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REMOVAL OF LOCAL SURFACE CONDUCTIVITY EFFECTS FROM LOW FREQUENCY MANTLE RESPONSE CURVES

J. C. LARSEN

JOINT TSUNAMI RESEARCH EFFORT PACIFIC MARINE ENVIRONMENTAL LABORATORY
ENVIRONMENTAL RESEARCH LABORATORIES, NOAA
HONOLULU, HAWAII

The effect of the surface conductivity is a static one, i.e. frequency independent with zero phase. On this basis the measured impedance tensor components can be split— by knowledge of a factor which can be deduced from geomagnetic deep soundings — into a part corresponding to the surface conductivity and into the mantle response.

The conductivity model treated here consists of a thin, two-dimensional inhomogeneous conducting surface layer insulated from a deeper layered conducting mantle. If we limit the study to low frequencies for which the response curve lies within the *h*-interval then the surface conductivity structure has an *S*-effect that is frequency independent with zero phase, that is, it is a static effect having the following simple description.

Let the complex *Fourier* transforms of the observed horizontal electric and magnetic field components be, respectively, E_x, E_y, B_x, and B_y with x north and y east. Further let $\bar{I}_x$ and $\bar{I}_y$ be the vertically integrated electric current induced in the homogeneous portion of the surface layer. The distortion of the horizontal electric field by the surface conductivity structure is describable by

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix} \begin{pmatrix} \bar{I}_x \\ \bar{I}_y \end{pmatrix} \tag{1}$$

and the distortion of the horizontal magnetic field is describable by

$$\begin{pmatrix} B_x - \bar{B}_x \\ B_y - \bar{B}_y \end{pmatrix} = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \begin{pmatrix} \bar{I}_x \\ \bar{I}_y \end{pmatrix} \tag{2}$$

where the a's and b's, although functions of location, are real and frequency independent for our low frequencies.

Let $\bar{S}$ be the vertically integrated conductivity of the surface layer in the homogeneous region, then

$$\begin{pmatrix} \bar{I}_x/\bar{S} \\ \bar{I}_y/\bar{S} \end{pmatrix} = \begin{pmatrix} 0 & Z \\ -Z & 0 \end{pmatrix} \begin{pmatrix} \bar{B}_x \\ \bar{B}_y \end{pmatrix} \tag{3}$$

where we seek the Z that is the response function at the top of the surface layer If B_x and B_y are observed in a region where the surface conductivity is not too rapidly varying, we can simplify equations (2) and (3) to

$$\begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} 1 - \mu h\bar{S}Z & 0 \\ 0 & 1 - \mu h\bar{S}Z \end{pmatrix} \begin{pmatrix} \bar{B}_x \\ \bar{B}_y \end{pmatrix} \tag{4}$$

where $0 < h < 1$. For a site at the center of a nonconducting island $h = 0.5$ Finally (1) can be rewritten to

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = Z_D \begin{pmatrix} A + C & B - 1 \\ B + 1 & A - C \end{pmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix} \tag{5}$$

where $Z_D = (Z/D)(1 - \mu h\bar{S}Z)^{-1}$ is a complex function of frequency and A, B, C, and D are real and frequency independent.

The thin sheet approximation shows that the response Z_D is the response just beneath a homogeneous surface layer of vertically integrated conductivity $h\bar{S}$. Thus if Z is given by a layer structure then Z_D will be directly invertable to a layered structure.

The response functions determined from the observed fields

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \begin{pmatrix} B_x \\ B_y \end{pmatrix} \tag{6}$$

can be simplified by the transformation $Z_1 = (Z_{xx} + Z_{yy})/2$, $Z_2 = (Z_{xy} + Z_{yx})/2$, $Z_3 = (Z_{yx} - Z_{xy})/2$, and $Z_4 = (Z_{xx} - Z_{yy})/2$. Then the surface parameters A, B, and C are related to the observables by

$$Z_1 = AZ_D,\ Z_2 = BZ_D,\ Z_3 = Z_D,\ Z_4 = CZ_D\,. \tag{7}$$

Applying equation (7) to the data, the surface parameters and Z_D, determined by least squares fit are $A = G\ \langle Z_1 Z_3^* \rangle$, $B = G\langle Z_2 Z_3^* \rangle$, $C = G\langle Z_4 Z_3^* \rangle$ for $G = \langle Z_3 Z_3^* \rangle^{-1}$ and $Z_D = (AZ_1 + BZ_2 + Z_3 + CZ_4)(1 + A^2 + B^2 + C^2)^{-1}$. Thus we have been able to eliminate the surface effect to within the factor D that will not be unity except for special surface conductivity structure. However, Z_D yields a conductivity profile, if logarithm of the conductivity σ is plotted versus the logarithm of depth z, that has a shape independent of D for the following reasons.

Our response function Z zatisfies the *Riccatti* equation

$$dZ/dz + \mu\sigma Z^2 + i\omega = 0 \tag{8}$$

for harmonic time dependence exp $(-i\omega t)$. One can show that (8) is invariant under the transformation $Z_D = Z/D$ provided $z_D = z/D$ and $\sigma_D = \sigma D^2$. Hence the profile log z_D vs. log σ_D is a simple translation of the profile log z vs. log σ. Also the ratio of the skin depth $(2/\mu\omega\sigma)^{1/2}$ over the layer thickness will be independent of D.

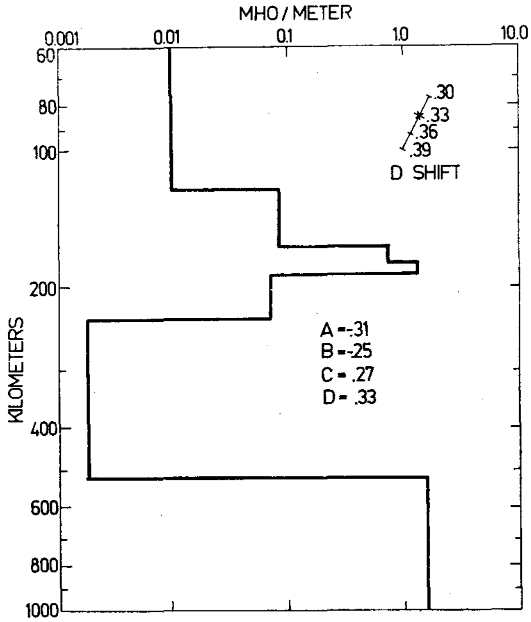

Fig. 1. Conductivity profile beneath Tucson, Arizona, USA

The value of D is probably best determined from a study of the extra low frequency response function determined by comparing the vertical and horizontal magnetic field. There is no surface disturbance of the magnetic field (see Eq. (4)) for low frequencies such that $\mu h\bar{S}Z \ll 1$. A preliminary estimate of the conductivity profile at Tucson, Arizona, from 11 years of hourly magnetic and electric field data, is given in Fig. 1. The magnetotelluric data show that the mantle response is describable by a horizontally layered mantle for the frequency range $0.035 < f < 6.0$ cpd. The magnetotelluric response functions are well determined because the coherence square exceeds 0.9. The vertical magnetic data for the frequency range $0.035 < f < 0.505$ cpd has coherence squared near 0.6 and the response functions are less accurately determined. However, the vertical magnetic data are adequate to determine D that is found to lie within the limits $0.30 < D < 0.39$. In the figure changing D from 0.33 means translating the profile parallel to the D shift line.

УСТРАНЕНИЕ ВЛИЯНИЯ ПОVЕРХНОСТНОЙ ПРОВОДИМОСТИ ИЗ МАЛОЧАСТОТНЫХ ОТВЕТНЫХ ФУНКЦИЙ МАНТИИ

И. С. ЛАРСЕН

РЕЗЮМЕ

Влияние распределения поверхностной проводимости статическое, т. е. независимо от частоты и имеет нулевую фазу. Исходя из этого, компонент тензора измеренных импедансов автор может отделить — при известной величине D, выводимой на основе геомагнитного зондирования — влияние поверхностной проводимости от ответной функции мантии.