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A Critique of the MTDIW2-PNG Data Set Based on Mohr Circles

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The MTDIW2-PNG magnetotelluric data are examined as Mohr circles. Within their error bounds, of the ten sites five may be classified as 1-D immediately. Another four sites are 2-D; the only strongly 3-D data-set is from site PNG121. Visual examination of this simple presentation of the data gives much of the information that comes from more sophisticated analyses and decompositions, and can be used to check the latter. For example, from these basic figures it is possible to give principal directions for the 2-D cases, and observe (where it is present) the characteristics of frequency-independent "static shift". Other characteristics also become evident from examining the data in real and quadrature pairs, notably that the real data are noisier than the quadrature data (a phenomenon attributed to lightning strikes, which would put noise predominantly into the real data). There is a suggestion of a systematic effect in the two-dimensionality of the data as recorded, as the sites were operated as pairs, and generally odd-numbered sites are two-dimensional in character, while even-numbered sites are one-dimensional.

1. Introduction

Estimates of magnetotelluric (MT) impedance tensors are the bridge between observed magnetotelluric data, as time-series, and conductivity models for Earth, arrived at by data inversion and interpretation. It is almost always necessary to perform some operation on the tensors, as first estimated, before their modelling and inversion. Rotation of a tensor has long been recognised as necessary, and other "de-distorting" processes are now becoming established as routine.

While it is straight-forward to visualise a quantity which is a vector, it is difficult to visualise a tensor, and especially the changes to a tensor as it is rotated, and when it is subjected to other de-distorting processes. For this reason tensors depicted as circles, by the technique first used by Mohr for the mechanical stress tensor, are a valuable adjunct for an interpreter who is making judgements on tensor interpretation. The use of Mohr circles to display other tensor quantities is reviewed by Means (1990, 1992), and examples for structural geology in particular are given by Lister and Williams (1983), DePaor and Means (1984), Treagus (1987) and Passchier (1993).

Mohr circles of a magnetotelluric impedance tensor, taking real and quadrature parts separately, show all the information known about a tensor, and also therefore many of the major characteristics commonly sought. These latter include skew (often now referred to as twist), anisotropy, degree of one-, two- or three-dimensionality (1-D, 2-D or 3-D), and angles of rotation to the principal axes.

The MTDIW2-PNG data are from a sedimentary basin, and show (as might be expected) relatively simple MT tensor characteristics. This simplicity is evident in the circles presented in this paper. The circles provide a good set of examples for which the parameters of skew (or twist), anisotropy, and principal MT directions, can be determined by inspection. To emphasize the simplicity of this point, the angles given in Sections 5, 6 and 7 below have been first read from the figures by protractor. Where a number of circles indicate the same parameter a mean is taken, with the standard deviation from the mean giving an error estimate.

2. The PNG Data Set

The PNG data set are from a petroleum prospect over a sedimentary basin in the southern highlands of Papua-New Guinea. They are part of the 1991 Kube Kabe (PPL-100) Magnetotelluric Survey, being the Wara line of that survey. The ten observing sites were spaced at approximately 500 m intervals along a line striking approximately S30°W, and circles for the magnetotelluric tensors determined for the ten sites are presented in this paper. They would, from the outset, be expected to be 1-D or perhaps 2-D (and not complicated 3-D), and this situation is demonstrated to pertain. The main purpose of this paper, intended as a companion to the others in the MTDIW2 volume, is to present the tensor data as Mohr circles, and to list the characteristics which are then immediately evident.

For the MTDIW2 Workshop, the PNG MT data were supplied referred to axes directed geographic north (X) and east (Y). In the text below, rotations "of the measuring axes" thus refer to (clockwise) rotations from axes oriented north and east.

3. The Circles

The essence of the Mohr circle method is to take the real and quadrature parts of an MT impedance tensor separately, and for each part to plot the variation of Zxx against Zxy as the observing axes rotate through 180°. In this way two circles are generated, which display all the information known about the impedance tensor.

For an observed magnetotelluric impedance tensor with real parts

$$\begin{bmatrix} Zxx_r & Zxy_r \\ Zyx_r & Zyy_r \end{bmatrix}$$

which changes to

$$\begin{bmatrix} Z'xx_r & Z'xy_r \\ Z'yx_r & Z'yy_r \end{bmatrix}$$

upon rotation of the measuring axes θ' clockwise, a circle may be drawn on a plot of $Z'xx_r$ against $Z'xy_r$, as shown in Lilley (1976, 1993a). The circle is of radius R given by

$$R = \frac{1}{2}\left[(Zxx_r - Zyy_r)^2 + (Zxy_r + Zyx_r)^2\right]^{1/2}$$

and is centred on the $Z'xy_r$, $Z'xx_r$ co-ordinate axes at point

$$Z'xy_r = \frac{1}{2}\left(Zxy_r - Zyx_r\right)$$

$$Z'xx_r = \frac{1}{2}\left(Zxx_r + Zyy_r\right)$$

where the subscript r denotes the real part of the appropriate tensor element.

For one-dimensional data, the circle is simply a point on the horizontal axis. For two-dimensional data there is a circle of finite radius, with its centre on the horizontal axis. For three-dimensional data there is again a circle, the centre of which is offset from the horizontal axis; the offset is a measure of the degree of three-dimensionality. When the tensor at a site is known for a range of frequencies, all the circles for that site may be plotted together.

If a radial arm is drawn from the centre of a circle to the observed pair of values at the point (Zxy_r, Zxx_r), then rotation of that arm by angle $2\theta'$ anticlockwise takes its outer end to the

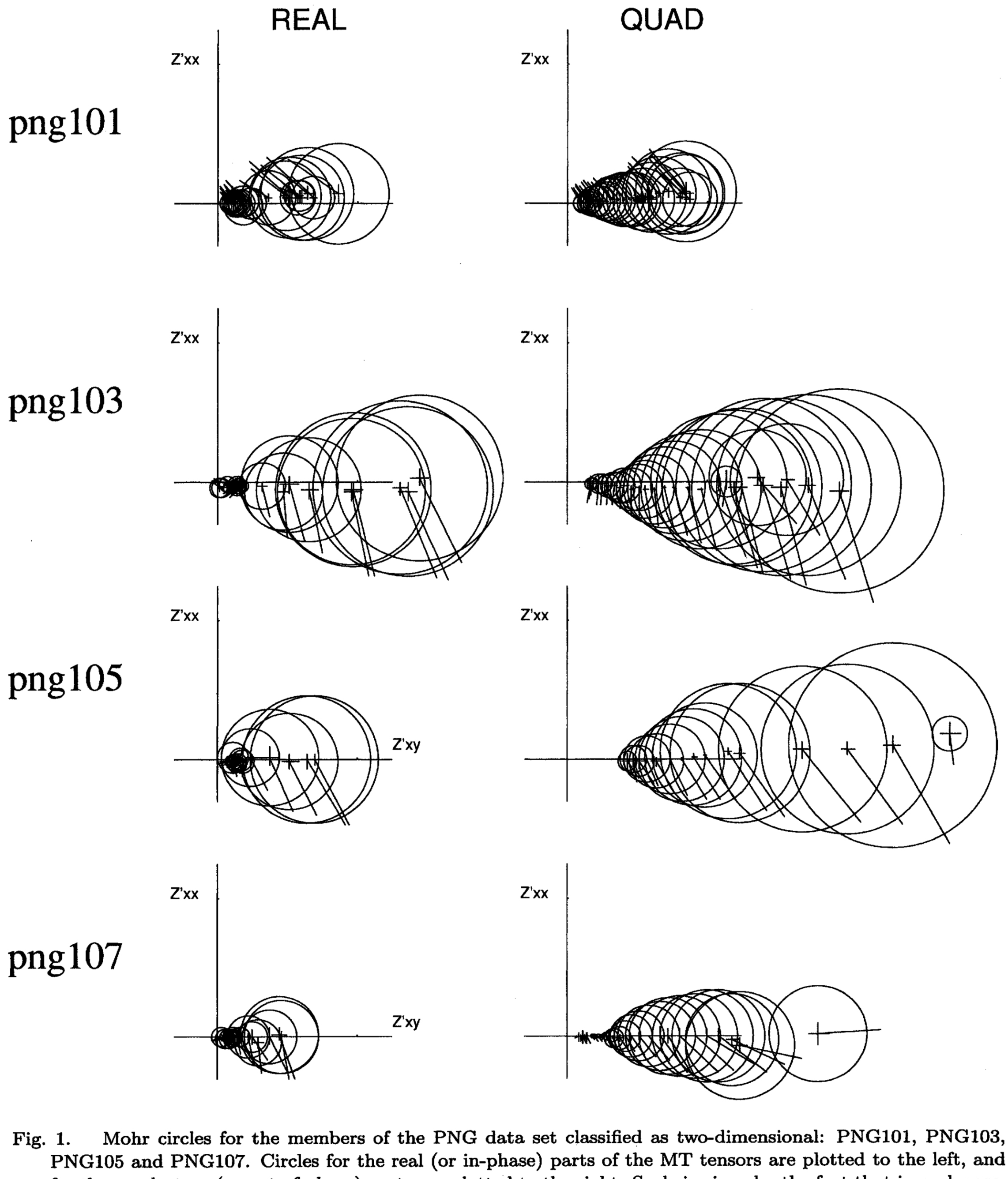

Fig. 1. Mohr circles for the members of the PNG data set classified as two-dimensional: PNG101, PNG103, PNG105 and PNG107. Circles for the real (or in-phase) parts of the MT tensors are plotted to the left, and for the quadrature (or out-of-phase) parts are plotted to the right. Scale is given by the fact that in each case, the origin of the axes for the quadrature circles is displaced 62.5 normalised impedance units from the origin of the axes for the real circles. A group of circles may contain up to 40 different members, one for each of the different frequencies (in the range 384 Hz–1820s) for which the MT impedance tensor is given in the PNG data set.

point on the circle corresponding to $(Z'xy_r, Z'xx_r)$. It is this property which is used in section 5 below to obtain strike angles. Thus where, as will be seen in Fig. 1, rotation of a radial arm anticlockwise by some angle (say 2θ) takes its end point to the horizontal axis, then rotation by θ clockwise of the measuring axes will bring about their alignment with the principal MT directions.

Thus the 2-D strike is at bearing θ (with a 90° ambiguity). The underlying reason is that a circle will cut the horizontal axis at points where $Z'xx_r$ is zero. For 2-D, these are the points where $Z'xy_r$ takes its two principal impedance values.

To demonstrate the determination of strike direction by inspection, angle 2θ can be read from diagrams like Fig. 1 by protractor.

4. Errors

The PNG data set include standard errors for the impedance tensor elements. In computing the parameters for a particular circle, these errors may be carried through to give standard errors for the coordinates of the circle centre, and for the radius of the circle. In the present paper the procedure described by Lilley (1993b) is followed.

Generally in this paper, a plot is omitted if the error in a centre coordinate, or radial arm, falls above a certain discriminant level (taken here to be 2.5 normalised impedance units). This discrimination procedure has been adopted so that scattered and often large circles from poorly-determined data will not confuse the better-defined pattern of a data-set, as shown by its well-determined members.

5. The PNG Magnetotelluric Circles

Full sets of Mohr circles for the PNG sites are presented in Figs. 1, 2 and 3. In the simplest case of magnetotelluric data recorded above a uniform half-space, the tensor elements are proportional to (period)$^{-1/2}$, and so decrease with increasing period. It is therefore helpful when drawing circles to first normalize the impedance tensor data by multiplying each element value by $T^{1/2}$, where T denotes period in seconds. All the tensor element data for Figs. 1, 2, 3 and 4 have been normalised in this way. With this normalisation, and because of an increase of conductivity with depth in the ground, the progression from high to low frequency generally corresponds to the circle centres moving from right to left in each diagram.

Figures 1, 2 and 3 show characteristics for the PNG sites which will now be described. In the figures, sites with major characteristics in common have been grouped together.

Set 1. PNG101 (Fig. 1)

For both real and quadrature sets of circles, the circle centres, imagined joined together, make lines which are rotated no more than 6° from the horizontal axes. Thus the skew or twist is at most 6°, and the data are basically two-dimensional.

The angles for the principal MT directions for the PNG101 site are given clearly and consistently by the parallel nature of the radial arms of the plotted circles. The directions of these radial arms indicate that the reference or "measuring" axes should be rotated clockwise 23° and 113° from north (with error 7°), for the axes to then be aligned with, and so indicate, the principal MT directions.

In fact, both the real and quadrature sets of circles fall within even envelopes, in the pattern predicted for static-shifted 1-D structure in an earlier paper (Lilley, 1993c). Defining a (tensor) anisotropy parameter as half the angle which the envelopes subtend at the origin of axes, the envelopes for the PNG101 data indicate a frequency-independent anisotropy of 24°. Thus PNG101

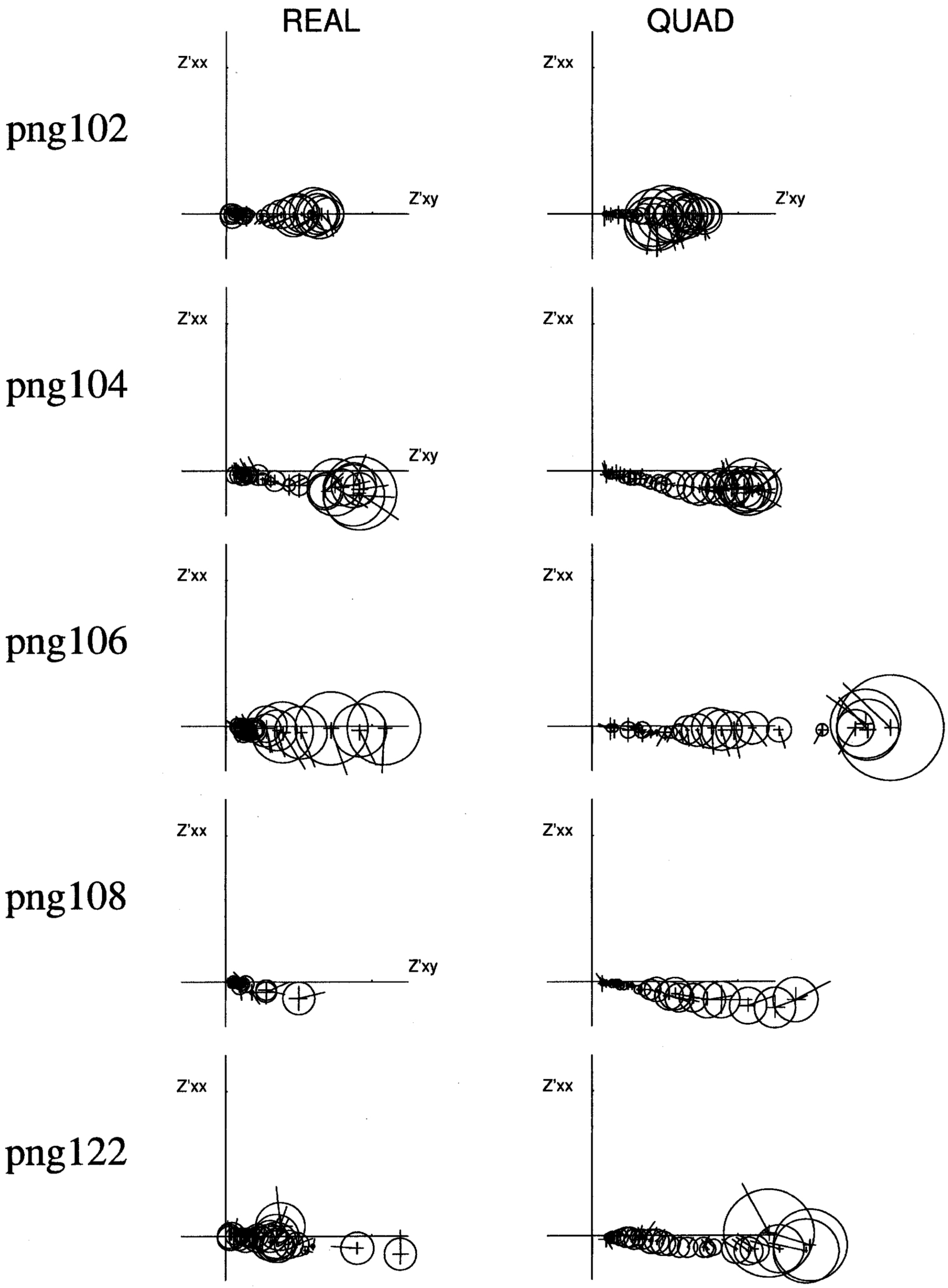

Fig. 2. Mohr circles for the members of the PNG data set classified as one-dimensional: PNG102, PNG104, PNG106, PNG108 and PNG122. The arrangement of the circles is as for Fig. 1.

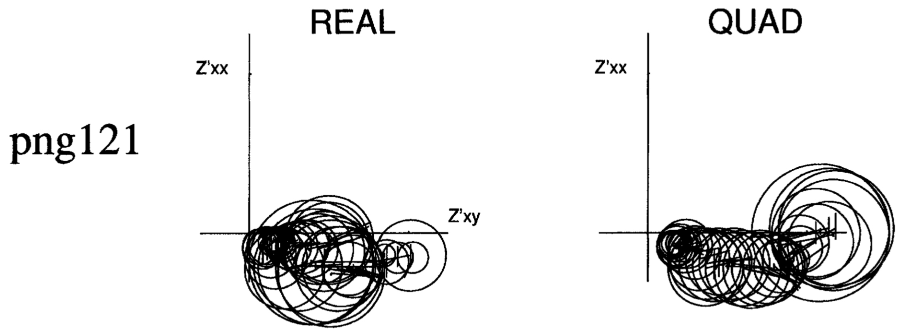

Fig. 3. Mohr circles for PNG121, the member of the PNG data set not classifiable directly as 1-D or 2-D. The arrangement of the circles is as for Fig. 1.

may be an example of data which have been recorded above 1-D geologic structure, and which have been affected by static shift.

Set 2. PNG102 (Fig. 2)

This data set has circles which lie closely along the horizontal axes, and (given the small size of the circle radii) are closely 1-D. An additional indication of 1-D behaviour is given by the observation that the radial arms for the circles are not consistently parallel, so that there are no preferred MT principal directions. The non-zero radii, and scattered radius directions, are interpreted as 1-D data with noise.

Set 3. PNG103 (Fig. 1)

The PNG103 data set has characteristics similar to the PNG101 set, with a skew of at most −3°, and an anisotropy angle of 25° (except at long periods, when the data are more closely 1-D). The directions to rotate the reference axes clockwise from north, to give the principal MT directions, are 37° and 127° (with error 4°).

Set 4. PNG104 (Fig. 2)

These data resemble PNG102, and are closely 1-D with perhaps a skew or twist of −11°. The random nature of the directions of the radial arms, especially in the real circle set, indicates that the departures from 1-D (albeit twisted 1-D) are a consequence of experimental noise.

Set 5. PNG105 (Fig. 1)

The PNG105 data are very similar in characteristics to PNG103. The PNG105 data show nil twist or skew; anisotropy variable around 30°; and rotations of the measuring axes of 31° and 121° (with error 2°) for alignment with the principal MT directions.

Set 6. PNG106 (Fig. 2)

The PNG106 data are a straight-forward case of 1-D structure (which would be lines of points along the horizontal axes), but with noise (to produce the circles seen in the plot).

Set 7. PNG107 (Fig. 1)

The circles indicate rotations of the measuring axes of 26° and 116° (with error 11°) for alignment with the principal MT directions. At long periods the data are 1-D, with circles of small radii on the horizontal axes. The real circles are noisier than the quadrature circles.

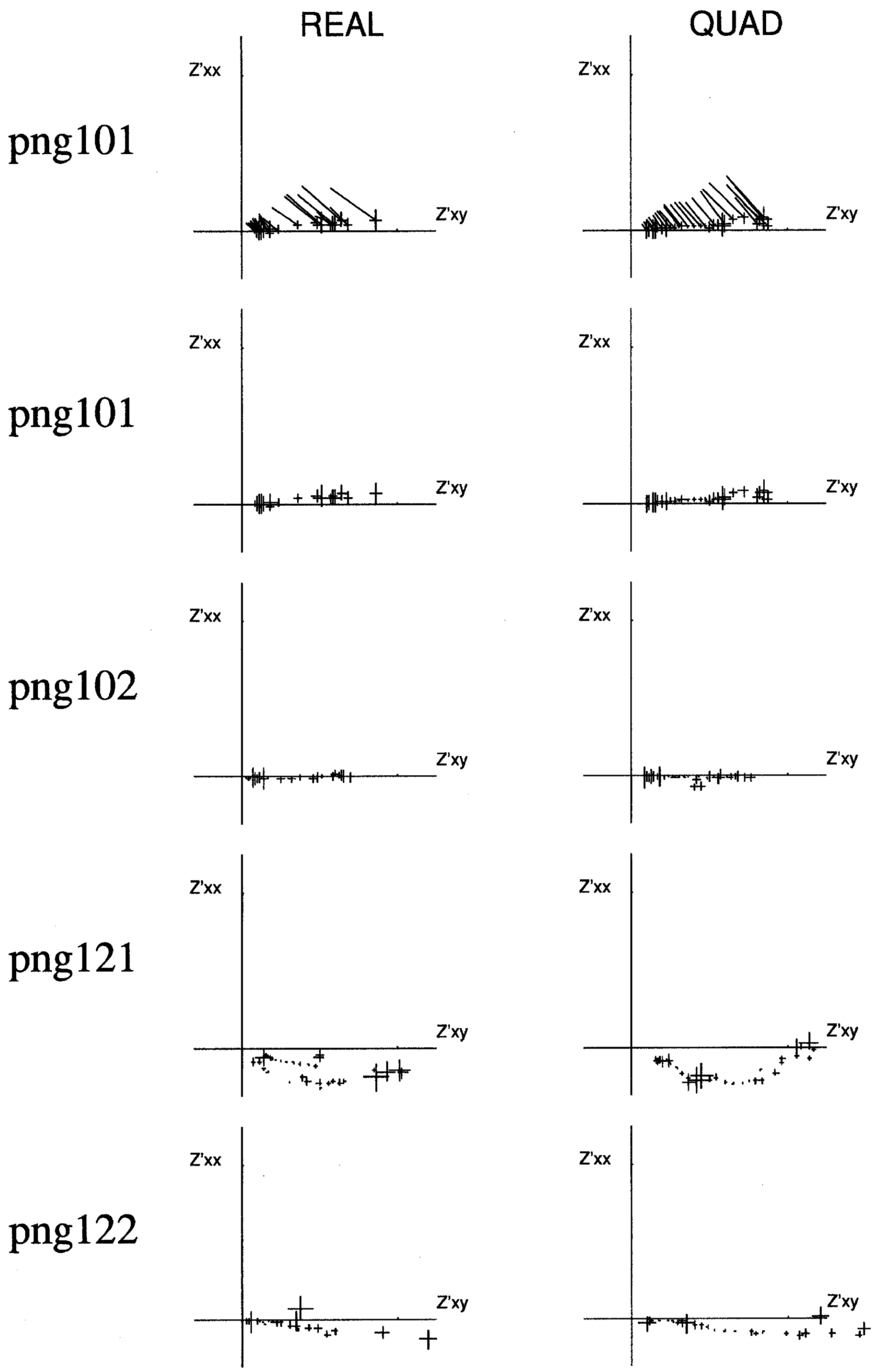

Fig. 4. Figure showing some of the elements of the circles in Figs. 1, 2 and 3 above, plotted as for Fig. 1. The top line shows the PNG101 circle centres and radial arms only, the latter clearly indicating a well-determined rotation of the axes for alignment with the MT principal directions. The next line shows the PNG101 circle centres, without the radial arms, and the next lines show similar plots for the sites PNG102, PNG121 and PNG122. The 1-D nature of the PNG102 data-set can be clearly seen (lines of points along the horizontal axes), as can the complicated plot of the PNG121 circle centres, and the straight line (of constant skew or twist) for the PNG122 data.

Set 8. PNG108 (Fig. 2)

The PNG108 data are similar to the PNG104 data in characteristics. The PNG108 data are 1-D, with a slight negative skew or twist (−8°).

Set 9. PNG121 (Fig. 3)

The PNG121 data are the most complex of the present set. Site PNG121 is three-dimensional in behaviour, and not easily interpreted. Disregarding the noisy data at high frequency, a frequency-independent skew of approximately −20° is evident. Anisotropy and strike direction are variable.

Set 10. PNG122 (Fig. 2)

The PNG122 data are like PNG108, and so also like PNG104. The PNG122 data are 1-D, with a slight negative skew (−7°).

6. Some Circle Elements Plotted Separately

In many cases, some of the information displayed in Figs. 1, 2 and 3 can be seen more clearly if elements of the circle diagrams are plotted separately. Examples of such elements plotted separately are given in Fig. 4. The top line shows the PNG101 circle centres and radial arms plotted without the circle circumferences, and the parallel nature of the radial arms shows well-determined rotation angles for alignment of the measuring axes with the principal MT directions.

The next line of Fig. 4 shows plots of just the circle centres of PNG101 data. A consistent skew or twist is shown clearly, as (for both the real and quadrature cases) the circle centres lie on a straight line which is rotated some 6° from the horizontal axis.

The remaining lines of Fig. 4 show circle-centre plots for PNG102, PNG121 and PNG122 respectively. These examples form a contrasting set, as the PNG102 plot shows circle centres lying on the horizontal axes in an 1-D (and 2-D) manner, while the PNG121 circle centres have a complicated locus, and the PNG122 centres again form straight lines, indicating a skew or twist of some −7°.

7. Conclusions

The PNG data, by the plots of their circles, can be seen to fall into two groups, and have been plotted this way in Figs. 1 and 2 (with Fig. 3 showing the exception). The groups have the following characteristics:

In Fig. 1 the "odd" sites, PNG101, 103, 105 and 107, are 2-D in character, with negligible skew or twist. In the case of site PNG101 in particular, the 2-D character may result from 1-D data being affected by static shift.

In Fig. 2 the "even" sites, PNG102, 104, 106, 108 and 122, are basically 1-D in character, with (in three cases) a slight negative skew.

The above observation may indicate some systematic effect in the MT field equipment or data reduction procedures, as the sites are understood to have recorded in pairs (i.e. site 101 simultaneous with site 102, site 103 simultaneous with site 104, etc.). The main point to note here is that displaying tensor data diagrammatically enables such characteristics to be noticed.

A second general observation is that the real circles, in many cases, can be seen to be noisier than the quadrature circles, in that the pattern formed by the latter is more even. This phenomenon is attributed to the effects of lightning strikes, which are sharp in time and so will affect the magnetic and electric time series in phase, (and thus also the real, rather than the quadrature, tensor elements). Once alerted to such a phenomenon, reduced vulnerability to it may be possible by particular remote referencing procedures, and by special time-series analysis techniques. Also a weighting of the quadrature data, relative to the real data, may be feasible in data inversion routines.

A third observation is that the similarity of the real and quadrature sets of circles means that generally the magnetotelluric phase will reside in the first quadrant, for all rotations of the measuring axes. There are some instances where circles go to the left-hand side of the vertical axis (the characteristic which shows a phase out of the first quadrant) but, with the exception of PNG121, all these circles can be seen to be noise effects associated with circles with centres already close to the origin. It is likely the same is true of PNG121, but this data set is more distinctly 3-D, and skew can directly cause phases to go out of quadrant by moving circles so that they cut to the left of the vertical axis.

Finally, some information accompanying the PNG data advises that the prevailing geologic structures in the area trend approximately 120°. This bearing can now be compared with the directions found for the principal MT directions in Section 5 above. Taking the usual 90° ambiguity favorably, sites 101, 103, 105 and 107 indicate strike directions of $113° \pm 7°$, $127° \pm 4°$, $121° \pm 2°$ and $116° \pm 11°$, respectively, in good agreement with the stated 120°.

In summary, portraying the data as Mohr circles has shown that it is straightforward to recognise the data as 1-D and 2-D, with strike angles immediately evident in the case of the latter. Also, before other interpretation procedures are invoked, a familiarity is gained with aspects of data quality.

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