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# WALDIM: A code for the dimensionality analysis of magnetotelluric data using the rotational invariants of the magnetotelluric tensor $\overset{\scriptscriptstyle \times}{}$

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# ABSTRACT

In the magnetotelluric (MT) method, the analysis of geoelectric dimensionality has acquired special importance in the last years, because numerical codes have made it possible to model and invert data using either one-dimensional (1D), two-dimensional (2D) or three-dimensional (3D) approaches. We present a FORTRAN code termed WALDIM to perform the dimensionality analysis of a set of MT data, according to the WAL rotational invariants criteria. These criteria are based on the possible annulment of the invariants of the MT tensor, which allow retrieval of as much information as possible from this tensor, without taking any a priori dimensionality assumption. When determining the dimensionality of real and therefore noisy data, two problems arise. The first is due to the data errors, which propagate into the invariants values, and therefore, to the determination of the dimensionality. The second is the fact that the invariants are rarely precisely zero, and the definition of a threshold is necessary. To solve these problems, WALDIM takes into account the data errors. Additionally, the dimensionality results can be grouped into frequency bands. Thus, we provide a software utility that allows providing a robust description of the dimensionality, and the parameters necessary for data correction prior to modeling. Given its completeness at analyzing the MT tensor for both individual and bands of frequencies, this code is meant to be a practical tool for MT data analysis.

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#### 1. Introduction

The magnetotelluric method (MT) uses natural electromagnetic energy to image the electrical resistivity distribution of the Earth,  $\rho(x, y, z)$ , based on the simultaneous measurement of the total electromagnetic field time variations at the Earth's surface (e.g. Simpson and Bahr, 2005). In the frequency domain  $(\omega)$ , the horizontal electric  $E(\omega)$  and magnetic  $H(\omega)$  fields can be related through the complex  $2 \times 2$  impedance tensor  $Z(\omega)$  (ohm):

$$\begin{pmatrix} E_{x}(\omega) \\ E_{y}(\omega) \end{pmatrix} = \begin{pmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{pmatrix} \cdot \begin{pmatrix} H_{x}(\omega) \\ H_{y}(\omega) \end{pmatrix}.$$
 (1)

Analogously, the magnetotelluric tensor (MT tensor),  $M(\omega)$  (m/s), is defined as the relationship between the electric field  $\mathbf{E}(\omega)$  and the magnetic induction  $\mathbf{B}(\omega)$ :  $M(\omega) = (1/\mu_0)Z(\omega)$ .

Assuming that the electromagnetic field source is a plane wave that propagates diffusively into the Earth, the MT tensor is independent of the natural source. Hence, at each frequency it only contains information of geoelectrical structure. Given that low frequency signals (or long period,  $T = 2\pi/\omega$ ) can propagate deeper than the high ones, the measuring frequency determines the investigation depth, related to the skin depth ( $\delta \approx 500(\rho T)^{1/2}$ ).

From each complex component of the MT tensor, the apparent resistivity and phase can be computed. These quantities provide information on the average resistivity and its variations, and can be used to characterize the geoelectrical conductivity  $\rho(x, y, z)$ . The character of the spatial distribution of the conductivity is known as geoelectric dimensionality, which can be described as 1D, 2D or 3D.

The MT tensor and, particularly, the relationships between its components, are reduced to specific expressions dependent on the dimensionality of the geoelectric structures being imaged.

In 1D (e.g. stratified medium), the MT tensor is a non-diagonal tensor (diagonal elements = 0) with its two components equal in modulus but with opposite signs:

$$M_{1D}(\omega) = \begin{pmatrix} 0 & M(\omega) \\ -M(\omega) & 0 \end{pmatrix}.$$
 (2)

In 2D, when one of the measurement axes coincides with the direction of the geoelectrical strike, the MT tensor is non-diagonal,

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with different values of the two components:

$$M_{2D}(\omega) = \begin{pmatrix} 0 & M_{xy}(\omega) \\ M_{yx}(\omega) & 0 \end{pmatrix},$$
(3)

whereas if measured along other directions,  $\theta$ , which is the general case, the measured MT tensor  $M'(\omega)$  has the general expression (1) but can be decomposed as

$$M'(\omega) = R_{\theta} \cdot M_{2D}(\omega) \cdot R_{\theta}^{I}, \tag{4}$$

where  $R_{\theta}$  and  $R_{\theta}^{T}$  are a clockwise rotation matrix and its transpose.

In 3D cases, the MT tensor takes the general form (1) and, in general, there is no orientation for which the diagonal components can vanish.

Other particular expressions of the MT tensor can be obtained when data are affected by galvanic distortion of the electric field, a phenomenon caused by local 3D inhomogeneities near the Earth's surface (Kaufman, 1988). Inductive effects due to small nearsurface structures rapidly decay with increasing period and will not be considered in this work (West and Edwards, 1985). Mathematically, galvanic distortion can be described as a  $2 \times 2$ real frequency independent matrix, *C*. Groom and Bailey (1989) describe this matrix as the contribution of four effects, represented by the gain (*g*) parameter, which accounts for the static shift, and the twist ( $\varphi_t$ ), shear ( $\varphi_e$ ) and anisotropy ( $\varphi_s$ ) angles or their tangents (*t*, *e* and *s* respectively):

$$C = g \cdot \begin{pmatrix} (1+s)(1-te) & (1-s)(e-t) \\ (1+s)(e+t) & (1-s)(1+te) \end{pmatrix}.$$
 (5)

The measured tensor is then expressed as

$$M_m(\omega) = R_\alpha \cdot C \cdot M_R(\omega) \cdot R_\alpha^1, \tag{6}$$

where  $M_{R}(\omega)$  is the tensor corresponding to the regional structure and  $\alpha$  is the measurement angle with respect to the regional reference frame. Depending on the type of regional structure affected by galvanic distortion, 1D, 2D or 3D, the dimensionality is referred to as 3D/1D, 3D/2D or 3D/3D, respectively.

Dimensionality analysis of the MT data is a necessary step that determines which type of approach is more adequate to perform the modeling, inversion or interpretation: 1D, 2D or 3D. At the same time, it provides information such as variation of the geoelectrical strike direction with depth, which can be correlated with different processes and structure of the subsurface. Moreover, the dimensionality analysis should be a tool to determine whether data are affected or not by galvanic distortion, so data can be appropriately corrected (e.g. Groom and Bailey, 1989; Smith, 1995).

Nowadays, common methods used to perform dimensionality analysis/distortion correction are Bahr parametrization (Bahr, 1988, 1991), Groom and Bailey decomposition (Groom and Bailey, 1989), applied as the Strike Decomposition code (McNeice and Jones, 2001) and, more recently, the Phase Tensor (Caldwell et al., 2004). Nevertheless, these methods present some limitations: Bahr criteria are based on the values and relationships of 4 invariants; however, they have not always been applied adequately and the method may fail in some specific dimensionality cases (as shown in Ledo et al., 2002 and Martí et al., 2005). Strike Decomposition code assumes a type of dimensionality (2D or 3D/2D), examines how well the data fit this model, and extracts the best fitting regional 1D or 2D impedances. Hence, it is a good tool when the regional structure is 2D (or one wants to assess the hypothesis that it is 2D). The Phase Tensor is defined as the relationship between the real and imaginary parts of the MT tensor. Not affected by galvanic distortion, the Phase tensor is a practical tool to easily obtain information about the dimensionality of the regional structure. However, it is not possible to correct data and recover the regional responses.

Weaver et al. (2000), based on Fischer and Masero (1994) and Szarka and Menvielle (1997), presented a dimensionality study based on the rotational invariants of the MT tensor, i.e., sets of scalars computed from the observed MT tensor that do not depend on the orientation of the axes considered.

The set of eight invariants (WAL hereafter), seven independent  $(I_1, I_2, I_3, I_4, I_5, I_6, I_7)$  and a dependent one (Q), was defined in a way that the invariants can be represented in a Mohr circle diagram, with the exception of two  $(I_1 \text{ and } I_2)$  they are non-dimensional and normalized to unity, and their vanishing has a physical interpretation, specifically related to the geoelectric dimensionality.

Invariants  $I_1$  and  $I_2$  serve to normalize the remaining invariants, and provide information on the 1D magnitude and phase of the geoelectrical resistivity.

Invariants  $I_3$  to  $I_7$  and Q make it possible to establish criteria (Weaver et al., 2000; Weaver, pers. comm.) that determine the type of dimensionality and identify galvanic distortion (Table 1). The use of these invariants provides some of the same features of the Strike Decomposition code and the Phase Tensor, by providing both a full dimensionality analysis and the parameters necessary to correct galvanic distortion. For this reason, we consider this set of invariants as the most suitable to perform the dimensionality analysis of MT data. However, some considerations must be taken when dealing with real data.

## 2. WAL dimensionality criteria using real data

The main problem when WAL invariants criteria are implemented on real, therefore noisy data is that the invariants are

Table 1

Dimensionality criteria according to the WAL invariants values of the magnetotelluric tensor (modified from Weaver et al., 2000).

Case	I <sub>3</sub> to I <sub>7</sub> and Q values	Geoelectric dimensionality
1	$I_3 = I_4 = I_5 = I_6 = 0$	<b>1D</b> $\rho_{\rm 1D} = \mu_0((l_1^2 + l_2^2)/\omega), \ \varphi_{\rm 1D} = \arctan(l_2/l_1)$
2	$I_3 \neq 0$ or $I_4 \neq 0$ ; $I_5 = I_6 = 0$ ; $I_7 = 0$ or $Q = 0$ ( $\xi_4 \neq 0$ and $\eta_4 \neq 0$ )	$p_{1D} = \mu_0((1_1, 1_2), 0_2), \ \phi_{1D} = \arctan(1_2/1_1)$ <b>2D</b>
3a	$I_3 \neq 0$ or $I_4 \neq 0$ ; $I_5 \neq 0$ ; $I_6 = 0$ ; $I_7 = 0$	3D/2D twist
3b	$I_3 \neq 0$ or $I_4 \neq 0$ ; $I_5 \neq 0$ ; $I_6 = 0$ ; $Q = 0$	2D affected by galvanic distortion (only twist) <b>3D/1D 2D</b>
3c	$I_3 \neq 0$ or $I_4 \neq 0$ ; $I_5 = I_6 = 0$ ; $I_7 = 0$ or $Q = 0$ ( $\xi_4 = 0$ and $\eta_4 = 0$ )	Galvanic distortion over a 1D or 2D structure (non-recoverable strike direction) <b>3D/1D 2D diag</b>
4	$l_3 \neq 0$ or $l_4 \neq 0$ ; $l_5 \neq 0$ ; $l_6 \neq 0$ ; $l_7 = 0$	Galvanic distortion over a 1D or 2D structure resulting in a diagonal MT tensor <b>3D/2D</b>
-	13/ 0 01 14/ 0, 13/ 0, 16/ 0, 17 = 0	General case of galvanic distortion over a 2D structure
5	$I_7 \neq 0$	<b>3D</b> (affected or not by galvanic distortion)

rarely precisely zero. Thus, the geoelectric dimensionality of noisy data may be found to be 3D, although other evidences (such as relatively small magnitude of diagonal impedance components or single preferred strike direction among different sites and periods) may suggest that 1D or 2D interpretation would be valid for modeling. Weaver et al. (2000) addressed this problem by introducing a threshold value, beneath which the invariants are taken to be zero.

The threshold value ( $\tau$ ) suggested was 0.1, which, although subjective, was tested using a synthetic model with 2% noise. Since experimental data usually have a higher percentage of error which propagates to the invariants, it was necessary to redefine this threshold value, taking into consideration the invariant values and their errors.

Using WAL criteria with the threshold defined, if the dimensionality obtained is 2D or 3D/2D (cases 2, 3a and 4 in Table 1), the strike directions and distortion parameters (obtained from distortion matrix *C*) must also be estimated with their errors.

The estimation of the errors of the invariants and related parameters and the choice of the threshold were addressed by Martí et al. (2004) and Martí (2006). After performing several tests on the error treatment and threshold value, the following recommendations were established:

- (1) Determine the dimensionality using WAL criteria for a range of threshold values between  $\tau = 0.1$  and  $\tau = 0.2$  for  $I_3$ – $I_7$  and  $\tau_Q = 0.1$  for invariant Q; and considering the invariant errors (computed using classical error propagation). With these threshold values, dimensionality will be well determined when relative errors in the off-diagonal components of *M* are not greater than approximately 30%. For any particular dataset, the maximum acceptable threshold would be a value for which 2D cases present strike directions with acceptable errors (see e.g. Martí et al., 2004).
- (2) Compute the strike directions and/or distortion angles corresponding to 2D and 3D/2D cases and their errors, using a Monte Carlo approach with addition of Gaussian noise in the data.

#### 3. WALDIM code

A FORTRAN application, named WALDIM, was created. It is a tool to perform the dimensionality analysis from a set of raw or synthetic MT data, based on WAL criteria and considering noise in the data. The program can be compiled using GNU f77/g77 and g95 under Cygwin on a Windows XP PC; f77 and f90 (Compaq Visual Fortran Optimizing Compiler v6.5) under Windows XP on a PC; and F77 and F95 (GNU Fortran 95 compiler) on a Linux Workstation (HP xw9400); and f77 and f90 under Unix on a Sun Solaris Server.

The main functions of this program are to compute WAL invariants corresponding to each MT tensor and to determine the dimensionality, following WAL criteria, according to the errors and a threshold value. The estimation of error on the invariants is done using classical error propagation (Martí et al., 2004) and the errors of the related parameters (strike direction and distortion parameters), using addition of Random Gaussian Noise (Function Gasdev, Press et al., 1992). In 1D cases, the resistivity and phase ( $\rho_{1D}$  and  $\varphi_{1D}$ , see Table 1) with their errors are computed. In cases related to two-dimensional structures and/or the presence of distortion, the strike and distortion parameters with their errors are also provided as relevant information (Weaver et al., 2000). In 2D cases, the strike angle is computed from the real and imaginary parts of the tensor ( $\theta_1$  and  $\theta_2$  respectively), which in an ideal 2D should have the same value ( $\theta_1 = \theta_2$ ). In 3D/2D cases, the

recovered parameters are the strike angle (computed considering both real and imaginary parts and referred to as  $\theta_3$ ) and the distortion parameters  $\phi_1$  and  $\phi_2$  (Smith, 1995), which are linear combinations of the Groom and Bailey (1989) angles, twist and shear:

$$\phi_1 = \frac{\varphi_t + \varphi_e}{2},\tag{7}$$

$$\phi_2 = \frac{\varphi_t - \varphi_e}{2}.\tag{8}$$

In 3D cases, invariant  $I_7$  is provided as an additional indicator, which can be useful to assess the strength of such 3D effects on the data.

WALDIM also allows classifying the dimensionality into bands of periods for each site, in order to have a more stable estimate of the dimensionality therein. In this classification, a specific period range is divided into bands made up of multiples or fractions of the decades contained within this range. For each site, the dimensionality of a band is the mathematical mode (i.e., the most occurring dimensionality response) of the data in the band. In the case that the mode has more than one dimensionality type, priority is given to the lowest dimensionality. If the strike and distortion parameters are to be computed, these and their errors are obtained as the arithmetic average and the corresponding classical error propagation of the data with that type of dimensionality. This method of averaging over period bands departs from that used in the Strike Decomposition code, with the main difference that, whereas Strike provides least-squares fit to data from a given period band, WALDIM averages the individual results within the band.

The program also solves some inconsistencies that can appear regarding two-dimensionality and strike directions: in some cases, WAL criteria indicate 2D dimensionality, whereas  $\theta_1$  and  $\theta_2$  strike angles have significantly different values. This is a consequence of having defined a threshold value lower than required. The program solves this by changing the dimensionality of these cases into 3D/2D and assigning  $\theta_3$ ,  $\phi_1$  and  $\phi_2$  as the strike direction and distortion parameters respectively. The difference between  $\theta_1$  and  $\theta_2$  is set to a maximum of 10°, or a minimum of 80°, given that one of the angles can have a 90° ambiguity.

The FORTRAN sources for the program WALDIM are the main code in WALDIM.f, the complementary code in inoutdata.f, which reads the inputs and generates the outputs; and the code in external.f, which contains the functions called from the main code. The main inputs for the program are standard EDI files (Wight, 1988) containing impedances.

Fig. 1 schematizes the general flowchart of the program. A detailed description of the files and parameters involved in the program can be found in the Appendix.

#### 4. Test examples

In order to illustrate the application of the WALDIM software we present two examples. The first is from a synthetic dataset from the model used in Weaver et al. (2000), which represents the main types of dimensionality, and for which we have tested the threshold values and different noise levels to prove their validity range and the overall robustness of the code. The second example is from site 85\_314 of the COPROD2 (Jones, 1993), which is a dataset known by the MT community, for which we retrieve the dimensionality description taking into account the data errors.

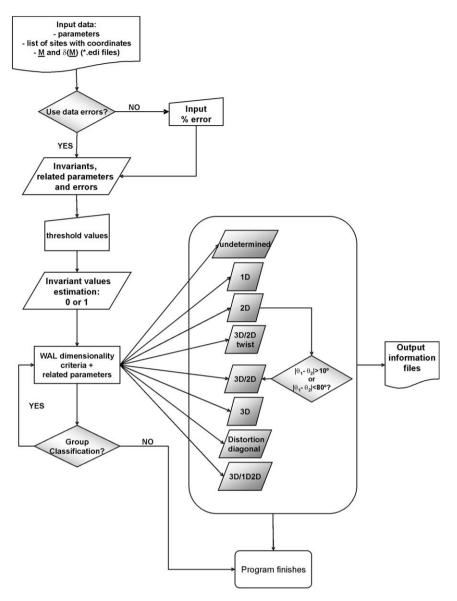


Fig. 1. General overview flowchart of WALDIM code.

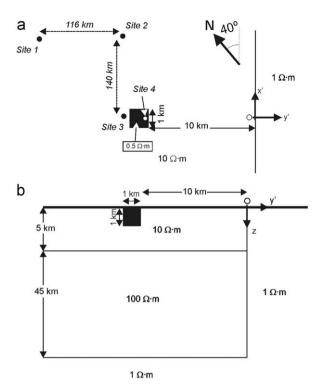
#### 4.1. Example 1

The data comprise seven magnetotelluric tensors (**A-G**) selected from the synthetic model used in Weaver et al. (2000). This model is oriented 40°E from the North and consists of a cubic conductive anomaly (0.5  $\Omega$ m) embedded on the surface of an otherwise 2D structure, formed by a vertical fault that separates a layered medium of 10, 100 and 1 $\Omega$ m from a homogeneous medium of 1 $\Omega$ m (Fig. 2). The magnetotelluric tensors were computed at four sites at different periods. One of these tensors (**E**) was computed after having rotated the electric field 10°, as to simulate a misalignment of the electrodes. These seven tensors exemplify the different types of dimensionality that can be identified using the WAL criteria (Table 2).

WALDIM was first run without considering noise in the data, and the same pattern of dimensionality and values of the parameters as in Table 2 were obtained, for threshold values ranging from  $\tau = 0.03$  to  $\tau = 0.10$ . For thresholds between  $\tau = 0.10$ to  $\tau = 0.20$ , dimensionality pattern of tensor **F**, theoretically 3D/2D, should change according to WAL criteria because invariant  $I_6 = 0.142$  would be below the threshold, and it would be considered 2D instead. However, the difference between  $\theta_1$  and  $\theta_2$  is higher than 10° ( $\theta_1 = 67.8^\circ$  and  $\theta_2 = 2.2^\circ$ ), hence WALDIM changes the dimensionality case to 3D/2D, with  $\theta_3 = 42.2^\circ$ ,  $\varphi_{\text{twist}} \approx 0^\circ$  and  $\varphi_{\text{shear}} = -20^\circ$ . For threshold values higher than  $\tau = 0.20$ , the program leads to a wrong description of the dimensionality cases, which may be far too simple for most of the tensors. For invariant *Q*, a threshold of  $\tau_Q = 0.10$  is valid, although, given that for this example *Q* values are either lower than 0.03 or higher than 0.3, a broader range of  $\tau_Q$  would be valid as well.

The code was also run considering errors in the data, after applying Gaussian noise to the components of the magnetotelluric tensors, in order to prove the limitations in the range of thresholds at different noise levels. Error levels of 5%, 10%, 20% and 30% were used, with both thresholds  $\tau$  and  $\tau_Q$  ranging between 0.05 and 0.30.

The dimensionality patterns retrieved from the different combinations of errors and thresholds were identified as "right dimensionality", if the dimensionality pattern was the same as in Table 2, "wrong dimensionality", if at least the dimensionality of one of the tensors differed from the expected, and "right



**Fig. 2.** Synthetic model from Weaver et al. (2000). (a) Plane view at z = 0, (b) vertical cross section at x' = 0.

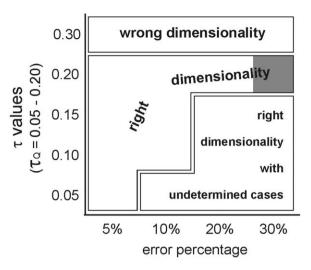
Table 2Dimensionality obtained for tensors A–G (example 1), according to Weaver et al.(2000) criteria.

Tenso	or Site, period	Dimensionality
А	Site <b>1</b> , 100 s	1D $\rho_{1D} = 29.5 \Omega \mathrm{m},  \varphi_{1D} = 28^{\circ}$
В	Site <b>2</b> , 1000 s	2D, $\theta = 40^{\circ}$
С	Site <b>2</b> , 100 s	2D, $\theta = 40^{\circ}$
D	Site <b>3</b> , 1000 s	3D/1D2D
Е	Site <b>2</b> , 100 s (rotating <i>E</i> field	3D/2D (only twist)
	10°)	$ heta=40^\circ$ , $\phi_{ m twist}=-10^\circ$ , $arphi_{ m shear}pprox0^\circ$
F	Site <b>3</b> , 100 s	3D/2D
		$ heta=42.2^\circ$ , $arphi_{ m twist}=-0.1^\circ$ ,
		$arphi_{ m shear}=-20^\circ$
G	Site <b>4</b> , 1 s	3D

dimensionality with undetermined cases", if the dimensionality could not be depicted in at least one tensor (undetermined because of error bars of the invariants), but was right in the others.

Using  $\tau_Q = 0.30$ , the dimensionality pattern obtained was "wrong dimensionality" for any value of the threshold  $\tau$  and error level. Fig. 3 summarizes the results from the rest of the combinations, which are valid for any value of  $\tau_Q$  between 0.05 and 0.20, with only one exception (grey shadowed zone, only valid for  $\tau_Q = 0.20$ ). For low values of the threshold  $\tau$  and relatively high error levels, the dimensionality might become undetermined in some cases because of the error bars of the invariant values, but keep the right pattern in the rest of cases. Using a threshold value  $\tau = 0.30$ , the dimensionality pattern is wrong independently of the error level. The rest of cases allow retrieving the right dimensionality pattern, with the exception already mentioned, corresponding to an error percentage of 30%.

From this simple distribution, it can be concluded that threshold values between  $\tau = 0.05$  and  $\tau = 0.20$  allow retrieving



**Fig. 3.** Dimensionality pattern obtained for tensors A–G (Example 1), using different error levels (horizontal axis) and threshold values ( $\tau$ , vertical axis), valid for any value of  $\tau_Q$  between 0.05 and 0.20. Grey shadowed area indicates that dimensionality pattern is right only for  $\tau_Q = 0.20$ .

a correct (or at the most undetermined) dimensionality pattern, when error levels are lower than 30%.

#### 4.2. Example 2

The COPROD2 dataset was collected along a 400 km EW profile in southern Saskatchewan and Manitoba (Canada), at 49°N, and from 106°W to 100°W, crossing the thick Paleozoic sediments of the Williston basin. Within the basement beneath the sediments lies one of the world's longest and most enigmatic crustal conductivity features: the North American Central Plains (NACP) conductivity anomaly. At the eastern extreme of the profile there is a second basement anomaly (TOBE) interpreted as being associated with the Thompson Nickel Belt at the Superior– Churchill boundary (Jones and Craven, 1990).

Data have a wide bandwidth (2.6 ms to 1820 s) and are of high quality (errors in the MT tensor typically <2%) (Jones, 1993). These data were made available to the MT community and are commonly used to test and compare 2D inversion codes, as in general these have a 2D behavior.

Site 85\_314 is located in the central part of the COPROD2 profile, at the top of the NACP anomaly. Fig. 4 displays the MT tensor components, apparent resistivities and phases, with the corresponding error bars, computed at this site.

WAL dimensionality analysis was performed using data errors and the threshold values  $\tau = 0.15$  and  $\tau_0 = 0.10$ . Results were averaged in six decade bands (from 0.001 to 1000 s). Fig. 5 shows the dimensionality pattern for each period (squared symbols) and for the six bands. In the later, also the related strike and distortion angles, when appropriate, are presented. The dimensionality is mainly 1D for short periods up to 10s; and 3D/2D, mixed with some 3D cases, from 10 to 1000 s. The strike, distortion angles and the corresponding error bars for each period are displayed in Fig. 6, differentiated in two groups. In grey, the short periods, where dimensionality is 1D and hence the strike angle is meaningless, with large error bars and steep variations, and distortion parameters close to zero. In black, periods from 10 to 1000 s, where the strike angle changes smoothly around 80°, distortion angles are non-zero, and, with the exception of the longest periods, error bars are low (less than 3°), which agrees with the dimensionality determined. It is worth noting that the dimensionality at these long periods (3D/2D) is very close to 2D,

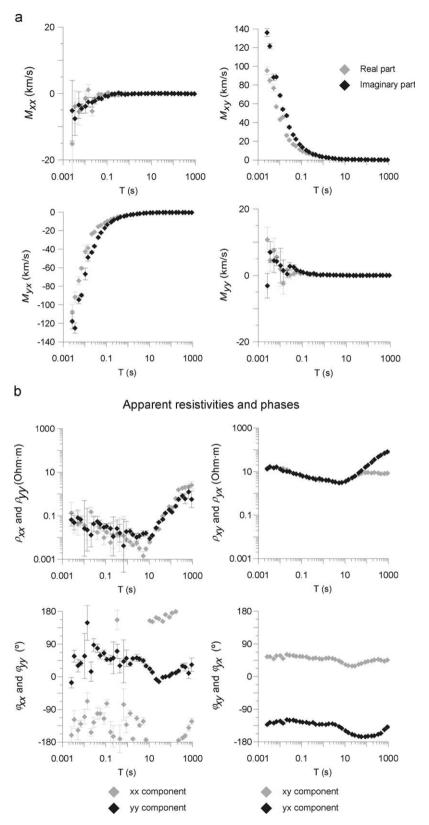


Fig. 4. (a) Magnetotelluric tensor components of site 85\_314 from COPROD2 dataset. (b) Apparent resistivity and phase computed from MT tensor components at this site.

as the low values of the distortion parameters show. However, the strike angle is better determined considering distortion than considering a pure 2D approach, for which angles computed from real and imaginary parts would not match.

Using cases with clearly differentiated types of dimensionality, these examples have illustrated how the WALDIM code can be a robust tool to analyze datasets with a more complex distribution of errors and dimensionalities.

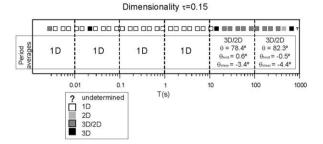
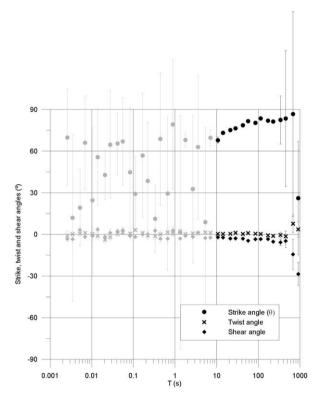


Fig. 5. Dimensionality cases for site 85\_314 from the COPROD2 dataset for each period and for data grouped in 6 decade bands.



**Fig. 6.** Strike, twist and shear angles determined for each period. In grey, periods up to 10 s, regarded as 1D. In black, periods regarded as 3D/2D or 3D.

## 5. Conclusions

A FORTRAN code, WALDIM, to perform the dimensionality analysis of MT data using Weaver et al. (2000) criteria, and considering data errors, has been presented. The code computes the type of dimensionality corresponding to each data site and frequency, and computes related parameters, such as strike direction and distortion parameters, with their errors. It is also possible to classify the results in bands of frequencies. Two data examples have been used to successfully test the program and to test the limits of the threshold values and the error level.

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#### 6. Appendix

Description of files and parameters: A: input parameters and options, B: input files, C: output parameters and dimensionality types, D: output files and E: screen outputs.

#### 6.1. Inputs

6.1.1. A. Input parameters (input files and program options): these can be read from the keyboard as they are asked at the beginning of the program, or can be read from the file **param.cfg** 

These input parameters are:

- File with EDI files names and coordinates (input file no. 1) [Default: list.dat].
- Units for impedances in EDI files (M = m/s; F = km/s ( = (mV/ km)/nT)(field units); Z = ohm) [Default: F (km/s) (the most usual)].
- Threshold value for Invariants *I*<sub>3</sub> to *I*<sub>7</sub>: τ. (Recommended value between 0.1 and 0.2) [default: 0.15]
- Threshold value for *Q*:  $\tau_Q$  (recommended value 0.1 or lower) [default: 0.1].
- EDI files contain errors (Y/N)? (Usually Y for raw data and N for synthetic data) [default: Y].

If **Y**: using errors in EDI files (**y**/**n**)? [default: Y]. If **n**: error percentage (%)? [default: 5].

If N: error percentage (%)? [default: 5].

Average in bands (**y**/**n**)?: [default: Y].

If y:

Minimum period to average? [Default: 0.001]. Maximum period to average? [Default: 10000].

- Number of bands per decade? [Default: 1].
- Root name for output files [Default: OUT].
- Writing file with all parameters and errors (Y/N)? [Default: Y]

#### 6.1.2. B. Input files (two files: list file+x edifiles, one for each site)

1. File containing a list of all sites with their coordinates, in the following format:

1st row: header information.

2nd row: number of sites (free format).

3rd and following rows: EDI file name (WITHOUT EXTENSION ".EDI"), latitude and longitude (both in decimal degrees or sexagesimal degrees).

<b>Example:</b> INFO 1		
site001	41:31:00	1:41:00

2. EDI files (Wight, 1988) containing impedances (as opposed to spectra): one file for each site (file names read from file **1**).

#### 6.2. Outputs

6.2.1. *C. List of output parameters and dimensionality types* (*displayed in output files listed in* **D**, *and*/ *or on the screen*)

 $I_1$ ,  $I_2$ ,  $I_3$ ,  $I_4$ ,  $I_5$ ,  $I_6$ ,  $I_7$ , Q (referred to in the program as  $I_8$ ). ( $I_1$ ,  $I_2$  in km/s; the rest are dimensionless).

- St<sub>1</sub>: 1D apparent resistivity (ohm m) computed from  $I_1$  and  $I_2$  assuming a 1D model.
- St<sub>2</sub>: 1D phase computed from  $I_1$  and  $I_2$  assuming a 1D model.

- $St_3$  and  $St_4$ : strike directions (degrees) corresponding to a 2D model.
- St<sub>5</sub>: strike directions (degrees) corresponding to a 3D/2D model.
- $St_6$  and  $St_7$ :  $\phi_1$  and  $\phi_2$  distortion angles (degrees) (linear combinations of twist and shear).

St<sub>8</sub> and St<sub>9</sub>: twist and shear angles (degrees).

errSt<sub>1</sub> to errSt<sub>9</sub>: errors of parameters St<sub>1</sub> to St<sub>9</sub>.

All strike directions are determined positive clockwise from the north, taking into account the rotation angle (ROT) read from EDI files.

Dimensionality types (DIM):

- **0**: UNDETERMINED,
- 1: 1D,
- **2**: 2D,
- 3: 3D/2D only twist,
- 4: 3D/2D general,
- **5**: 3D,
- **6**: 3D/2D with diagonal regional tensor
- **7**: 3D/2D or 3D/1D indistinguishable.

Statistical parameters (optional output, file OUT+\_STATS\_ + $\tau$ +.dat):

- True: Parameter value computed directly.
- Err: Parameter error using classical error propagation.
- Sta: Parameter value computed randomly.
- Dev: Parameter error computed statistically.
- Bias: Difference between true and sta, normalized.

If averaging in bands (files OUT+\_BAND 1\_+ $\tau$ +.dat, OUT+\_BAND 2\_\_+ $\tau$ +.dat, etc.):

Band: number of band

- T1: minimum period in a band for a specific site.
- T2: maximum period in a band for a specific site.
- Nper: number of periods in a band for a specific site (excluding undetermined cases).
- Cont: parameter stating whether strike angle has been determined (1) or not (0). Useful to display strike angles.
- Scale: length of the strike arrows (inversely proportional to strike error). Useful for plotting.
- Strikecomp: strike value \* (-1). Useful for certain plotting packages.

Other indicators (file OUT+\_other\_+ $\tau$ +.dat):

- Impxy\_rot and Impyx\_rot: impedances rotated to the strike direction.
- Skew and ph\_s\_skew: Skew and Phase Sensitive Skew (Bahr (1988) parameters).

6.2.2. D. Output files (5 files+n files (one for each band)+x files (one for each site))

ROOT: Root name for output files [Default: OUT].

- $\tau$ : threshold value used for dimensionality analysis.
- 1. ROOT+"\_INV\_"+ $\tau$ +".dat" [Default: OUT\_INV\_0.15.dat]: table with invariant values and errors for all sites and periods: Site, Longitude, Latitude, *F*(Hz), *T*(s), *I*(1 to 7), *Q*, err*I*(1 to 7) and err*Q*.
- ROOT+"\_DIM\_"+τ+".dat" [Default: OUT\_DIM\_0.15.dat]: file with dimensionality for all sites and periods with relevant

information if 2D or 3D/2D: Site, Longitude, Latitude, *F*(Hz), *T*(s), DIM, St<sub>3</sub>, errSt<sub>3</sub>, St<sub>4</sub>, errSt<sub>4</sub>, St<sub>5</sub>, errSt<sub>5</sub>, St<sub>6</sub>, errSt<sub>6</sub>, St<sub>7</sub>, errSt<sub>7</sub>.

- ROOT+"\_ERR\_"+τ+".dat" [Default: OUT\_ERR\_0.15.dat]: file with dimensionality, invariants, related parameters (St) and errors for all sites and periods: Site, Longitude, Latitude, F(Hz), log (F), Per (s), ROT, DIM, *I*(1 to 7), *Q*, St(1 to 9), err*I*(1 to 7), errQ, errSt(1 to 9).
- 4. Optional file: ROOT+"\_STATS\_"+ $\tau$ +".dat" [Default: OUT\_STATS\_ 0.15.dat]: file with true values, statistical values, errors, and biases for all invariants and related parameters for all sites and periods: Site, Longitude, Latitude, *F*(Hz), Per (s)+for each parameter: true, err, stat, dev, bias.
- 5. ROOT+"\_BANDCLASS\_"+ $\tau$ +".dat" [Default: OUT\_BAND-CLASS\_0.15.dat]: file with dimensionality results classified in period bands, for all sites and all bands. Site, Longitude, Latitude, BAND, T1, T2, number of periods in the band, strike, errstrike, twist, errtwist, shear, errshear, cont, scale, strikecomp.
- 6. ROOT+"\_BAND"+B+ $\tau$ +".dat" [Default: OUT\_BANDCLASS\_0.15. dat]: file with dimensionality results for band "B" for all sites: Site, Longitude, Latitude, nper, strike, errstrike, twist, errtwist, shear, errshear, cont, scale, strikecomp. (ONE FILE FOR EACH BAND).
- 7. EDISITENAME+"\_RES\_"+ $\tau$ +".dat" (e.g.:"site001\_RES\_0.15.DAT): file with dimensionality results for each site. This file is divided in two blocks: 1) dimensionality results for each period, and 2) dimensionality averages in bands. (ONE FILE FOR EACH SITE). Block 1: Site, Longitude, Latitude, *F*(Hz), Per (s), DIM, strike, errstrike, twist, errtwist, shear, errshear. Block 2: Site, Longitude, Latitude, BAND, Tmin, Tmax, number

of periods in the band, strike, errstrike, twist, errtwist, shear, errshear, cont, scale, strikecomp.

 ROOT+"\_other\_"+τ+".dat" [Default: OUT\_other\_0.15.dat]: file with other indicators used to assess 2D, 3D/2D or 3D dimensionalities: Site, Per (s), DIM, strike (2D or 3D/2D), Impxy\_rot (2D), Impyx\_rot (2D), twist (3D/2D), shear (3D/2D), I7 (3D), skew, ph\_s\_skew.

#### 6.2.3. E. Screen outputs

- Error or warning messages regarding input parameters, EDI files, overwriting existing files.
- Summary of dimensionality results: total number of periods (from all sites) and total number of cases for each dimensionality type.

If band averaging:

• Minimum and maximum period, total number of bands and period range for each band.

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