Comprehensive maps of geomagnetic and magneto-telluric transfer functions in EM array studies

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EM array studies provide a multitude of transfer function estimates, six for geomagnetic depth sounding (GDS) and four for magneto-telluric sounding (MTS). Their complete presentation in real and imaginary parts, or in amplitudes and phases, requires 20 maps for each frequency or the same number of sounding curves for every site. Some of these displays are redundant and it is the purpose of this contribution to concentrate on a few key presentation, in the case of MTS also separately for the induction TE mode and the galvanic TM mode of geo-electric variations (TE for "tangential electric" and TM for "tangential magnetic"). The galvanic TM mode is largely responsible for the so-called "static distortion" of MTS results.

Field vectors are split into their normal and anomalous parts in reference to a layered surrounding structure and deviations from it within an anomalous domain: $\underline{B} = \underline{B}_n + \underline{B}_a$ and $\underline{E} = \underline{E}_n + \underline{E}_a$. Cartesian (x,y,z) co-ordinates are used with z down. Induction is by a quasi-uniform TE source field in a conducting lower half-space $z \geq 0$ in two polarisations :

x-polarisation with $\underline{E}_n = (0, -E_{ny}, 0)$ and $\underline{B}_n = (B_{nx}, 0, 0)$,
y-polarisation with $\underline{E}_n = (\ E_{nx}, 0, 0)$ and $\underline{B}_n = (0, B_{ny}, 0)$.

Demonstrations are for COMMEMI model 3.1(a) as shown in **Fig. 1**. Only the in-phase parts of the fields with respect to $\underline{B}_n$ are displayed, normalised to $B_{nx} = B_{ny} = 1\, nT$.

GDS results: The observable magnetic variation field on and above ground is a potential field in quasi-stationary approximation. The potential V for their anomalous part is of internal origin only and therefore $\underline{B}_a = -grad\ V$ can be formally regarded as the magnetic field of equivalent thin-sheet currents flowing at zero depth $z = +0$ just below ground. Their tangential current density vector is given by

$$\underline{j}_a = \hat{\underline{z}} \times grad\ F = 2/\mu_0 \cdot (B_{ay}, -B_{ax}), \qquad (1)$$

with $\hat{\underline{z}}$ as unit vector, and $F = 2/\mu_0 \cdot V$ as current function. **Fig. 2** provides contour lines of the current function together with current vectors $\underline{j}_a$ for a complete representation of the three components of $\underline{B}_a$ for a given polarisation. They indicate the concentration of induction currents in conductive domains and their dilution outside, and vice versa for resistive domains. Because these are fictitious currents, no direct connection exists between them and physical currents below ground driven by electric fields as observed in magneto-tellurics.

MTS results: Non-divergent vector fields can be described by superpositions of toroidal and poloidal field, derivable from scalar potentials P as $rot(\hat{\underline{z}}P)$ and $rot\, rot(\hat{\underline{z}}P)$, respectively; with $\hat{\underline{z}}$ as unit vector in Cartesian co-ordinates. Here a potential Ψ is introduced for the TE modes of vector fields $\underline{B}_a$ and $\underline{E}_a$ and a potential Φ for their TM modes:

$$\underline{B}_a = rot\,(\underline{\hat{z}}\,\Phi) + rot\,rot\,(\underline{\hat{z}}\,\Psi)\,, \quad \mu_0\,\sigma\,\underline{E}_a = rot\,rot\,(\underline{\hat{z}}\,\Phi) - rot\,(\underline{\hat{z}}\,\nabla^2\Psi) \tag{2}$$

with $\nabla^2\Psi = i\omega\,\mu_0\,\sigma\,\Psi$ in uniform domains to satisfy Maxwell's equations. In air the TM mode of $\underline{B}_a$ can be neglected and the TM mode of $\underline{E}_a$ can be regarded as the field of a potential U due to electric charges, for example on internal domain boundaries. This converts eqs (2) for $z \le -0$ into

$$\underline{B}_a = rot\,rot\,(\underline{\hat{z}}\,\Psi)\ , \quad \underline{E}_a = -grad\,U - i\omega\ rot\,(\underline{\hat{z}}\,\Psi)\ . \tag{3}$$

With notations $\Psi_x = \partial\Psi / \partial x$, $\Psi_{xz} = \partial\Psi^2 / \partial x\,\partial z\,,\ldots$ the anomalous field components follow as

$$\begin{aligned} &B_{ax} = \Psi_{xz}\,,\ \ B_{ay} = \Psi_{yz}\,,\ \ B_{az} = -\Psi_{xx} - \Psi_{yy}\,, \\ &E_{ax} = -U_x - i\omega\,\Psi_y\,,\ \ E_{ay} = -U_y + i\omega\,\Psi_x\,.\ \ E_{az} = -U_z\,, \end{aligned} \tag{4}$$

which implies that the tangential components of $\underline{E}_a$ have the spatial derivatives

$$\begin{aligned} &\partial E_{ax} / \partial x = -U_{xx} - i\omega\,\Psi_{yx}\,,\ \ \partial E_{ax} / \partial y = -U_{xy} - i\omega\,\Psi_{yy}\,, \\ &\partial E_{ay} / \partial x = -U_{yx} + i\omega\,\Psi_{xx}\,,\ \ \partial E_{ay} / \partial y = -U_{yy} + i\omega\,\Psi_{xy}\,. \end{aligned} \tag{5}$$

Their observable combinations

$$\partial E_x / \partial y - \partial E_y / \partial x = -i\omega\ (\Psi_{xx} + \Psi_{yy}) = i\omega\,B_{az} \tag{6}$$

and

$$\partial E_x / \partial x + \partial E_y / \partial y = -U_{xx} - U_{yy} = U_{zz} = -\partial E_{az} / \partial z\,|_{z=-0} \tag{7}$$

decompose the *total* electric field according to TE and TM modes in their spatial derivatives, noting that their normal parts are sole functions of depth.

Contour lines for B_{az} in the left part of **Fig. 3** visualise the *induction mode* of the anomalous electric field in response to a changing depth of penetration and thus changing conductivity with depth. Contour lines for $-\partial E_{az} / \partial z\,|_{z=-0}$ in the right part of the same illustration represent the *galvanic mode* of the anomalous electric field in terms of equivalent electric surface charges. They provide a smoothed image of physical charges, for example on vertical domain boundaries, where the internal resistivity changes in horizontal direction. In practice field components are replaced by their respective transfer functions, but here in reference to the horizontal components of $\underline{B}_n$ rather than to the horizontal components of the locally observed magnetic field as in single-site operations. This suggests the following iterative procedure to interpret EM array data: A layered normal structure is found from the Berdichevsky averages $(Z_{xy} - Z_{yx})/2$, later on with correction factors for 3-D effects. Then deviations are determined from this normal structure which reproduce firstly B_{az} and secondly $-\partial E_{az} / \partial z$, not necessarily with the same model and not necessarily for the same frequencies.

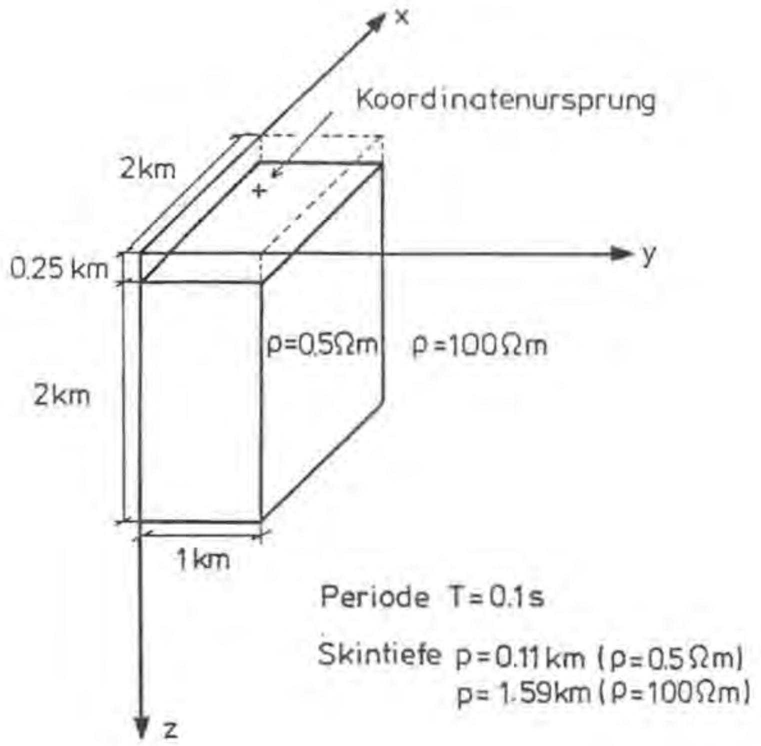

Fig. 1: COMMEMI model 3.1 (a). – (Comparison of Modelling Methods in Electro-Magnetic Induction 1986-88)

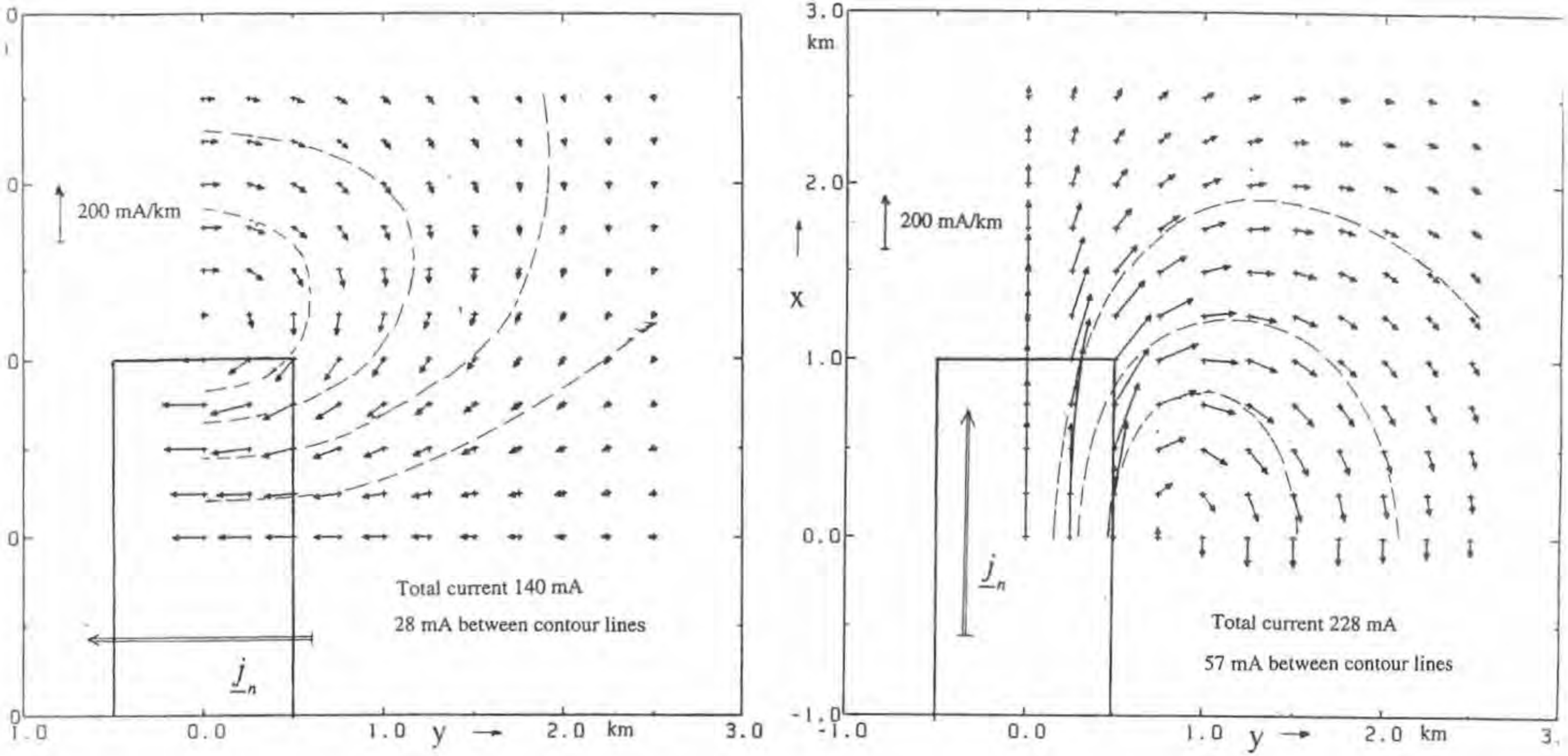

Fig. 2: Equivalent thin-sheet currents at zero depth and contour lines of their current function, representing the anomalous part of geomagnetic variations according to **eq. (1)**. – Left: COMMEMI model 3.1 (a) for x-polarisation. Right: The same for y-polarisation.

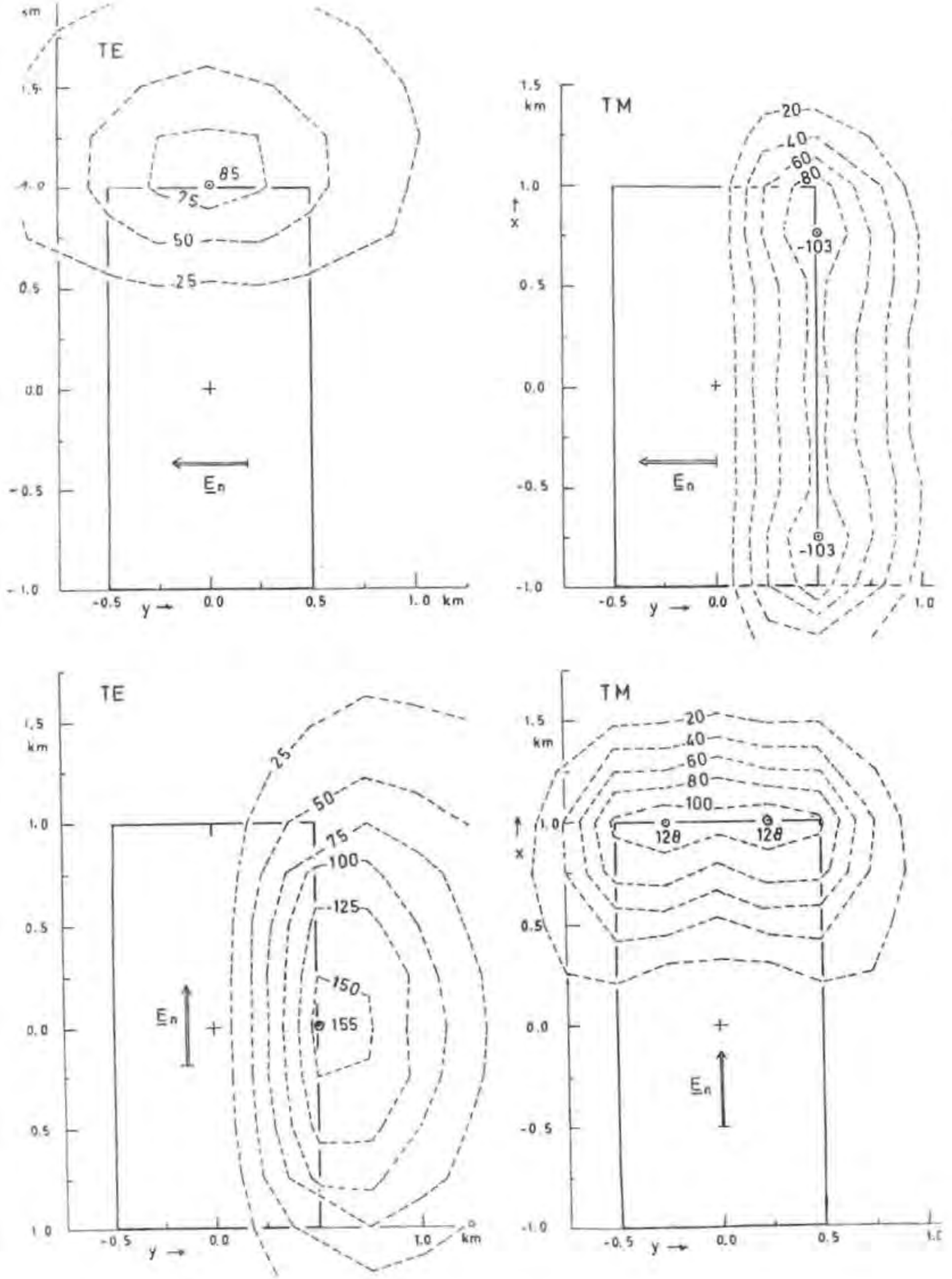

Fig.3: Left: Contour lines B_{az} for the anomalous vertical component of geomagnetic variations, in Picotesla, representing the *induction mode* of anomalous geo-electric variations according to **eq. (6)**. - Right: Contour lines $-\partial E_{az}/\partial z|_{z=0}$ for the derivative of the anomalous vertical electric component with respect to height just above the ground, in Millivolt per kilometre squared, representing the *galvanic mode* of anomalous geo-electric variations according to **eq. (7)**. - Top: COMMEMI model 3.1 (a) for x-polarisation. Bottom: The same for y-polarisation.