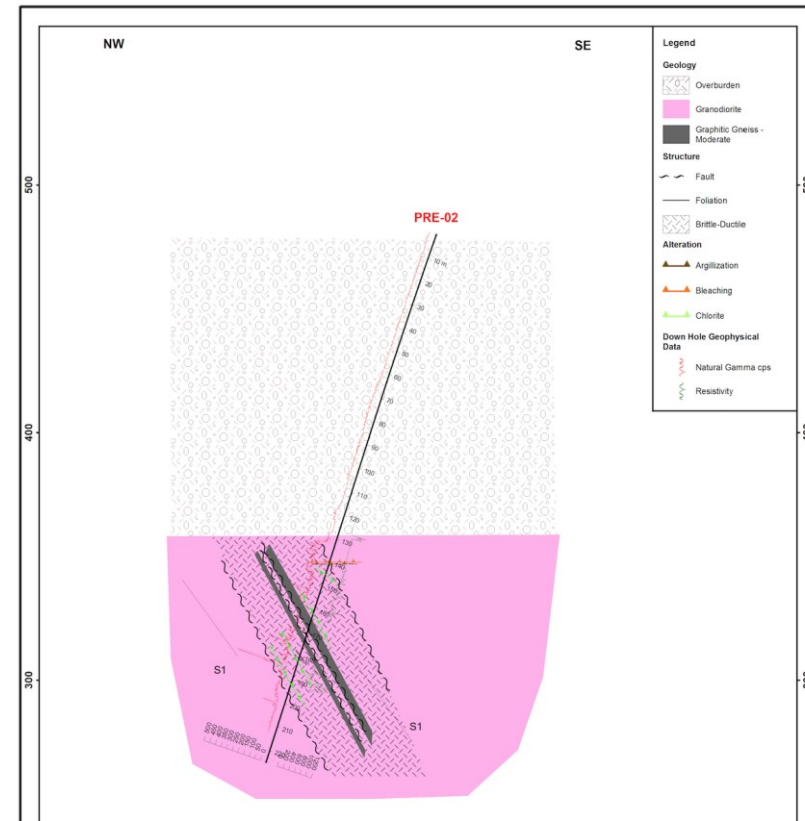
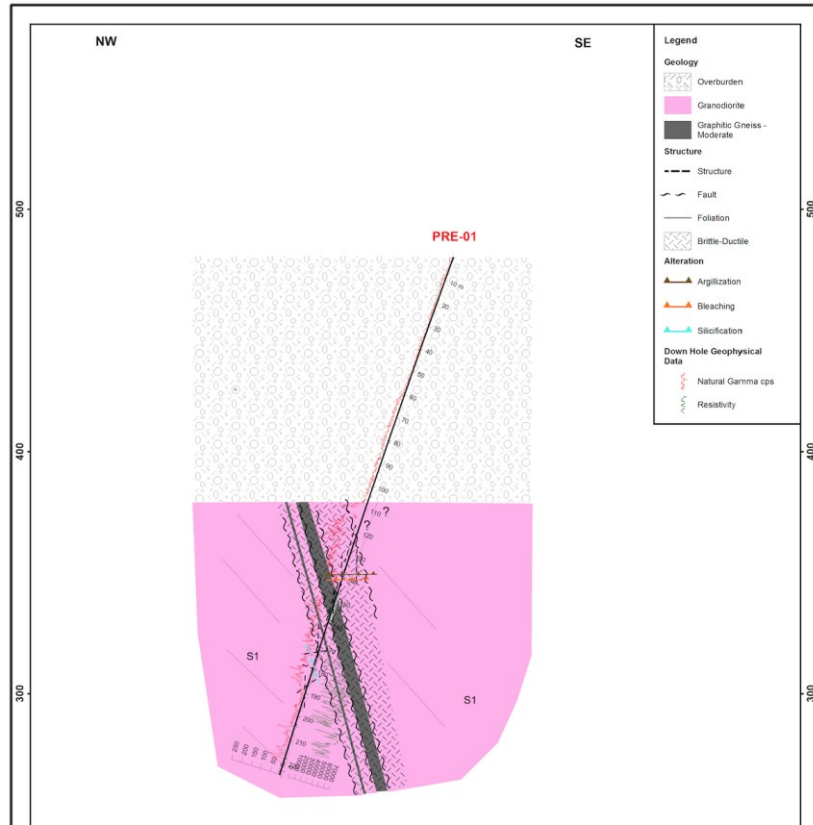
# Preston Lake project: survey configuration



- Only invert data from L2400E & L3200E
- Drill hole PRE-01 & PRE-02 intersected graphite

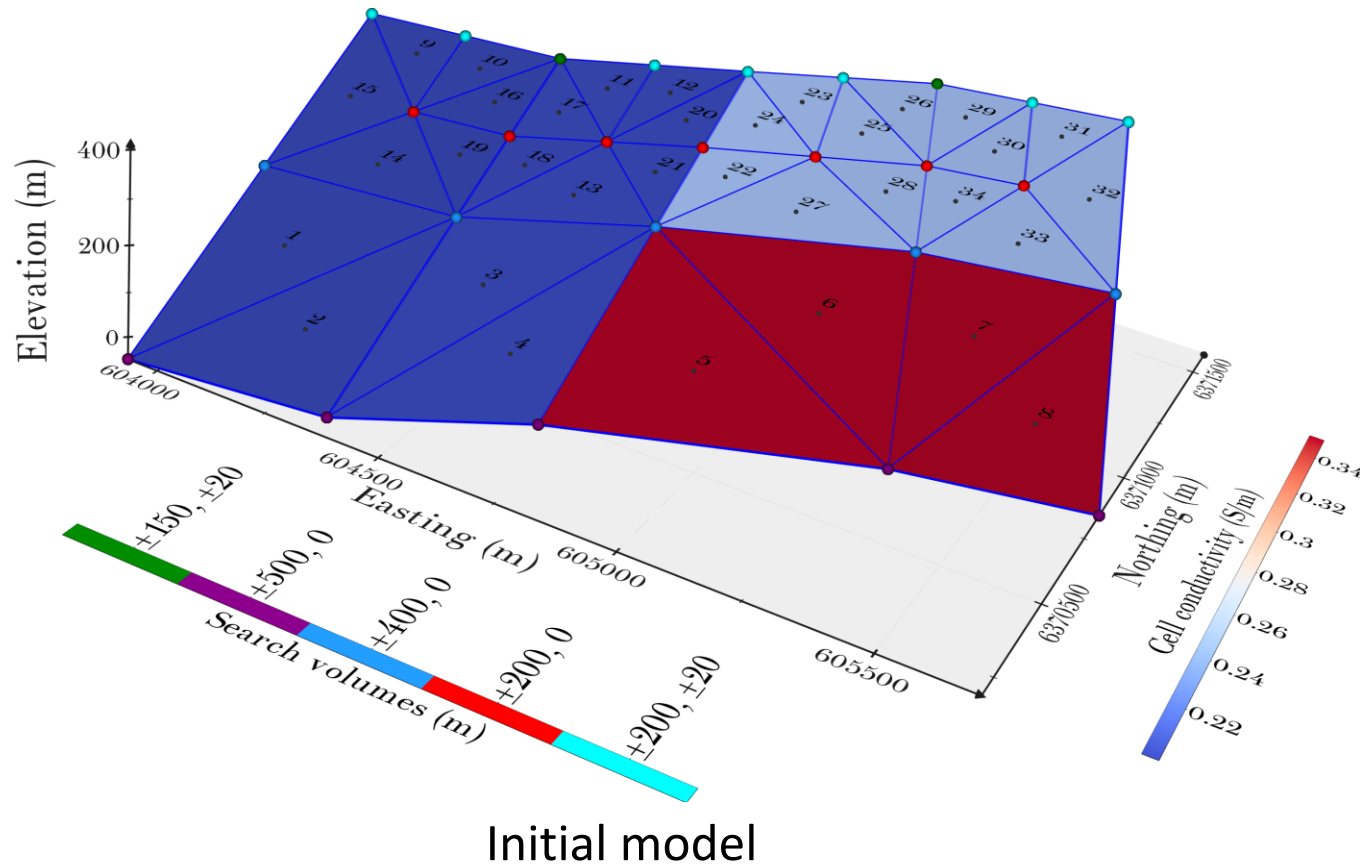


# Preston Lake project: survey configuration



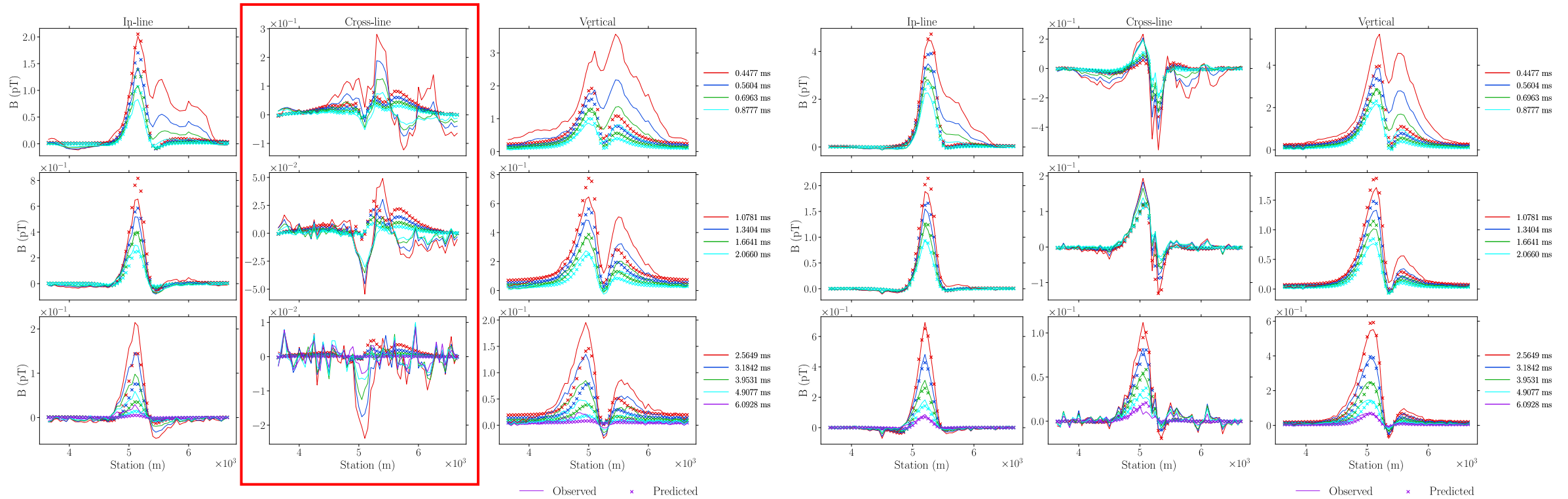
Basement: crystalline metamorphic basement rocks of the Taltson domain

# SGI of Preston Lake data: model setup

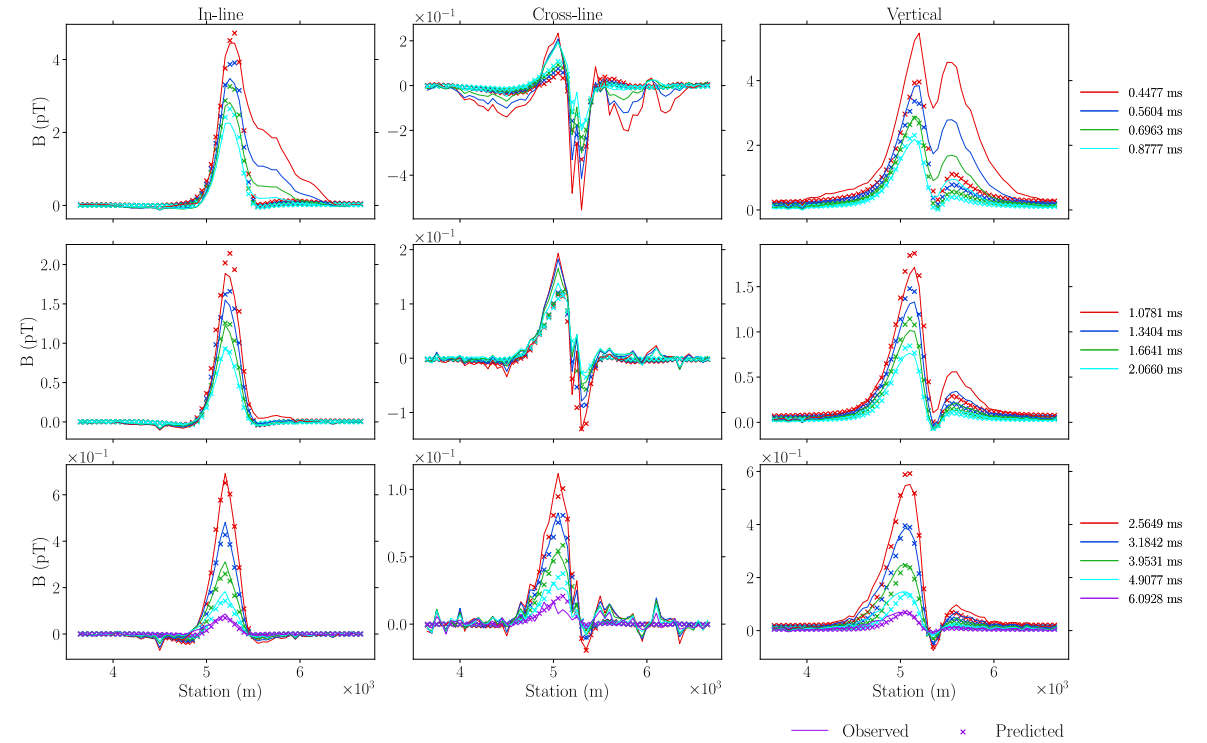


- Background conductivity model obtained from trial-and-error modelling
- # parameters: 69 (26 nodes moving along strike, 9 nodes moving vertically, and 34 regions)
- GA population size: 599
- Data uncertainties: max(std, 2% data)
- 15 nodes with 600 Intel® Xeon® Gold 6248 Processor @ 2.5 GHz
- 1 CPU for each model (1 MPI process with 1 OMP thread)

# Data fitting

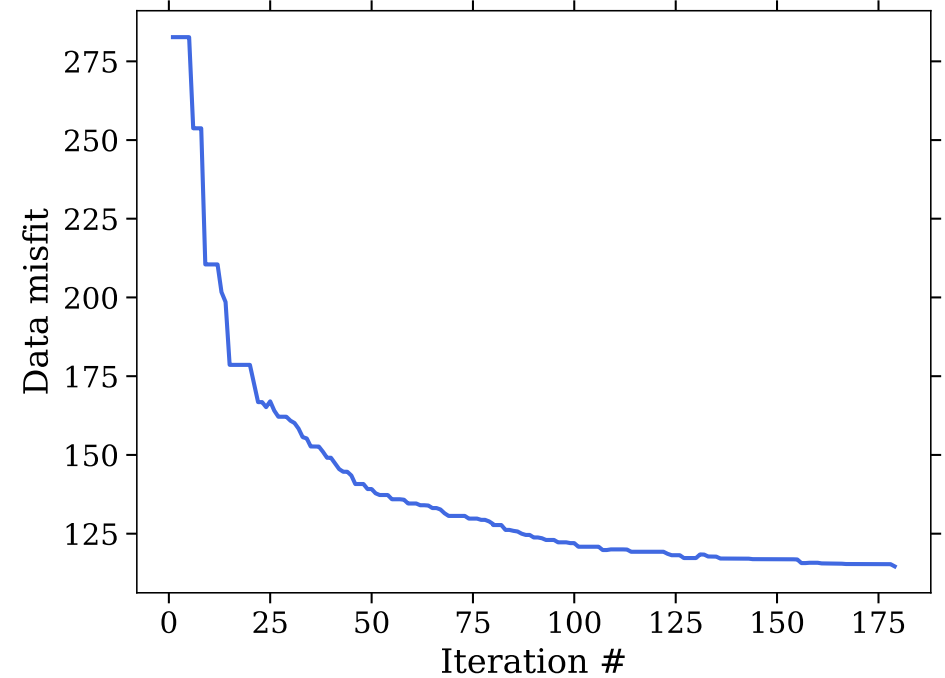
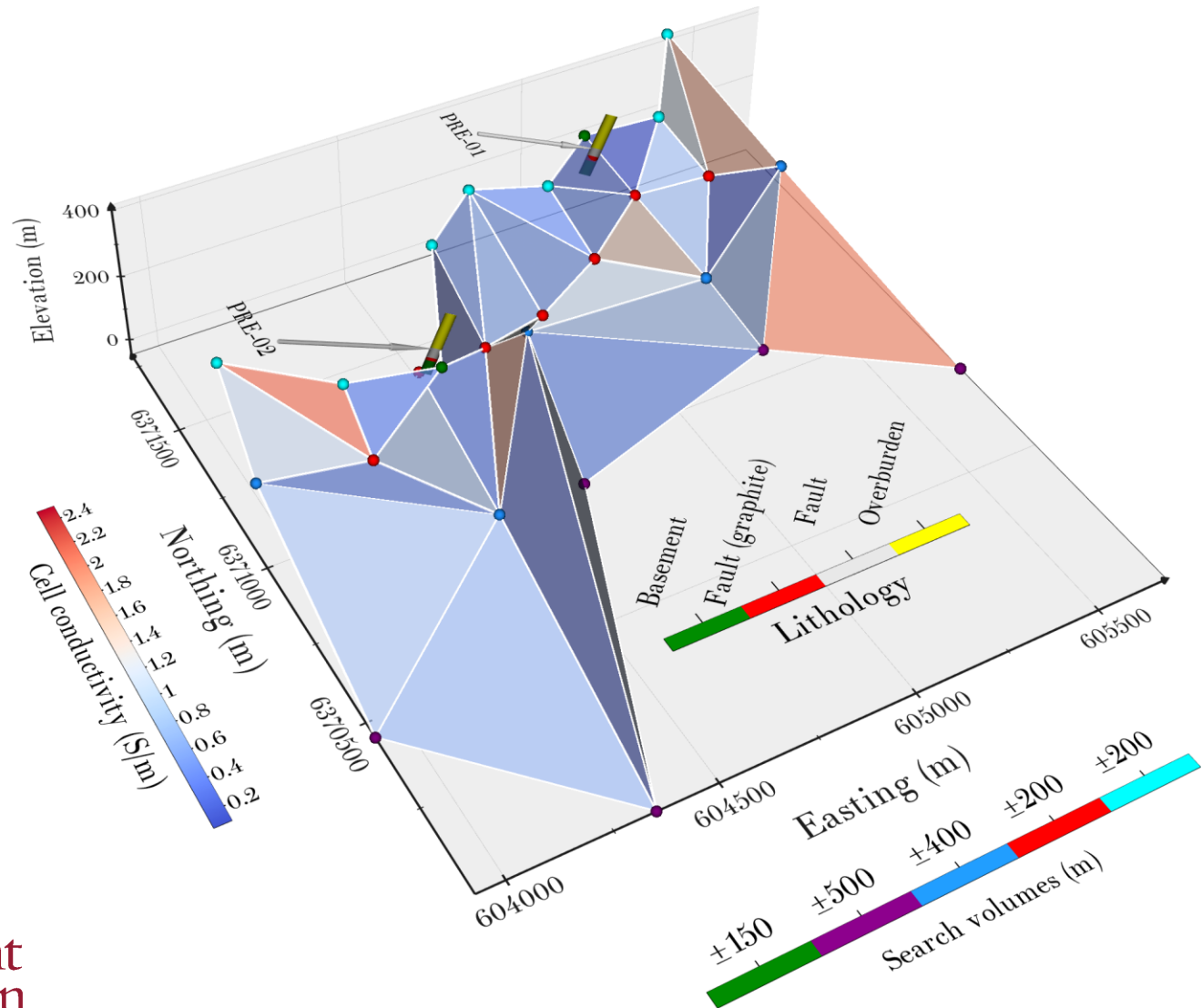


Data fitting of L2400E

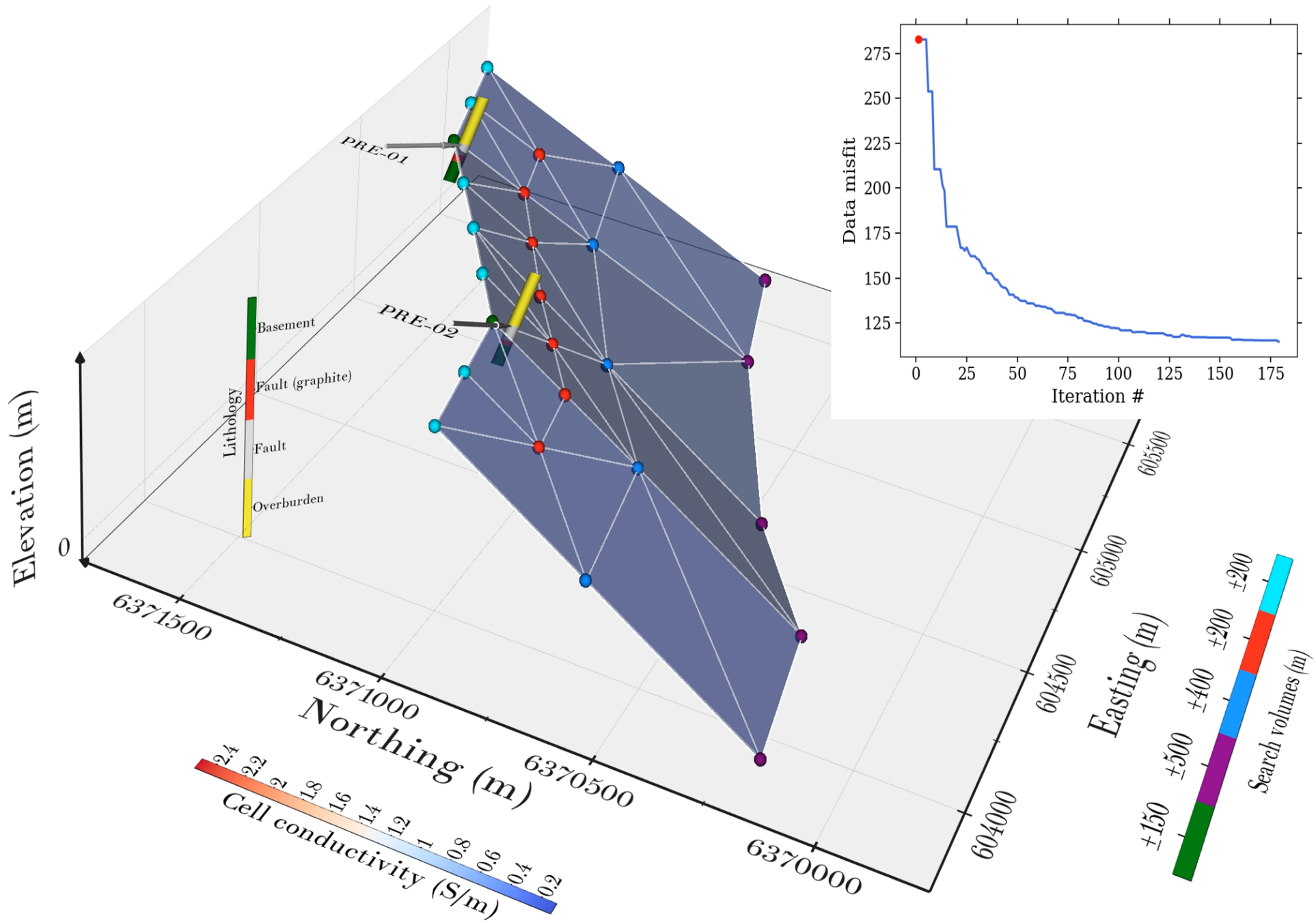


Data fitting of L3200E

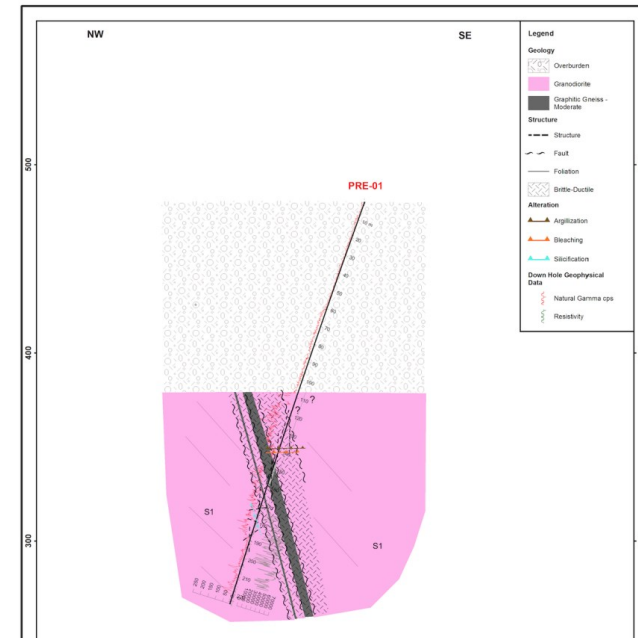
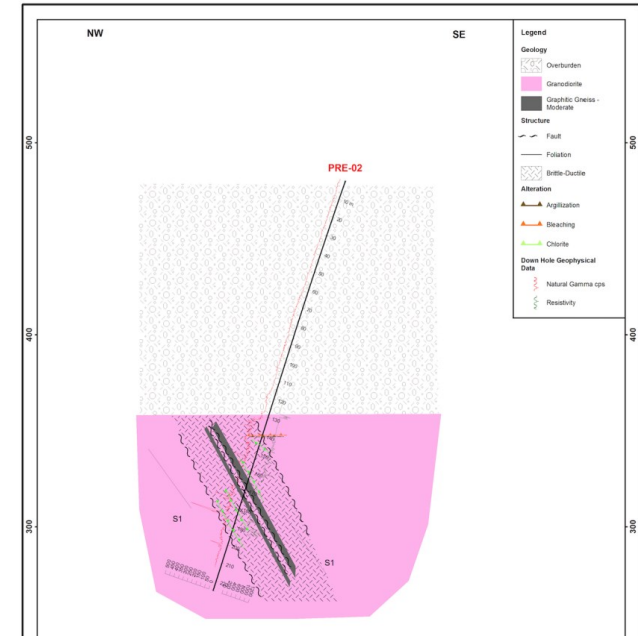
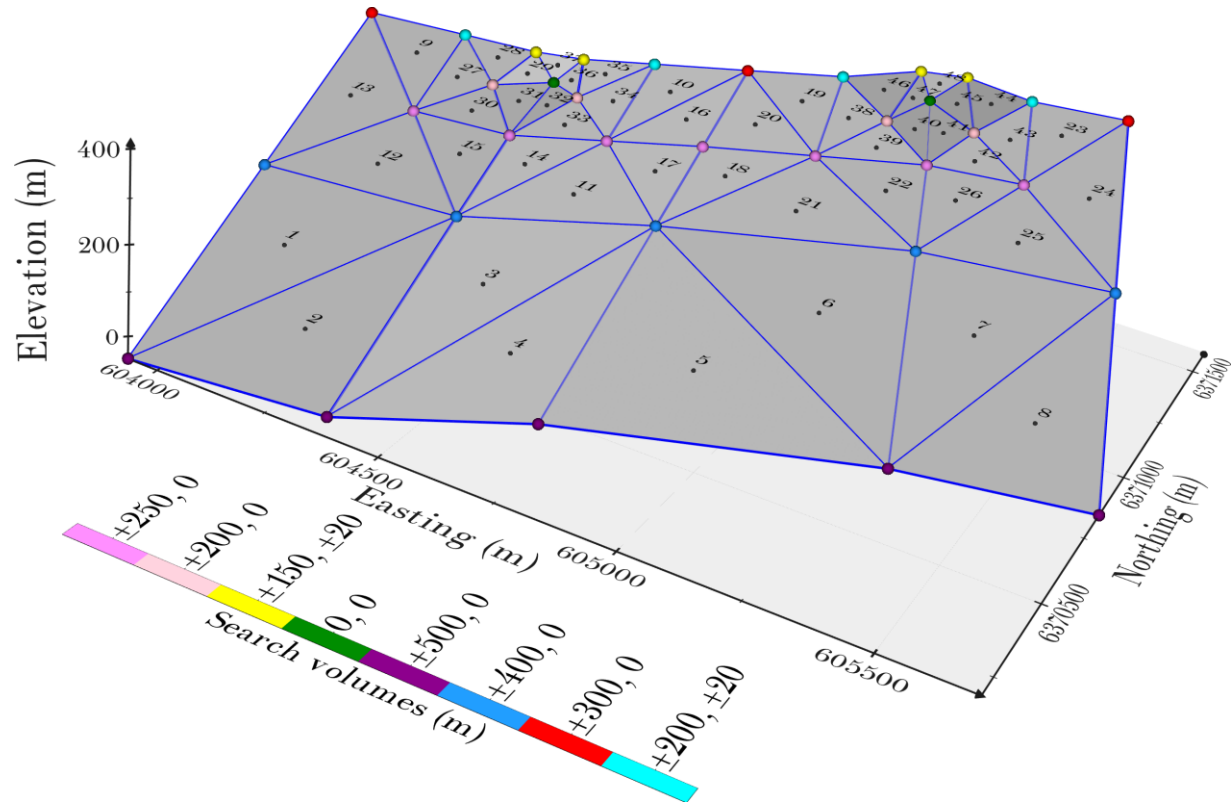
# Constructed model and convergence



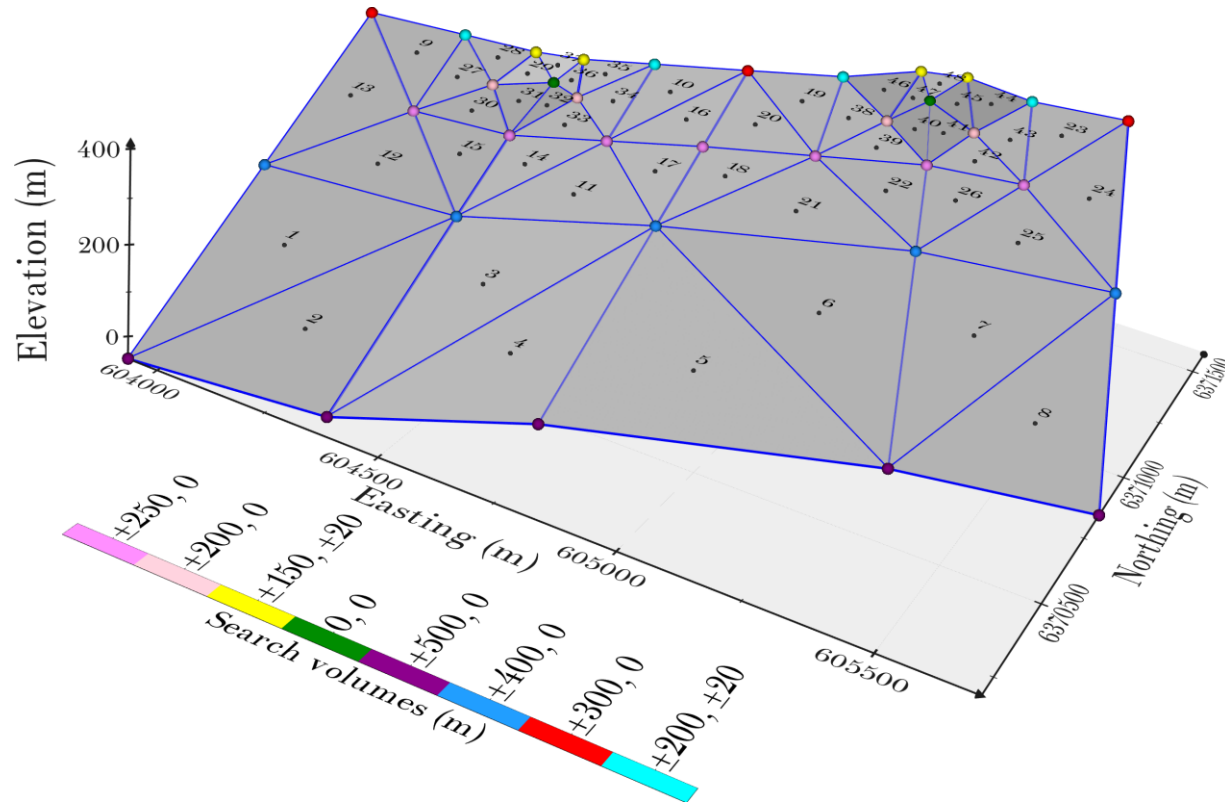
Each iteration takes about 1 hour to finish



# Constrained inversion

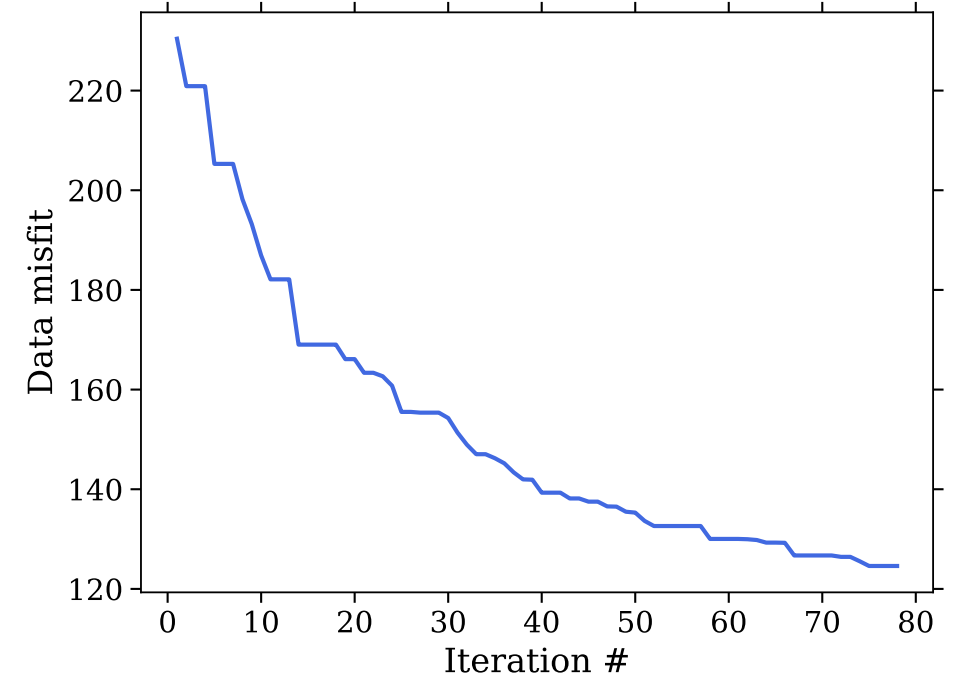
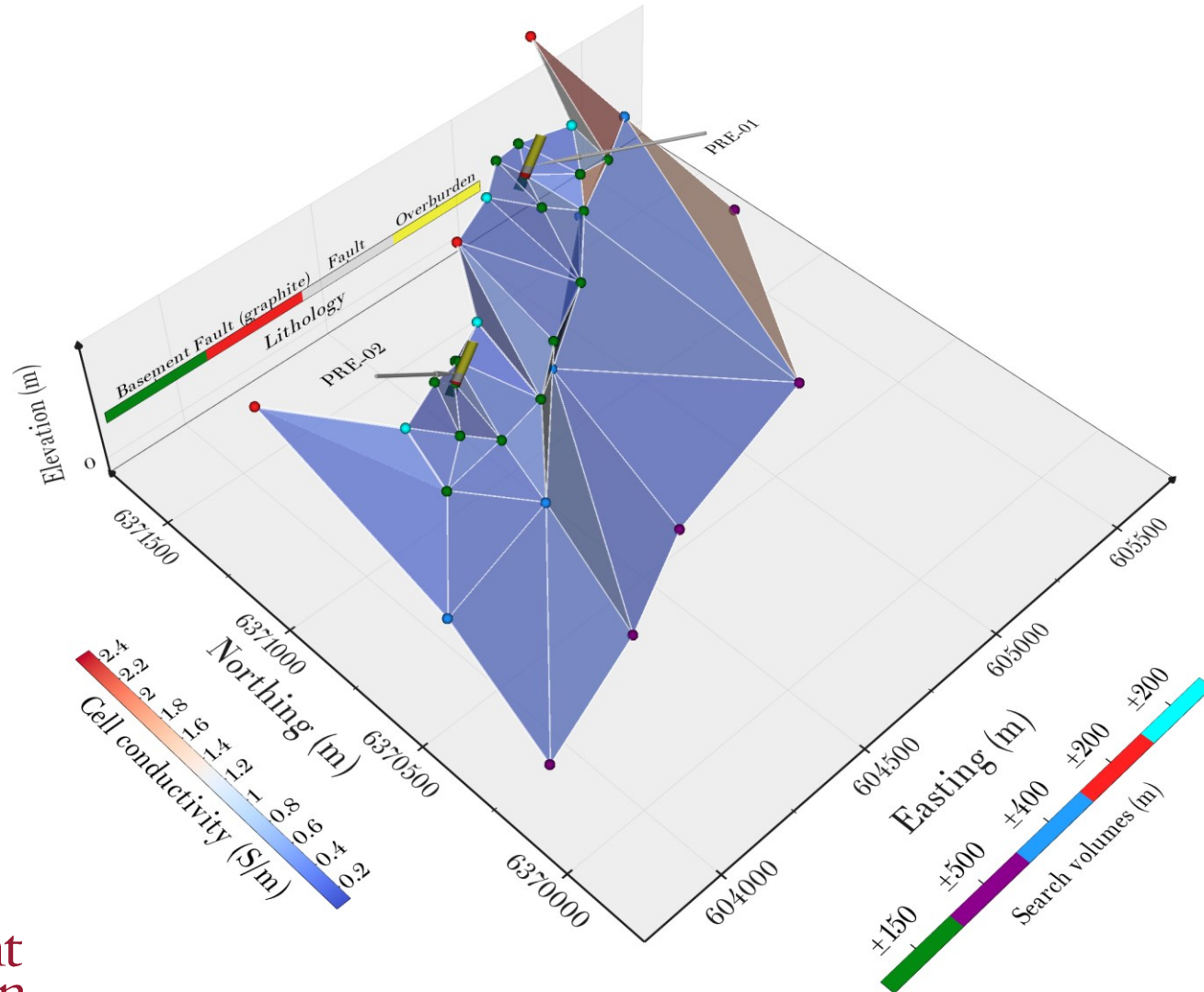


# Constrained inversion



- # parameters: 88 (32 nodes moving along strike, 8 nodes moving vertically, and 48 regions)
- GA population size: 599
- Data uncertainties: max(std, 2% data)
- 15 nodes with 600 CPU: Intel® Xeon® Gold 6248 Processor @ 2.5 GHz
- 1 CPU for each model (1 MPI process with 1 OMP thread)

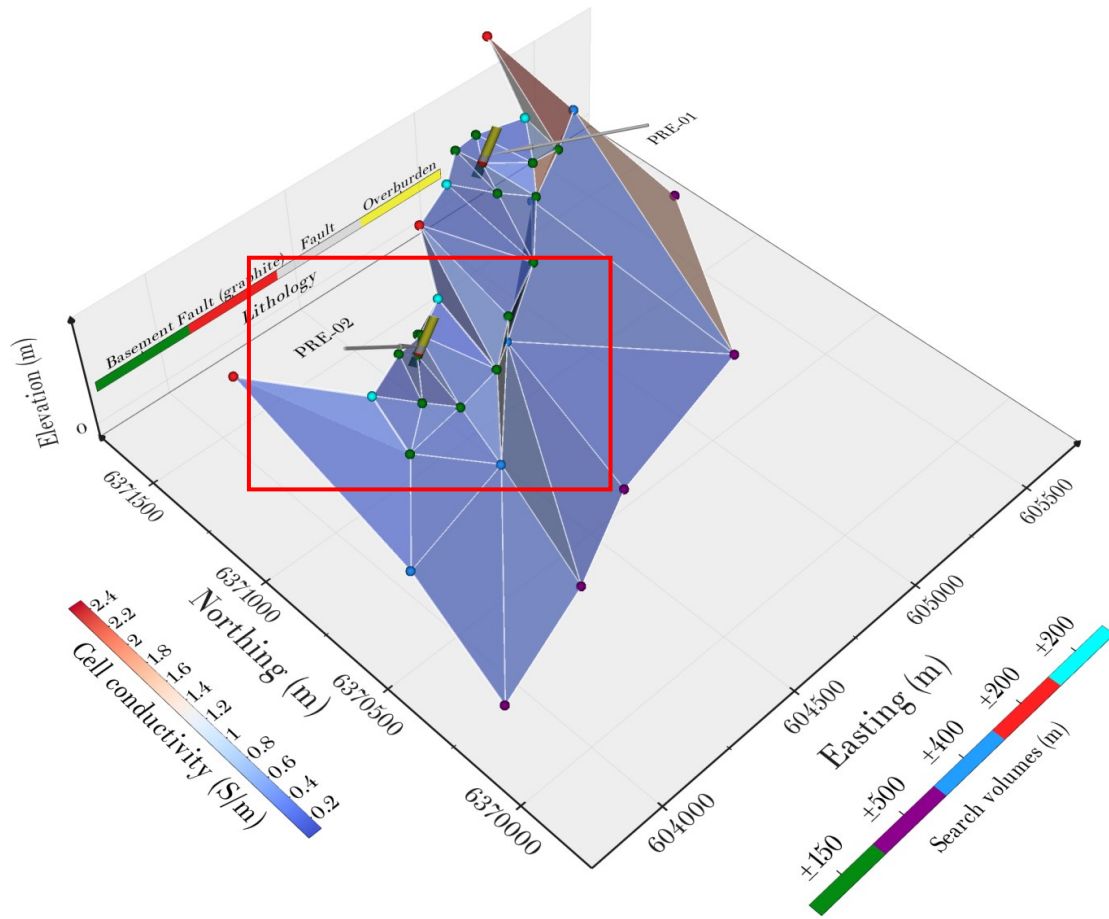
# Constructed model and convergence



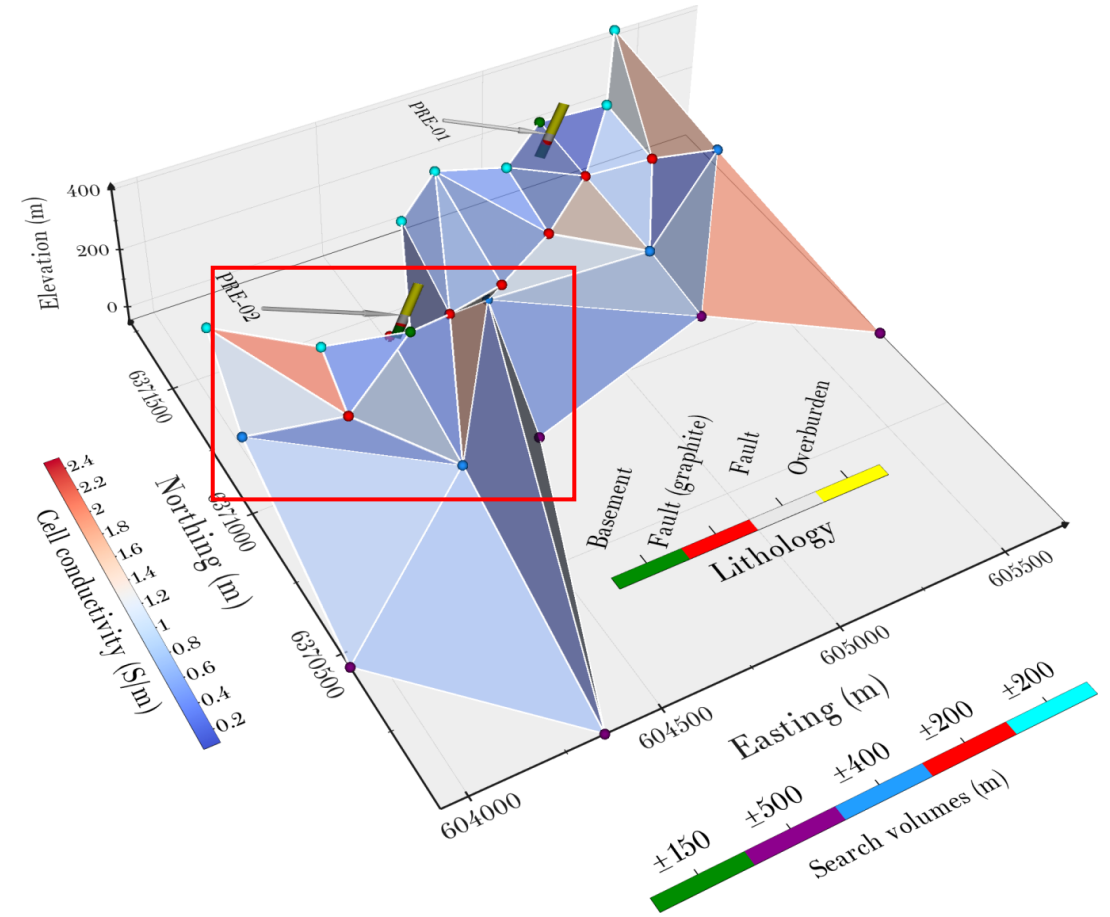
Each iteration takes about 1 hour to finish



# Constrained VS unconstrained

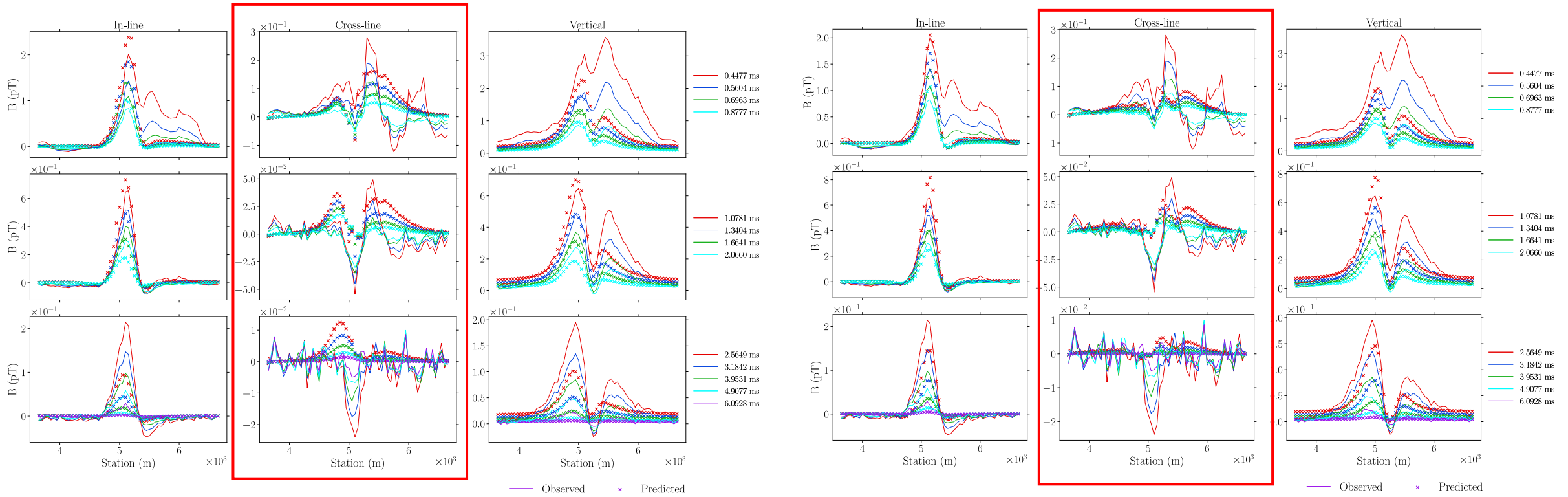


Constrained



Unconstrained

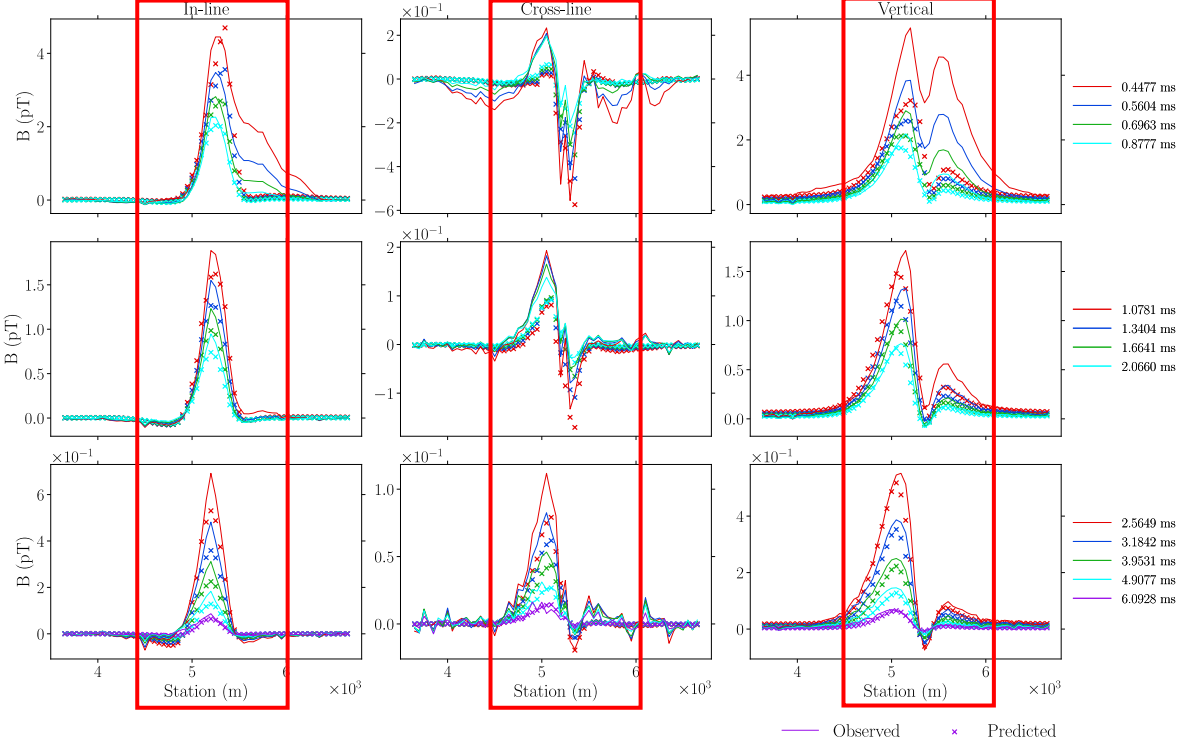
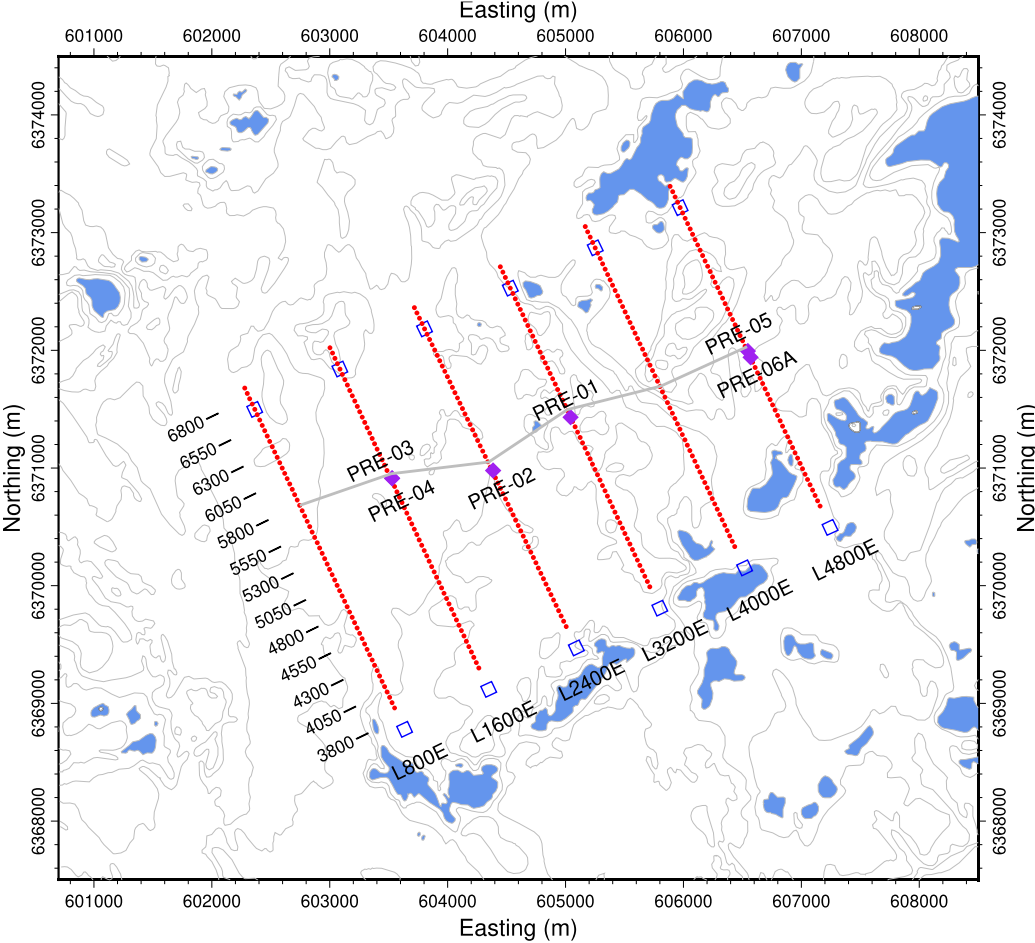
# Constrained VS unconstrained (L2400E)



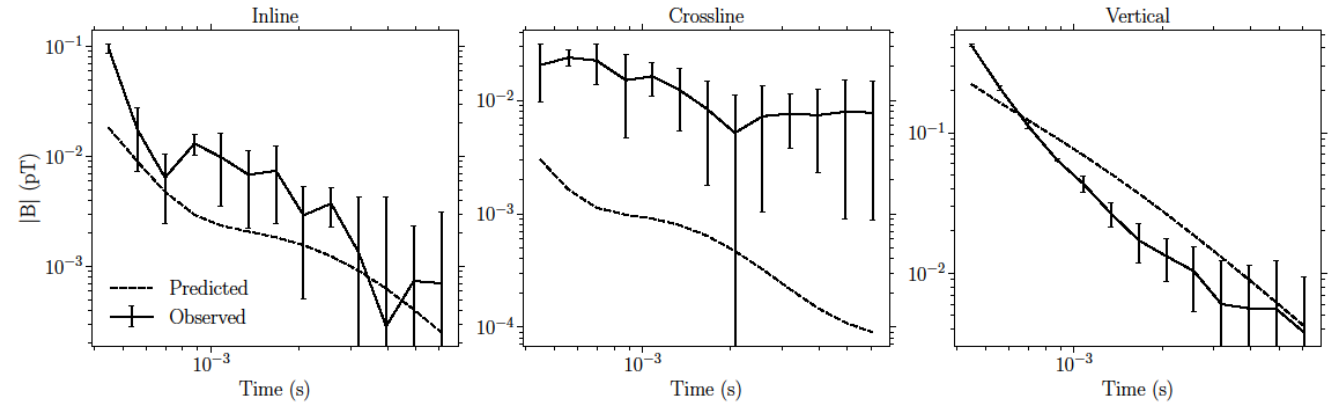
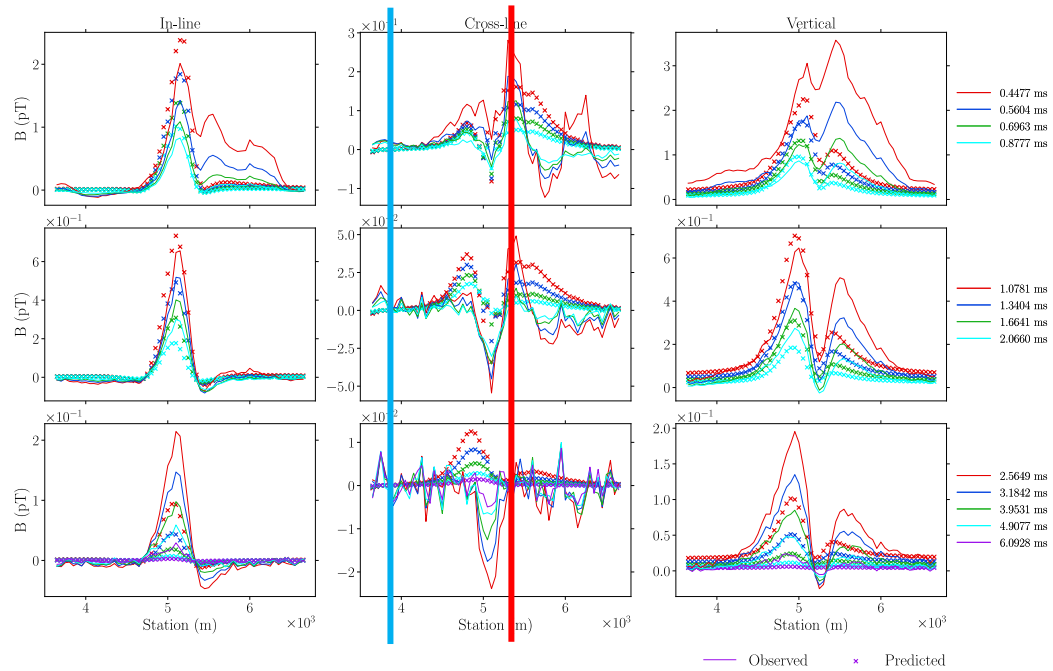
Constrained

Unconstrained

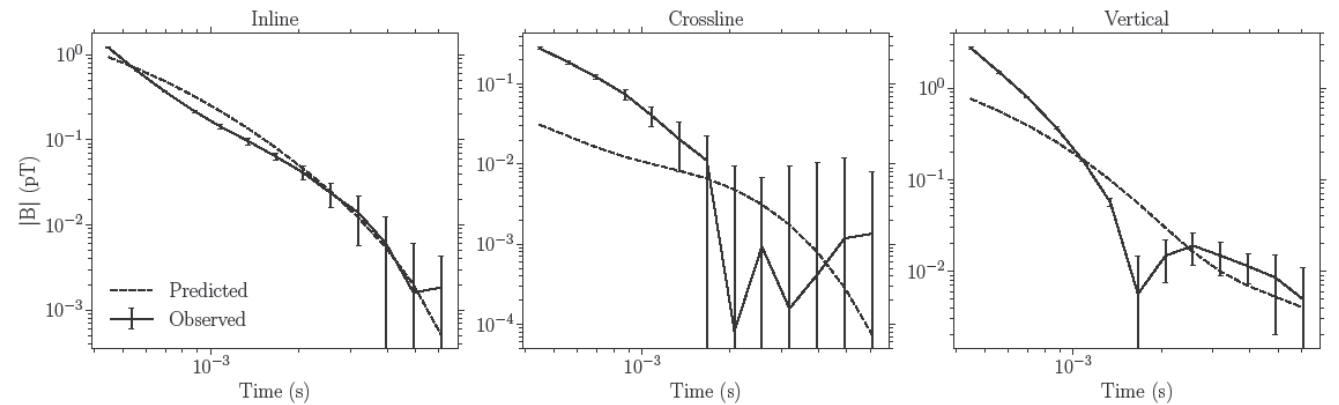
# Decimated data inversion



# Decimated data inversion



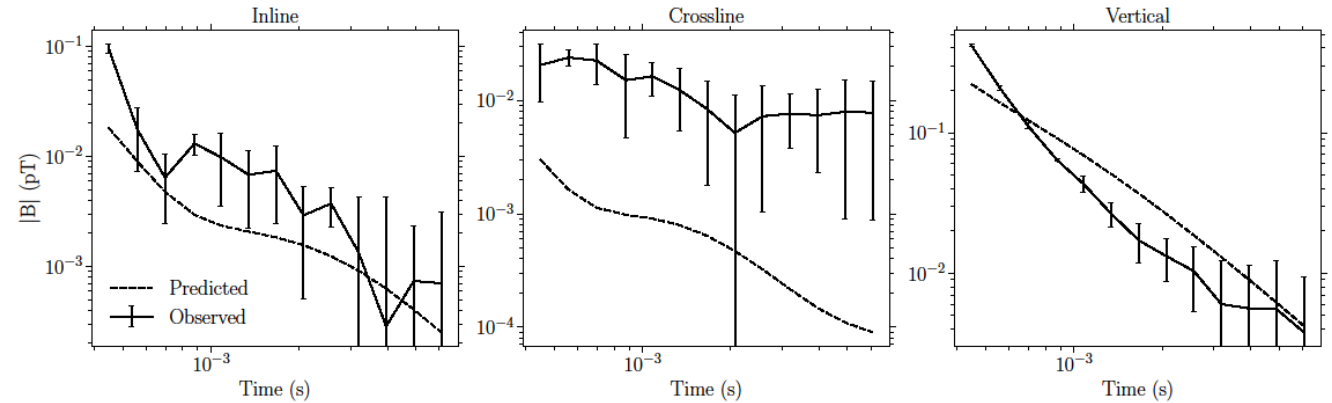
Station 3900S



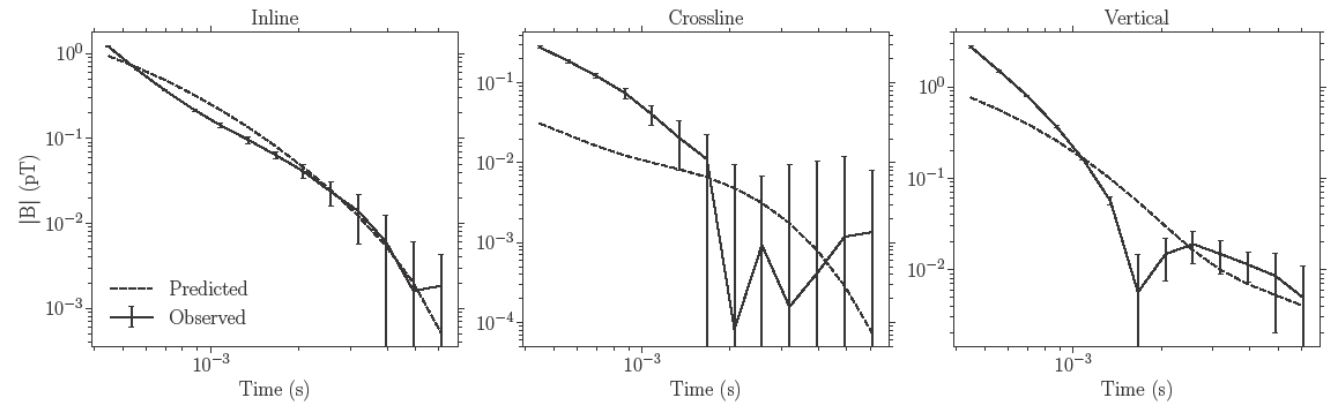
Station 5450S

# Uncertainty calculation

- Uncertainty calculation:
- Std from 3 measurements
  - Max(std, 2% of data)
  - No noise floor used



Station 3900S



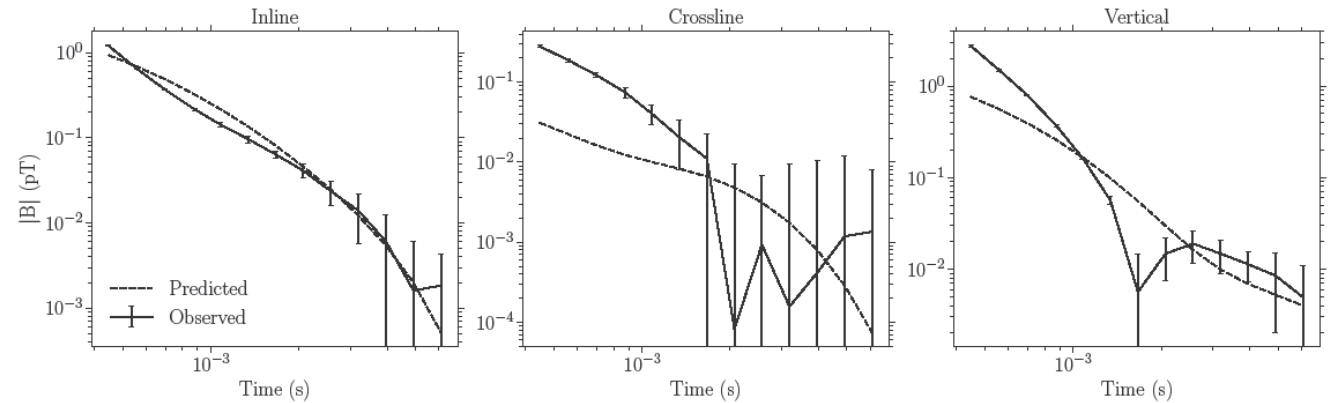
Station 5450S

# Updated uncertainty calculation

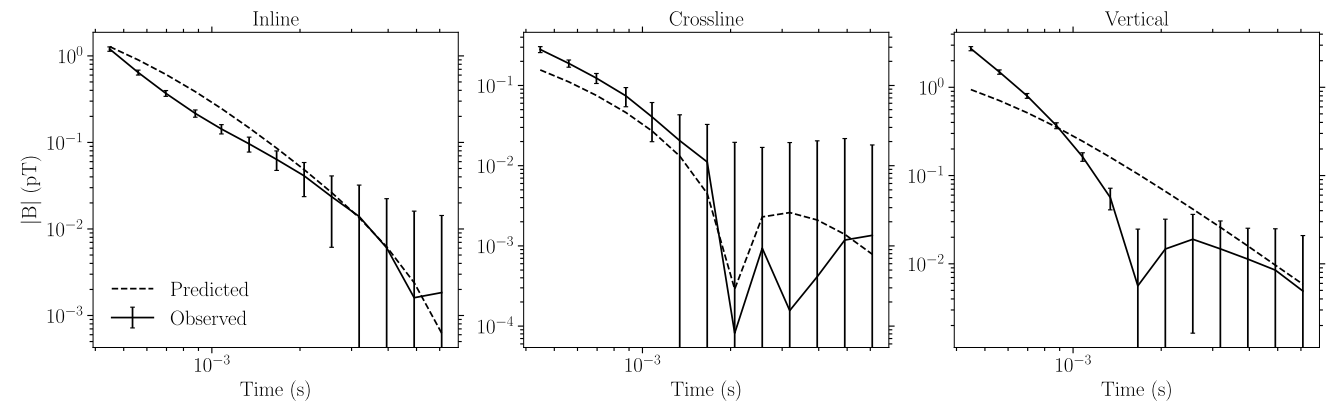
- Uncertainty calculation:
  - Std from 3 measurements
  - Max(std, 2% of data)
  - No noise floor used

- Updated uncertainty calculation

- Std from 3 measurements
- Max(std, 5% of data)
- Noise floor: 0.001 pT

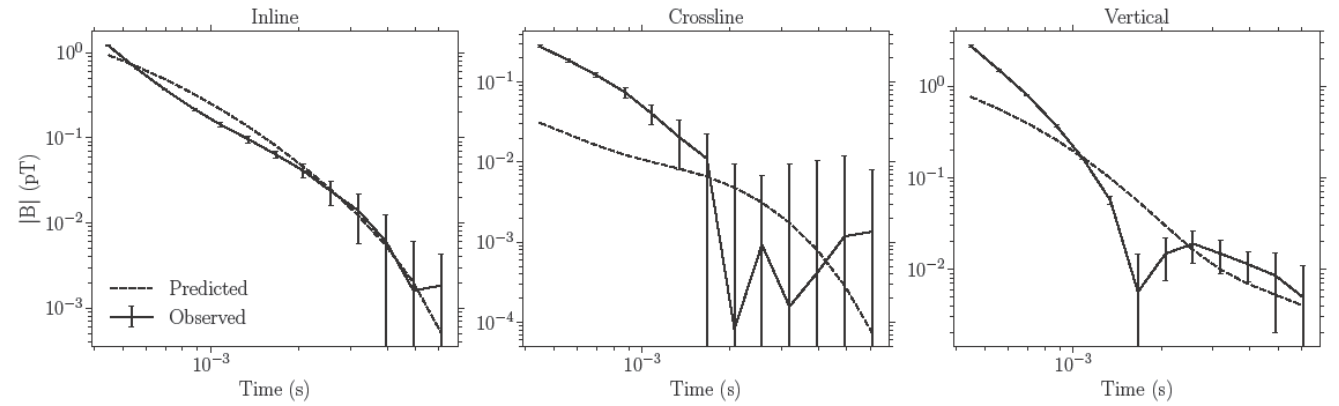
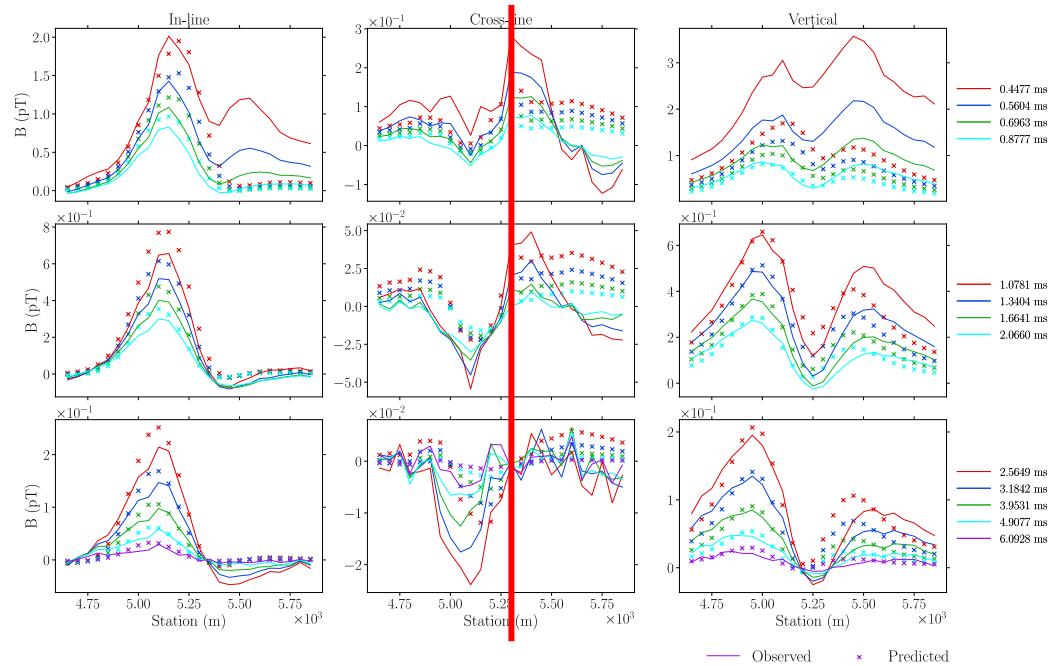


Original Station 5450S

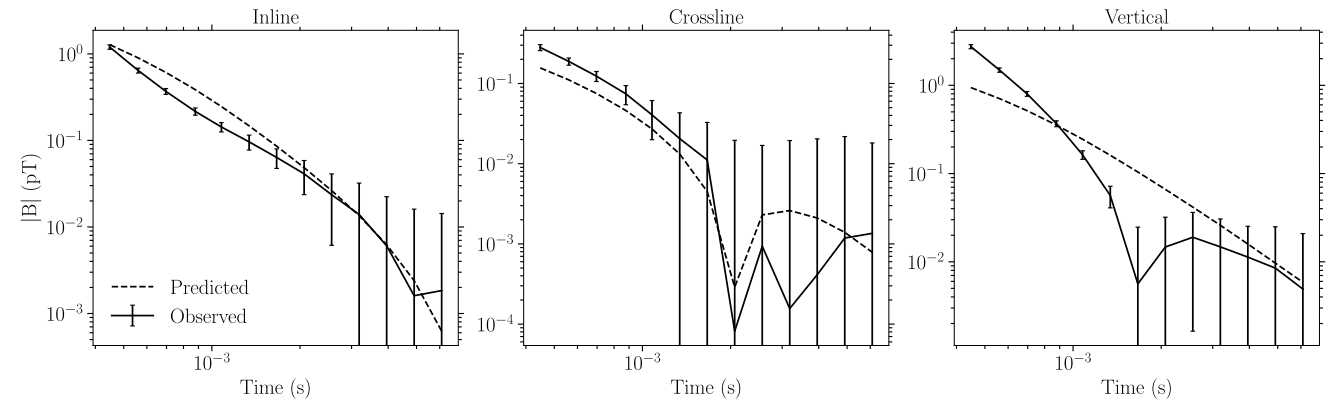


Decimated Station (5450S)

# Decimated data inversion

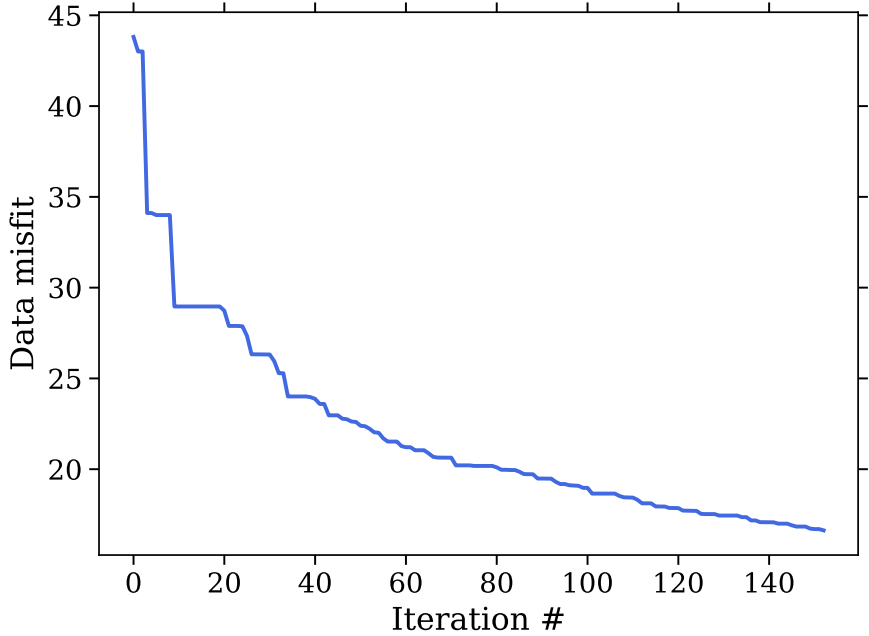


Original Station 5450S

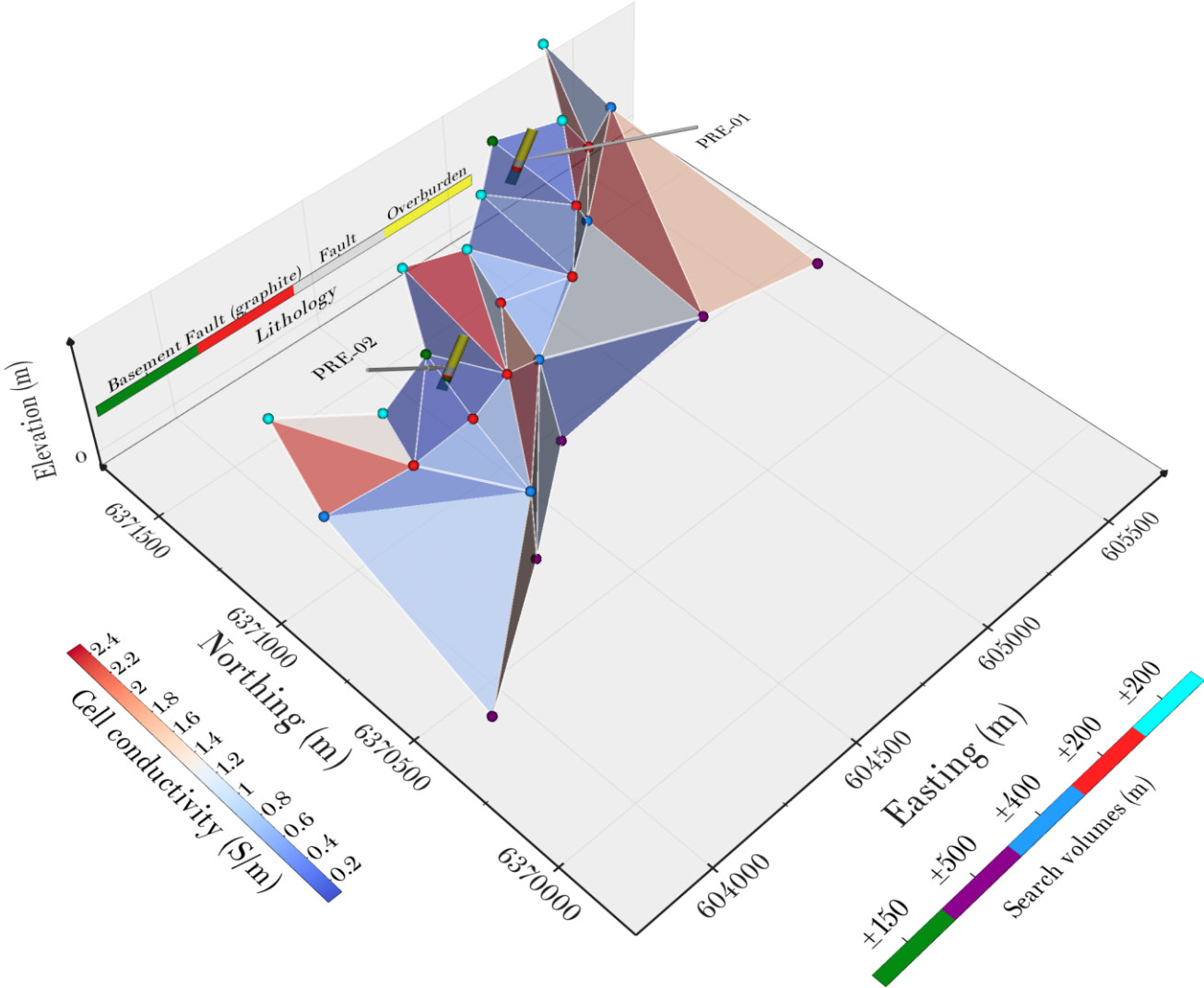


Decimated Station (5450S)

# Decimated data inversion

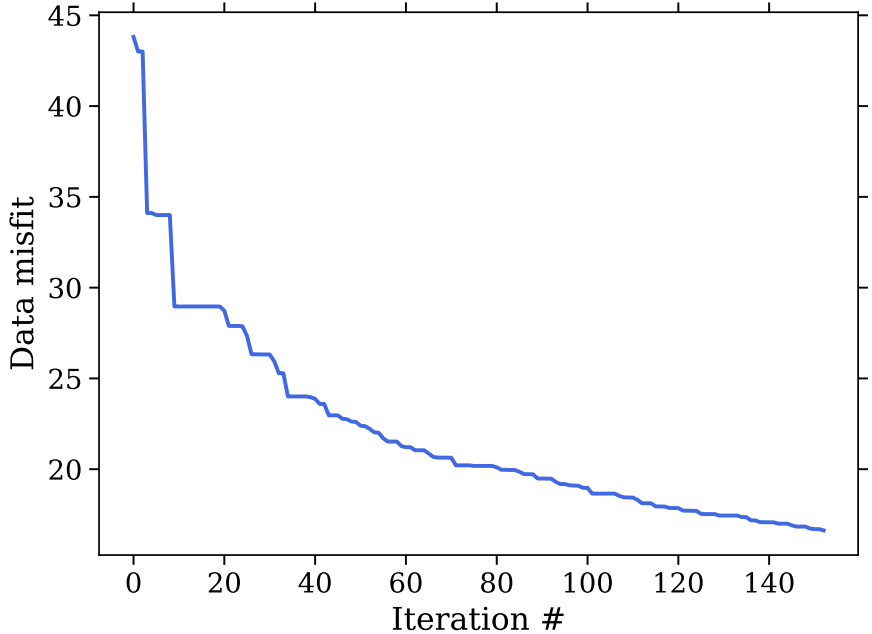


Each iteration takes about **40 minutes** to finish

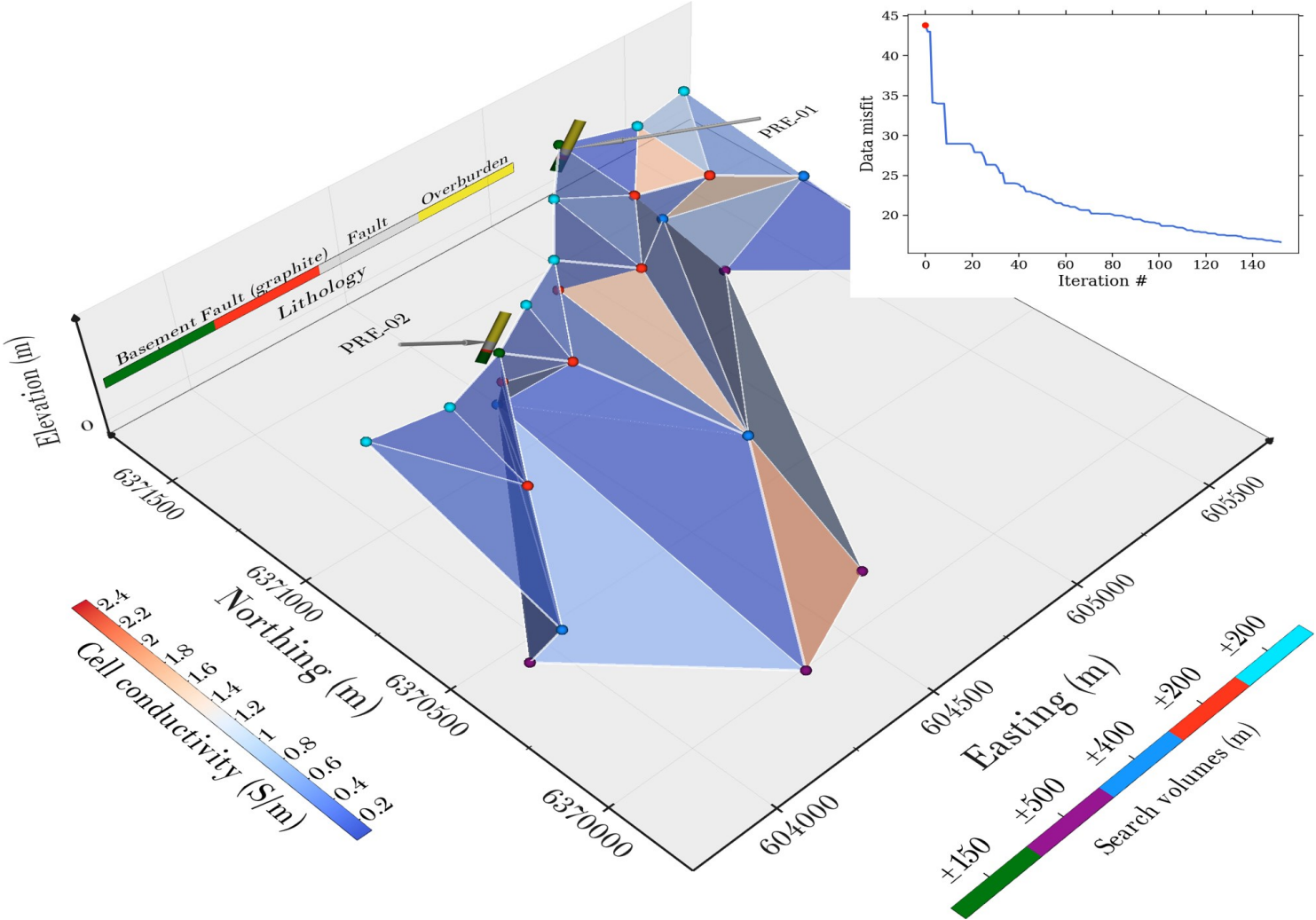




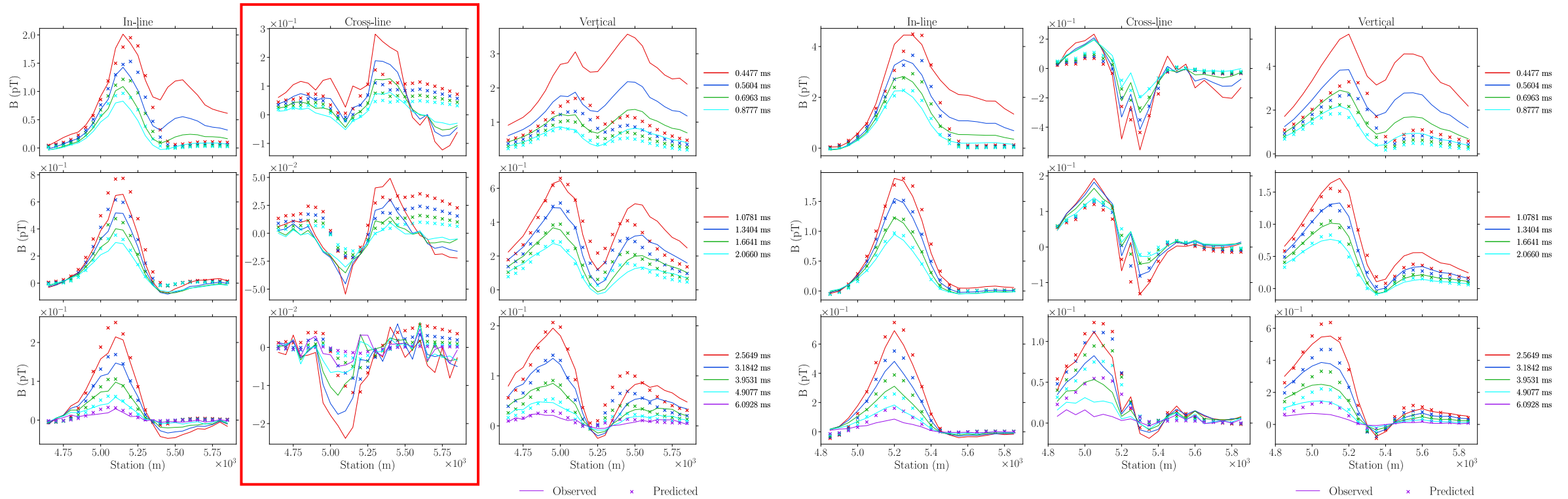
# Decimated data inversion



Each iteration takes about **40 minutes** to finish



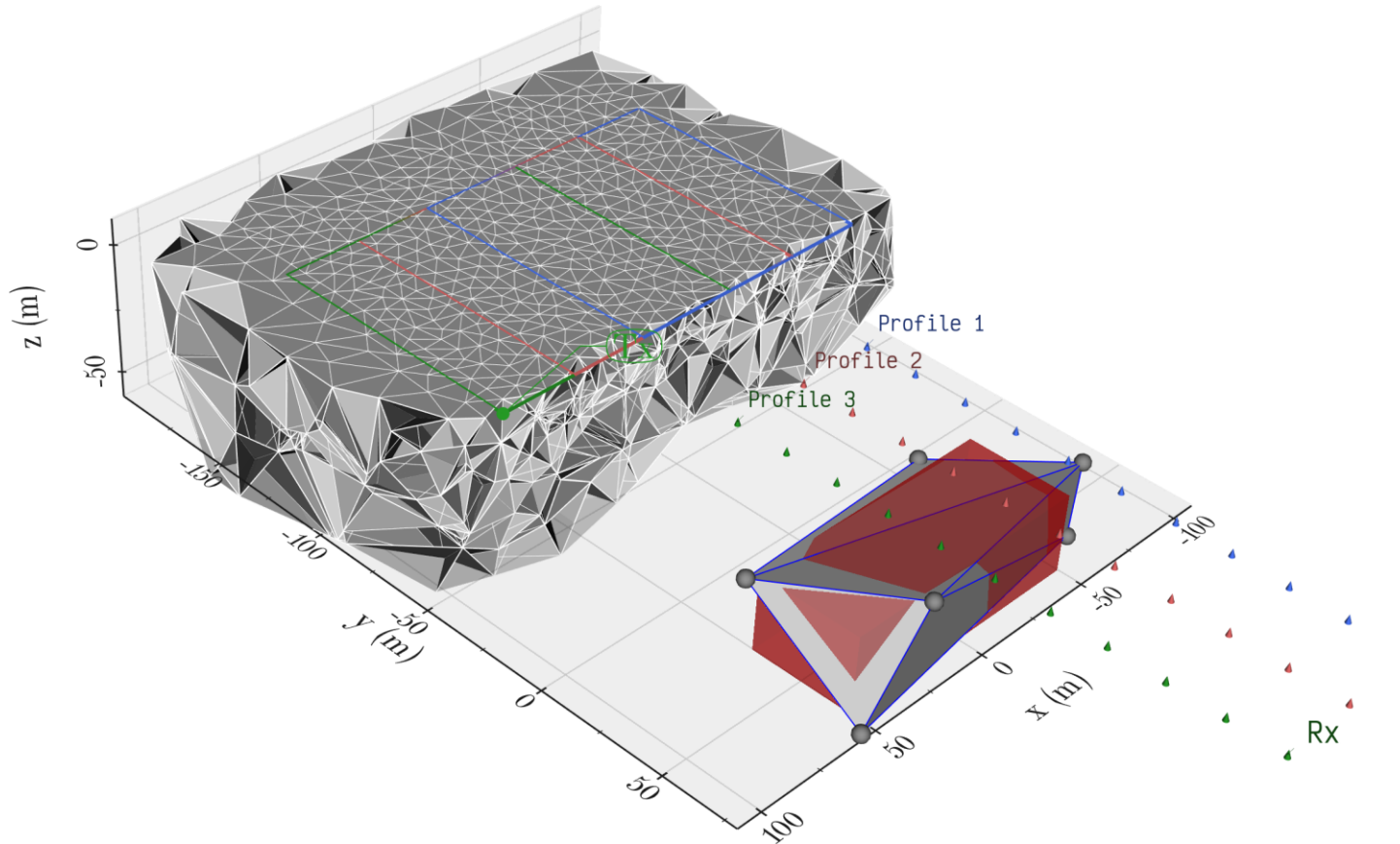
# Data fitting



Data fitting of L2400E

Data fitting of L3200E

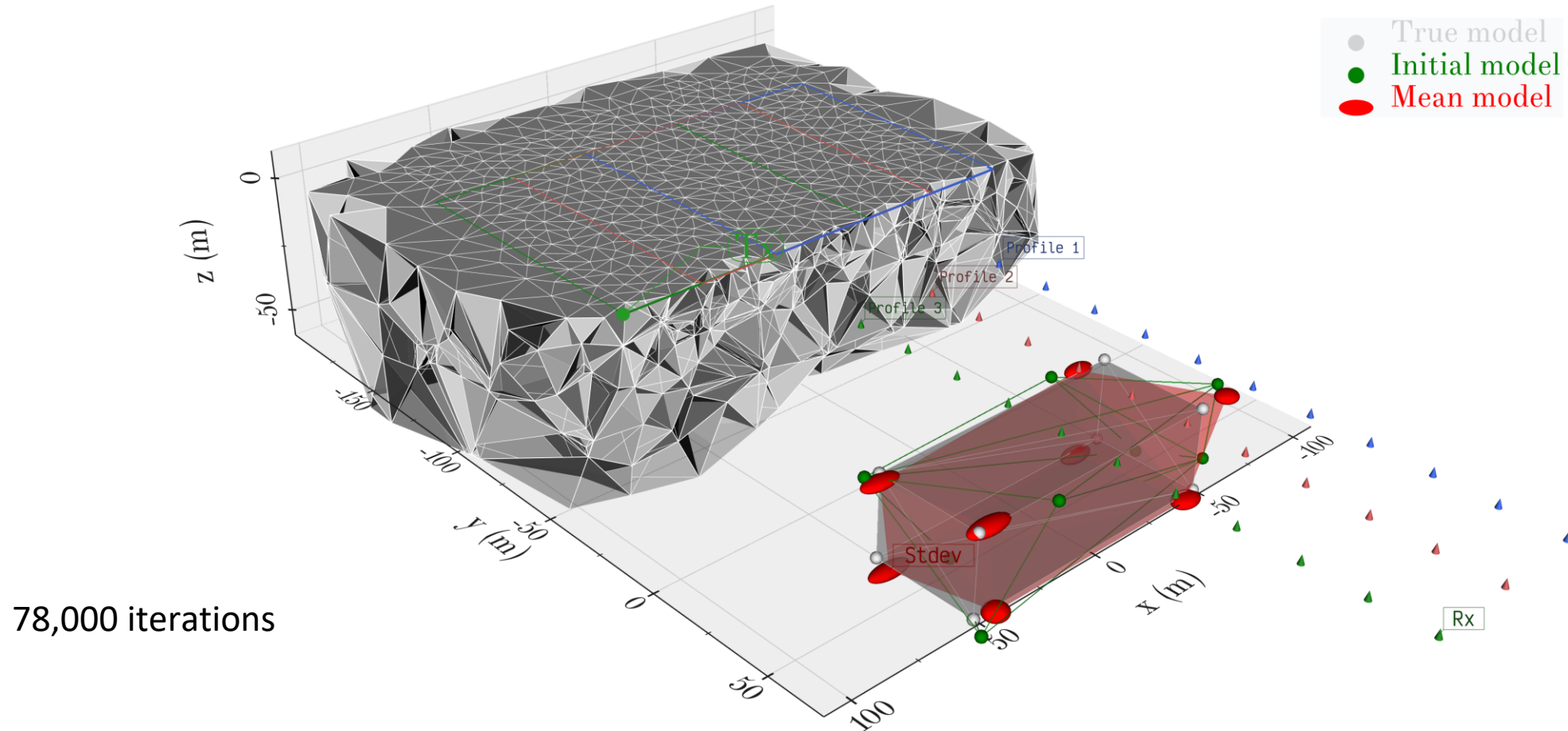
# Uncertainty quantification: MCMC sampling



- Block in half-space
- Moving loop survey
- Three profiles
- Background: 0.01 S/m
- Block: 2 S/m
- Parameterization: 8 nodes
- # parameters: 24
- Population size: 239
- Search volume: +/-30 m, +/-15 m, +/- 15 m in x-, y-, and z-direction

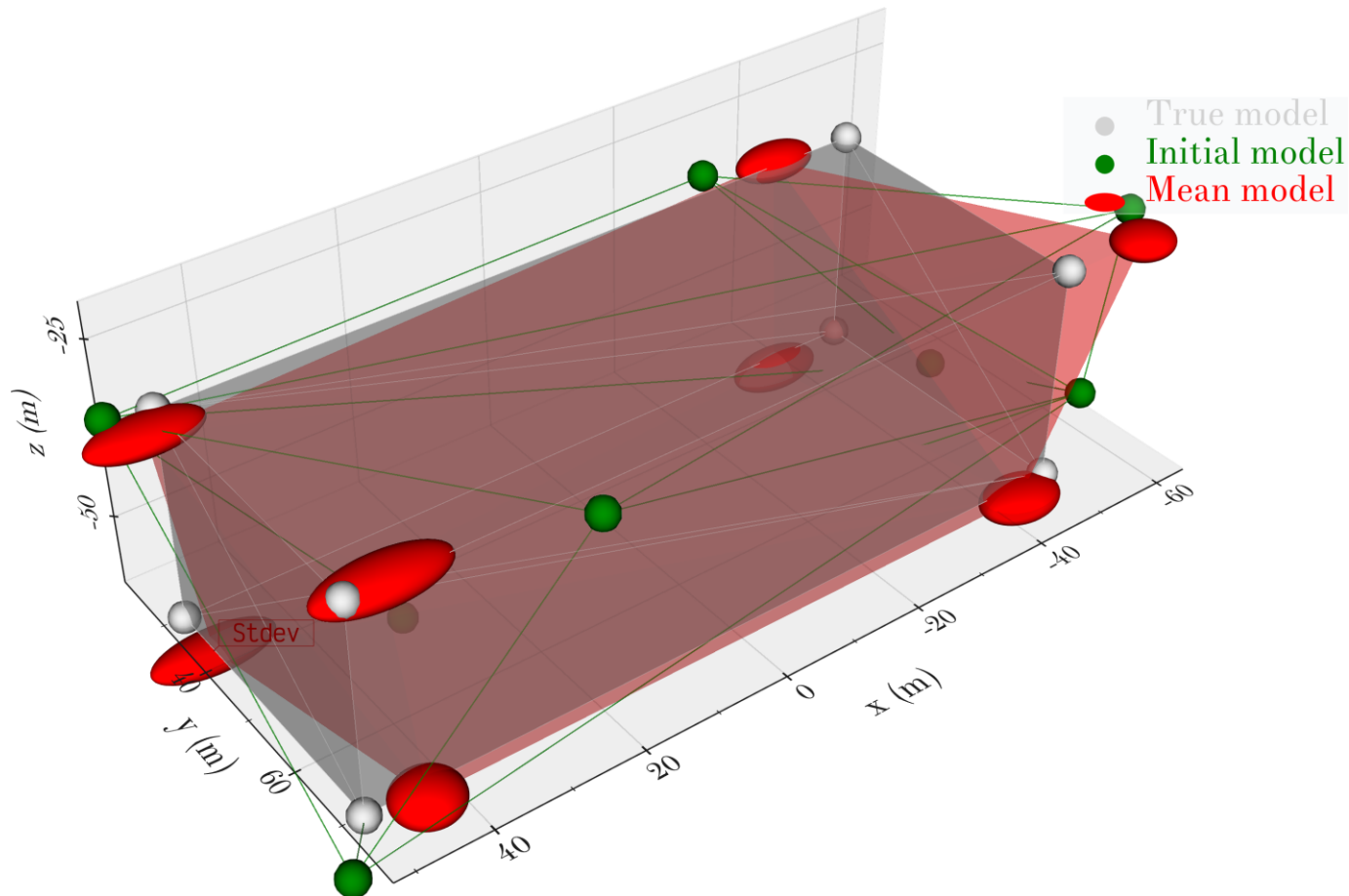
True model (red); Recovered model (gray)

# Uncertainty quantification: MCMC sampling



Mean model (red) is much closer to the true model (gray)

# Uncertainty quantification: MCMC sampling



- Mean model is **closer** to the true model
- **Uncertainty** (standard deviation) is the **largest** in the **x-direction**
- In general, **bottom nodes** have **larger uncertainty**
- Uncertainty is also related to the initial model
- Uncertainty is the smallest in the z-direction

# Outline

- Motivation
- Surface geometry inversion
- Marine CSEM examples
- TEM examples
- **Conclusions**

# Conclusions

- We have implemented a SGI algorithm for EM data
- The SGI algorithm works with both blocky and thin, plate-like anomalies
- The SGI algorithm has been tested using both synthetic and real-data examples
- Data uncertainties can significantly affect the inversion results
- Cross-line component of a MLTEM survey is also important
- MCMC sampling can be used for model uncertainty quantification

# Acknowledgements

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- Dr. Jianbo Long

Email: [xl0762@mun.ca](mailto:xl0762@mun.ca)



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