Decomposition of the Magnetotelluric Impedance Tensor Which is Useful in the Presence of Channeling

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Summary

It is believed that there are many occasions when the magnetotelluric impedance tensor is the result of local galvanic distortion of electric currents which arise from induction in a structure which is approximately two-dimensional on a regional scale. Procedures for rotating the impedance tensor, such as minimizing the mean square modulus of the diagonal elements, cannot in general recover the principal axes of induction nor do they recover the principal impedances but rather linear combinations of them.

This paper presents a decomposition of the impedance tensor which separates the effects of channelling from those of induction in these cases. That such a decomposition is possible in principle is implicit in the work of Bahr (1985). When the impedance tensor is actually the result of regional one or two-dimensional induction combined with local, frequency- independent telluric distortion this method correctly recovers the principal axes of induction and except for a constant, independent of frequency, the principal impedances. Also obtained are two parameters (twist and shear) which partially describe the current distortion.

The method is applied to field data from a site in northern Canada. A normalized residual error is used as an estimator of fit to the model. The ability to invert the data for channelling parameters which are independent of frequency is a test of the hypothesis of current distortion coupled with two-dimensional induction. The error of fit, by this technique, was found to be significantly less than that by the standard method.

Introduction

If the Earth has a two-dimensional conductive structure on a regional scale, then magnetotellurics assumes a source field such that

$$\mathbf{e}_r = \begin{pmatrix} 0 & a \\ -b & 0 \end{pmatrix} \mathbf{h}_r.$$

$\mathbf{e}_r$ and $\mathbf{h}_r$ represent the regional electric and magnetic fields parallel to the Earth's surface with respect to the principal axes of the two-dimensional structure. Local inhomogeneities will distort regional telluric currents and prohibit meaningful one or two-dimensional interpretations even if the large-scale (regional) geological structure is not strongly three-dimensional. If these inhomogeneities are small compared to the electromagnetic skin-depth in either the host or the heterogeneity, the effect will mostly be a galvanic, frequency independent distortion of the large scale currents. In this case, the magnetotelluric impedance tensor can be written, in the principal co-ordinate system, as

$$\mathbf{Z} = \mathbf{C}\,\mathbf{Z}_2$$

where $\mathbf{C}$ is a real tensor (matrix) operator of rank two acting on the two-dimensional regional (large-scale) impedance tensor ($\mathbf{Z}_2$). Zhang *et al* (1986) have considered the special case where $\mathbf{C}$ is due to "two-dimensional channelling" and thus is symmetric. The channelling tensor, $\mathbf{C}$, will be frequency independent over some range. It is worth noting that it is only when $\mathbf{C}$ is diagonal that the channelling produces what is normally described as a "static shift" of the apparent resistivities.

A mathematical decomposition of the channelling tensor is developed which separates the effects of channelling into three physically meaningful parameters (shear, twist and channelling anisotropy). A non-linear system of equations governing the relation between the measured impedance tensor and physical parameters (induction and channelling) is derived. A discussion of the method for solving the non-linear system and the practical application of the method follows, illustrated by actual field data. As well, a comparison of results with the more standard decomposition method is given.

Factorization of the Channelling Tensor:

The model of current channelling coupled with at most two-dimensional induction decomposes the measured impedance tensor, $\mathbf{Z}_m$, as

$$\mathbf{Z}_m = \mathbf{R}\,\mathbf{C}\,\mathbf{Z}_2\,\mathbf{R}^T$$

where $\mathbf{R}$ represents the rotation from the measured co-ordinate system to the principal co-ordinate system. This decomposition is clearly not unique as there are nine real parameters but only eight real equations. A more useful decomposition is outlined below. As well, a means of testing the hypothesis of model is given and the determined parameters are at least approximately interpretable.

It can be shown (Bailey and Groom, 1986), for all channelling tensors, there is a factorization

$$\mathbf{C} = g\,\mathbf{T}\,\mathbf{S}\,\mathbf{A}$$

where $\mathbf{T}$, $\mathbf{S}$ and $\mathbf{A}$ are tensors and g is a scaling factor. The effects of these tensors are illustrated in Figure 1.

The splitting (anisotropy) tensor, $\mathbf{A}$, stretches the horizontal field components produced by regional induction by different factors. Figure 1(a) represents an arbitrary regional electric field while (b) represents the result after the vector is modified by the splitting tensor. The shear tensor, however, develops anisotropy on an axes which bisects the regional induction principal axes. The effect of $\mathbf{S}$ can be seen in Figure 1(c) where now the regional current vector has been operated on by the shear tensor. The twist tensor, $\mathbf{T}$, rotates the electric field vectors produced by $\mathbf{S}\ \mathbf{A}\ \mathbf{Z}_2$ through an angle termed the twist angle. This effect is easily imagined.

The advantage of this factorization now becomes apparent. It is clear both on physical and mathematical grounds that an effect which develops anisotropy on the same axes as the regional anisotropy is not separable from the regional impedance tensor. That is

$$\mathbf{Z}_2' = g\ \mathbf{A}\ \mathbf{Z}_2$$

is an equally valid two-dimensional impedance tensor. This alternate impedance tensor, however, maintains the correct regional strike, principal impedance phases and the same shape of the apparent resistivity curves. Information is needed to fix the scaling of the apparent resistivities; this a problem of static shift correction for which solutions have already been proposed.

Decomposition of the Impedance Tensor

The decomposition has now become

$$\mathbf{Z}_m = \mathbf{R}\ \mathbf{T}\ \mathbf{S}\ \mathbf{Z}_2'\ \mathbf{R}^T.$$

The decomposition can be performed analytically. However, in the presence of data errors a more appropriate means of solution is by a non-linear, least squares solver. The determined parameters, both channelling and regional induction, can be used to construct an estimated impedance tensor based on the model. As a measure of fit, a relative error is used

$$\epsilon^2 = \frac{\sum_{i=1}^{2}\sum_{j=1}^{2}\left|\hat{Z}_{ij} - Z_{ij}\right|^2}{\sum_{i=1}^{2}\sum_{j=1}^{2}\left|Z_{ij}\right|^2} \qquad (1)$$

where Z_{ij} and $\hat{Z}_{ij}$ are the measured and modeled impedance tensor elements respectively.

It is emphasized that the use of the decomposition does not require that the above physical model be true. If the model is true , then the parameters obtained will be meaningful. However with its more general nature, the decomposition will encompass a larger range of field situations. The ability to obtain a decomposition in which the channelling parameters are frequency independent, over the range of interest, is one means of testing the hypothesis that the model is true.

An Example with Real Data

The data are long period data obtained at a site on the Canadian Shield in northern Ontario about 30 km north of the town of Chapleau. The upper crustal rocks are granitic and have resistivities of the order of 10^4 ohm-meters. The topographic relief of a few tens of meters is extensively but erratically covered with glacial debris ranging from clays (resistivity of the order of 10 ohm-meters) to gravels and sands (resistivity of the order of 10^3 ohm-meters). The likelihood of strong channelling with a length scale of several hundred meters is high.

The impedance tensors used are averages of impedance estimates. Each impedance estimate is obtained from a different time series with averaging in a spectral window. Error bars are determined from the scatter of the parameter within the set of impedance estimates.

The following is not meant to be an exhaustive discussion of all aspects of the method in terms of this data set. The data set illustrates many interesting facets but there is not sufficient space in which to describe them all so we will restrict our discussion to just a few features.

Figure 2 contains two columns of information. The left column contains the parameters; regional strike, apparent resistivities and phases obtained from the data by the standard method (Swift, 1967). As well, the square root of the error of fit obtained by Equation 1 is included. The right column contains the same information but by the new decomposition. It is important to note that, for this presentation, the data has been inverted allowing the channelling parameters to vary with frequency. The discussion of the inversion for full frequency independent channelling and its implications will be presented in another publication. However, some main points can be made here with this presentation.

There are two main inferences that could be made from the standard method of interpretation. First, the regional azimuth varies only slightly with frequency with respect to the measurement axes. Second, the principal impedances appear to be real, scalar multiples of each other and there is little to distinguish the phases. This appears to be an example of a one-dimensional geology with "static shift" with a relatively unchanging regional strike. However, the new decomposition suggests the structure is more likely two-dimensional. The local channelling azimuth (a parameter already defined for "two-dimensional channelling" by Zhang *et al* (1986)), an estimate of the direction of the local currents, is relatively frequency independent and agrees closely with standard azimuth as predicted by this method (Bailey and Groom, 1986). The impedance phases by the new method indicate the regional structure is not one-dimensional. Also to note, the error of fit is approximately an order of magnitude better for this decomposition.

The channelling parameters; shear and channelling azimuth can be held fixed with frequency for this data set. A set of decompositions is obtained with little loss of error of fit over the frequency dependent decompositions. This supports a model of frequency independent channelling for this field data.

Conclusions

This decomposition is likely to be useful (i.e. physically meaningfful) in those field situations where the earth is approximately two-dimensional on a large scale, but three dimensional on a small scale. (Scale size here is determined by comparison with relevant skin depths.) This class of earth models is very much larger than the class for which the conventional rotation method is meaningful and it includes the conventionaal case as a special case.

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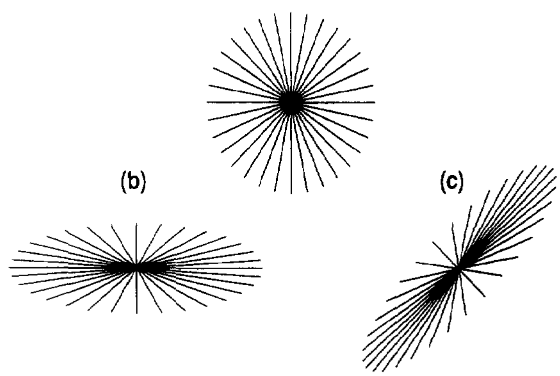

FIG. 1. Effects of splitting and shear tensors.

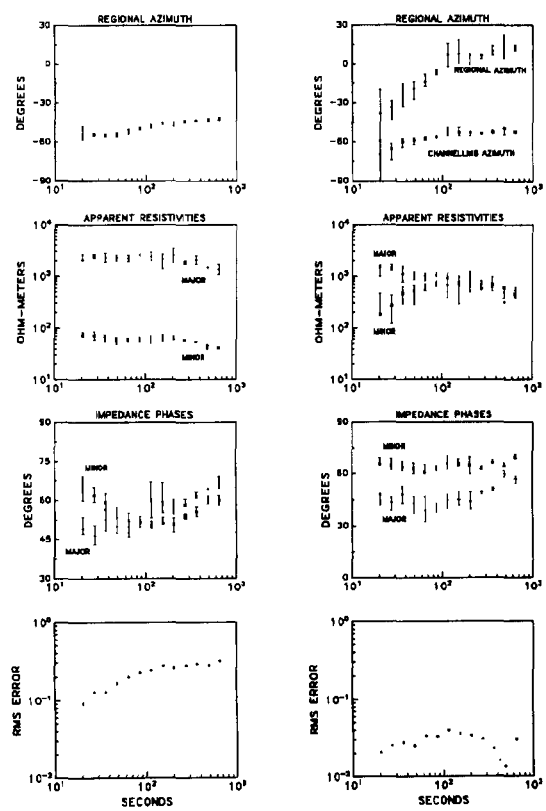

FIG. 2. Classical (left) and new (right) parameters from field data.