APPENDIX

A SIMPLE ALMOST EXACT METHOD OF MT ANALYSIS

by

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A simple approximate method of MT analysis based on the asymptotic behavior of the apparent resistivity curves at low frequencies is briefly described. Reference is first made to Fig. 1. In that figure the exact apparent resistivity curve and its low frequency asymptote are plotted for an earth model consisting of a single section of uniform resistivity overlying a perfectly conducting substrate. In a more general context it can be shown that the asymptote depends only on the depth to substrate and is independent of the conductivity of the resistive section even if that conductivity is a function of depth.

Figure 2 shows the apparent resistivity curve together with its low frequency asymptote for an earth model consisting of the same resistive section as in the model of Figure 1, but overlying a perfectly insulating substrate of infinite thickness. It can be demonstrated that the asymptote for this model depends only on the S value defined in Fig. 2.

If Figures 1 and 2 are superimposed the apparent resistivity values at frequencies higher than the one corresponding to the point of intersection of the asymptotes are seen to be little affected by an infinite change in the conductivity of the substrate. Furthermore, at the frequency of the asymptote intersection the corresponding apparent resistivity value appears to be a reasonable single value central estimate of the small range of values over which the exact apparent resistivity varies in response to changes in the substrate conductivity. Expressing each apparent resistivity value in terms of the equations describing the asymptotes intersecting at that resistivity value forms the basis of the approximate MT analysis described here. It is interesting to note that expressed in this fashion the apparent resistivity value depends only on the earth conductivity at depths shallower than D.

Figure 3 shows a plot of the asymptotes discussed above for a sequence of values of both S and D. The equations for these asymptotes are repeated for reference immediately below the plot. The plotted asymptotes form an orthogonal coordinate system rotated 45° with respect to the logorithmic axes of ρA and The S and D values associated with each point on an arbitrary ρA vs ω curve drawn on this grid can be determined conveniently from the S and D coordinates of the point. In the lower half of Figure 3 it is shown that simultaneous solution of the asymptote equations implies that the MT apparent resistivity at any frequency w is the reciprocal of the average conductivity to the depth D, where D is uniquely defined in terms of ρA and ω.

Figure 4 presents an example of a forward MT problem worked within the framework of the asymptote intercept approximations. The values of S and a sequence of increasing values of D are computed and tabulated for the three layer earth model shown in the figure. The corresponding ρA and ω values can be computed from the asymptote expressions or can be obtained graphically. To find the values graphically the S and D values are plotted on the combined S,D; ρA,ω grid. Such a plot for the example three layer earth model is shown at the top of Fig. 4. Where the resulting approximate ρA curve is also compared with the exact one-dimensional curve computed for the model.

In addition to providing a technique for working the forward MT problem the asymptote intercept approximation also yields a simple direct procedure for the inverse problem. Figure 5 presents the derivation of the additional equation used for the direct inverse. This equation yields well behaved resistivity vs depth estimates when used with ρA vs ω curves theoretically generated. The derivative term does, however, cause the inverse to be sensitive to the scatter inevitably present in experimental curves. This sensitivity can be greatly reduced if the necessary derivative is estimated from phase measurements. As indicated in Fig. 6 the Hilbert transform provides the necessary equations. The zeroth order approximation of the Hilbert Transform provides the particularly simple equation expressing the relationship between phase and the required logarithmic derivative of the apparent resistivity curve.

The approximate MT analysis procedures described above have been used for approximately three years in a wide variety of both theoretical and experimental situations. For the most part the results obtained in those applications have been remarkably accurate. In addition, the simplicity of the technique provides for considerable insight into the inherent capabilities and limitations of the MT method.

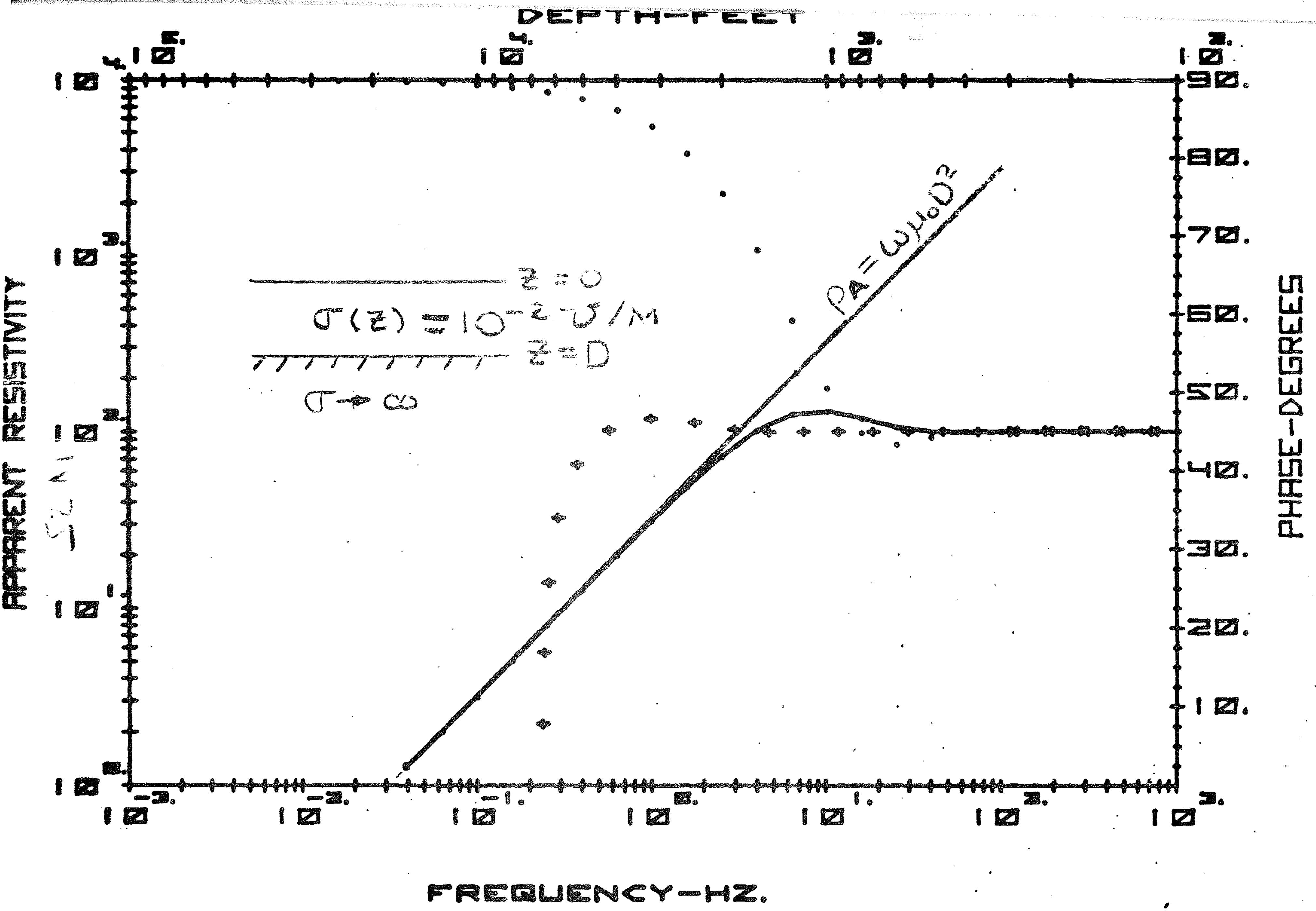

DEPTH–FEET
APPARENT RESISTIVITY
PHASE–DEGREES
FREQUENCY–HZ.
$\rho_A = \omega \mu_0 D^2$
$z = 0$
$\sigma(z) = 10^{-2}$ ℧/M
$z = D$
$\sigma \to \infty$

FIG 1

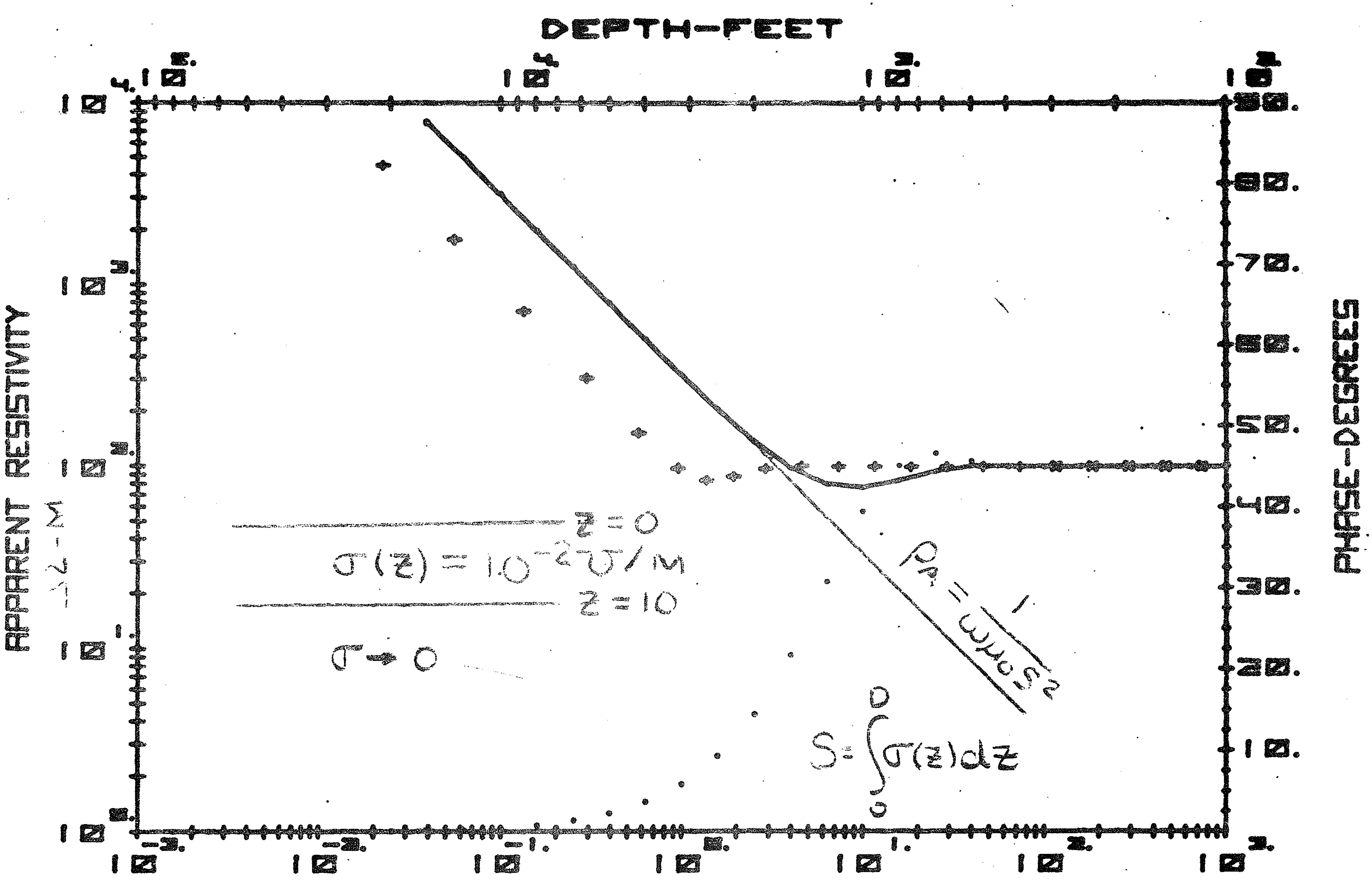
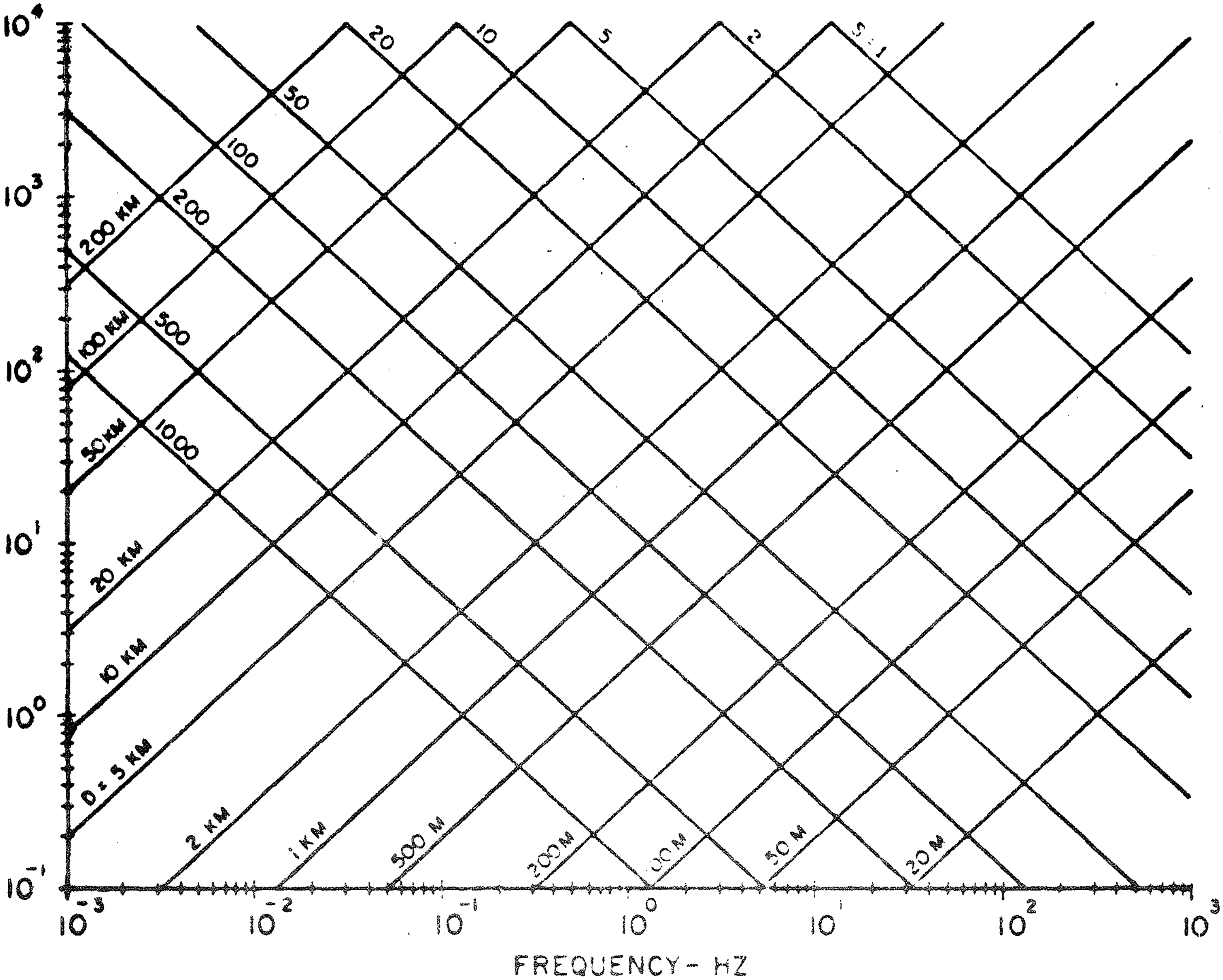

DEPTH-FEET
APPARENT RESISTIVITY
Ω-M
PHASE-DEGREES
FREQUENCY-HZ.
Z = 0
σ(Z) = 1.0^-2 ℧/M
Z = 10
σ → 0
ρA = 1 / ωμ₀S²
S = ∫₀^D σ(z)dz

FIG 2



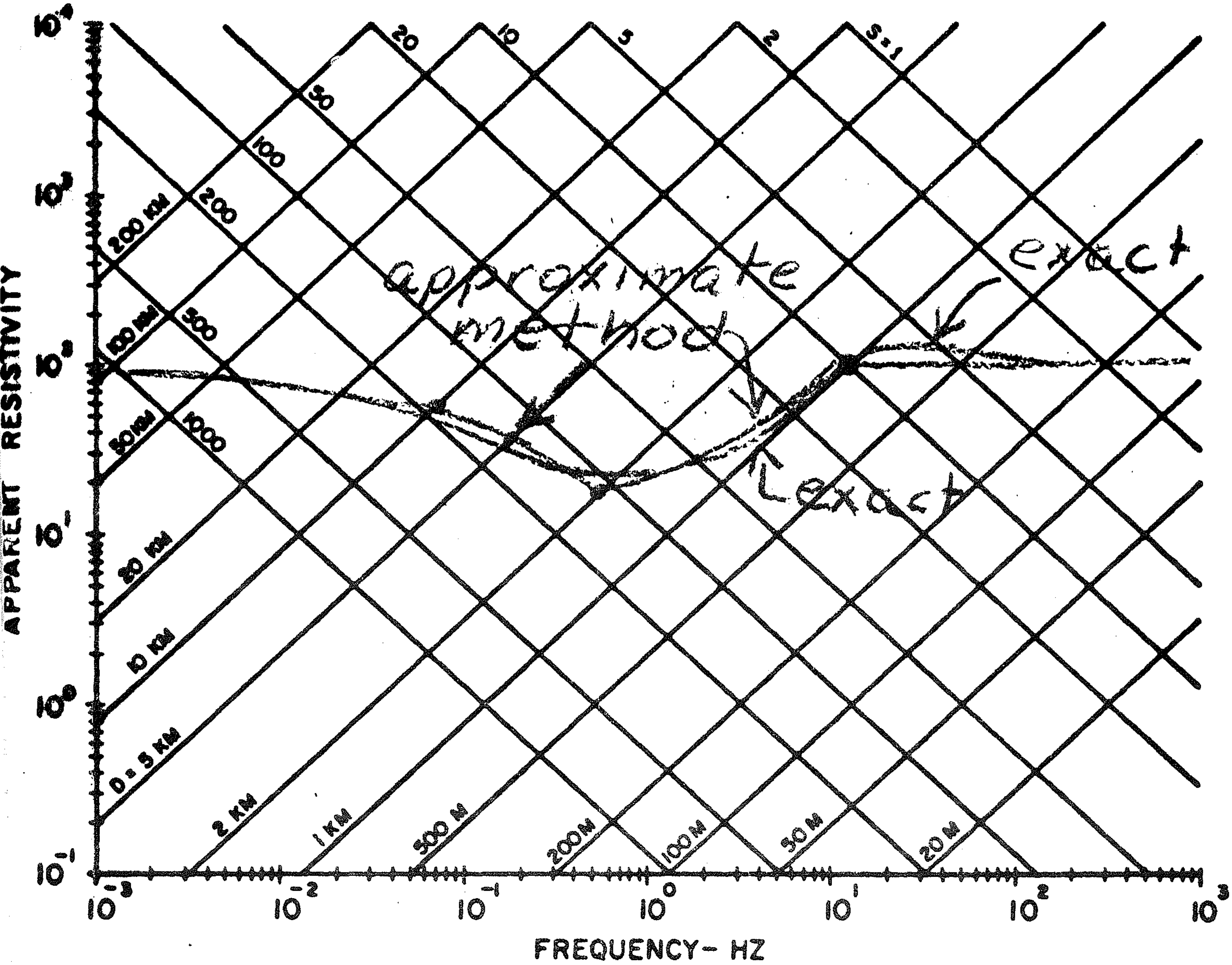
$\rho_A = \omega\mu D^2$ (D-lines)

$\rho_A = \dfrac{1}{\omega\mu S^2}$ (S-lines)

Eliminate $\omega\mu$

$$\rho_A = \frac{D}{S}, \quad \sigma_A = \frac{1}{\rho_A} = \frac{1}{D}\int_0^D \sigma(z)\,dz$$

FIG 3

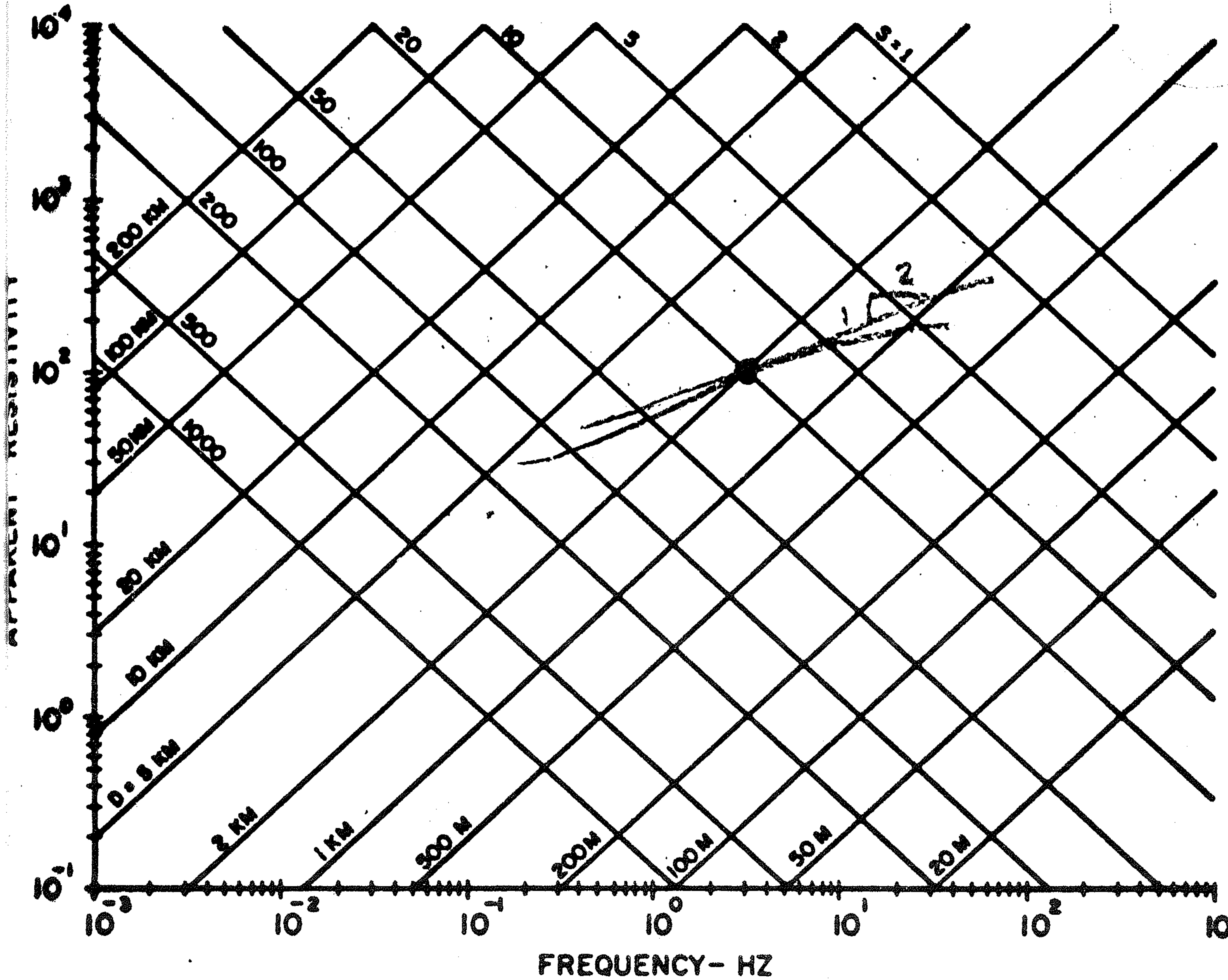


The Foward Problem - An Example

	$z = 0$
$\sigma = 10^{-2}$	
	$z = 10^3$
$\sigma = 10^{-1}$	
	$z = 2\times10^3$
$\sigma = 10^{-2}$	

D	ΔD	σ	ΔS	S
10^3	10^3	10^{-2}	10	10
2×10^3	10^3	10^{-1}	100	110
5×10^3	3×10^3	10^{-2}	30	140
10^4	5×10^4	10^{-2}	50	190

FIG4



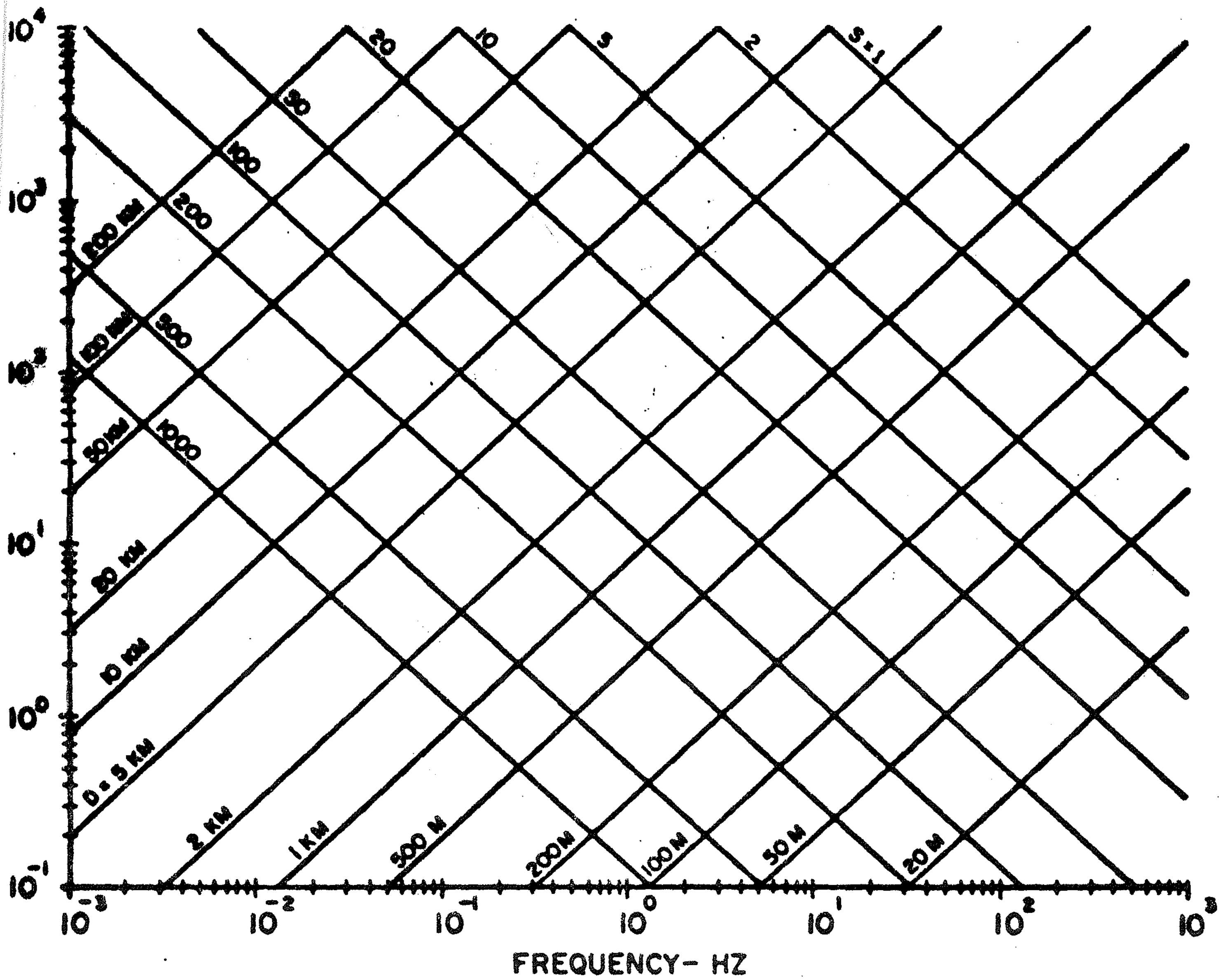
The Inverse Problem

$$S = \int_0^D \sigma \, dz, \quad \rho_A = \omega \mu D^2, \quad \rho_A = \frac{1}{\omega \mu S^2}$$

$$\frac{dS}{dD} = \sigma, \qquad \sigma = \frac{\frac{dS}{d\omega}}{\frac{dD}{d\omega}}$$

$$\sigma = \frac{1}{\rho_A} \, \frac{1 + \frac{d \log \rho_A}{d \log \omega}}{1 - \frac{d \log \rho_A}{d \log \omega}}$$

FIG 5



The Hilbert Transform
Dominant Operation

$$\phi = \frac{\pi}{4} + \frac{\pi}{4}\frac{d \log \rho_A}{d \log \omega}$$

$$\frac{d \log \rho_A}{d \log \omega} = \frac{4}{\pi}\phi - 1$$

$$\log \rho_A = \frac{4}{\pi}\int \phi \, d\log \omega + C$$

FIG 6