Comment on 'The magnetotelluric phase tensor' by T. Grant Caldwell, Hugh M. Bibby and Colin Brown

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SUMMARY

We demonstrate a minor inconsistency in the appendix of 'The magnetotelluric phase tensor' by Caldwell *et al.* that can lead to incorrect results, and provide a simple solution for this problem.

Key words: magnetotellurics.

Since its publication the phase tensor has become an important tool to analyse magnetotelluric data. The reason for this success is, that it is not affected by galvanic distortion and does not make any assumptions about the conductivity structure in the subsurface (Caldwell *et al.* 2004). However the discussion in the appendix on how to treat the problematic case of a negative determinant is misleading and leads to incorrect expressions when used to calculate the two invariants $\mathbf{\Phi}_{\max}$ and $\mathbf{\Phi}_{\min}$.

Given the magnetotelluric transfer function $\mathbf{Z} = \mathbf{X} + i\,\mathbf{Y}$ the phase tensor is given by

$$\mathbf{\Phi} = \boldsymbol{X}^{-1}\boldsymbol{Y} = \frac{1}{\det \boldsymbol{X}} \begin{pmatrix} X_{22}Y_{11} - X_{12}Y_{21} & X_{22}Y_{12} - X_{12}Y_{22} \\ X_{11}Y_{21} - X_{21}Y_{11} & X_{11}Y_{22} - X_{21}Y_{12} \end{pmatrix}. \quad (1)$$

This real second rank tensor can also be expressed in term of three rotational invariants and one rotation angle. In the appendix of Caldwell *et al.* (2004) the formulae to calculate the invariants $\mathbf{\Phi}_{\max}$, $\mathbf{\Phi}_{\min}$ and β are given as

$$\Phi_1 = \mathrm{tr}(\mathbf{\Phi})/2, \quad (2)$$

$$\Phi_2 = \sqrt{\det(\mathbf{\Phi})}, \quad (3)$$

$$\Phi_3 = \mathrm{sk}(\mathbf{\Phi})/2, \quad (4)$$

$$\Phi_{\max} = \left(\Phi_1^2 + \Phi_3^2\right)^{1/2} + \left(\Phi_1^2 + \Phi_3^2 - \Phi_2^2\right)^{1/2}, \quad (5)$$

$$\Phi_{\min} = \left(\Phi_1^2 + \Phi_3^2\right)^{1/2} - \left(\Phi_1^2 + \Phi_3^2 - \Phi_2^2\right)^{1/2}, \quad (6)$$

$$\beta = \frac{1}{2}\tan^{-1}\left(\frac{\Phi_3}{\Phi_1}\right), \quad (7)$$

an equivalent definition for $\mathbf{\Phi}_{\max}$ and $\mathbf{\Phi}_{\min}$ is given by Bibby (1986, 2005)

$$\Pi_1 = \frac{1}{2}\left[(\Phi_{11} - \Phi_{22})^2 + (\Phi_{12} + \Phi_{21})^2\right]^{\frac{1}{2}}, \quad (8)$$

$$\Pi_2 = \frac{1}{2}\left[(\Phi_{11} + \Phi_{22})^2 + (\Phi_{12} - \Phi_{21})^2\right]^{\frac{1}{2}}, \quad (9)$$

$$\Phi_{\max} = \Pi_2 + \Pi_1, \quad (10)$$

$$\Phi_{\min} = \Pi_2 - \Pi_1. \quad (11)$$

In certain cases the determinant of the phase-tensor $\det(\mathbf{\Phi})$ can become negative. The recommendation by Caldwell et al. is to use the absolute value $|\det(\mathbf{\Phi})|$ in these cases instead to avoid problems while calculating the intermediate quantity $\mathbf{\Phi}_2$ which should be real. Through this manipulation the expressions for $\mathbf{\Phi}_{\max}$ and $\mathbf{\Phi}_{\min}$ become incorrect for $\det(\mathbf{\Phi}) < 0$. Short algebraic manipulation of eq. (5) shows that in this case

$$\Phi_{\max} = \underbrace{\left(\Phi_1^2 + \Phi_3^2\right)^{1/2}}_{\Pi_2} + \left[\frac{1}{4}\left(\Phi_{11} + \Phi_{22}\right)^2 + \frac{1}{4}\left(\Phi_{12} - \Phi_{21}\right)^2 \underbrace{+\Phi_{11}\Phi_{22} - \Phi_{12}\Phi_{21}}_{|\det(\mathbf{\Phi})|}\right]^{1/2} \quad (12)$$

$$= \Pi_2 + \left[\frac{1}{4}\left(\Phi_{11}^2 + \Phi_{12}^2 + \Phi_{21}^2 + \Phi_{22}^2\right) + \frac{3}{2}\det(\mathbf{\Phi})\right]^{1/2} \quad (13)$$

$$\neq \Pi_2 + \Pi_1, \quad (14)$$

and similarly for $\mathbf{\Phi}_{\min}$. There are two simple ways to avoid problems with negative determinants: Either use the formulae (10) and (11) given by Bibby (2005) that avoid the calculation of the determinant altogether, or circumvent the calculation of $\mathbf{\Phi}_2$ by inserting (3) into (5) and (6). The resulting expressions are

$$\Phi_{\max} = \left(\Phi_1^2 + \Phi_3^2\right)^{1/2} + \left[\Phi_1^2 + \Phi_3^2 - \det(\mathbf{\Phi})\right]^{1/2}, \quad (15)$$

$$\Phi_{\min} = \left(\Phi_1^2 + \Phi_3^2\right)^{1/2} - \left[\Phi_1^2 + \Phi_3^2 - \det(\mathbf{\Phi})\right]^{1/2}, \quad (16)$$

and are correct in all cases.

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