their paper) whose properties are not known to me. To summarize, the interpretation procedures developed by Mohan et al. (1982) for analyses of the magnetic fields is not based on the Hilbert Transform.

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B. N. P. Agarwal

Reply by the author to B. N. P. Agarwal

N. L. Mohan feels that he adequately addressed points raised earlier by Nagendra and Babu regarding the same

Discussion

On: "The magnetotelluric method in the exploration of sedimentary basins," by K. Vozoff (February 1972 GEOPHYSICS, 37, pages 98-141)

The paper by Vozoff (1972) is largely responsible for the variety of applications of the magnetotelluric method that we are witnessing today. The paper discusses field procedures, data processing techniques, modeling results, and, for the first time, important definitions and properties of basic magnetotelluric quantities. One of these definitions concerns the transfer function that relates the vertical magnetic field H_z to the horizontal components H_x and H_y. The general relationship between these components at any given frequency can be written as

$$H_z = AH_x + BH_y. \tag{1}$$

The complex numbers A and B represent the transfer function of the earth. A third complex number $T=(A, B)$ is called the "Tipper," and its magnitude is defined by Vozoff (1972) as

$$|T| = \left(|A|^2 + |B|^2\right)^{\frac{1}{2}}. \tag{2}$$

The original definition of the phase of T given by Vozoff (1972) was later corrected by Jupp and Vozoff (1976) and is given as

$$\phi_T = \frac{|A|^2 \phi_a + |B|^2 \phi_b}{|A|^2 + |B|^2}, \tag{3}$$

where ϕ_a and ϕ_b represent the phases of A and B, respectively. All phases, ϕ_a, ϕ_b, and ϕ_T, are restricted to the domain (-90°, 90°).

The above definitions are currently considered invariants under rotation (e.g., Vozoff, 1991). The subject of our discussion concerns the definition of the phase ϕ_T in relation to this property. Our experience with equation (3) is that the phase varies with the rotation angle. Similar experiences are reported by users of commercial software (Geotools, personal communication). There is the feeling that the phase should be invariant and that something must be wrong with the computer codes. We considered that the matter deserved a closer examination, given the importance of invariants when one is interested in eliminating the directional characteristics of the measurements. This is currently accomplished in the case of the impedance tensor by using its determinant (Berdichevski and Dmitriev, 1976).

The matter of the invariance of (3) can be settled by considering that A and B transform to new complex numbers A' and B', when the cartesian coordinate system rotates an angle θ with respect to the original system. The new complex numbers are given as

$$A' = A\cos\theta + B\sin\theta \tag{4}$$

and

$$B' = -A\sin\theta + B\cos\theta. \tag{5}$$

It is a simple matter to show that $|T|$ is invariant under rotation. To do this, simply substitute A' and B' into equation (2) for A and B. The result is again $|A|^2+|B|^2$. Thus the magnitude as defined by equation (2) is invariant under rotation. The proof is very simple considering that $A=(a_r, a_i)$ $B=(b_r, b_i)$, $A'=(a'_r, a'_i)$ and $B'=(b'_r, b'_i)$.

We did the same exercise with equation (3) for the phase. The new formula is a lot more complicated and does not seem to reduce to the unrotated case. We gave up after a few attempts. We decided to take a different approach. Instead of trying to demonstrate that equation (3) is invariant, we decided to demonstrate that it is not. It was sufficient to find a simple and clear example for which the phase changes with the rotation angle, but keeping away from computers.

Consider the following case:

$$A = \left(\sqrt{3}, 3\right)$$

and

$$B = \left(\sqrt{3}, 1\right).$$

From these we obtain $|A|^2$, $|B|^2$, ϕ_a and ϕ_b. These are

$$|A|^2 = 12,$$

$$|B|^2 = 4,$$

$$\phi_a = 60^{\circ},$$

and

$$\phi_b = 30^{\circ}.$$

Applying equation (3) we find ϕ_T=52.5°.

Let us now compute ϕ_T for the rotated case. Choose θ=45° in equations (4) and (5). The new complex numbers are

$$A' = \left(\sqrt{6}, 2\sqrt{2}\right)$$

and

$$B' = \left(0, -\sqrt{2}\right).$$

The new magnitudes and phases are

$$|A'|^2 = 14,$$

$$|B'|^2 = 2,$$

$$\phi_{a'} = tan^{-1}\frac{2\sqrt{3}}{3} \approx 49^{\circ}$$

and

$$\phi_{b'} = -90^{\circ}$$

Applying equation (3) we find that ϕ_T=31.7°, which is different from the unrotated value of 52.5°. This example shows that the phase ϕ_T as given by equation (3) is not invariant under rotation.

An invariant definition for the phase is

$$\phi_T = tan^{-1}\left(\frac{a_i^2 + b_i^2}{a_r^2 + b_r^2}\right)^{\frac{1}{2}}. \qquad (6)$$

A formula similar to this, with the numerator and denominator interchanged was originally proposed by Vozoff (1972). However, in the later communication (Jupp and Vozoff, 1976) the formula was discarded and replaced by equation (3), which is the currently accepted version (e.g., Vozoff, 1991).

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Francisco J. Esparza and Enrique Gómez-Treviño

Reply by the authors to F. J. Esparza and E. Gómez-Treviño

Despite the time that has passed since the original short discussion, I think some useful points can be made regarding the note by Esparza and Gómez-Treviño.

First, the authors are quite correct to point out that (3) of their note is not a rotation invariant definition of phase as was claimed in the original discussion. This slip most likely carried into later texts unchallenged. The fact is, however, that (3) is rotation invariant for a 2-D earth. It was in this context that the change was made to the definition in Vozoff (1971). The reason for the change was as follows:

As noted in all the discussions, during MT processing, the vertical magnetic field vector (H_2) is modeled as:

$$H_z=AH_z+BH_y$$

The (complex) model may not fit very well and the processing should report when it does not. If the fit is good enough, it is also the case that for a 2-D earth there is a rotation of (x, y) coordinates such that:

$$H_z=A'H_{z'}$$

When the earth is not 2-D but has a significant regional strike that can be approximated as a 2-D earth the method is used to estimate that strike by rotation that in some sense makes $|B'|$ small.

The real issue of the 1986 discussion was the estimation of this rotation angle in the field collected data situation and the way non2-D effects alter the estimates. To resolve the (interesting) regional estimate of the rotation and the "Tipper Direction" unambiguously in a way that, for a vertical contact, the tipping vector will be pointing downward from the conductive to the resistive side of the contact, we used the phase of A' which was denoted δ.

Unfortunately, the Tipper phase in the original paper by Vozoff (1972) was not unambiguous (despite being rotation invariant) as even for a truly 2-D earth it is actually:

$$\phi_r=tan^{1}|tan\delta|\cdot$$

To obtain an estimate that could be resolved in (-90, 90) rather than (0, 90) we changed the definition to (3) in the Esparza and Gómez-Treviño discussion which, for a 2-D earth is equal to δ and is rotation invariant.

The fact that it is not rotation invariant for arbitrary complex numbers A and B was not considered to be as crucial as the sign resolution but in the 1976 Discussion it was mistakenly claimed that this was so. If someone can find an expression that unambiguously resolves δ in the case of a 2-D earth and is also rotation invariant for arbitrary complex A and B, it would be better — and highly meritorious. In the meantime, a possible improvement to the situation would be to evaluate the phase estimate of Jupp and Vozoff (1976) {equation (3) in the Esparza and Gómez-Treviño discussion} *after* tensor rotation. That is, when aligned as near as possible to a 2-D structure. The rotation invariant phase of Vozoff (1972) with the correction noted by Esparza and Gómez-Treviño could also be usefully calculated.

David L. B. Jupp and Keeva Vozoff